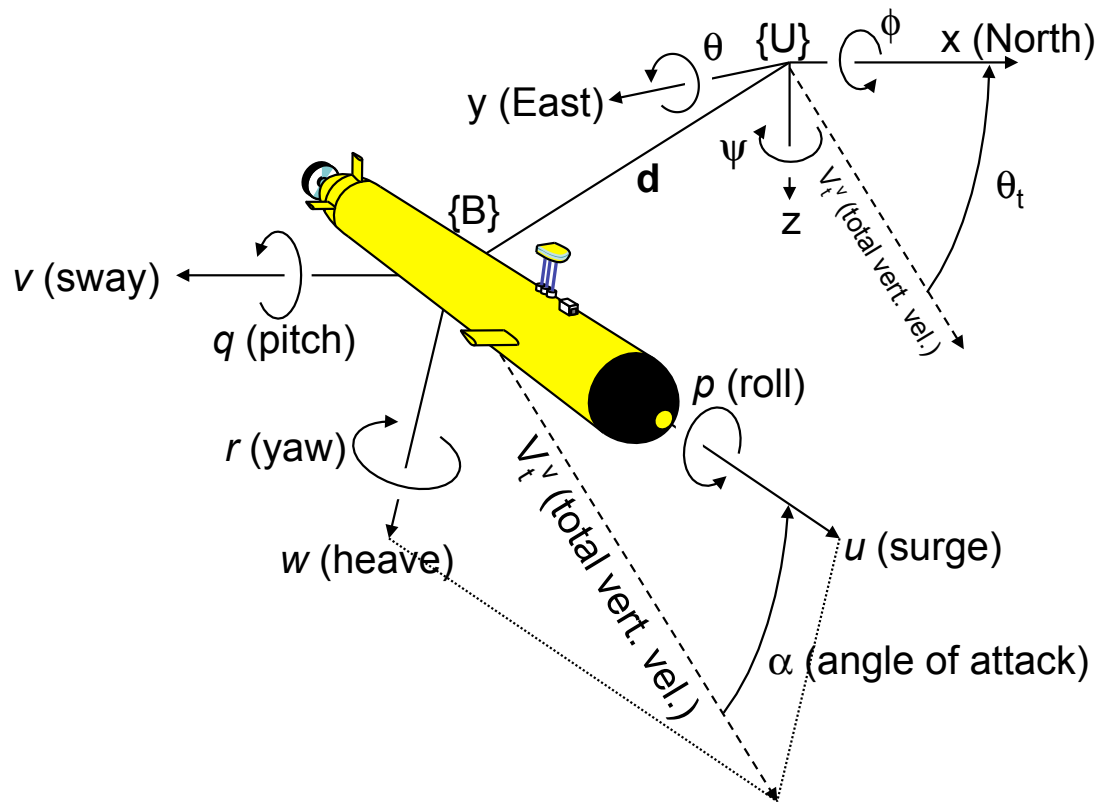


The use of dynamic model

Notation



$$\eta = \begin{bmatrix} \text{Position w.r.t } \{U\} \\ x & y & z & \phi & \theta & \psi \end{bmatrix}^T$$

$$v = \begin{bmatrix} \text{Velocity w.r.t } \{B\} \\ u & v & w & p & q & r \end{bmatrix}^T$$

Kinematic relation

$$\dot{\eta} = R(\eta) \cdot v$$

Dynamic relation

$$\mathbf{F}_B = f_{Dyn}(\dot{v}, v, \eta, \mathbf{P})$$

$$\mathbf{P} = [m_u \quad m_v \quad m_z \quad \dots]^T$$

Actuation system

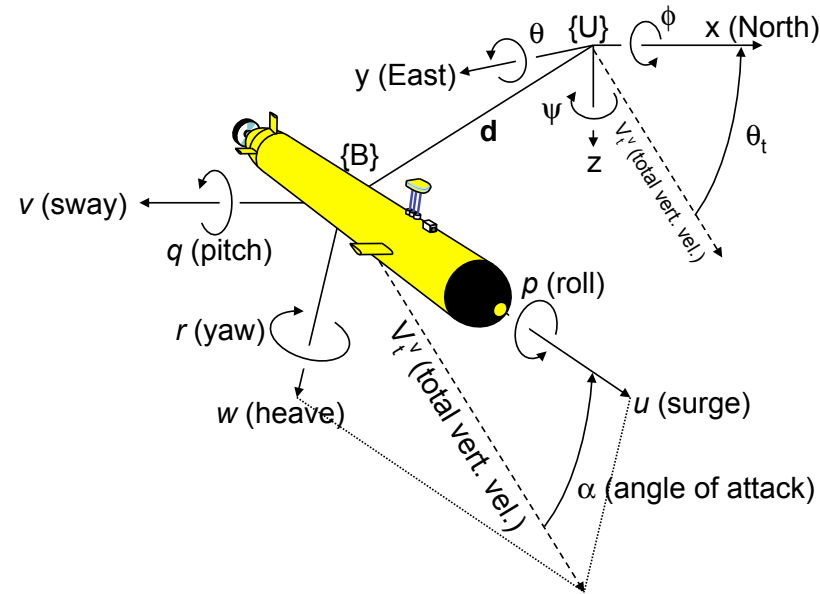
$$\mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

Actuators model

$$\mathbf{F}_m = \mathbf{M}(\chi)$$

Kinematic Model

$$\underbrace{\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}}_{\eta}$$



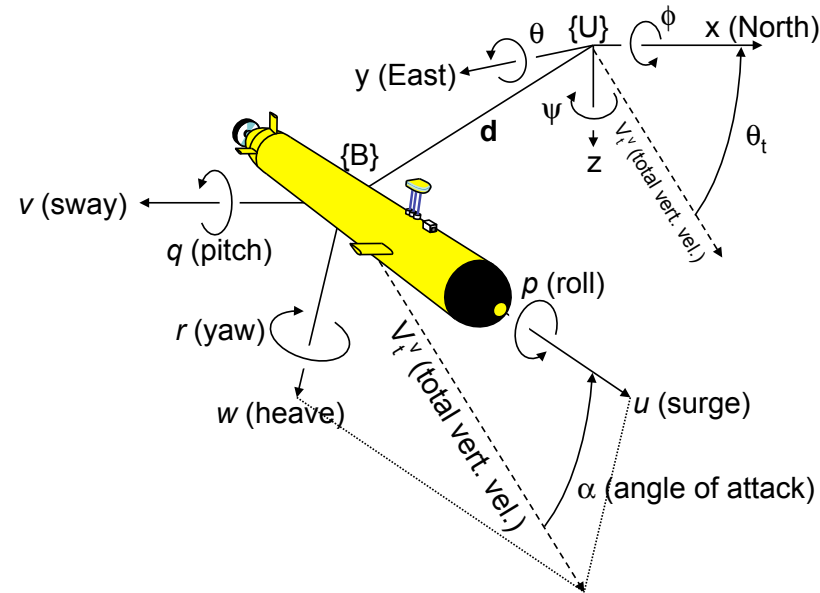
$$\underbrace{\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}}_v$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi \cdot \cos \theta & \cos \psi \cdot \sin \theta \cdot \sin \phi - \sin \psi \cdot \cos \phi & \cos \psi \cdot \sin \theta \cdot \cos \phi + \sin \psi \cdot \sin \phi & 0 & 0 & 0 \\ \sin \psi \cdot \cos \theta & \sin \psi \cdot \sin \theta \cdot \sin \phi + \cos \psi \cdot \cos \phi & \sin \psi \cdot \sin \theta \cdot \cos \phi - \cos \psi \cdot \sin \phi & 0 & 0 & 0 \\ -\sin \theta & \cos \theta \cdot \sin \phi & \cos \theta \cdot \cos \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin \phi \cdot \tan \theta & \cos \phi \cdot \tan \theta \\ 0 & 0 & 0 & 0 & \cos \phi & -\sin \phi \\ 0 & 0 & 0 & 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

$$\dot{\eta} = R \cdot v$$

Kinematic Model

$$\underbrace{\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix}}_{\eta}$$



$$\underbrace{\begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}}_v$$

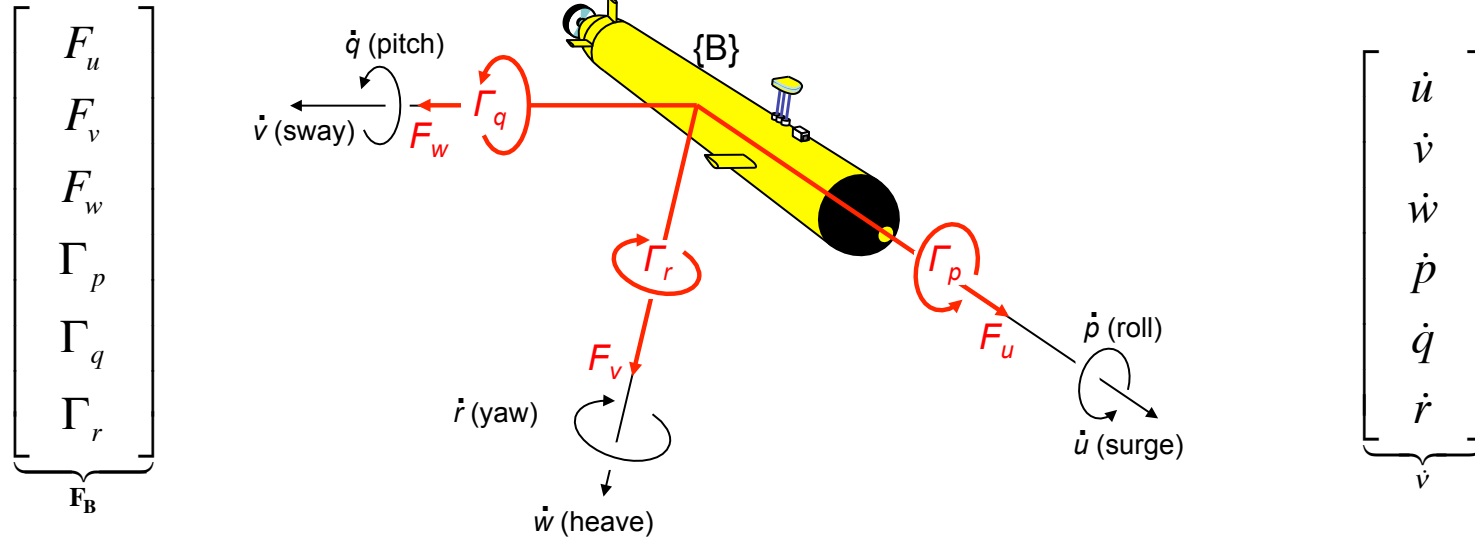
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi \cdot \cos \theta & \cos \psi \cdot \sin \theta \cdot \sin \phi - \sin \psi \cdot \cos \phi & \cos \psi \cdot \sin \theta \cdot \cos \phi + \sin \psi \cdot \sin \phi & 0 & 0 & 0 \\ \sin \psi \cdot \cos \theta & \sin \psi \cdot \sin \theta \cdot \sin \phi + \cos \psi \cdot \cos \phi & \sin \psi \cdot \sin \theta \cdot \cos \phi - \cos \psi \cdot \sin \phi & 0 & 0 & 0 \\ -\sin \theta & \cos \theta \cdot \sin \phi & \cos \theta \cdot \cos \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin \phi \cdot \tan \theta & \cos \phi \cdot \tan \theta & 0 \\ 0 & 0 & 0 & \frac{\sin \phi}{\cos \theta} & -\frac{\cos \phi}{\cos \theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

$$\theta = \pm \frac{\pi}{2}, \phi = \pm \frac{\pi}{2}$$

$$\dot{\eta} = R \cdot v$$

Singularities !

Dynamic Model



$$F_u = X_{\dot{u}} \cdot \dot{u} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = Y_{\dot{v}} \cdot \dot{v} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + Y_{w \cdot p} \cdot w \cdot p + Y_G(\eta)$$

$$F_w = Z_{\dot{w}} \cdot \dot{w} + Z_{w \cdot |w|} \cdot w \cdot |w| + Z_{u \cdot q} \cdot u \cdot q + Z_{v \cdot p} \cdot v \cdot p + Z_G(\eta)$$

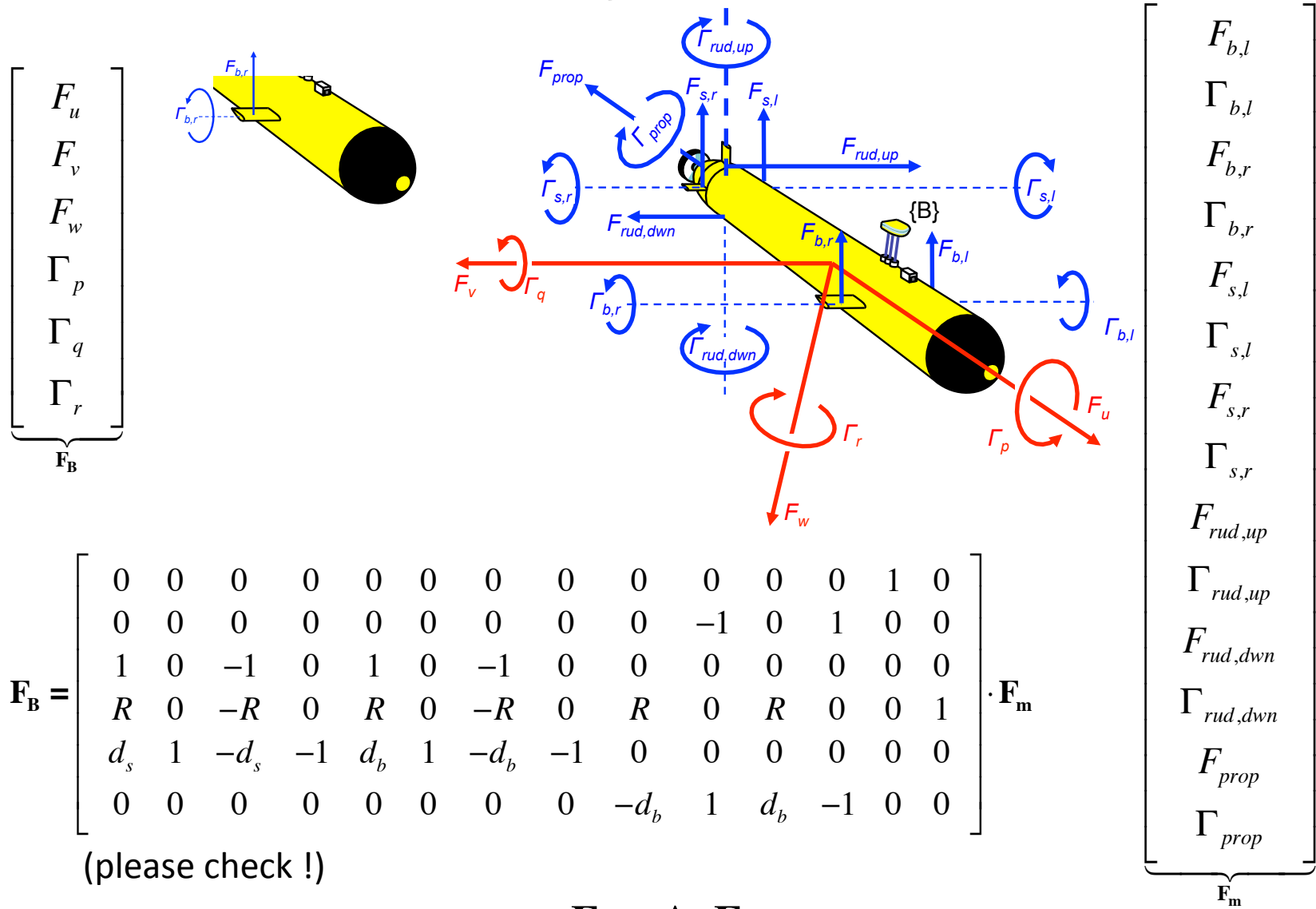
$$\Gamma_p = K_{\dot{p}} \cdot \dot{p} + K_{p \cdot |p|} \cdot p \cdot |p| + K_{q \cdot r} \cdot q \cdot r + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = M_{\dot{q}} \cdot \dot{q} + M_{q \cdot |q|} \cdot q \cdot |q| + M_{p \cdot r} \cdot p \cdot r + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = N_{\dot{r}} \cdot \dot{r} + N_{r \cdot |r|} \cdot r \cdot |r| + N_{p \cdot q} \cdot p \cdot q + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B = f_{Dyn}(\dot{\mathbf{v}}, \mathbf{v}, \boldsymbol{\eta}, \mathbf{P})$$

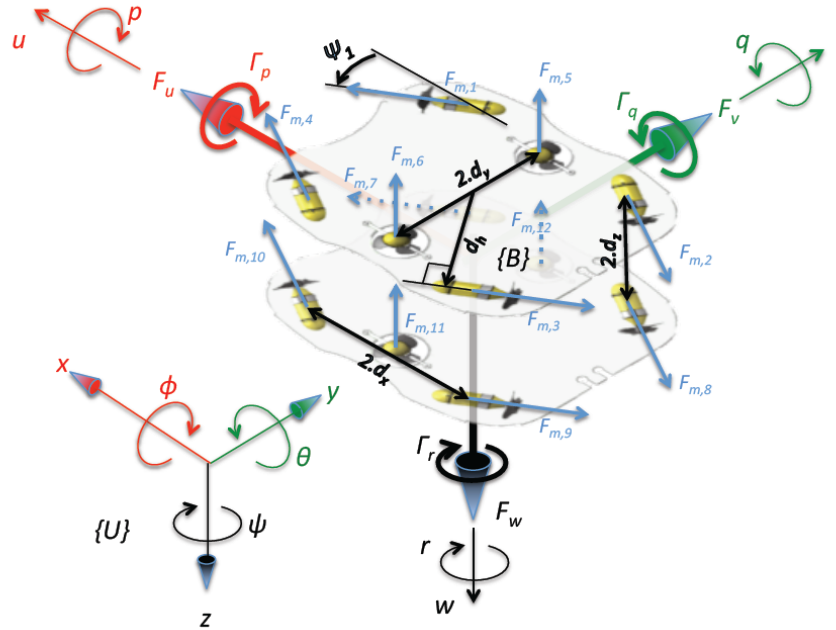
Actuation System Model



$$\mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

Actuation System Model

$$\underbrace{\begin{bmatrix} F_u \\ F_v \\ F_w \\ \Gamma_p \\ \Gamma_q \\ \Gamma_r \end{bmatrix}}_{\mathbf{F}_B}$$



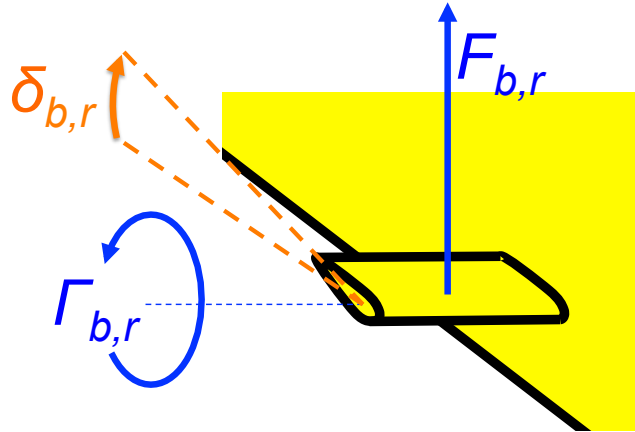
$$\underbrace{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{bmatrix}}_{\mathbf{F}_m}$$

$$\mathbf{A}_{5 \times 8} = \begin{bmatrix} -.866 & -.866 & .866 & .866 & -.866 & -.866 & .866 & .866 \\ -.5 & .5 & .5 & -.5 & -.5 & .5 & .5 & -.5 \\ .1 & -.1 & -.1 & .1 & -.1 & .1 & .1 & -.1 \\ .173 & .173 & -.173 & -.173 & -.173 & -.173 & .173 & .173 \\ .226 & -.226 & .226 & -.226 & .226 & -.226 & .226 & -.226 \end{bmatrix}$$

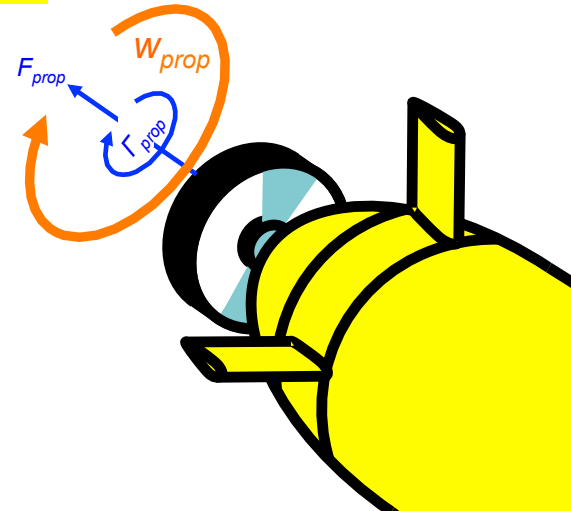
$$\mathbf{F}_B = \mathbf{A} \cdot \mathbf{F}_m$$

Actuators Model

$$\underbrace{\begin{bmatrix} F_{b,l} \\ \Gamma_{b,l} \\ F_{b,r} \\ \Gamma_{b,r} \\ F_{s,l} \\ \Gamma_{s,l} \\ F_{s,r} \\ \Gamma_{s,r} \\ F_{rud,up} \\ \Gamma_{rud,up} \\ F_{rud,dwn} \\ \Gamma_{rud,dwn} \\ F_{prop} \\ \Gamma_{prop} \end{bmatrix}}_{\mathbf{F}_m}$$

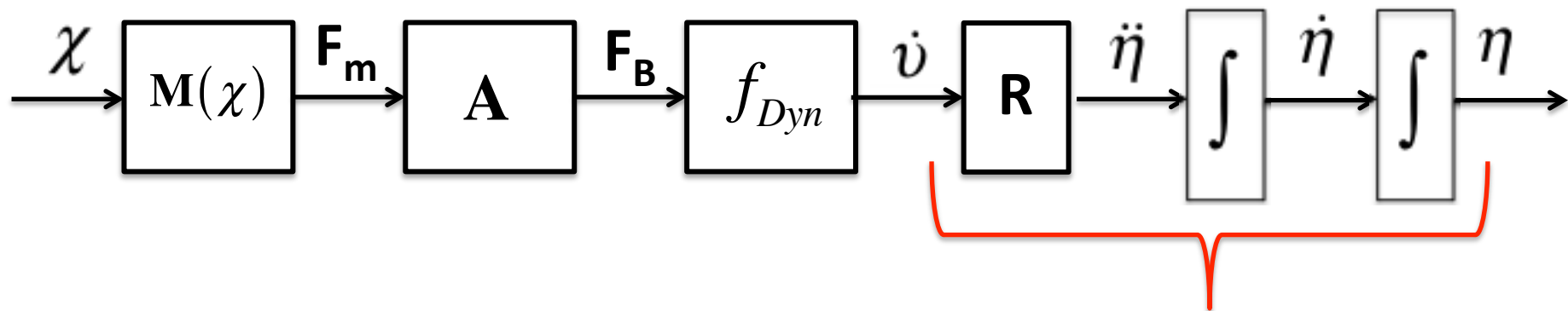


$$\mathbf{F}_m = \mathbf{M}(\chi)$$



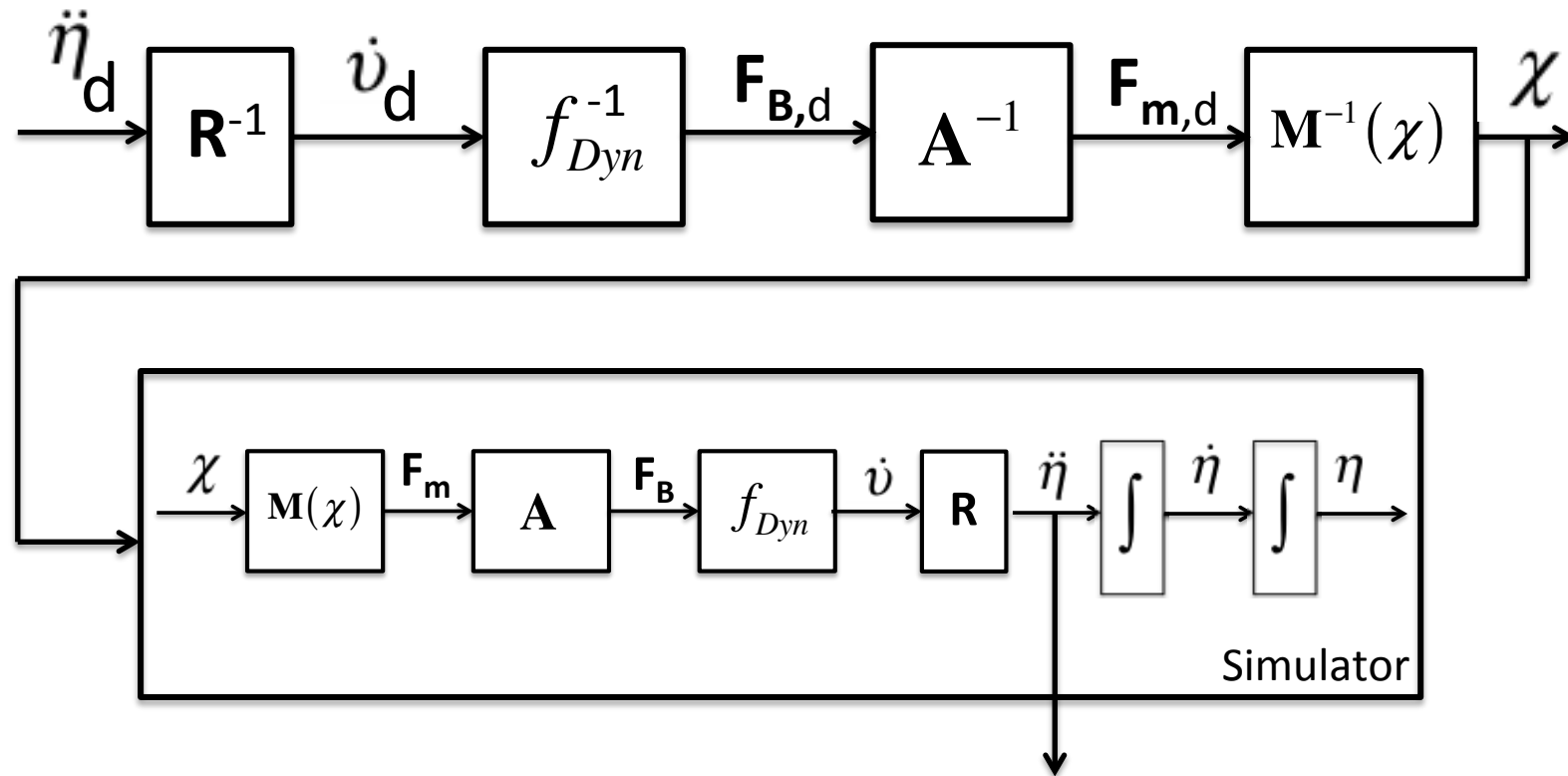
$$\underbrace{\begin{bmatrix} \delta_{b,l} \\ \delta_{b,r} \\ \delta_{s,l} \\ \delta_{s,r} \\ \delta_{rud,up} \\ \delta_{rud,dwn} \\ w_{prop} \end{bmatrix}}_{\chi}$$

Dynamic Simulation



Integration has to be performed in the fixed (gallilean) frame !

Control as an inverse problem



The 'computed torque' solution : $\ddot{\eta} = \ddot{\eta}_d$

Everything works fine in simulation...

Closed loop identification

- Station keeping test (null velocity stabilisation)
 - Station keeping around η_d
 - Once static regime is achieved, for $\chi^\infty : v = 0$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + \cancel{X_{u \cdot |u|} \cdot u \cdot |u|} + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + \cancel{Y_{v \cdot |v|} \cdot v \cdot |v|} + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + \cancel{Z_{w \cdot |w|} \cdot w \cdot |w|} + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + \cancel{K_{p \cdot |p|} \cdot p \cdot |p|} + \cancel{K_{q \cdot r} \cdot q \cdot r} + \cancel{K_v \cdot v} + \cancel{K_w \cdot w} + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + \cancel{M_{q \cdot |q|} \cdot q \cdot |q|} + \cancel{M_{p \cdot r} \cdot p \cdot r} + \cancel{M_u \cdot u} + \cancel{M_w \cdot w} + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + \cancel{N_u \cdot u} + \cancel{N_v \cdot v} + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = [\ X_G(\eta_d) \quad Y_G(\eta_d) \quad Z_G(\eta_d) \quad K_G(\eta_d) \quad M_G(\eta_d) \quad N_G(\eta_d) \]^T$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control u to u_d . Once static regime is achieved,
 - for χ^∞ : $v = [u_d \ 0 \ 0 \ 0 \ 0 \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + \cancel{Y_{v \cdot |v|} \cdot v \cdot |v|} + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + \cancel{Z_{w \cdot |w|} \cdot w \cdot |w|} + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + \cancel{K_{p \cdot |p|} \cdot p \cdot |p|} + \cancel{K_{q \cdot r} \cdot q \cdot r} + \cancel{K_v \cdot v} + \cancel{K_w \cdot w} + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + \cancel{M_{q \cdot |q|} \cdot q \cdot |q|} + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + \cancel{M_w \cdot w} + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + \cancel{N_v \cdot v} + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control v to v_d . Once static regime is achieved,
 - for χ^∞ : $v = [0 \quad v_d \quad 0 \quad 0 \quad 0 \quad 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + \cancel{Z_{w \cdot |w|} \cdot w \cdot |w|} + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + \cancel{K_{p \cdot |p|} \cdot p \cdot |p|} + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + \cancel{K_w \cdot w} + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + \cancel{M_{q \cdot |q|} \cdot q \cdot |q|} + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + \cancel{M_w \cdot w} + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control w to w_d . Once static regime is achieved,
 - for χ^∞ : $v = [0 \ 0 \ w_d \ 0 \ 0 \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + \cancel{K_{p \cdot |p|} \cdot p \cdot |p|} + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + \cancel{M_{q \cdot |q|} \cdot q \cdot |q|} + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control p to p_d . Once static regime is achieved,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ p_d \ 0 \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + K_{p \cdot |p|} \cdot p \cdot |p| + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + \cancel{M_{q \cdot |q|} \cdot q \cdot |q|} + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control q to q_d . Once static regime is achieved,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ 0 \ q_d \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + K_{p \cdot |p|} \cdot p \cdot |p| + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + M_{q \cdot |q|} \cdot q \cdot |q| + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + \cancel{N_{r \cdot |r|} \cdot r \cdot |r|} + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Independant velocity tests (along each DOF)
 - Control r to r_d . Once static regime is achieved,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ 0 \ 0 \ r_d]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + \cancel{Y_{u \cdot r} \cdot u \cdot r} + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + K_{p \cdot |p|} \cdot p \cdot |p| + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + M_{q \cdot |q|} \cdot q \cdot |q| + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control u to u_d and r to r_d . Once static reg. is achieved,
 - for χ^∞ : $v = [u_d \ 0 \ 0 \ 0 \ 0 \ r_d]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + \cancel{X_{v \cdot r} \cdot v \cdot r} + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + K_{p \cdot |p|} \cdot p \cdot |p| + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + M_{q \cdot |q|} \cdot q \cdot |q| + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control v to v_d and r to r_d . Once static reg. is achieved,
 - for χ^∞ : $v = [0 \quad v_d \quad 0 \quad 0 \quad 0 \quad r_d]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + \cancel{X_{w \cdot q} \cdot w \cdot q} + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

$$F_w = \cancel{Z_{\dot{w}} \cdot \dot{w}} + Z_{w \cdot |w|} \cdot w \cdot |w| + \cancel{Z_{u \cdot q} \cdot u \cdot q} + \cancel{Z_{v \cdot p} \cdot v \cdot p} + Z_G(\eta)$$

$$\Gamma_p = \cancel{K_{\dot{p}} \cdot \dot{p}} + K_{p \cdot |p|} \cdot p \cdot |p| + \cancel{K_{q \cdot r} \cdot q \cdot r} + K_v \cdot v + K_w \cdot w + K_G(\eta)$$

$$\Gamma_q = \cancel{M_{\dot{q}} \cdot \dot{q}} + M_{q \cdot |q|} \cdot q \cdot |q| + \cancel{M_{p \cdot r} \cdot p \cdot r} + M_u \cdot u + M_w \cdot w + M_G(\eta)$$

$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control w to w_d and q to q_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \ 0 \ w_d \ 0 \ q_d \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + \cancel{Y_{w \cdot p} \cdot w \cdot p} + Y_G(\eta)$$

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$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control w to w_d and p to p_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \ 0 \ w_d \ p_d \ 0 \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

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$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control v to v_d and p to p_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \quad v_d \quad 0 \quad p_d \quad 0 \quad 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + Y_{w \cdot p} \cdot w \cdot p + Y_G(\eta)$$

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$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control u to u_d and q to q_d . Once static reg. is ach.,
 - for χ^∞ : $v = [u_d \ 0 \ 0 \ 0 \ q_d \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = \cancel{Y_{\dot{v}} \cdot \dot{v}} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + Y_{w \cdot p} \cdot w \cdot p + Y_G(\eta)$$

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$$\Gamma_r = \cancel{N_{\dot{r}} \cdot \dot{r}} + N_{r \cdot |r|} \cdot r \cdot |r| + \cancel{N_{p \cdot q} \cdot p \cdot q} + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control q to q_d and r to r_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ 0 \ q_d \ r_d]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

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$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control p to p_d and r to r_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ p_d \ 0 \ r_d]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

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$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- cross velocity tests (along each DOF)
 - Control p to p_d and q to q_d . Once static reg. is ach.,
 - for χ^∞ : $v = [0 \ 0 \ 0 \ p_d \ q_d \ 0]^T$



$$F_u = \cancel{X_{\dot{u}} \cdot \dot{u}} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

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$$\mathbf{F}_B^\infty = \mathbf{M}(\chi^\infty) = \dots$$

Closed loop identification

- Added mass (along each DOF)
 - Requires an acceleration control
 - Simpler to rely on theoretical models...



$$F_u = X_{\dot{u}} \cdot \dot{u} + X_{u \cdot |u|} \cdot u \cdot |u| + X_{w \cdot q} \cdot w \cdot q + X_{v \cdot r} \cdot v \cdot r + X_G(\eta)$$

$$F_v = Y_{\dot{v}} \cdot \dot{v} + Y_{v \cdot |v|} \cdot v \cdot |v| + Y_{u \cdot r} \cdot u \cdot r + Y_{w \cdot p} \cdot w \cdot p + Y_G(\eta)$$

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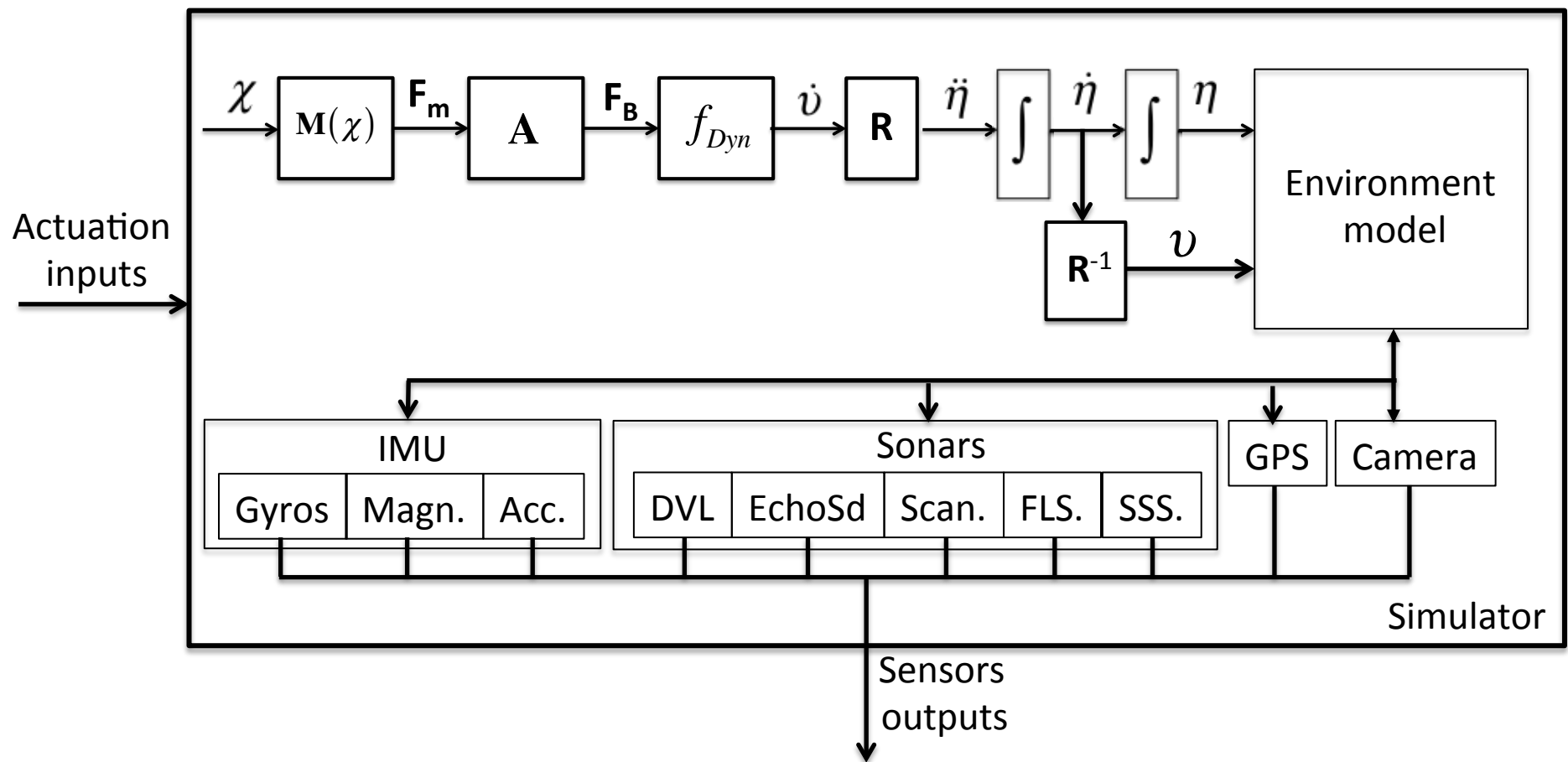
$$\Gamma_r = \underbrace{N_{\dot{r}} \cdot \dot{r} + N_{r \cdot |r|} \cdot r \cdot |r|}_{\text{Theoretical models}} + N_{p \cdot q} \cdot p \cdot q + N_u \cdot u + N_v \cdot v + N_G(\eta)$$

Theoretical models

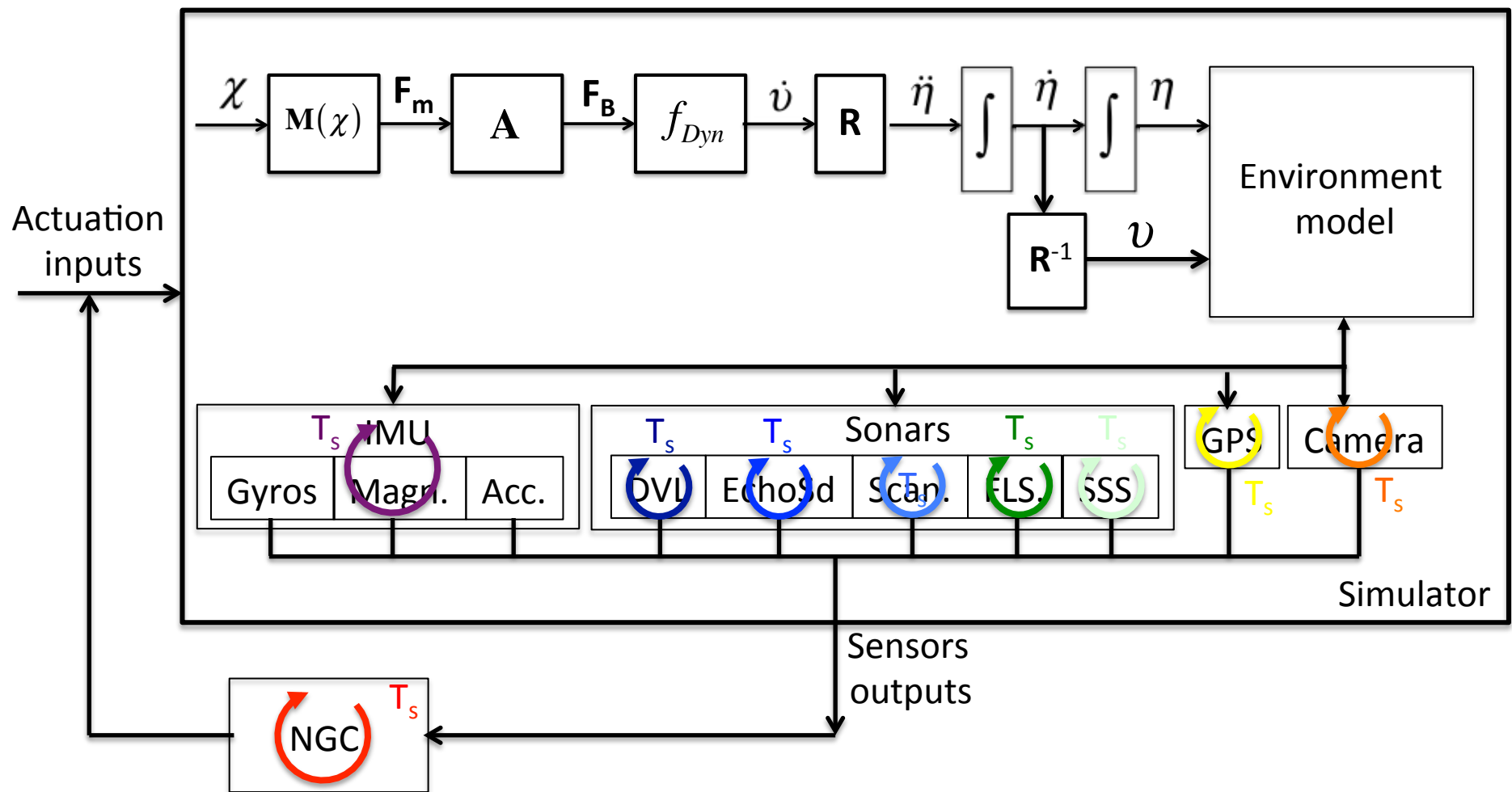
-> 16 experimentations !

Simplify the problem considering system symmetry

Simulation



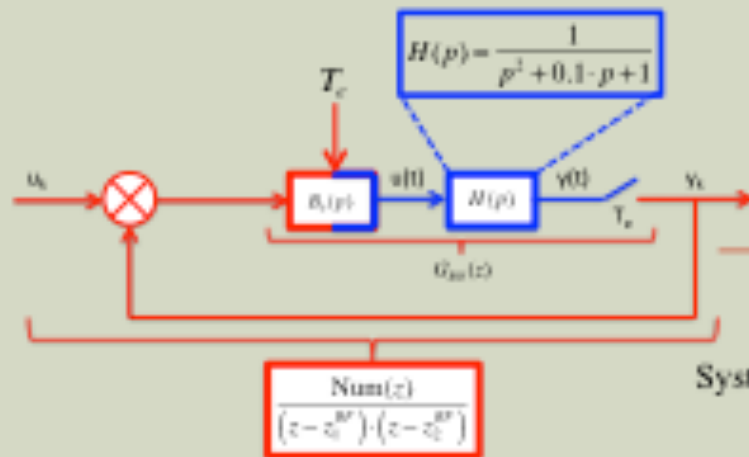
Simulation



Open issue : Discrete Control Problem

■ Automatique discrète non linéaire

$$\begin{cases} \dot{s} = u \cdot \cos \theta + k_1 \cdot s_1 \\ r = \dot{\delta} - \gamma \cdot y_1 \cdot u \cdot \frac{\sin \theta - \sin \delta}{\theta - \delta} - k_2 \cdot (\theta - \delta) \\ u > 0 \end{cases}$$



Système stable si $\|z_{1,2}^{RP}(T_c)\| < 1$

