ARTICLE IN PRESS

Ocean Engineering xxx (xxxx) xxx

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Karst exploration: Unconstrained attitude dynamic control for an AUV

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ARTICLE INFO

Keywords:
Attitude control
Quaternion
Karstic robotic exploration

ABSTRACT

This paper introduces a new design for the control of the unconstrained attitude of an underwater mobile robot, dedicated to exploration of karstic environment. The approach followed in this study considers a quaternion formalism and proposes a dynamic control design that copes with the classic two-steps Lyapunov and Backstepping design, that remains within the quaternion formalism. Hence the control expression considers quaternion expressions without (as classically done in literature) requiring the decomposition of the quaternion into vectors and angle, which heavies the control expression and may introduce singularities. This control is then experimented on the Cube robot, showing the performance of the method.

1. Introduction

1.1. Applicative context

This study takes part in the research initiative REK (Robots for Karst Exploration) dedicated to the development of robotic systems to explore karstic environments. Karst generally denotes networks of underground natural conduits resulting from the dissolution of soluble rocks, limestone, dolomite and gypsum, that drain groundwater on a large scale, cf. Fig. 1-a. Karst aquifers already supply drinking water to millions of people worldwide, and are still considered as a potential gigantic reservoir of renewable fresh water, if a deeper investigation would provide a precise characterisation: location, geomorphology and dynamics. This is a key step towards an *active groundwater management*, enabling deep drilling, seasonal management and also allowing reliable prediction of dramatic sudden floods which may occur in karstic regions. This is a major and urgent issue for public authorities involved in prospection, protection and management of the groundwater resource and hydrogeologic risk prevention. Assessing the geometry of flow paths network in karst, which drives the dynamics of groundwater and transport processes, is an ambitious scientific objective that requires *in situ* information, which may be difficult to acquire. Cave divers are heroic, but face obvious physiological limitations. The use of an autonomous robotic solution in this context will induce a significant breakthrough, in its capacity to go further and deeper in the karst maze and acquire objective and dense information on this environment. This objective raises many scientific challenges. Among them, the one addressed in this paper is related to the control of the system's attitude. The chaotic nature of karstic environment implies an unconstrained system manoeuvrability, since the system has to be compact and able navigate in any direction, including vertical, cf. Fig. 1-b.

1.2. Control structure

In the following, we adopt the NGC-A (Navigation, Guidance, Control - Actuation system, Fig. 2) structure, where the design process is fourfold:

- Design a Navigation system to provide a reliable estimation of system and target states (denoted $\hat{\eta}$ in Fig. 2), which is not under the scope of this study. In this paper, we assume that an Inertial Measurement Unit (IMU) provides the necessary measurements to estimate the current value of the attitude state: the current orientation quaternion and the body-frame rotational velocities.
- Design a Guidance strategy to the objective. This step does not explicitly consider system dynamics, apart actuation properties (nonholonomy, underactuation). Indeed, the guidance strategy has to produce an achievable reference (denoted W_c in Fig. 2) to be tracked by the control system.
- The third step consists in the explicit consideration of system's dynamic model with a backstepping approach (Krstic et al., 1995) in order to insure the tracking of the guidance reference. It outputs a desired effort, expressed in the body frame (denoted F_B^d in Fig. 2). At this stage, the effect of the uncertainties of model parameters on the

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https://doi.org/10.1016/j.oceaneng.2020.108321
Received 9 March 2020; Received in revised form 7 September 2020; Accepted 28 October 2020

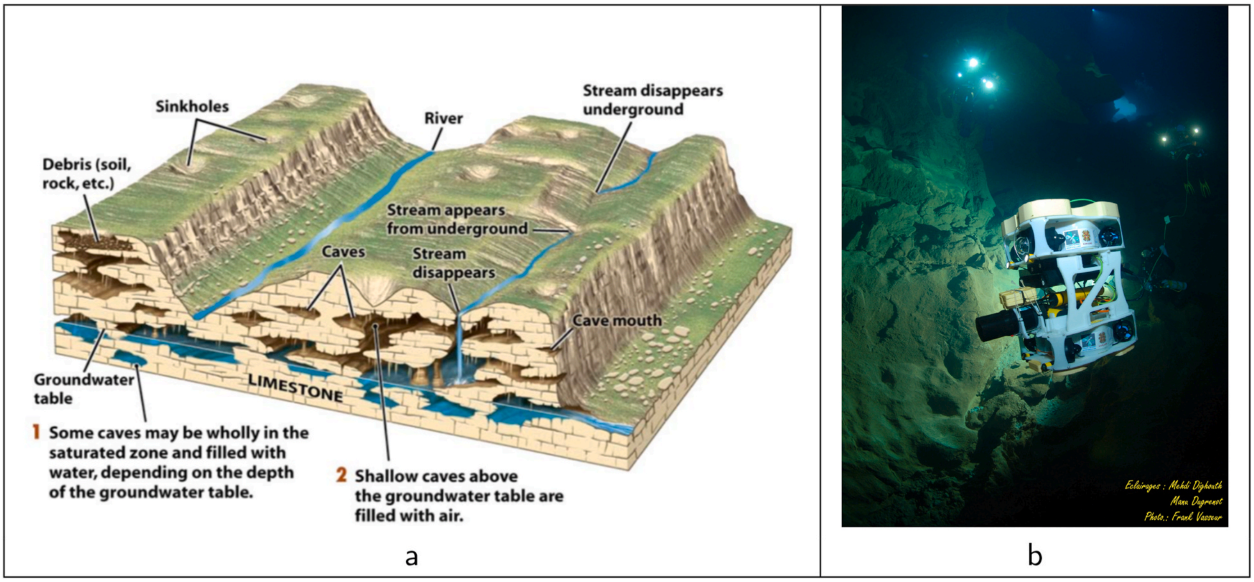

Fig. 1. a) a karstic environment (from (Jeremiah Chukwunonso, 2016)) and b) the Ulysse system in the chasm of Gourneyras.

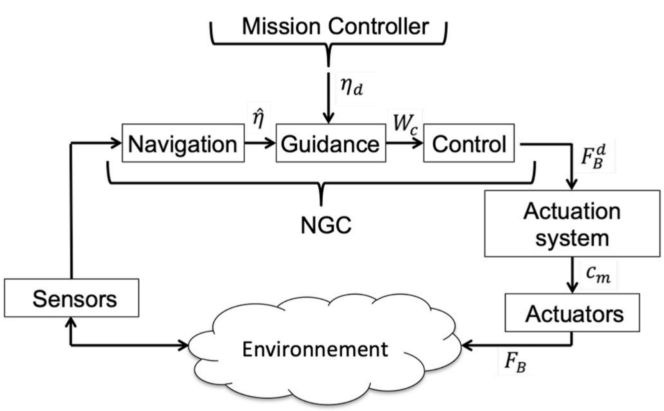

Fig. 2. NGC-A control structure.

convergence guarantee can be explicitly tackled. This question will not be treated in this paper.

- The last step considers the system's actuators, in order to produce previous actuation demand (denoted c_m in Fig. 2). The actuators structural configuration affords the system with different (under/iso/over) actuation property, which has to be explicitly considered during the Guidance design. The over-actuation property, case of our demonstrator, induces interesting redundancy management and robustness property (Ropars et al., 2018).

1.3. Control of the unconstrained attitude

Recent applications in the field of underwater robotics require an unconstrained attainability of the attitude, from hull inspection (Vaganay et al., 2006) to localised environmental observation or karstic/flooded galleries/well exploration (Stone, 2007). This of course implies that the system is equipped with enough actuators to afford the system with a full maneuverability, and also to choose a formalism to describe the attitude (the 3 rotations) without restriction nor singularity.

Three main formalisms are used to address the question of attitude control of unconstrained 6 Degrees of Freedom (DOF) systems as UAV, ASC, AUV or ROV.

- Euler angles, i.e roll, pitch and yaw, provides a meaningful interpretation of system's attitude, largely used in Naval Engineering. But this formalism suffers from nonlinearities and inconvenient singularities for control design process. For small variation of pitch and roll angles around zero, it is possible to design attitude control using Euler angles (Zheping et al., 2019), or Rodrigues' angles (Jolla, 1999), (Dai, 2006). These approaches assume a decoupling between horizontal and vertical plane, which is valid for a large number of applications where the mobile robot is asked to perform slow and near-horizontal trajectories.
- Three-dimension rotation matrices of $SO(3)$ can also be used for control design (Wang et al., 2019) and requires the manipulation of three-dimension skew-symmetric matrice, of the Lie algebra $so(3)$. This formalism is singular-free.
- Quaternion formalism is singular-free and allows for describing any 3D attitude of a rigid body as a (1×4) vector with a unity norm constraint. Attitude control using quaternion has been initially addressed for satellite control (Liu and Yang, 2019), (Egeland and Godhavn., 1994) and (Minh-Duc Hua et al., 2013), these systems requiring an unconstrained attainability of the 3D orientation space. Application to underwater systems has been more recently addressed in (Lekkas and Fossen, 2016), (Rodriguez et al., 2020) and (Xiangke and Changbin, 2010).

It has to be noted that any of these formalisms are subject to the 'unwinding' phenomenon (Bhat and Bernstein, 2000) that prevents to design a Globally Uniform Asymptotic Convergent continuous control for stabilizing the rotational motion of a rigid body. The effects of this unwinding phenomenon will be shown with the control proposed in this paper in the sequel.

An interesting point that differentiates the previous solutions using quaternion formalism, is the way which error function is computed. Indeed the control design requires the expression of an error function, from which convergence to zero will be provided by the control. Two options are reported in the literature:

- Attitude quaternion are normalized 4D vectors and error function can be expressed as the difference between two vectors, current and desired quaternions (Rodriguez et al., 2020). This approach allows for a direct application of Lyapunov-based design. The problem is that this vector difference is no more a normalized quaternion and results, after derivation, in a control expression which do not cope

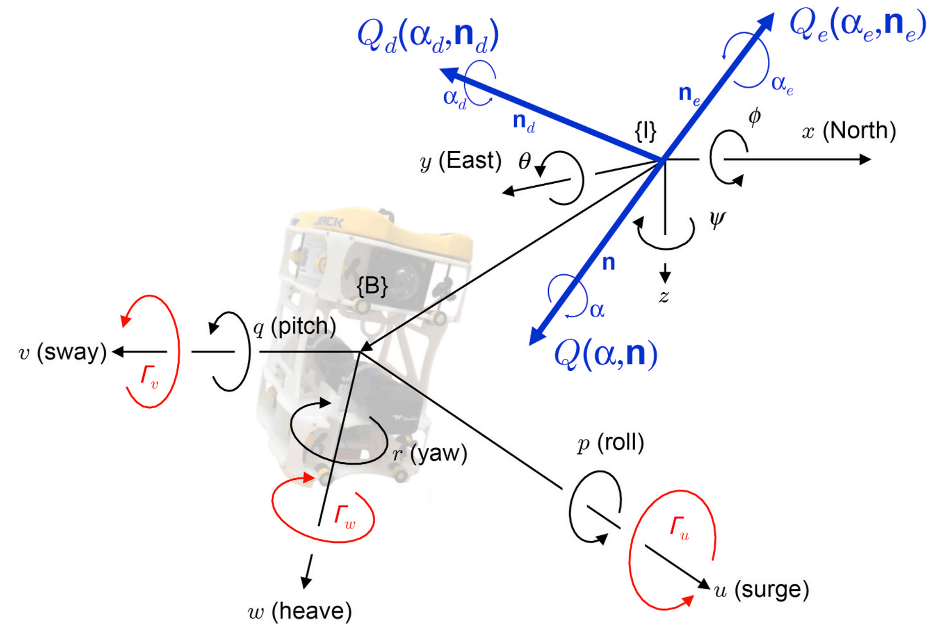

Fig. 3. Frame definition.

with the quaternion formalism. Note that a final arbitrary normalisation of the vectorial error function allows to avoid a drifting effect of this approximation, in the control computation, but prevents the building of a rigorous Proof of convergence.

- The quaternion difference, denoted Q_e in (2) remains on the unitary sphere (as done in (Fresk and Nikolakopoulos, 2013), (Guerrero-Castellanos et al., 2011), (Jorgensen and Gravdahl, 2011) and (Kim and Woolsey, 2007)), i.e. is still a rotation quaternion. This choice, as the function error, generally implies a decomposition of the quaternion into meaningful elements as an angle α_e around a unitary 3D vector $\mathbf{n}_e$. The control is then expressed using these error variables, and convergence is proven in (Dapeng et al., 2007) for a station keeping control objective and illustrated for tracking. These approaches result in complicate control expression, which does not remain within the quaternion formalism.

We are interested here, in applying the 'classic' Lyapunov-based dynamic control design, within the quaternion formalism, without requiring $(\alpha, \mathbf{n})$ decomposition. The work of (Kristiansen et al., 2009) reports a similar approach, applied to satellite control, but is applied to stabilisation (constant reference). In the sequel, we propose a (formally proven) tracking solution resulting in a compact and simple control expression, coping with a classic backstepping process for dynamics extension. This solution is then applied to the dynamic model of an underwater system (AUV Cube), and a solution for actuators allocation is proposed, concluding the control design process. Then, this solution is experimented on one of the LIRMM's AUV: the Cube Robot (cf Fig. 4). Please note that this paper focuses on attitude control only. Position, or longitudinal velocity control is, of course, also required, but do not present any difficulty since the system is omni-directional. In fact, the relevant question about position control is more related to the sensors question: how to obtain an accurate position, or longitudinal velocity, estimation in this confined karstic environment. This question is presently studied and is not addressed in this paper.

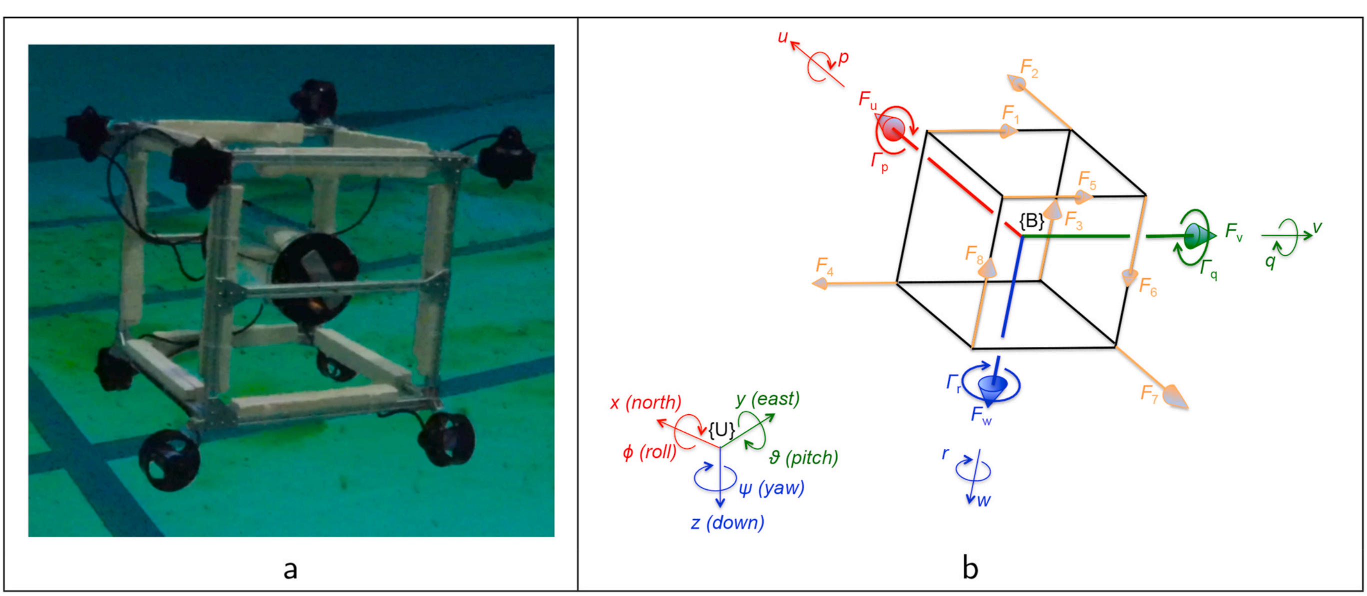

Fig. 4. The underwater Cube System.

1.4. Paper contribution

The contribution of this work is threefold:

- Design a dynamic control law in the quaternion space with classic Lyapunov-based and backstepping design, whose expression is compact and remains in quaternion formalism. The interest is that this design techniques will allow for extending solutions for classic control problems (path following, path tracking, robust control, obstacle avoidance …) to the quaternion formalism.
- Expose the complete attitude control chain, from control design to the computation of actuators' input for a new underwater redundant vehicle: the Cube AUV. Note that, as detailed in section 6.1, the dynamic model of the Cube system has not been identified yet. Hence we will consider the trivial (and false) dynamic model of equation (28). Also note that the control in translation is not exposed in this paper, since it does not present any originality.
- Address these question under the scope of a new ambitious applicative context: Karstic exploration with robots.

1.5. Paper organisation

After some preliminaries and the definition of the notation used in the sequel (section 2), the paper is organised as follows. Section 3 proposes the design of a kinematic guidance law which respects the quaternion formalism and the system actuation capabilities (fully actuated in our case). Section 4 considers the system dynamics in order to design a control expression that tracks the previously designed guidance reference. Section 5 is related to the consideration of the actuation system, in order to compute actuation inputs. Finally, section 6 proposes some simulation and experimental results to show the performances of the solution proposed.

2. Preliminaries and notation

In the sequel, we will use the following quaternion definition: a quaternion Q denotes the attitude of a body frame $\{B\}$ w.r.t a reference frame $\{I\}$, as a rotation of angle α around a normalized vector $\mathbf{n}$, whose coordinates are expressed w.r.t $\{I\}$, as illustrated at Fig. 3. Fig. 3 also denotes the body-frame velocities, classically written $\mathbf{v} = [u, v, w]^T$ and $\mathbf{w} = [p, q, r]^T$ for linear and rotational velocities, expressed in the body-frame $\{B\}$, where $\mathbf{x}^T$ denotes the transpose of vector $\mathbf{x}$. $\mathbf{\Gamma} = [\Gamma_u\ \Gamma_v\ \Gamma_w]^T$ denotes the resulting torques, w.r.t. $\{B\}$, provided by the actuation system. Euler angles $\{\varphi, \theta\ \psi\}$ are illustrative and will not be used in the sequel.

Quaternion Q is expressed as the (4×1) dimension vector:

$$Q = \left[\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\cdot\mathbf{n}^T\right]^T \tag{1}$$

The current error between Q and Q_d is expressed as:

$$Q_e = Q^* \otimes Q_d \tag{2}$$

where Q^* designs the conjugate of Q and $\otimes$ is the quaternion product.

Two other important relations link the quaternion derivative with the associated angular velocities expressed in the reference frame, $W = 2\cdot Q^* \otimes \dot{Q}$, where W is the imaginary quaternion expression of vector $\mathbf{w} = [p,\ q,\ r]^T$ designing the angular velocities expressed in the body frame, $W = [0, \mathbf{w}^T]^T$.

We introduce the matrix $\mathbf{Q_v}$, that expresses the relation between a (3×1) vector $\mathbf{x}$ to its associated imaginary (4×1) quaternion X, such that: $X = \mathbf{Q_v}\cdot\mathbf{x} = [0,\ \mathbf{x}^T]^T$ and $\mathbf{x} = \mathbf{Q_v}^T\cdot X$ where:

$$\mathbf{Q_v} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

We also introduce the following matrix $\mathbf{Q_q}$,

$$\mathbf{Q_q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where if $X = [x_1,\ x_2,\ x_3,\ x_4]^T$ is any quaternion, then $\mathbf{Q_q}\cdot X = [0,\ x_2,\ x_3,\ x_4]^T$. Note that if $y = [0,\ y_2,\ y_3,\ y_4]^T$ is any imaginary quaternion, then $Y = \mathbf{Q_q}\cdot Y$. Also note that $\mathbf{Q_v}\cdot\mathbf{Q_v}^T = \mathbf{Q_q}$ and $\mathbf{Q_v}^T\cdot\mathbf{Q_v} = \mathbf{I}_3$, where $\mathbf{I}_3$ stands for the (3×3) identity matrix.

Another necessary expression, initially introduced in (Markley, 2003), expresses the velocity of the error quaternion as:

$$\dot{Q}_e = \frac{1}{2}(-W \otimes Q_e + Q_e \otimes W_d) \tag{5}$$

Finally, in the sequel, as $1_Q = [1, 0, 0, 0]^T$ denotes the identity quaternion, $0_Q = [0, 0, 0, 0]^T$ denotes the null quaternion.

3. Guidance design

The design of the kinematic guidance reference is a control objective that drives the current orientation of a frame $\{B\}$, denoted with the quaternion Q, to a desired attitude Q_d, while tracking the evolution of the reference $\dot{Q}_d$. At this stage, the considered system can be expressed as the derivation relation of a normalized quaternion Q as:

$$\dot{Q} = \frac{1}{2}\cdot Q \otimes W \tag{6}$$

where, according to the general expression $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$, Q denotes the state vector and W the input.

The kinematic guidance reference will be the result of the following control problem:

Guidance Problem:*Consider system (6) and find an expression for W that asymptotically and uniformly drives Q to Q_d.*

Rewriting (5) gives us a candidate for the structure of W as:

$$W = Q_e \otimes W_d \otimes Q_e^* - W_e \tag{7}$$

where $W_d = 2\cdot Q_d^* \otimes \dot{Q}_d$ and, as justified in the sequel, $W_e = -K\cdot\widehat{Q}_e$. Consider now the following Lyapunov function candidate, originally introduced by (Mahony et al., 2008) on SO(3)::

$$V = \frac{1}{2}\cdot(1_Q - Q_e)^T\cdot(1_Q - Q_e) \tag{8}$$

Direct derivation yields:

$$\dot{V} = -\dot{Q}_e^T\cdot(1_Q - Q_e) = -\dot{Q}_e^T\cdot 1_q - \underbrace{\dot{Q}_e^T\cdot Q_e}_{0} \tag{9}$$

where $\dot{Q}_e^T\cdot Q_e = 0$ since vectors $\dot{Q}_e$ and Q_e are perpendicular. Hence, considering the kinematic differential equation of the error quaternion, $\dot{Q}_e = \frac{1}{2}\cdot W_e \otimes Q_e$, the simple choice:

$$W_e = K\cdot\left[0, \sin\frac{\alpha_e}{2}\cdot\mathbf{n}_e^T\right]^T = -K\cdot\mathbf{Q_q}\cdot Q_e = -K\cdot\widehat{Q}_e \tag{10}$$

yields

$$\dot{V} = \frac{K}{2}\cdot\left(\widehat{Q}_e \otimes Q_e\right)^T\cdot 1_Q = -\frac{K}{2}\cdot\sin^2\alpha_e < 0 \tag{11}$$

for any arbitrary positive gain K and where α_e is the rotation angle of

Q_e around vector $\mathbf{n}_e$, as generally expressed at Equation (1). Note that control expression (10) can be directly computed using Q_e, without explicitly computing α_e and $\mathbf{n}_e$. In (10), $\widehat{Q}_e = \mathbf{Q_q} \cdot Q_e$ is simply Q_e where its first component has been nullified.

We are now ready to state our second proposition:

Proposition 1. *Consider the system* (6), *and references* Q_d *and* $\dot{Q}_d$. *The following control expression* :

$$W = Q_e \otimes W_d \otimes Q_e^* + K \cdot \widehat{Q}_e \tag{12}$$

where Q_e *denotes the quaternion error, cf.* Equation (2), *and where* $\widehat{Q}_e$ *is* Q_e *where first component has been nullified, uniformly and asymptotically drives* Q *to* Q_d, *and tracks* $\dot{Q}_d$, *hence solving the Guidance Problem.* □

Proofof Proposition 1: *Consider the Lyapunov function candidate expressed in* (8). *The choice of the control expression* (12) *yields to the tracking error expression* (10), *considering* (7), *and equivalently implies that* $\dot{V} < 0$, *as expressed in* Equation (11). *Note that* $\ddot{V}$ *is easily computable and is bounded. Hence, we can now invoke the Barbalat's lemma* (*as done in* (Lapierre and Soetanto, 2007)) *to prove the asymptotic and uniform convergence of* $\dot{V}$ *to* 0, *hence proving the asymptotic and uniform convergence of* α_e *to* 0, *as well for* Q_e *to* 1_q *since* Q_e *is normalized. Moreover, considering now the control expression* (12), *it is clear that since* $\widehat{Q}_e$ *asymptotically and uniformly vanishes to* 0_Q, *then* W *asymptotically tracks* W_d.

Not that, as stated in (Bhat and Bernstein, 2000), *the globality of the convergence cannot be achieved since the system's trajectories evolve on a vector bundle over* $SO(3)$ *which is compact - irrespective to* R^3, *which is the state space when translations are considered and where the globality of the convergence is proven with the argument that* $V(\infty) = \infty$.

A kinematic velocity control $\mathbf{w} = [p, q, r]^T$ *is extracted from* W, *as* $\mathbf{w} = \mathbf{Q_v} \cdot W$.

4. Control design

The previous guidance design process resulted in the expression of a desired velocity profile which, if tracked by a physical system, will ensures a Global, Uniform and Asymptotic Convergence of system's attitude to its target.

The sequel exposes a backstepping design to consider system's dynamic model, in order to provide the expression for the desired forces and torques (expressed in the body-frame) that the actuation system has to produce in order to follow its reference.

Consider now a physical system, asked to track a desired orientation reference in terms of Q_d and $\dot{Q}_d$, with rotational dynamic model:

$$\mathbf{\Gamma} = \mathbf{J} \cdot \dot{\mathbf{w}} + \mathbf{f}(\mathbf{w}, \mathbf{v}, \eta) \tag{13}$$

where $\mathbf{J}$ is the invertible inertia matrix, $\mathbf{f}(\mathbf{w}, \mathbf{v}, \eta)$ includes damping, cross velocity terms and potential effects (weight and buoyancy), and where $\mathbf{w} = [p, q, r]^T$ and $\mathbf{v} = [u, v, w]^T$ denote the velocities (angular and linear, resp.) of $\{B\}$ w.r.t $\{I\}$, expressed in $\{B\}$, and $\mathbf{\Gamma} = [\Gamma_u, \Gamma_v, \Gamma_w]^T$ denotes the 3 torques, expressed in $\{B\}$, resulting from actuation.

Hence, the control problem is expressed as:

Control Problem:*Consider system (13) and find an expression for* Γ *that asymptotically and uniformly drives* Q *to* Q_d *and* $\dot{Q}$ *to* $\dot{Q}_d$.

Consider the following Lyapunov function candidate:

$$V_2 = \frac{K_1}{2} \cdot (1_q - Q_e)^T \cdot (1_q - Q_e) + \frac{1}{2} \cdot (W_c - W)^T \cdot (W_c - W) \tag{14}$$

where K_1 is a positive gain and, as classically done in backstepping procedure, the reference W_c is chosen as the previous kinematic solution:

$$W_c = Q_e \otimes W_d \otimes Q_e^* + K_2 \cdot \widehat{Q}_e \tag{15}$$

where K_2 is an arbitrary positive gain. Direct derivation of (14) yields:

$$\dot{V}_2 = -K_1 \cdot \dot{Q}_e(1) + \left(\dot{W}_c - \dot{W}\right)^T \cdot (W_c - W) \tag{16}$$

where $\dot{Q}_e(1)$ denotes the first component of $\dot{Q}_e$. Consider now the expression of system's dynamics, expressed at Equation (13), in 16.

$$\dot{V}_2 = -K_1 \cdot \dot{Q}_e(1) + \left(\dot{W}_c - \mathbf{Q_v} \cdot (\mathbf{J}^{-1} \cdot (\mathbf{\Gamma} - \mathbf{f}(\mathbf{w}, \mathbf{v}, \eta)))\right)^T \cdot (W_c - W) \tag{17}$$

Hence, the choice

$$\mathbf{\Gamma} = \mathbf{J} \cdot \dot{\mathbf{w}}_\mathbf{R} + \mathbf{f}(\mathbf{w}, \mathbf{v}, \eta) \tag{18}$$

where

$$\dot{\mathbf{w}}_\mathbf{R} = \mathbf{Q_v}^T \cdot \left(\dot{W}_c + K_2 \cdot (W_c - W) + K_3 \cdot \widehat{Q}_e\right) \tag{19}$$

implies

$$\dot{V}_2 = -K_1 \cdot \dot{Q}_e(1) - K_2 \cdot (W_c - W)^T \cdot (W_c - W) - K_3 \cdot \widehat{Q}_e^T \cdot (W_c - W) \tag{20}$$

since W, W_c, $\widehat{Q}_e$ are imaginary quaternions. Considering (15) yields

$$\begin{aligned} \dot{V}_2 \quad = \quad & -K_1 \cdot \dot{Q}_e(1) - K_2 \cdot (W_c - W)^T \cdot (W_c - W) \\ & -K_3 \cdot K_2 \cdot \widehat{Q}_e^T \cdot \widehat{Q}_e - K_3 \cdot \widehat{Q}_e^T \cdot (Q_e \otimes W_d \otimes Q_e - W) \end{aligned} \tag{21}$$

since W is an imaginary quaternion, then $W = -W^*$. Consider now the following expression for (5):

$$Q_e \otimes W_d \otimes Q_e^* - W = 2 \cdot \dot{Q}_e \otimes Q_e^* \tag{22}$$

Tedious but direct computation shows that $\widehat{Q}_e^T \cdot (\dot{Q}_e \otimes Q_e^*) = \frac{\dot{\alpha}_e}{2} \cdot \sin\frac{\alpha_e}{2} = -\dot{Q}_e(1)$. Hence, without loss of generality, let for $K_3 = K_1/2$,

$$\dot{V}_2 = -K_2 \cdot (W_c - W)^T \cdot (W_c - W) - K_3 \cdot \widehat{Q}_e^T \cdot \widehat{Q}_e \tag{23}$$

which is strictly negative if any component of $\widehat{Q}_e$ is non null. Hence, $\widehat{Q}_e$ asymptotically and uniformly converges to 0_Q, inducing similar convergence of Q_e to 1_Q, since Q_e is normalized.

Proposition 2. *Consider system* (13), *and references* Q_d, $\dot{Q}_d$ *and* $\ddot{Q}_d$. *Then control expression* (18) *uniformly and asymptotically drives* Q *to* Q_d, $\dot{Q}$ *to* $\dot{Q}_d$, *and tracks* $\ddot{Q}_d$, *hence solving the Control Problem.* □

The Proof is similar to previous one and is omitted.

5. Actuation system

The role of the Actuation system is to provide a reliable physical realisation of (18), resulting from actuators effects. If the system has more actuators than degrees of freedom (case of our Cube system, equipped with 8 thrusters, as shown at Fig. 4-b), it is considered to have a redundant actuation system, and redundancy management can be done during the control allocation process. Note that this study is dedicated to the control of the attitude of the robot, which involves 3 degrees of freedom, without consideration for the other longitudinal degrees of freedom. Hence, our problem involves 8 thrusters to control 3 degrees of freedom, resulting is a highly redundant actuation system.

This question has been tackled in (Ropars et al., 2018) where the advantages of a redundant actuation system have been stated and experimented. The theoretical statement of this question is illustrated in the sequel, but not exploited here.

Two relations are characterizing the actuation model: the Configuration matrix $\mathbf{C}$ and the Dispatcher $\mathbf{D}$.

5.1. *Configuration matrix*

The configuration Matrix $\mathbf{C}$ expresses the body-frame resulting action of actuators. As classically done for underwater robots, we neglect

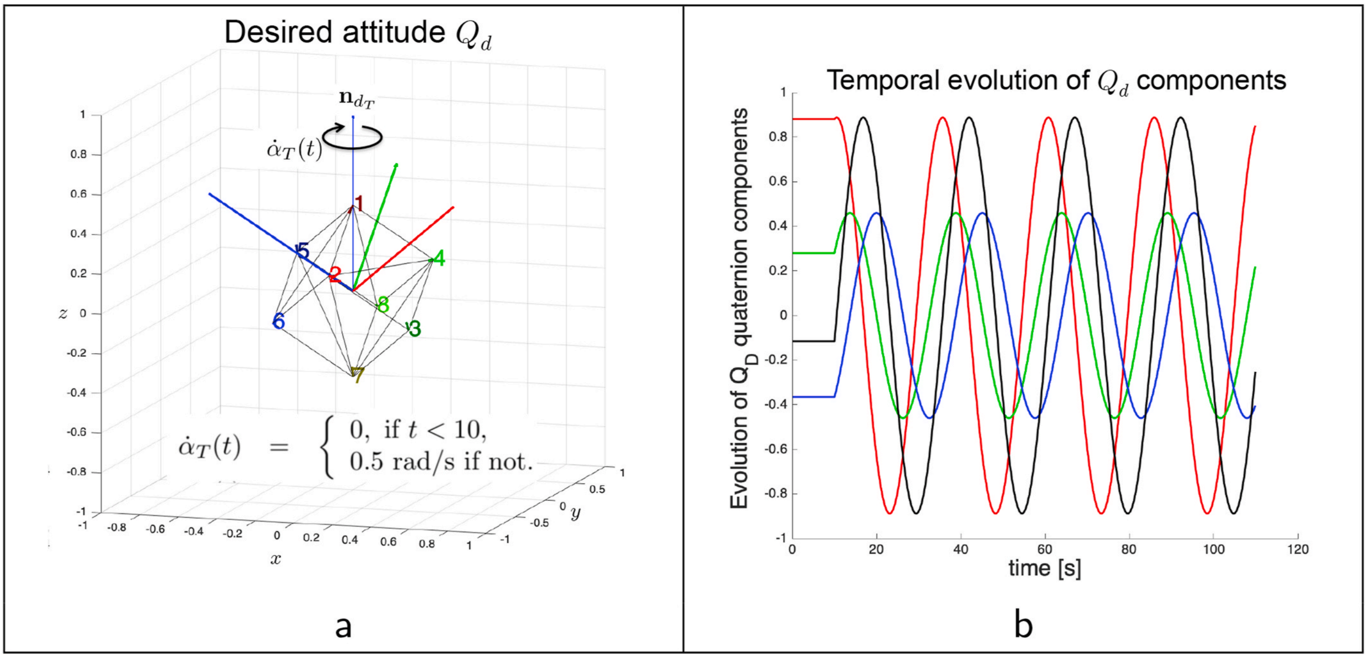

Fig. 5. a) Initial position, b) Guidance Reference, c) Temporal evolution of the components of vector Q_d.

the thrusters' torque generated around the axis of the propeller. Hence, given the location of origin and direction the force F_i w.r.t $\{B\}$, (cf. Fig. 4) generated by each actuators i, by vector $\mathbf{d}_i$ and its orientation w.r.t. $\{B\}$ with quaternion Q_i, the resulting action of n actuators is written as:

$$\boldsymbol{\Gamma} = \sum_{i=1}^{n} \mathbf{d}_i \times [F_i, 0, 0]^T \tag{24}$$

where $\times$ denotes the vectorial product. Hence, (24) can be written in a compact form with:

$$\boldsymbol{\Gamma} = \mathbf{C} \cdot \mathbf{F}_m \tag{25}$$

where $\mathbf{F}_m = [F_1\ F_2\ \dots\ F_n]^T$ denote the individual action of the n actuators and $\mathbf{C}$ is a $(3 \times n)$ configuration matrix, expressing the resulting action of n actuators in the body-frame $\{B\}$.

5.2. The dispatcher

Previous relation needs to be inverted, in order to compute the actuator inputs, according to the control demand $\boldsymbol{\Gamma}_d$ of Equation (18). Here, one can exploit redundancy, as done in (Ropars et al., 2018), where it is shown that a satisfying candidate for the inversion of (25) is:

$$\mathbf{F}_m = \underbrace{[\mathbf{C}^+\ \mathbf{M}_m]}_{\mathbf{D}} \cdot \begin{bmatrix} \boldsymbol{\Gamma}_d \\ \mathbf{r}_m \end{bmatrix} \tag{26}$$

where $\mathbf{C}^+ = \mathbf{C}^T \cdot (\mathbf{C} \cdot \mathbf{C}^T)^{-1}$ is the Moore-Penrose pseudo-inverse, $\mathbf{M}_m \in ker(\mathbf{C})$ and $\mathbf{r}_m$ is an arbitrarily chosen $(n-3 \times 1)$ vector. $\mathbf{D} = [\mathbf{C}^+\ \mathbf{M}_m]$ is called Dispatcher. Matrix $\mathbf{M}_m$ plays the role of a projector of $\mathbf{r}_m$ in the null space of $\mathbf{C}$, making its contribution to the actuators input independent from the main task: the use of the control (26) in system (25) implies $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_\mathbf{d}$.

The actuation redundancy management is performed with a judicious choice of $\mathbf{r}_m$ in order to minimise complementary criteria, i.g. energy, robustness to failure, reactivity …

6. Simulations and experimentations

This section proposes to illustrate the performances of the solution proposed on the Cube system, depicted at Fig. 4-a. The chosen reference is identical to (30).

6.1. System description

Consider now the system on which previous control solution will be implemented, i.e. the fully actuated underwater system Cube (cf. Fig. 4).

Cube is an AUV with a cubic shape of 0.52 cm length, equipped with 8 thrusters oriented as shown at Fig. 4-b. A Dropix embedded card carries an IMU and runs the controller. Its actuation configuration results in the following $\mathbf{C}$ configuration matrix of Equation (25), where actuation torque has been neglected:

$$\mathbf{C} = \begin{bmatrix} 0.27 & 0 & -0.27 & 0.27 & 0.27 & 0.27 & 0 & 0.27 \\ 0 & -0.27 & 0.27 & 0 & 0 & 0.27 & -0.27 & -0.27 \\ 0.27 & -0.27 & 0 & -0.27 & -0.27 & 0 & 0.27 & 0 \end{bmatrix} \tag{27}$$

The dynamic model of the lab-prototype Cube system has not been identified. This identification requires intensive experimental tests that are presently performed on the experimental robot Ulysse (Fig. 1-b). Hence, for the Cube system, we use in the control computation the following trivial rotational dynamic model:

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \mathbf{f}(\mathbf{w}, \mathbf{v}, \eta) = 0_3 \tag{28}$$

6.2. Reference design

We compute the reference as a stationary initial desired attitude Q_{d_0}, for the 10 first seconds:

$$\begin{array}{rcc} Q_{d_0} & = & Q_{d_1} \otimes Q_{d_2} \\ Q_{d_1} & = & [\cos(\alpha_1/2),\ \sin(\alpha_1/2),\ 0,\ 0] \\ \alpha_1 & = & \arctan\left(1/\sqrt{2}\right) \\ Q_{d_2} & = & [\cos(\alpha_2/2),\ 0,\ \sin(\alpha_2/2),\ 0] \\ \alpha_2 & = & -\pi/4 \end{array} \tag{29}$$

This reference will drive the cube to vertically align two opposite corners (1 and 7), as shown at Fig. 5-a. Then, after 10 s, a trajectory is given to the reference as:

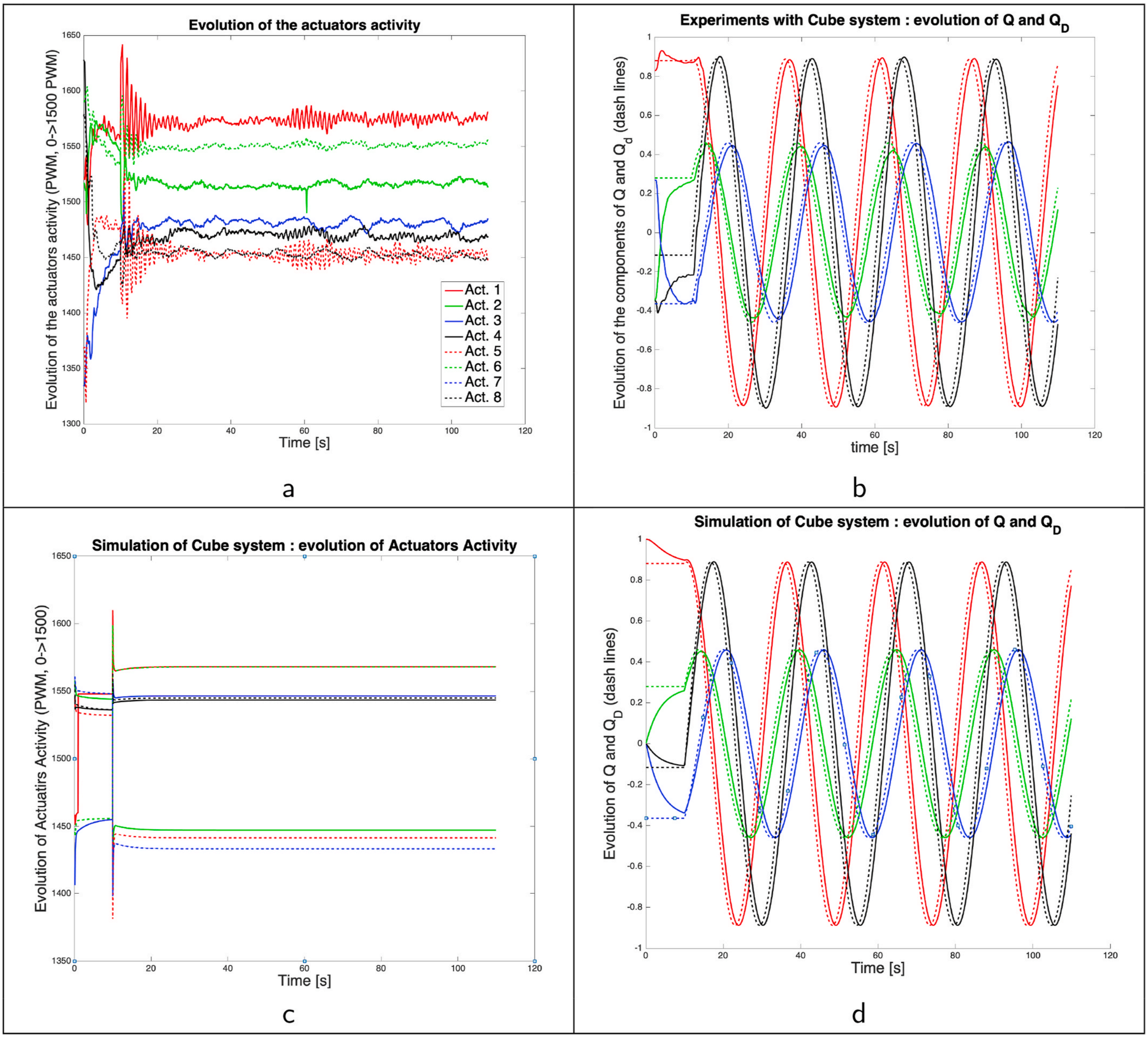

Fig. 6. Experimental results (a and b), simulated results (c and d).

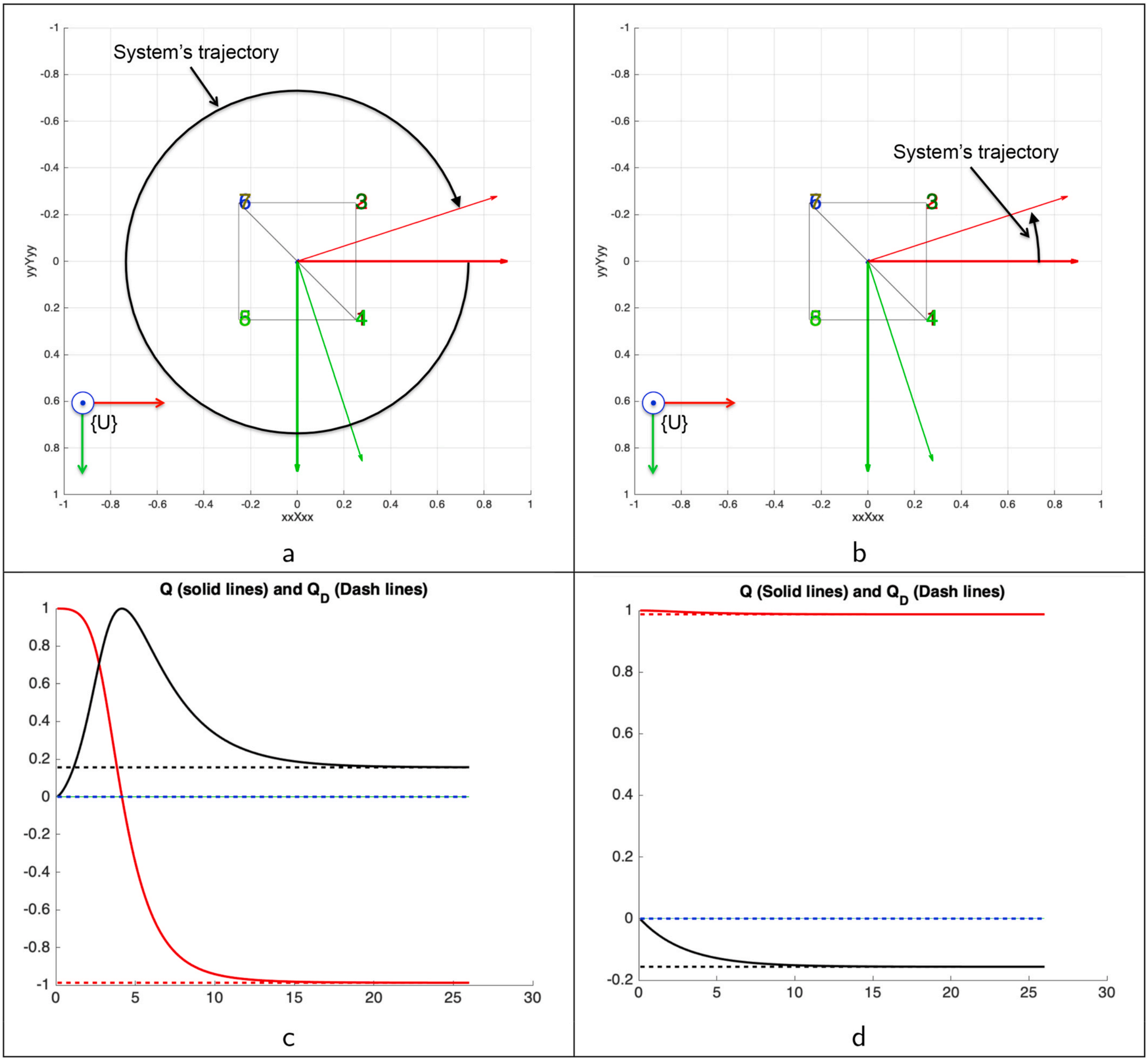

Fig. 7. Unwinding phenomenon, when $\alpha_D = 0.9{\cdot}2\pi$. When the angle used to design the reference is defines in $\alpha_D \in [0, 2\pi)$, we obtain the response of figures a & c. When the angle used to design the reference is defines in $\alpha_D \in [-\pi, \pi)$, we obtain the response of figures b & d.

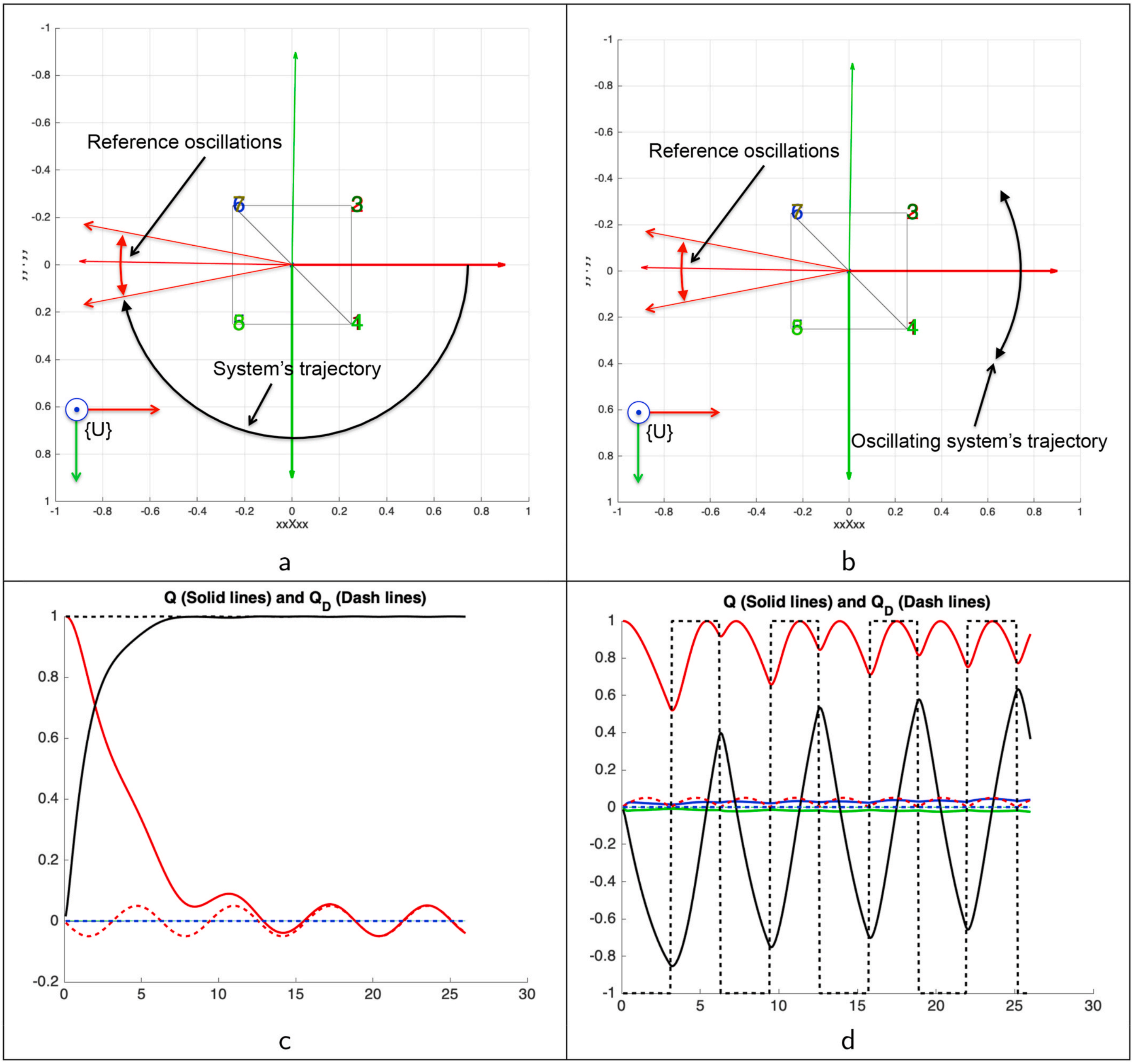

Fig. 8. Unwinding phenomenon, when $\alpha_D = A_t * \sin(K_t \cdot t) + \pi$. When the angle used to design the reference is defines in $\alpha_D \in [0, 2\pi)$, we obtain the response of figures a & c. When the angle used to design the reference is defines in $\alpha_D \in [-\pi, \pi)$, we obtain the response of figures b & d.

$$
\begin{aligned}
Q_d &= Q_{d_0} \otimes Q_{d_T} \\
Q_{d_T} &= \left[\cos(\alpha_T/2),\ \frac{\sin(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\sin(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\sin(\alpha_T/2)}{\sqrt{(3)}}\right] \\
\dot{Q}_d &= Q_{d_0} \otimes \dot{Q}_{d_T} \\
\dot{Q}_{d_T} &= \frac{\dot{\alpha}_T}{2}\cdot\left[-\sin(\alpha_T/2),\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}}\right] \\
\ddot{Q}_d &= Q_{d_0} \otimes \ddot{Q}_{d_T} \\
\ddot{Q}_{d_T} &= \frac{\ddot{\alpha}_T}{2}\cdot\left[-\sin(\alpha_T/2),\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}},\ \frac{\cos(\alpha_T/2)}{\sqrt{(3)}}\right] - \frac{\dot{\alpha}_T^2}{4}\cdot Q_{d_T} \\
\alpha_T(t) &= \begin{cases} 0, & \text{if } t < 10, \\ \int_0^t \dot{\alpha}_T(\tau)\cdot d\tau & \text{if not.} \end{cases} \\
\dot{\alpha}_T(t) &= \begin{cases} 0, & \text{if } t < 10, \\ 0.5 \text{ rad/s} & \text{if not.} \end{cases} \\
\ddot{\alpha}_T(t) &= 0.
\end{aligned}
\tag{30}
$$

This reference produces a rotation of $\dot{\alpha}_T = 0.5 rad/s$ around vector $\mathbf{n}_{d_T} = (1/\sqrt{3})\cdot[1,1,1]^T$, which is vertically aligned since previous transformation Q_{d_0}. The evolution of the components of Q_d is shown at Fig. 5-b.

6.3. Experimentations

Hence, the application of control (26), with reference (30), and a minimal-energy pseudo-inverse Dispatcher ($\mathbf{D} = \mathbf{C}^+$) provides the results of Fig. 6-a and -b.

Fig. 6-a indicates the evolution of actuators activity during test. Fig. 6-b shows the convergence of the actual system's attitude Q to the reference Q_D. Analysing these experimental results, we can extract 2 undesired phenomena:

- The control signal presents vibration of frequency of 0.5Hz, which is similar to the desired rotational velocity $\dot{\alpha}_T$. This is most likely due to an unbalance phenomenon due to the incomplete model.
- A non-negligible delay is present in the tracking of the reference. This can be explained with the approximated model we chose for control computation (Equation (28)). In order to confirm the implication of the misestimation of the dynamic model parameters on the tracking error, we perform a simulation where the control is unchanged while the dynamic model of the simulated system considers a damping effect with the form $\mathbf{f}_{sim}(\mathbf{w},\mathbf{v},\eta) = -2\cdot\mathbf{I}_3\cdot[p\cdot|p|\ ,\ q\cdot|q|\ ,\ r\cdot|r|]^T$, where $\mathbf{I}_3$ stands for the (3×3) identity matrix.

It is clear that this tracking error can be a consequence of dynamic parameters misestimation. Other sources of imprecision can be identified: a misplacement of the thrusters and/or the IMU, a difference between individual thruster's characteristic … This underline the necessity for a precise dynamic model or the design of an efficient robust control scheme. This warrants further research.

7. Unwinding phenomenon

This section enlights the effect of the unwinding phenomenon on the proposed control solution. Let's define the desired attitude as a quaternion expressing a rotation of angle $\alpha_D(t)$ around the z axis of the universal frame.

$$
\begin{aligned}
Q_D &= [\cos(\alpha_D/2)\ 0\ 0\ \sin(\alpha_D/2)] \\
\dot{Q}_D &= \dot{\alpha}_D \big/ 2\cdot[-\sin(\alpha_D/2)\ 0\ 0\ \cos(\alpha_D/2)] \\
\ddot{Q}_D &= \ddot{\alpha}_D \big/ 2\cdot[-\sin(\alpha_D/2)\ 0\ 0\ \cos(\alpha_D/2)] - \dot{\alpha}^2 \big/ 2\cdot Q_D
\end{aligned}
\tag{31}
$$

Let's consider two different domains of definition for α_D:

$$
\begin{aligned}
A &: \alpha_D \in [0, 2\pi) \\
B &: \alpha_D \in [-\pi, \pi)
\end{aligned}
\tag{32}
$$

Fig. 7 shows a simulated system's response to a constant reference where $\alpha_D = 0.9\cdot 2\pi$, while the initial attitude is $Q(0) = 1_Q$. When α_D is defined according to the domain A (Fig. 7-a and -c), the system rotates clockwise through large angle, while the counterclockwise rotation is obviously better. This counterclockwise is achieved when the domain of definition B is considered (Fig. 7-b and -d).

We then perform another simulation where $\alpha_D = A_t\cdot\sin(K_t\cdot t) + \pi$, where $A_t = 0.1$ and $K_t = 1$. Results are shown at Fig. 8. Figure Fig. 8-a and -c shows the system's behaviour when domain of definition A is considered for α_D. Clearly, convergence is achieved. Nevertheless, when domain of definition B is chosen, the system does not achieve convergence.

This unwinding phenomenon is due to a topological obstruction to the problem of attitude control, as stated in (Bhat and Bernstein, 2000). Some works have addressed this question, proposing an anti-unwinding attitude error function as a component of a potential function, avoiding problematic pointing situation (Hu et al., 2019), or discontinuous controllers (Tiwari et al., 2017). Based on the development proposed in this paper, we will tackle this question in redefining the trajectory-tracking objective to a path-following problem, with the objective to improve the system response to this unwinding phenomenon. This warrants further research.

8. Conclusion

In this paper, we propose a new compact expression for the control of the attitude of an AUV, based on the quaternion formalism without requiring the classic $(\alpha, \mathbf{n})$ decomposition. The approach proposed allows for applying backstepping technique, within classic Lyapunov design, to explicitly consider system dynamics in the control expression, and remains in the quaternion formalism, i.e. does not require any arbitrary normalisation. The convergence of the proposed control law is formally proven and experimentations on the AUV Cube illustrates the performances of the approach. The unavoidable unwinding phenomenon has been illustrated with the solution proposed.

The interest of this control design is that it provides a framework that can cope with a large class of control problems as path-following, adaptive robust control, obstacle avoidance … The extension of the method proposed here to these problems, on $SO(3)$, is the subject of our present research.

Indeed the applicative context (karst exploration with robots) requires robotics systems with a manageable reactivity and precise control of their movement with an unconstrained attainability of attitude. The next step will be to include translational control strategy, where difficulties occurs at the sensors level, in such a confined, hazardous and turbid environment. This promises a lot of exiting scientific challenges which warrants further research.

CRediT authorship contribution statement

Lionel Lapierre: Conceptualization, Methodology, Investigation, Writing - original draft. **Rene Zapata:** Conceptualization, Methodology, Data curation, Writing - review & editing. **Pascal Lepinay:** Software, Validation. **Benoit Ropars:** Software, Validation, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This project was supported by the LabEx NUMEV (ANR-10-LABX-0020) within the I-SITE MUSE (ANR-16-IDEX-0006) and the Region Occitanie (french FEDER funds). The authors would also like to express their gratefull thanks to Vincent Deconchy for his enthusiastic and helpful comments.

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