Localization of partially hidden targets using a fleet of UAVs via robust bounded-error estimation

J. Ibenthal[1], L. Meyer[1], H. Piet-Lahanier[1], and **M. Kieffer**[2]

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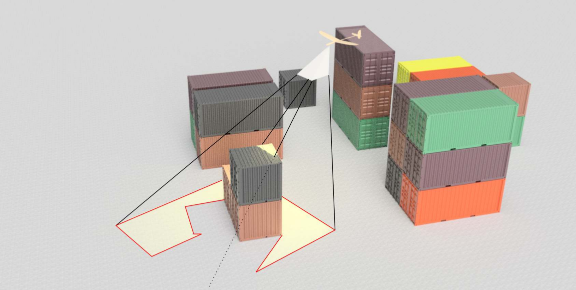
Séminaire ENSTA-Bretagne, 6 juillet 2022

Outline

Problem

Localization

- of partially hidden static targets
- in an unknown cluttered environment
- using a fleet of collaborative UAVs



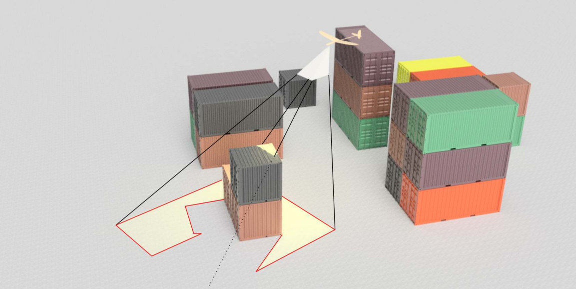
Difficulties encountered

UAVs have limited ability to detect targets due to

- limited field of view
- presence of obstacles

Introduction of probability of non-detection [Hu et al., 2014], [Li and Duan, 2017]

- poorly accounts for influence of local environment



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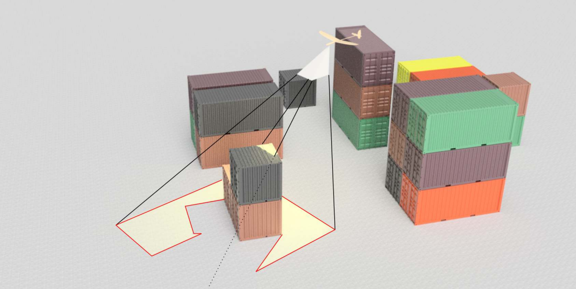


Proposed approach

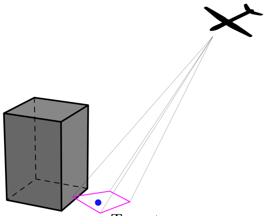
Here, consider

- UAVs equipped with optical seekers,
- static targets,
- obstacles with unknown location.

Target detected and identified when

- located within field of view of seeker,
- observed from some conic *detectability set*.

Set-membership estimation technique as in [Ibenthal et al., 2021, Ibenthal et al., 2020]



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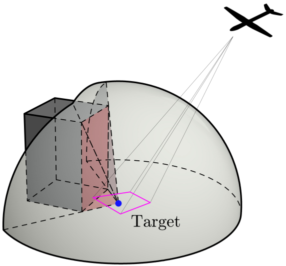

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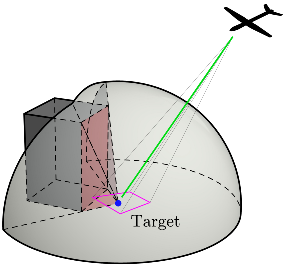

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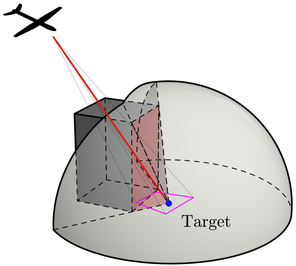

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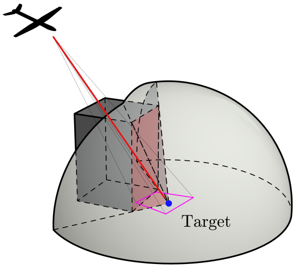

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Outline

UAVs and targets

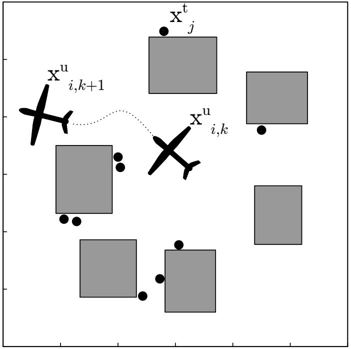
Region of interest (RoI) $\mathbb{X}_0$

At time instant $t = kT$

- state vector of UAV: $\mathbf{x}^{\mathrm{u}}_{i,k}$, $i = 1, \ldots, N_{\mathrm{u}}$ evolves as

$$\mathbf{x}^{\mathrm{u}}_{i,k+1} = \mathbf{f}^{\mathrm{u}}_{k}\left(\mathbf{x}^{\mathrm{u}}_{i,k}, \mathbf{u}_{i,k}\right),$$

  with control input $\mathbf{u}_{i,k} \in \mathbb{U}$.
- static target locations: $\mathbf{x}^{\mathrm{t}}_{j} \in \mathbb{X}_{\mathrm{T}}$, $j = 1, \ldots, N_{\mathrm{t}}$.



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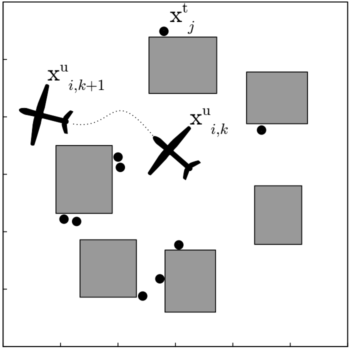

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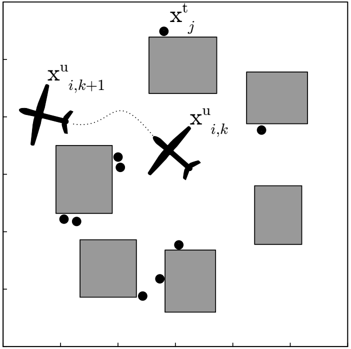
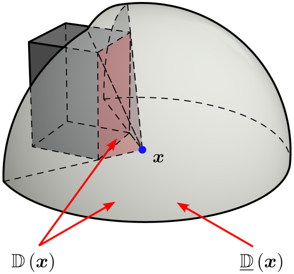


Detectability set

Obstacles may partly hide some points $\boldsymbol{x} \in \mathbb{X}_\mathrm{T}$.



Assumption: for all $\boldsymbol{x} \in \mathbb{X}_\mathrm{T}$, there exists (unions of) half-cones

$$\underline{\mathbb{D}}(\boldsymbol{x}) = \left\{ \boldsymbol{x} + a_1 \boldsymbol{v}_1(\boldsymbol{x}) + \dots + a_{n(\boldsymbol{x})} \boldsymbol{v}_{n(\boldsymbol{x})}(\boldsymbol{x}) \mid a_i \in \mathbb{R}^+, i = 1, \dots, n(\boldsymbol{x}) \right\}$$

such that $\underline{\mathbb{D}}(\boldsymbol{x}) \subset \mathbb{D}(\boldsymbol{x})$.

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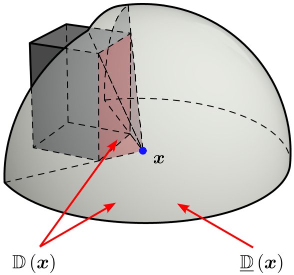

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Measurements

UAV i with state $\mathbf{x}^{\mathrm{u}}_{i,k}$ observes $\mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \subset \mathbb{X}_{\mathrm{T}}$.

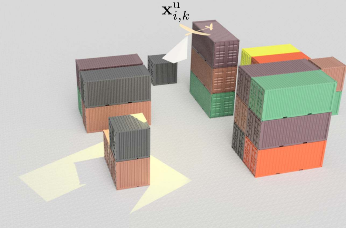
Indexes of detected targets: $\mathcal{D}_{i,k}$

$$j \in \mathcal{D}_{i,k} \Longleftrightarrow \mathbf{x}^{\mathrm{u}}_{i,k} \in \mathbb{D}\left(\mathbf{x}^{\mathrm{t}}_j\right) \text{ and } \mathbf{x}^{\mathrm{t}}_j \in \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right).$$

For each target $j \in \mathcal{D}_{i,k}$, measurement

$$\mathbf{y}_{i,j,k} = \mathbf{h}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}, \mathbf{x}^{\mathrm{t}}_j\right) + \mathbf{w}_{i,j,k},$$

with $\mathbf{w}_{i,j,k} \in [\mathbf{w}_{i,j,k}] \subset [\mathbf{w}_i]$ (bounded errors).



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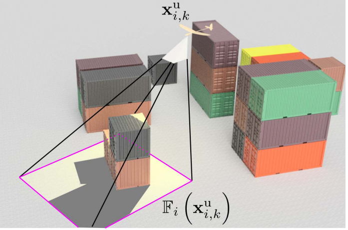

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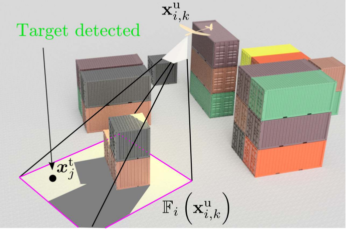

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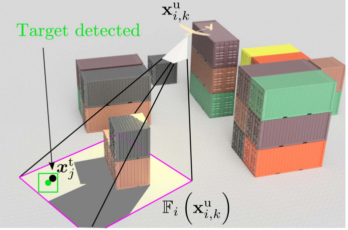

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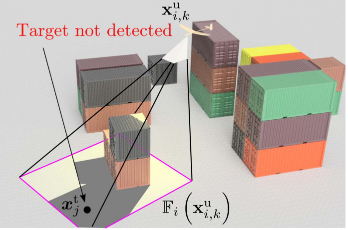

Communications

UAVs can exchange information

Communication condition

- accounts for distance between two UAVs
- may also consider relative orientation of UAVs

Problem formulation

For each UAV i

- explore $\mathbb{X}_0$
- build list $\mathcal{L}_{i,k}$ of detected targets
- obtain set estimates $\mathbb{X}_{i,j,k}$, $j \in \mathcal{L}_{i,k}$ of possible target locations

Set estimates consistent with

- observations collected by UAV i
- information received from its neighbors.

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In presence of uncharted obstacles

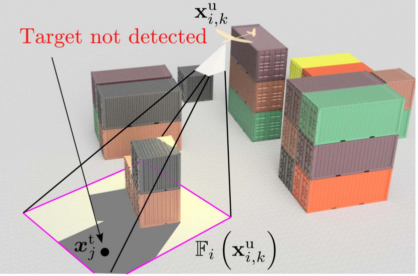
$$\mathcal{D}_{i,k} = \emptyset$$

does not necessarily imply

$$\mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \text{ clear from targets}$$

How to

- prove the absence of a target at location x?
- guarantee detection of all targets in $\mathbb{X}_0$?



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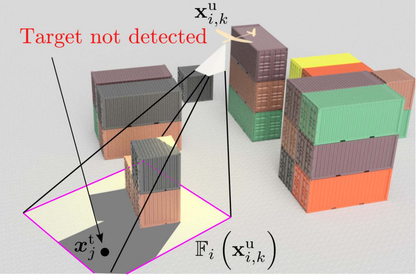

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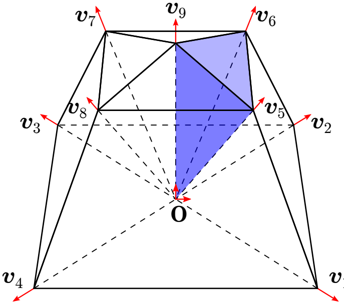
Outline

Conic observation subsets

Consider L non-zero volume cones $\mathbb{C}_\ell\left(\mathbf{O}\right) \in \mathbb{R}^3$ with apex $\mathbf{O}$.



For all $\boldsymbol{x} \in \mathbb{R}^3$ consider translated cones $\mathbb{C}_\ell\left(\boldsymbol{x}\right)$, $\ell = 1, \ldots, L$ of apex $\boldsymbol{x}$

$$\boldsymbol{x}' \in \mathbb{C}_\ell\left(\boldsymbol{x}\right) \Leftrightarrow \boldsymbol{x}' - \boldsymbol{x} \in \mathbb{C}_\ell\left(\mathbf{O}\right). \qquad (1)$$

Conic observation subsets

Main hypothesis: Assume that $\mathbb{C}_1(\boldsymbol{x}), \ldots, \mathbb{C}_{L-1}(\boldsymbol{x}), \mathbb{C}_L(\boldsymbol{x})$ satisfy

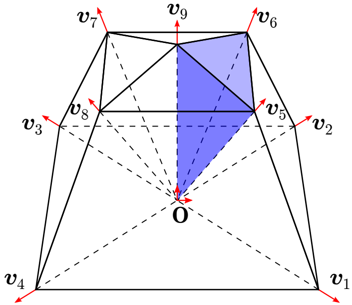
$$\forall \boldsymbol{x} \in \mathbb{X}_{\mathsf{T}}, \exists \ell \in \{1, \ldots, L\}, \mathbb{C}_\ell(\boldsymbol{x}) \subset \underline{\mathbb{D}}(\boldsymbol{x}). \tag{2}$$

For every $\boldsymbol{x} \in \mathbb{X}_{\mathrm{T}}$

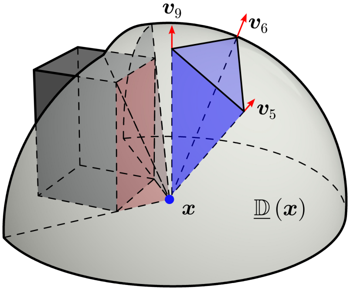
- there exists a cone $\mathbb{C}_\ell(\boldsymbol{x})$ included in $\underline{\mathbb{D}}(\boldsymbol{x})$,
- and thus also included in $\mathbb{D}(\boldsymbol{x})$ (since $\underline{\mathbb{D}}(\boldsymbol{x}) \subset \mathbb{D}(\boldsymbol{x})$

Conic observation subsets

Conic observation subsets $\mathbb{C}_\ell(\boldsymbol{x})$



Detectability set $\mathbb{D}(\boldsymbol{x})$



Partition of the field of view

Consider

- state $\mathbf{x}^{\mathrm{u}}_{i,k}$ of UAV i at time k,
- its FoV $\mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right)$.

Introduce for $\ell \in \{1, \ldots, L\}$

$$\mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) = \left\{\boldsymbol{x} \in \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \mid \boldsymbol{x}^{\mathrm{u}}_{i,k} \in \mathbb{C}_\ell\left(\boldsymbol{x}\right)\right\}, \tag{3}$$

subset of potential target locations $\boldsymbol{x} \in \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right)$ observed from UAV location $\boldsymbol{x}^{\mathrm{u}}_{i,k} \in \mathbb{C}_\ell\left(\boldsymbol{x}\right)$.

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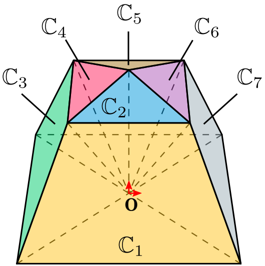
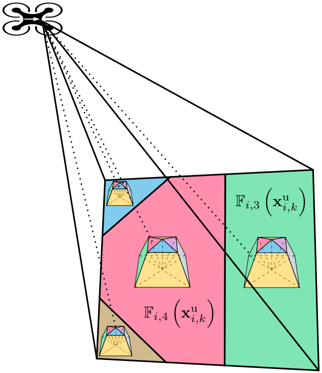
Conic observation subsets

$L = 8$ conic observation subsets $\mathbb{C}_\ell(\boldsymbol{x})$



FoV subsets $\mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right)$



Processing measurements

From $\mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right)$, UAV i gets

- set of detected targets $\mathcal{D}_{i,k}$,
- measurements $\mathbf{y}_{i,j,k}$, $j \in \mathcal{D}_{i,k}$ of target state.

No target is detected at $\boldsymbol{x}$ when observed from $\mathbf{x}^{\mathrm{u}}_{i,k} \in \mathbb{C}_{\ell}\left(\boldsymbol{x}\right)$

$$\text{if } \boldsymbol{x} \in \mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \text{ and } \mathcal{D}_{i,k} = \emptyset.$$

$$\text{if } \boldsymbol{x} \in \mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \text{ and } \forall j \in \mathcal{D}_{i,k}, \mathbf{h}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}, \boldsymbol{x}\right) \notin \mathbf{y}_{i,j,k} - \left[\mathbf{w}_i\right].$$

Proposition

If no target is detected when $\boldsymbol{x}$ is observed from at least one point of view belonging to each of the cones $\mathbb{C}_{\ell}\left(\boldsymbol{x}\right)$, $\ell = 1, \dots, L$, then there is no target located in $\boldsymbol{x}$.

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$$\text{if } \boldsymbol{x} \in \mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k}\right) \text{ and } \forall j \in \mathcal{D}_{i,k}, \mathbf{h}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k}, \boldsymbol{x}\right) \notin \mathbf{y}_{i,j,k} - [\mathbf{w}_i].$$

Proposition

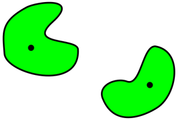
If no target is detected when $\boldsymbol{x}$ is observed from at least one point of view belonging to each of the cones $\mathbb{C}_\ell(\boldsymbol{x})$, $\ell = 1, \ldots, L$, then there is no target located in $\boldsymbol{x}$.

Set estimates

$\mathcal{L}_{i,k}$: *list* of indices of targets already detected / signaled

Estimates at time t_k:

- $\mathcal{X}_{i,k} = \{\mathbb{X}_{i,j,k}\}_{j \in \mathcal{L}_{i,k}}$: target set estimates,
- $\overline{\mathcal{X}}_{i,k} = \left\{\overline{\mathbb{X}}_{i,\ell,k}\right\}_{\ell=1,\dots,L}$: sets of potential target locations not yet observed from $\boldsymbol{x}^{\mathrm{u}}_{i,k} \in \mathbb{C}_\ell(\boldsymbol{x})$,
- $\overline{\mathbb{X}}_{i,k} = \bigcup_{\ell \in \{1,\dots,L\}} \overline{\mathbb{X}}_{i,\ell,k}$: Set of potential target locations $\boldsymbol{x}$ not yet observed from $\boldsymbol{x}^{\mathrm{u}}_{i,k}$ *each* cone $\mathbb{C}_\ell(\boldsymbol{x})$, $\ell = 1, \dots, L$.



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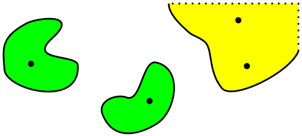

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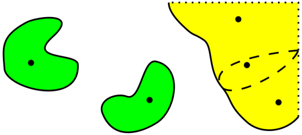

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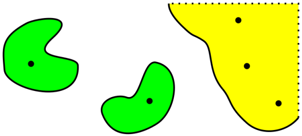
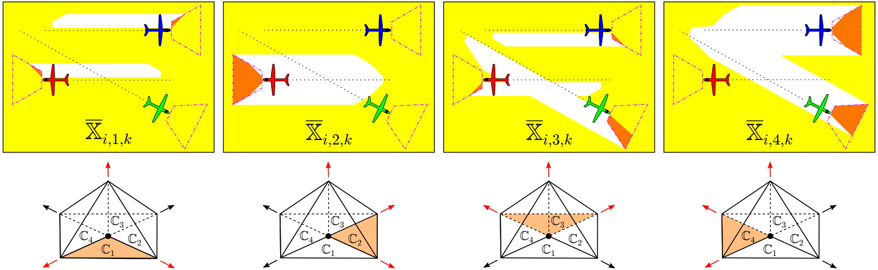
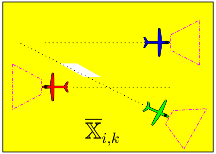


Set estimates





Evolution of the set estimates

Initialization

Recursive set-membership state estimator

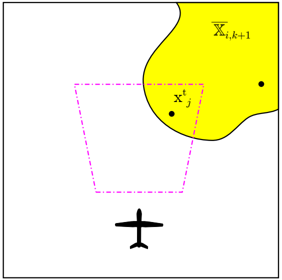
Initialization at $k = 0$:

- $\mathcal{L}_{i,0} = \emptyset$,
- $\mathcal{X}_{i,0} = \emptyset$,
- $\overline{\mathcal{X}}_{i,k} = \{\mathbb{X}_0\}_{\ell=1,\dots,L}$,
- $\overline{\mathbb{X}}_{i,0} = \mathbb{X}_0$ for $i = 1, \dots, N_{\mathrm{u}}$.

Evolution of the set estimates

Accounting for measurements I

When a new target $j \in \mathcal{D}_{i,k+1}$ (and $j \notin \mathcal{L}_{i,k}$) is detected



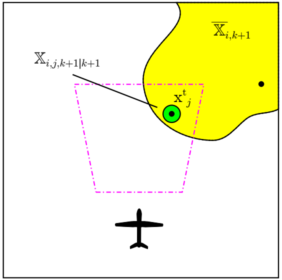
Evolution of the set estimates

Accounting for measurements I

When a new target $j \in \mathcal{D}_{i,k+1}$ is detected

$$\mathcal{L}_{i,k+1|k+1} = \mathcal{L}_{i,k} \cup \{j\}$$

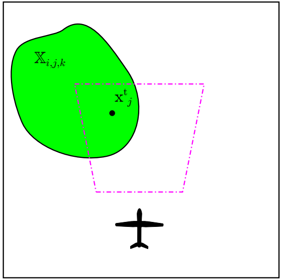
$$\mathbb{X}_{i,j,k+1|k+1} = \left\{ \boldsymbol{x} \in \overline{\mathbb{X}}_{i,k+1} \cap \mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right) \middle| \mathbf{h}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}, \boldsymbol{x}\right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}_i] \right\}$$



Evolution of the set estimates

Accounting for measurements II

When a detected target is seen again

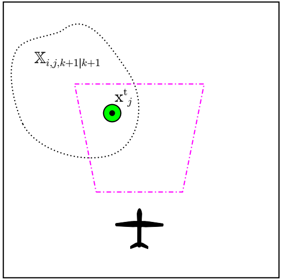


Evolution of the set estimates

Accounting for measurements II

When a detected target is seen again

$$\mathbb{X}_{i,j,k+1|k+1} = \left\{ \boldsymbol{x} \in \mathbb{X}_{i,j,k} \cap \mathbb{F}_i \left(\mathbf{x}^{\mathrm{u}}_{i,k+1} \right) \middle| \mathbf{h}_i \left(\mathbf{x}^{\mathrm{u}}_{i,k+1}, \boldsymbol{x} \right) \in \mathbf{y}_{i,j,k+1} - [\mathbf{w}_i] \right\}$$

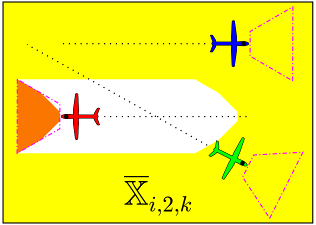


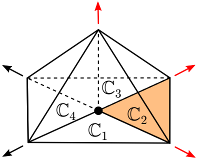
Evolution of the set estimates

Updating the explored region

When no target is detected

$$\overline{\mathbb{X}}_{i,\ell,k+1|k+1} = \overline{\mathbb{X}}_{i,\ell,k} \setminus \mathbb{F}_{i,\ell}\left(\mathbf{x}^{u}_{i,k+1}\right)$$





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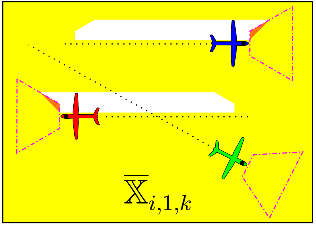

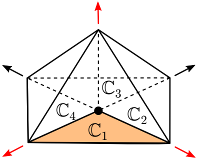

Evolution of the set estimates

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When no target is detected

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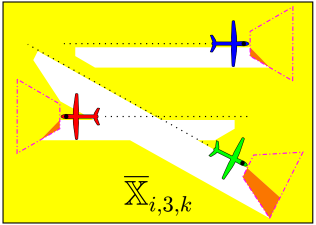
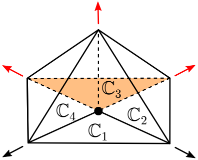


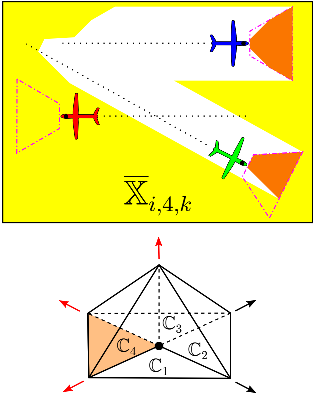


Evolution of the set estimates

Updating the explored region

When no target is detected

$$\overline{\mathbb{X}}_{i,\ell,k+1|k+1} = \overline{\mathbb{X}}_{i,\ell,k} \setminus \mathbb{F}_{i,\ell}\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right)$$

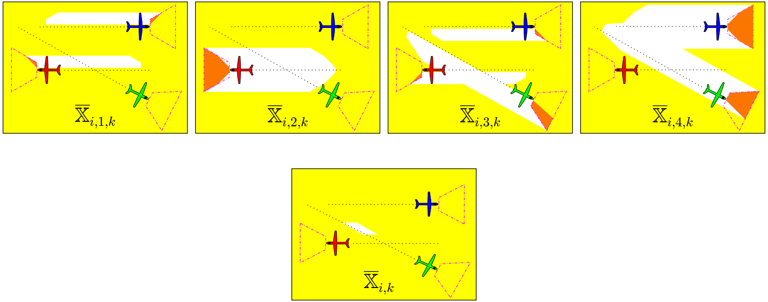


Evolution of the set estimates

Updating the explored region

Finally,

$$\overline{\mathbb{X}}_{i,k+1|k+1} = \bigcup_{\ell \in \{1,\ldots,L\}} \overline{\mathbb{X}}_{i,\ell,k+1|k+1},$$



Outline

MPC control input design

Design of control inputs of each UAV so as

$$\widehat{\mathbf{u}}_{i,k:k+h-1} = \arg \min_{\mathbf{u}_{i,k:k+h-1}} \sum_{\ell \in \{1,\dots,L\}} \phi\left(\overline{\mathbb{X}}^{\mathsf{P}}_{i,\ell,k+h}\right)$$

Accounting for

- Previously computed control inputs by neighbors
- Communication constraints

Outline

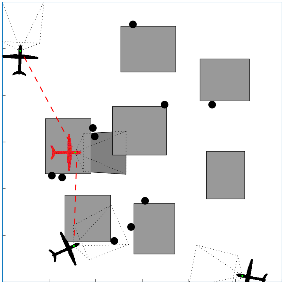
Simulation conditions

$\mathbb{X}_0$: box of $[0, 300] \times [0, 300] \times [0, 100]$ m^3.

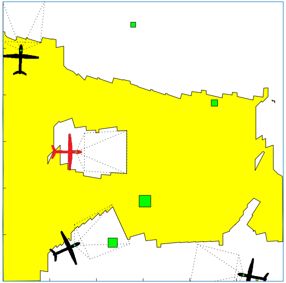
Configuration with

- 7 random placed obstacles
- 10 static ground targets
- 4 UAVs
- 8 conic observation subsets

Unknown environment



Estimates of the red UAV



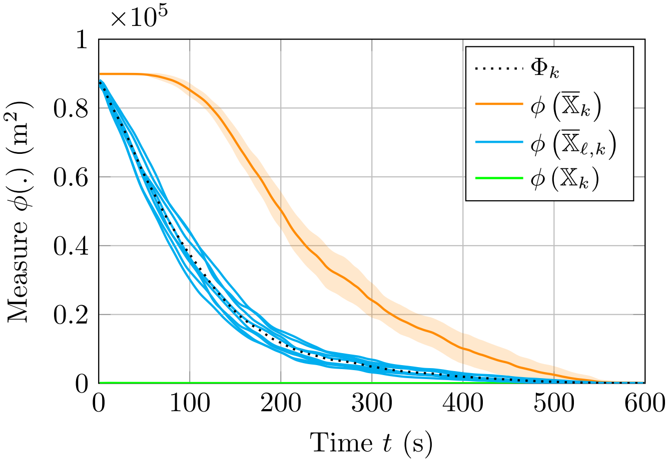
Evolution of set estimates

For video sequences, see

`drive.google.com/drive/folders/1djk7qQJCGBYPKVwD8nAey7C9f4bbQ65I`

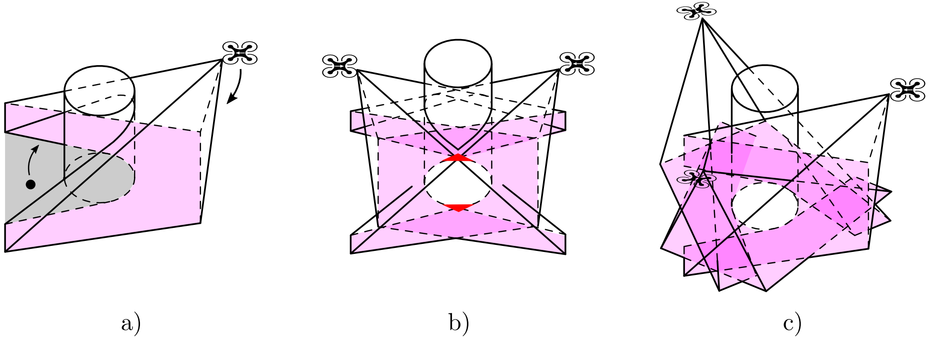
Evolution of size of set estimates

Evolution of the average size of set estimates over 30 simulations



Outline

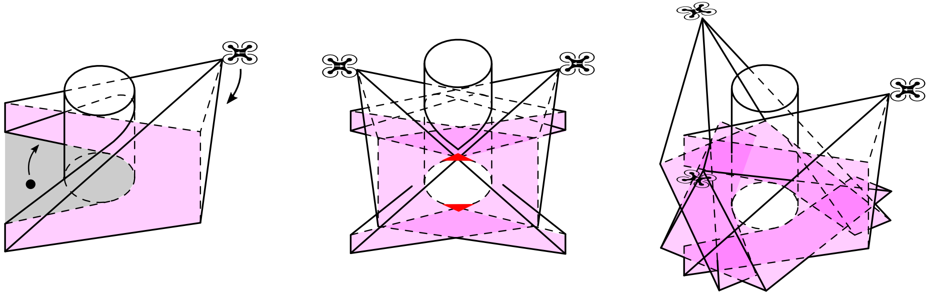
Moving targets



Moving target may remain undetected from single UAV

Simultaneous observations from L different points of view are necessary

Moving targets

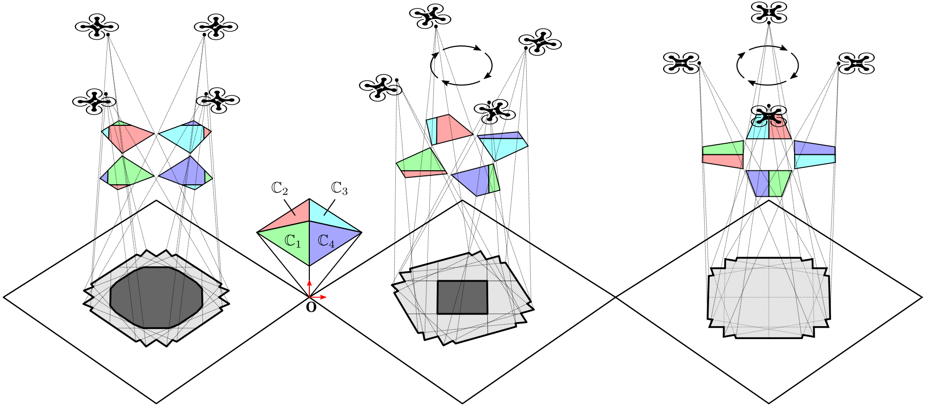


a) b) c)

Moving target may remain undetected from single UAV

Simultaneous observations from L different points of view are necessary

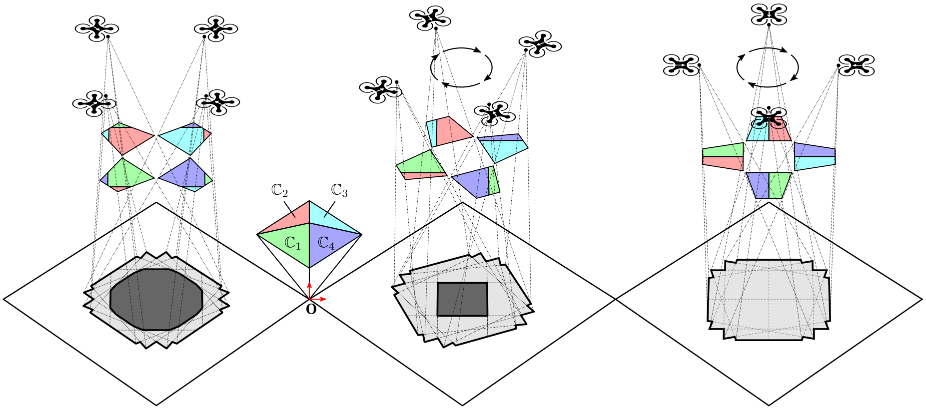
Organizing the drones



Drones organized in groups

Fully observed subset (black) depends on relative drone orientation

Organizing the drones



Drones organized in groups

Fully observed subset (black) depends on relative drone orientation

Simulations

For video sequences, see

drive.google.com/drive/folders/1SlM2FT6LQrNpfgtHBLhaysG8O1Hya1Hp

Outline

Conclusions

Development of several approaches solving the following issues:

- Cooperative target localization and tracking
- Static targets
- Presence of obstacles $\Longrightarrow$ Potential non-detection
- Detection of a target depends deterministically on the point of view
- Distributed UAV control scheme

Entensions

Results already obtained

- Presence of static and dynamic decoys $\Longrightarrow$ Potential false-detection
- Presence of moving targets and obstacles

Ongoing work

- Compare proposed approaches with stochastic approaches
- Improve displacement strategies of the UAVs
- Implement on test platform

Publication

Conferences

- J. Ibenthal, L. Meyer, M. Kieffer and H. Piet-Lahanier. "Bounded-Error Target Localization and Tracking in Presence of Decoys Using a Fleet of UAVs". In: IFAC-PapersOnLine. Vol. 53. 2020, pp. 9521–9528.
- J. Ibenthal, L. Meyer, H. Piet-Lahanier and M. Kieffer. "Target Search and Tracking Using a Fleet of UAVs in Presence of Decoys and Obstacles". In: Proc. IEEE CDC. 2020, pp. 188–194.
- J. Ibenthal, L. Meyer, H. Piet-Lahanier and M. Kieffer. "Localization of Partially Hidden Targets Using a Fleet of UAVs via Robust Bounded-Error Estimation". In: Proc. IEEE CDC. 2021.
- J. Ibenthal, L. Meyer, H. Piet-Lahanier and M. Kieffer. "Comparison of Set-Membership State Estimation and Bayesian State Estimation Techniques to search and track Targets using a Fleet of UAVs" . **In preparation**.

Journals

- J. Ibenthal, M. Kieffer, L. Meyer, H. Piet-Lahanier and S. Reynaud. "Bounded-Error Target Localization and Tracking Using a Fleet of UAVs". Automatica 132 (2021), p. 109809.
- J. Ibenthal, H. Piet-Lahanier, L. Meyer and M. Kieffer. "Localization of Partially Hidden Moving Targets Using a Fleet of UAVs via Robust Bounded-Error Estimation". **In preparation** for Automatica.

Correction from communication

$$k \;\rightarrow\; k+1|k \;\rightarrow\; k+1|k+1 \;\rightarrow\; k+1$$

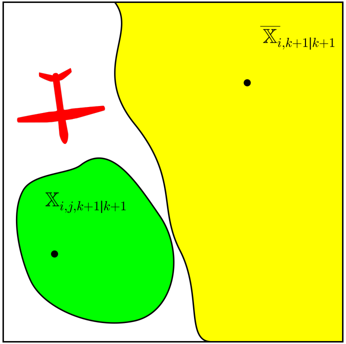
At the end of each time step k UAV i communicates with its neighbors

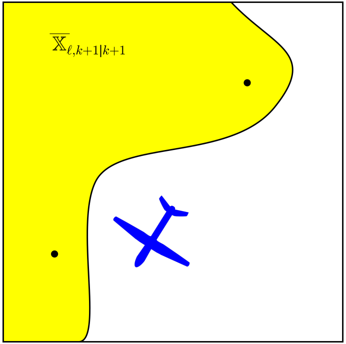
Exchanged information:

- Target set estimate
- Unexplored set
- Receiving the corresponding sets from its neighbors.

Correction from communication

$$k \rightarrow k+1|k \rightarrow k+1|k+1 \rightarrow k+1$$





Correction from communication

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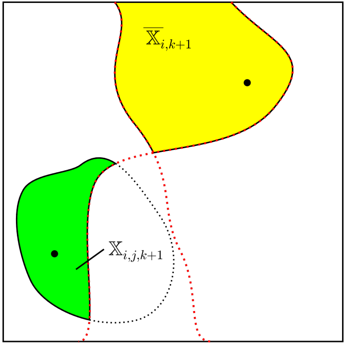
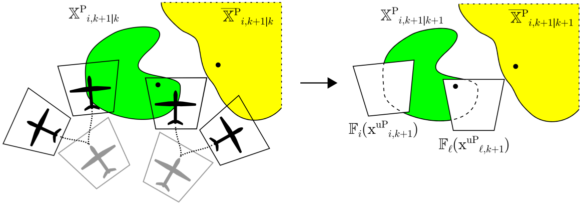


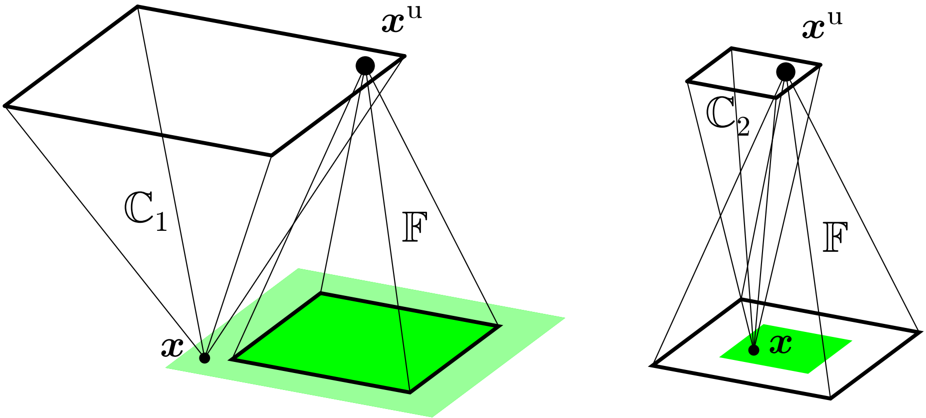
Cooperative control design

One step ahead prediction

Predicting the impact of $\mathbb{F}_i\left(\mathbf{x}^{\mathrm{u}}_{i,k+1}\right)$ on the set estimates $\mathbb{X}^{\mathrm{P}}_{i,j,k+1}$, $\overline{\mathbb{X}}^{\mathrm{P}}_{i,\ell,k+1}$, and $\overline{\mathbb{X}}^{\mathrm{P}}_{i,k+1}$

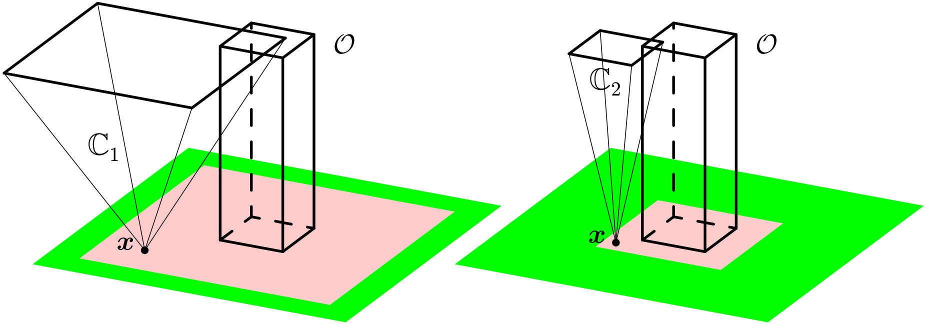


Conic observation subsets



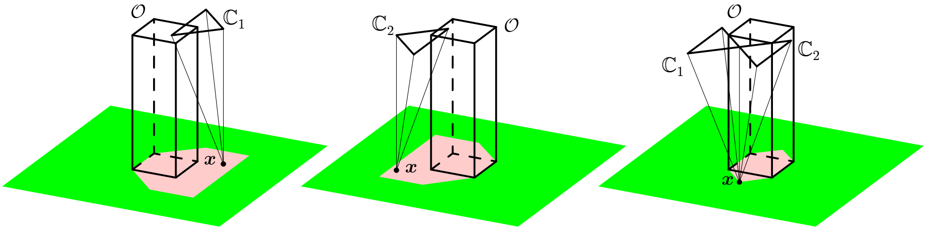
Impact of the aperture of cones

Conic observation subsets



Impact of the aperture of cones

Conic observation subsets



Impact of the number of cones

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Hu, J., Xie, L., Xu, J., and Xu, Z. (2014).
Multi-agent cooperative target search.
Sensors, 14(6):9408–9428.

Ibenthal, J., Kieffer, M., Meyer, L., Piet-Lahanier, H., and Reynaud, S. (2021).
Bounded-error target localization and tracking using a fleet of UAVs.
Automatica, 132:109809.

Ibenthal, J., Meyer, L., Kieffer, M., and Piet-Lahanier, H. (2020).
Bounded-error target localization and tracking in presence of decoys using a fleet of UAVs.
In *IFAC-PapersOnLine*, volume 53, pages 9521–9528.

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