Level Sets and Controls in a Two Pursuers One Evader Differential Game

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6th Small Workshop on Interval Methods
ENSTA Bretagne, Brest, France
June 5-7, 2013

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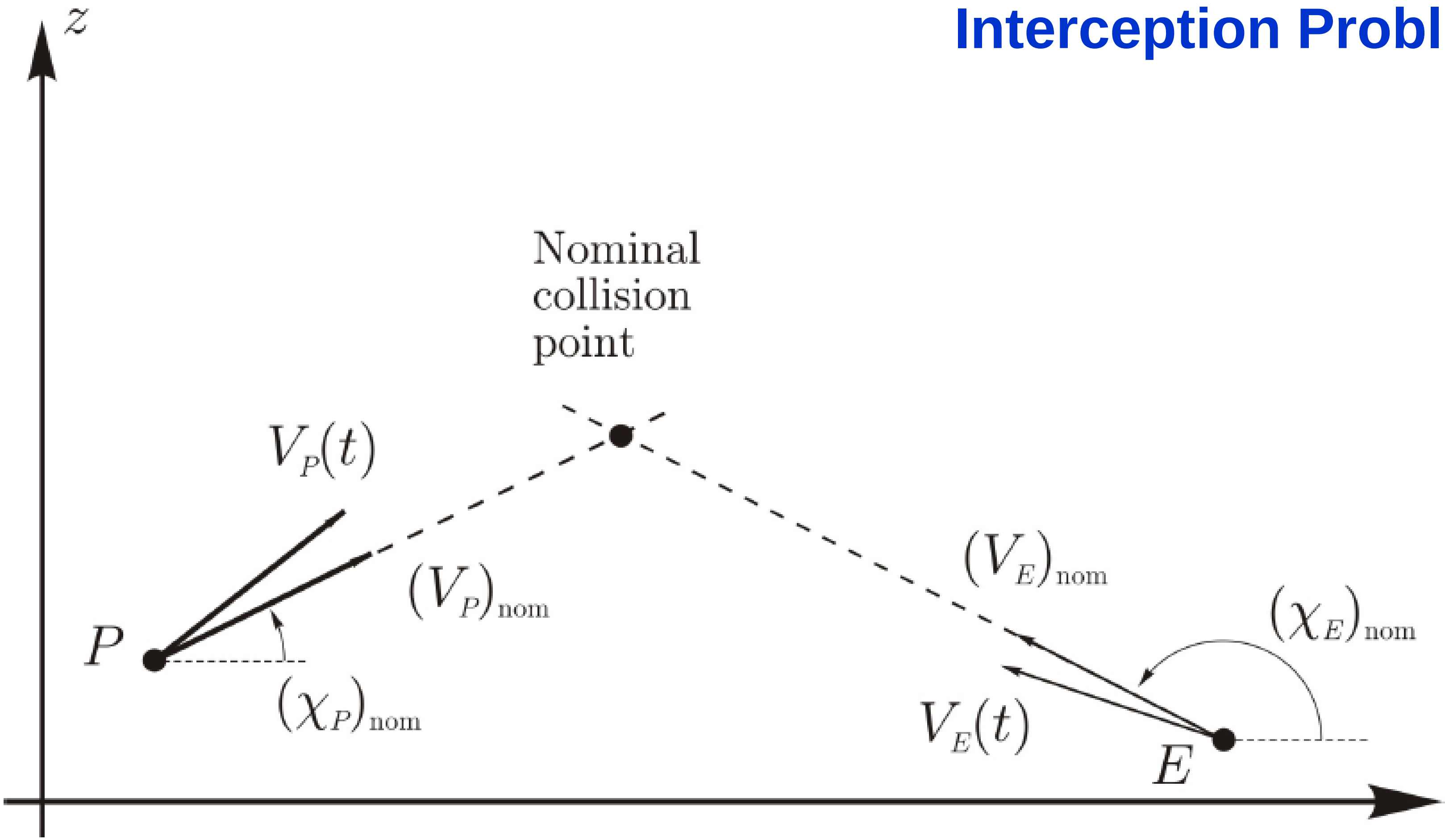
Interception Problem



Velocities along the horizontal axis are very large.

S.Gutman, G.Leitmann, J.Shinar, T.Shima, V.Glizer, V.Turetsky. . .

(1976 – up to now)

Linearized Dynamics

$$t \in [0, T], \quad z_P,\, z_E \in R,$$

$$\ddot{z}_P = a_P, \qquad \ddot{z}_E = a_E,$$

$$\dot{a}_P = (u - a_P)/l_P, \qquad \dot{a}_E = (v - a_E)/l_E,$$

$$|u| \le \mu,\ a_P(t_0) = 0, \qquad |v| \le \nu,\ a_E(t_0) = 0.$$

$$\text{Payoff } \varphi = \left| z_E(T) - z_P(T) \right|, \quad \min_u \max_v \varphi$$

l_P, l_E are the time constants;
μ, ν are the constraints for the controls u and v.

Shinar, J., Shima, T.: *Non-orthodox guidance law development approach for intercepting maneuvering targets.* Journal of Guidance, Control, and Dynamics **25**(4), 658–666 (2002)

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Change of variables: $y = z_E - z_P$

The new dynamics is

$$\begin{aligned} \ddot{y} &= -a_P + a_E, \\ \dot{a}_P &= (u - a_P)/l_P, \\ \dot{a}_E &= (v - a_E)/l_E. \end{aligned}$$

Constraints for the players controls:

$$|u| \leq \mu, \quad |v| \leq \nu$$

The payoff:

$$\varphi\big(y(T)\big) = \big|y(T)\big|$$

Dynamics in Forecasted Coordinate
ZEM, Zero Effort Miss Coordinate

Consider a coordinate x that is the value of y forecasted to the termination instant T under zero controls:

$$x = y + \dot{y}\tau - a_P l_P^2 h(\tau/l_P) + a_E l_E^2 h(\tau/l_E),$$
$$\tau = T - t, \qquad h(\alpha) = e^{-\alpha} + \alpha - 1$$

The dynamics:

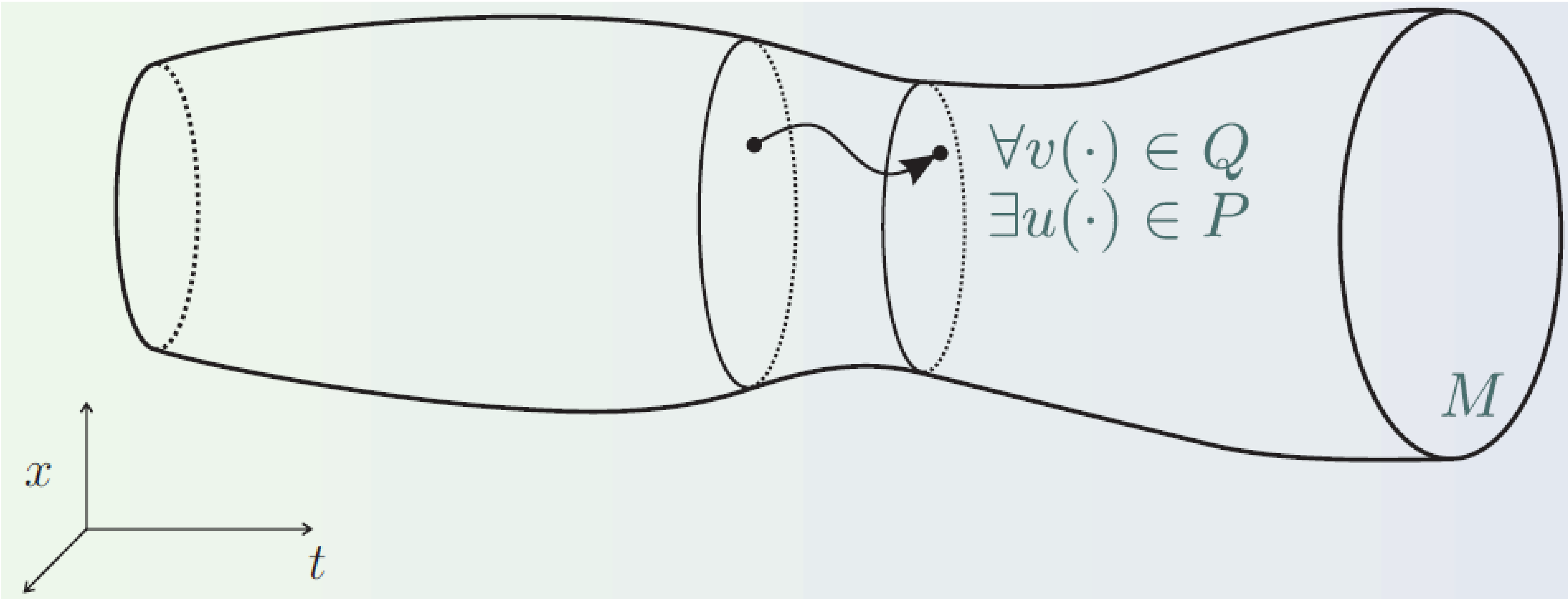
$$\dot{x} = -l_P h(\tau/l_P)u + l_E h(\tau/l_E)v$$

The parameters of the game (J.Shinar, T.Shima):

$$\eta = \frac{\mu}{\nu} \quad \text{— maximal relative acceleration}$$

$$\varepsilon = \frac{l_E}{l_P} \quad \text{— relative agility}$$

Differential Game Capture Zone
Stable Bridge / Minimum - Zero Level Set



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- Isaacs R.; *Differential Games; A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization*; John Wiley and Sons, Inc., New York; 1965
- …

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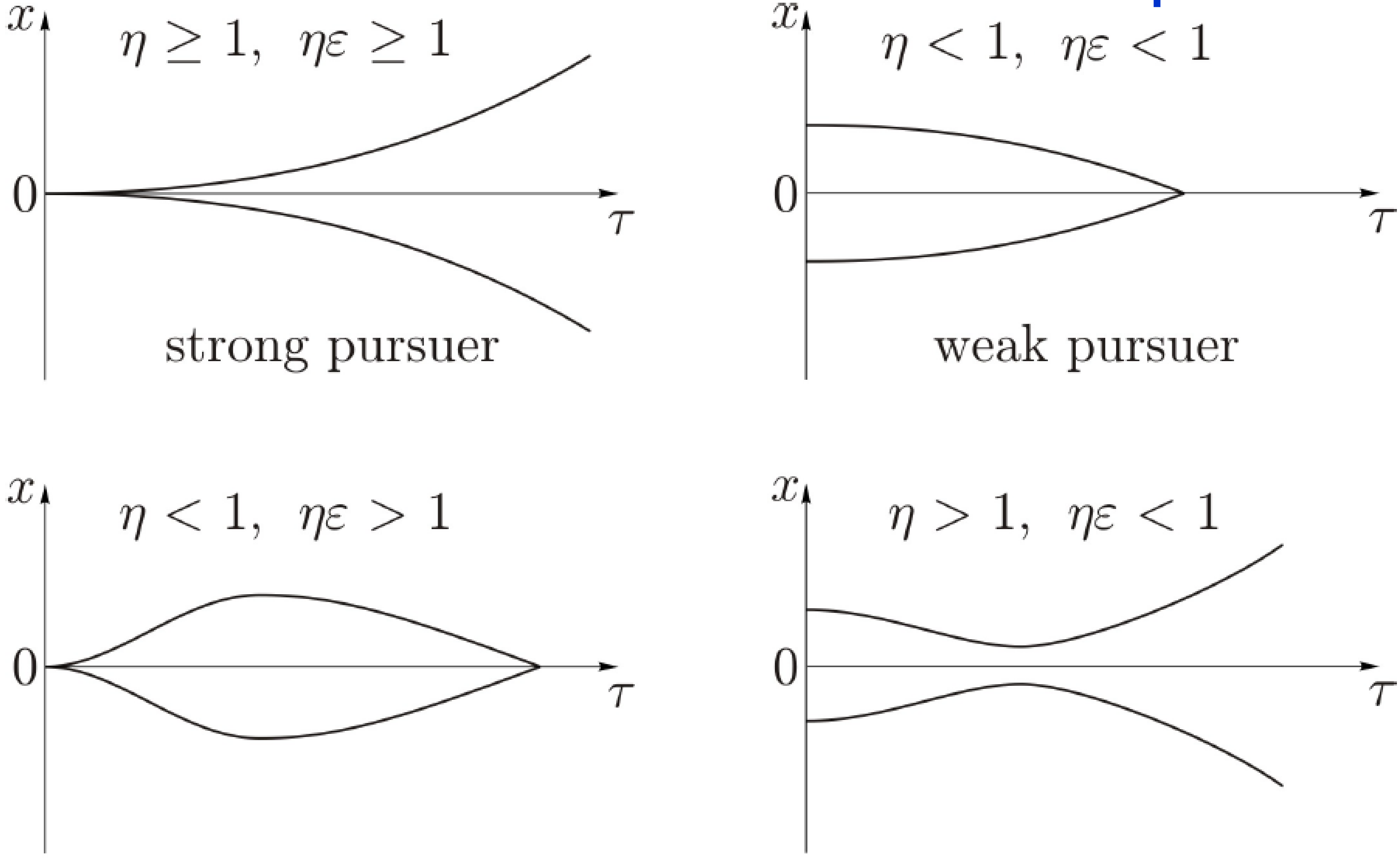
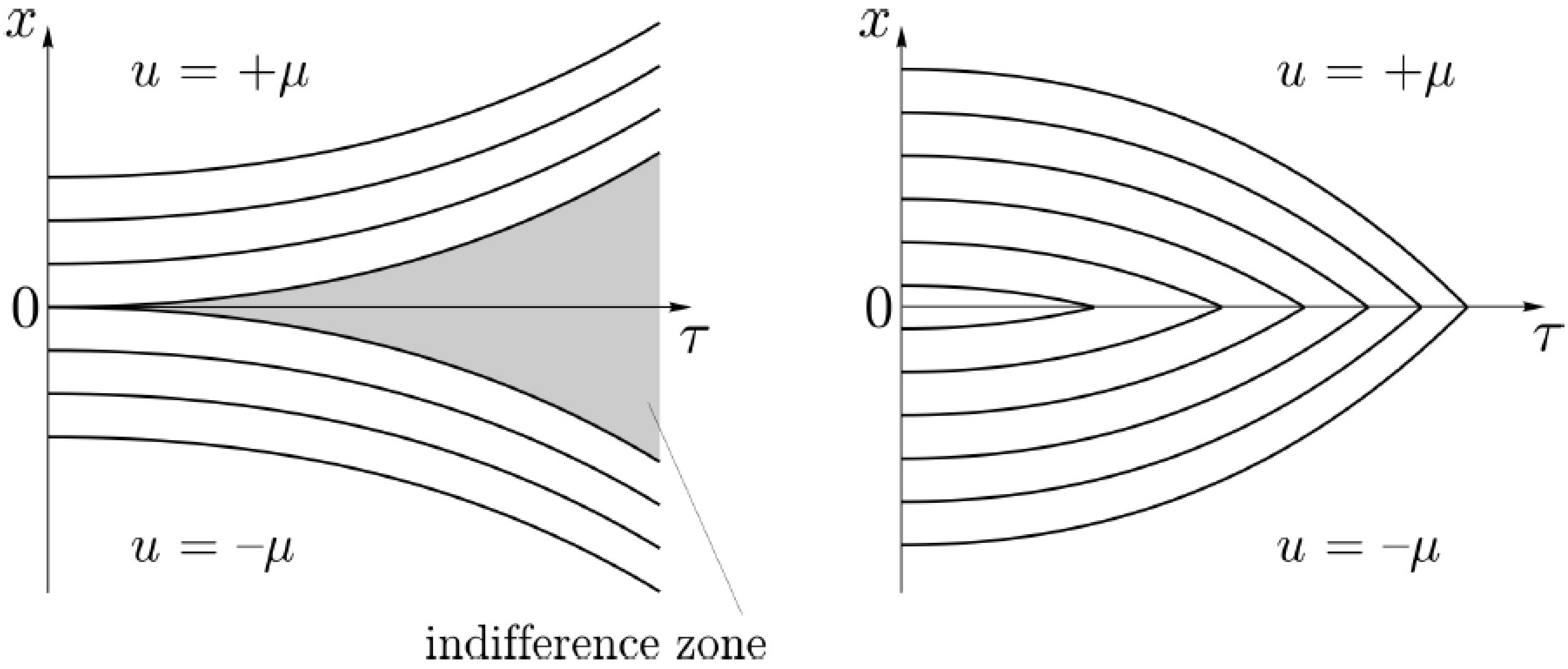
Variants of Capture Zones

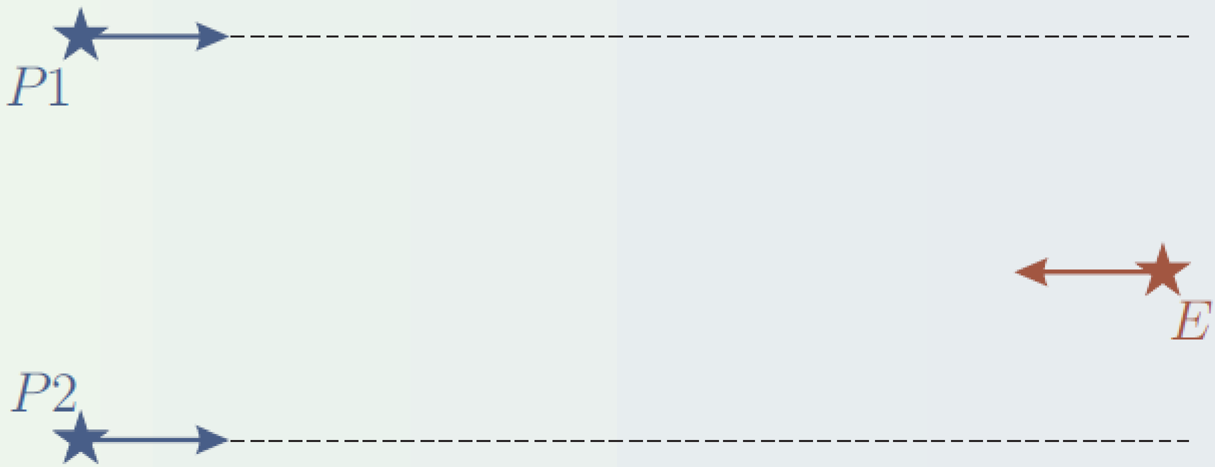


varying advantage

System of Level Sets



Formulation of the Problem



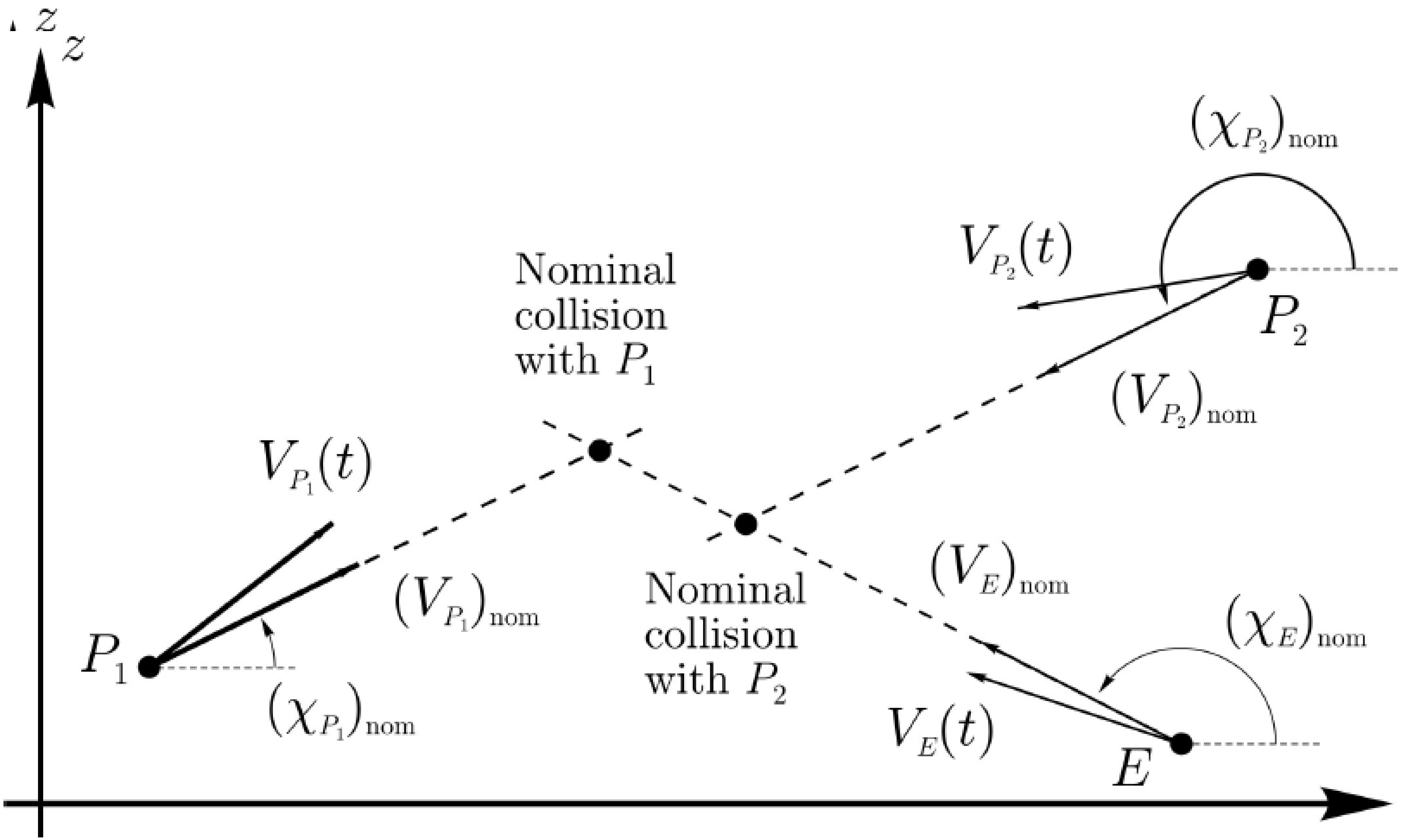
A game with two pursuers and one evader is considered in the plane.

Such a model problem can arise when two aircrafts or missiles pursue another one in the horizontal plane.

Purpose is to increase the resulting No Escape Zone

And to compute interval spacing (vertical / horizontal) between the pursuers

Scheme of Interception with Two Pursuers



Velocities along the horizontal axis are very large.

Two termination instants T_1 and T_2 for the corresponding pursuers. The dynamics:

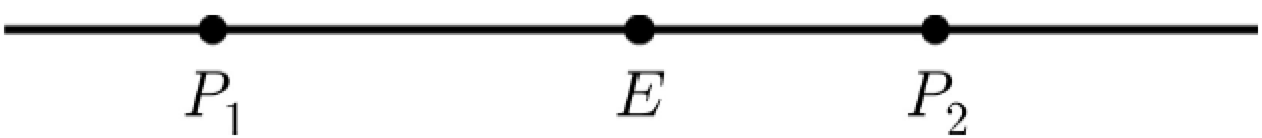
$$\ddot{z}_{P_i} = a_{P_i}, \qquad \ddot{z}_E = a_E,$$

$$\dot{a}_{P_i} = (u_i - a_{P_i})/l_{P_i}, \qquad \dot{a}_E = (v - a_E)/l_E,$$

$$|u_i| \le \mu_i,\ a_{P_i}(t_0) = 0, \quad i = 1, 2, \qquad |v| \le \nu,\ a_E(t_0) = 0.$$

The joint control $u = (u_1, u_2)$. The payoff:

$$\varphi = \min\{|z_E(T_1) - z_{P_1}(T_1)|, |z_E(T_2) - z_{P_2}(T_2)|\}, \quad \min_u \max_v \varphi$$



Le Menec, S.: *Linear Differential Game with Two Pursuers and One Evader*. In: Annals of the International Society of Dynamic Games, Vol.11, 209–226 (2011)

Change of variables: $y_1 = z_E - z_{P_1}$, $y_2 = z_E - z_{P_2}$

The new dynamics is

$$\begin{aligned} \ddot{y}_1 &= -a_{P_1} + a_E, \\ \dot{a}_{P_1} &= (u_1 - a_{P_1})/l_{P_1}, \\ \dot{a}_E &= (v - a_E)/l_E \end{aligned} \qquad \begin{aligned} \ddot{y}_2 &= -a_{P_2} + a_E, \\ \dot{a}_{P_2} &= (u_2 - a_{P_2})/l_{P_2}, \end{aligned}$$

Constraints for the players controls:

$$|u_1| \leq \mu_1, \quad |u_2| \leq \mu_2; \quad |v| \leq \nu$$

The payoff:

$$\varphi\big(y_1(T_1), y_2(T_2)\big) = \min\big(\big|y_1(T_1)\big|, \big|y_2(T_2)\big|\big)$$

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Dynamics in Forecasted Coordinates
ZEM, Zero Effort Miss Coordinates

Consider coordinates x_1 and x_2 that are the values of y_1 and y_2 forecasted to the corresponding termination instants T_i under zero controls:

$$x_i = y_i + \dot{y}_i \tau_i - a_{P_i} l_{P_i}^2 h(\tau_i / l_{P_i}) + a_E l_E^2 h(\tau_i / l_E),$$
$$\tau_i = T_i - t, \qquad i = 1,\, 2, \qquad h(\alpha) = e^{-\alpha} + \alpha - 1$$

The dynamics:

$$\dot{x}_1 = -l_{P_1} h(\tau_1 / l_{P_1}) u_1 + l_E h(\tau_1 / l_E) v,$$
$$\dot{x}_2 = -l_{P_2} h(\tau_2 / l_{P_2}) u_2 + l_E h(\tau_2 / l_E) v$$

The parameters of the game:

$$\eta_i = \frac{\mu_i}{\nu} \quad \text{— maximal relative acceleration}$$

$$\varepsilon_i = \frac{l_E}{l_{P_i}} \quad \text{— relative agility} \qquad i = 1,\, 2$$

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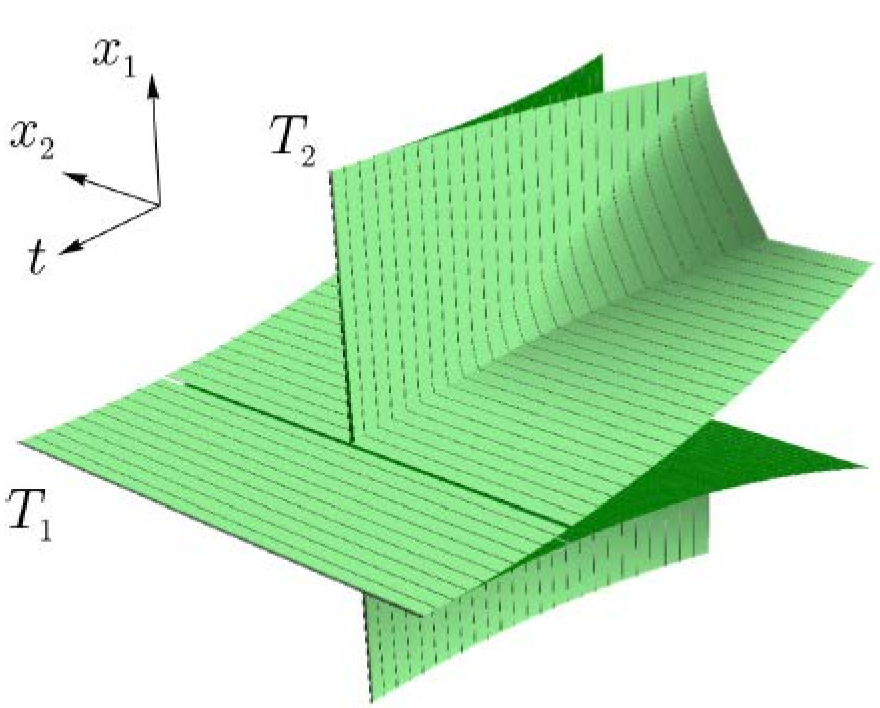
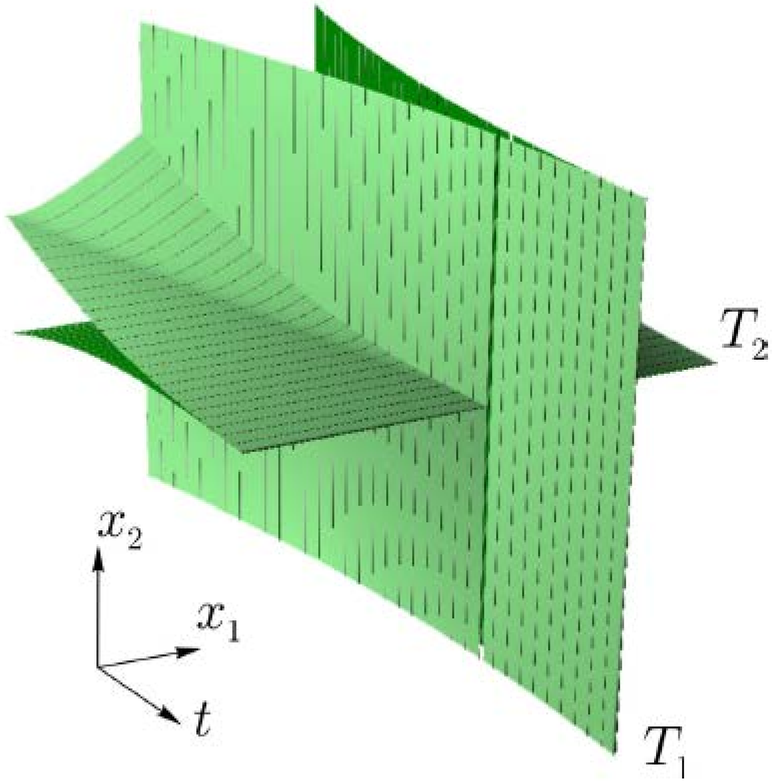
Solvability Set: « Strong » Pursuers (1)





$$\mu_1 = 2,\ \mu_2 = 3,\ \nu = 1,\ l_{P_1} = 1/2,\ l_{P_2} = 1/0.857,\ l_E = 1,$$
$$T_1 = 7,\ T_2 = 5,\ c = 0$$

Solvability Set: « Strong » Pursuers (2)

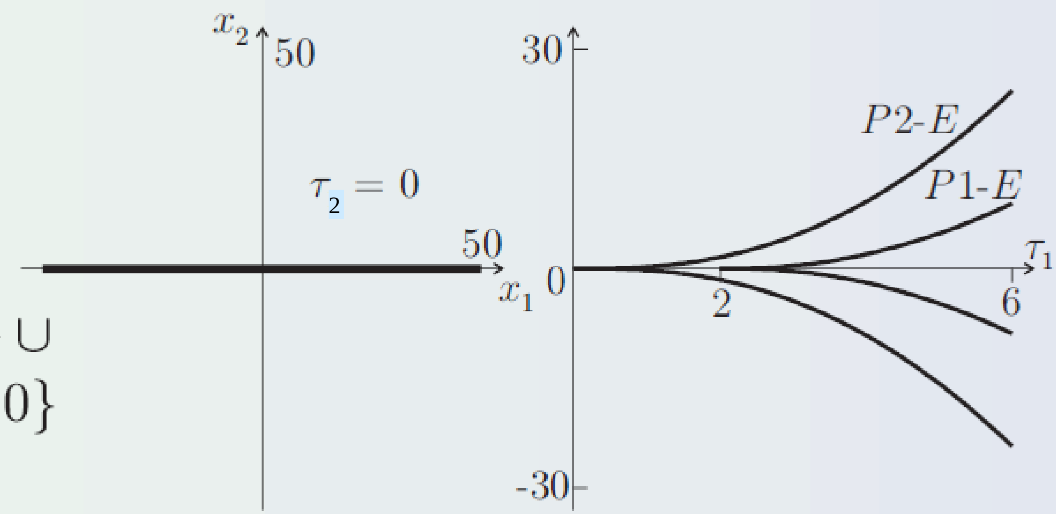
Case $T_{f1} \neq T_{f2}$, Strong Pursuers

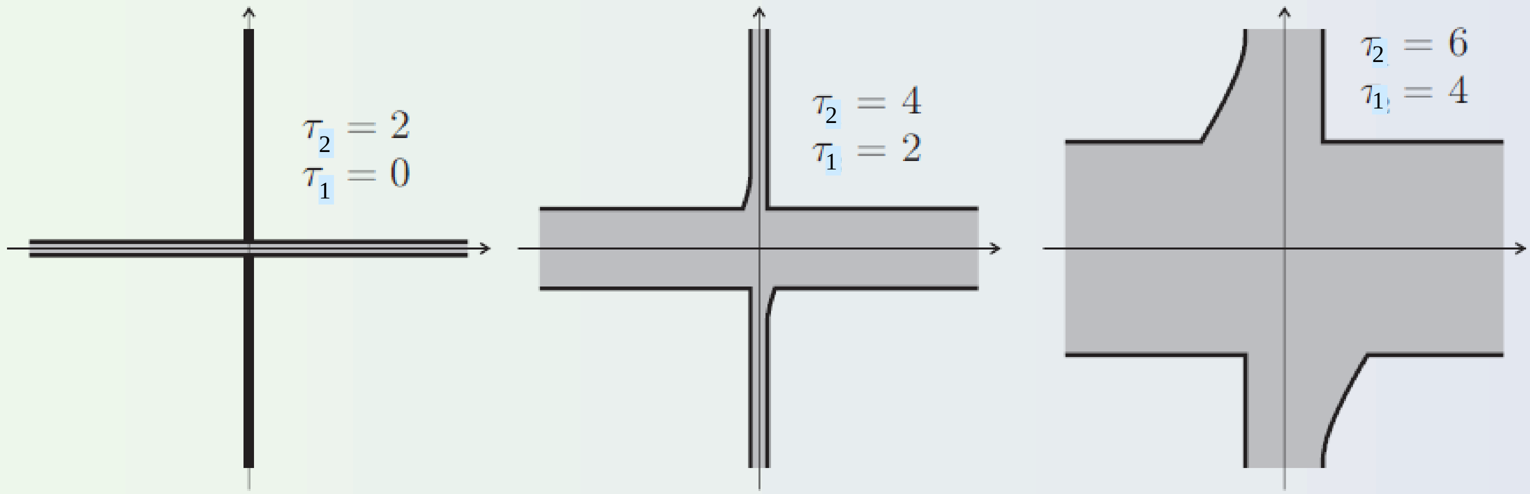
$T_{f1} = 4$

$T_{f2} = 6$

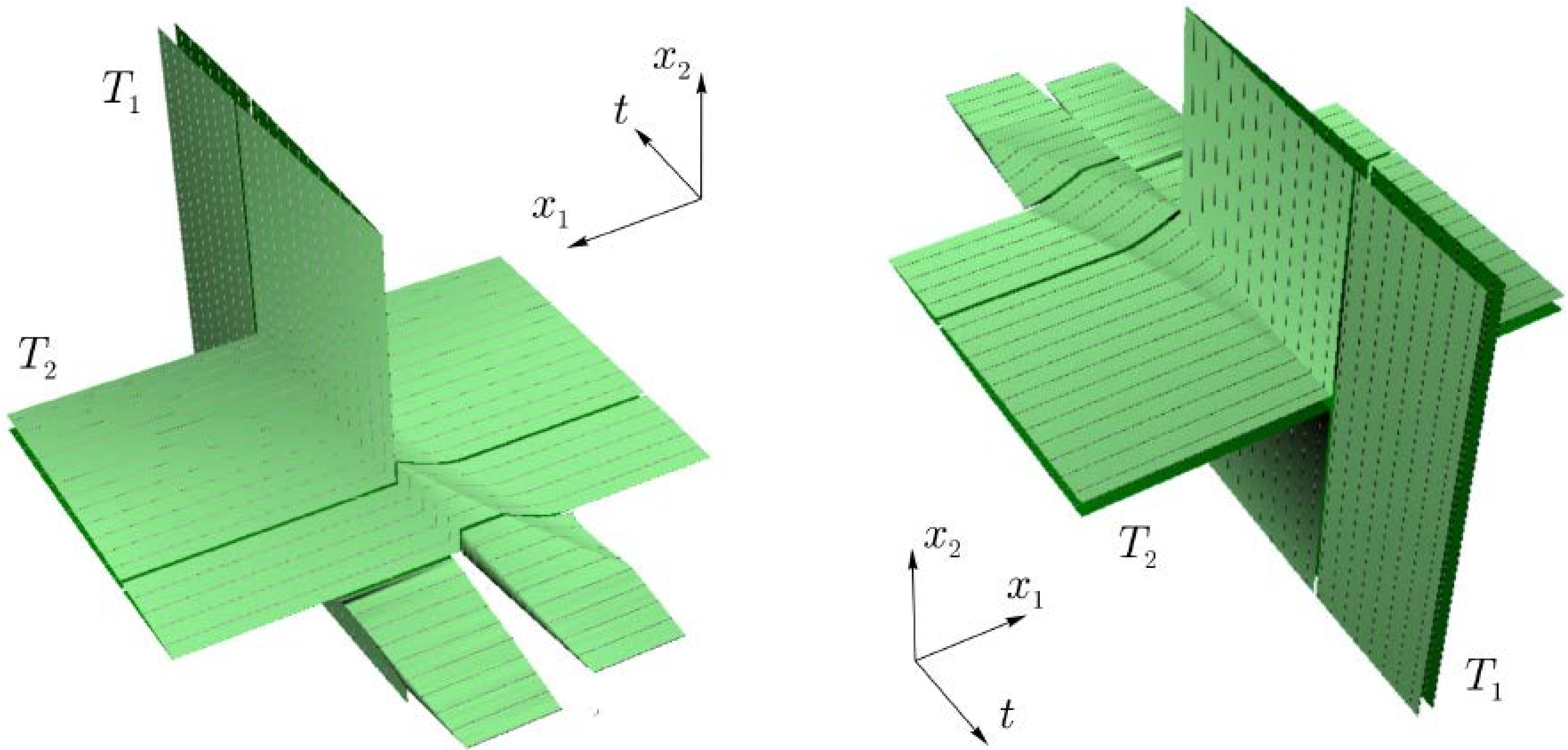
Guiding to the set

$$\{t = T_{f1},\ x_1 = 0\} \cup$$
$$\cup \{t = T_{f2},\ x_2 = 0\}$$



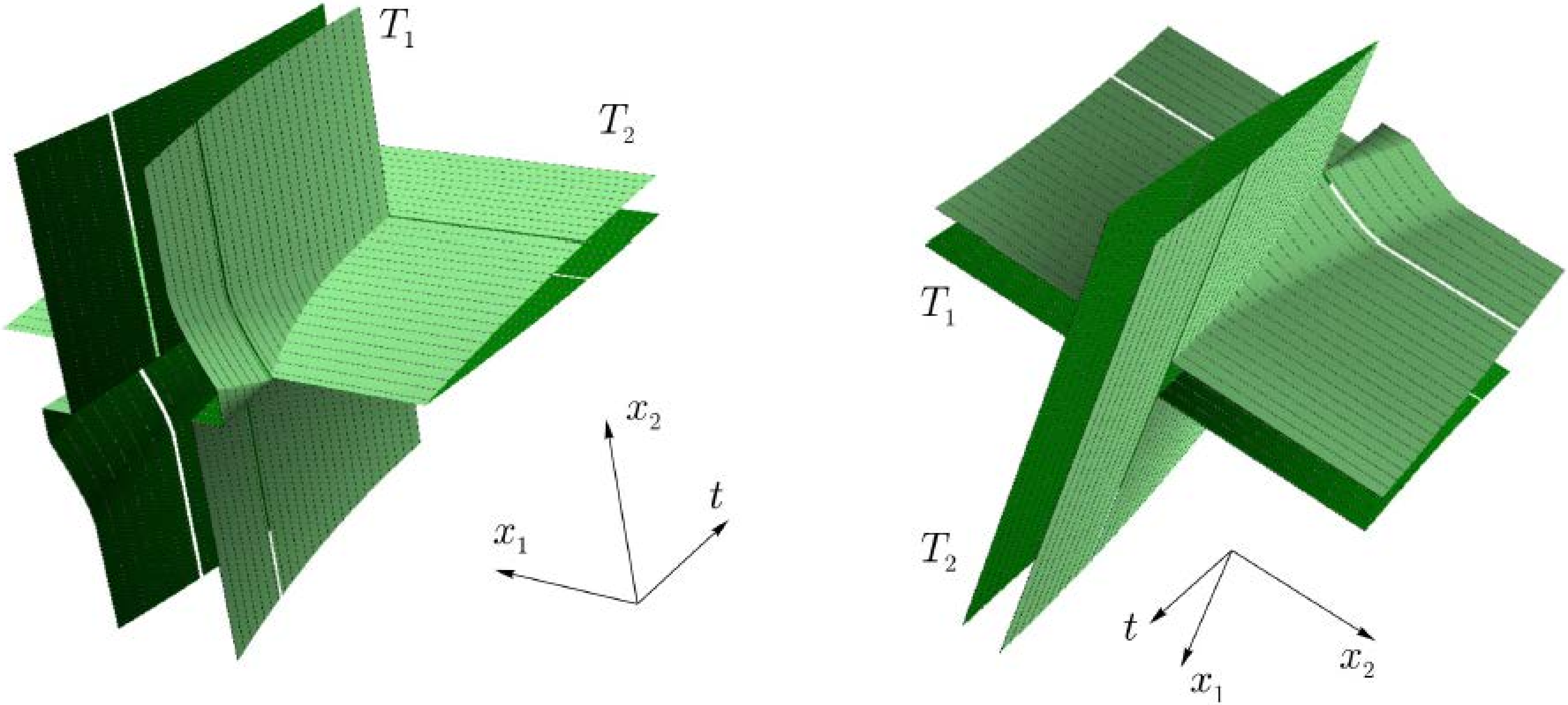


Solvability Set: « Weak » Pursuers



$\mu_1 = 0.9$, $\mu_2 = 0.8$, $\nu = 1$, $l_{P_1} = l_{P_2} = 1/0.7$, $l_E = 1$, $T_1 = 9$, $T_2 = 7$, $c = 2.0$

One Strong and One Weak Pursuers



$$\mu_1 = 2,\ \mu_2 = 1,\ \nu = 1,\ l_{P_1} = 1/2,\ l_{P_2} = 1/0.3,\ l_E = 1,$$
$$T_1 = 5,\ T_2 = 7,\ c = 5.0$$

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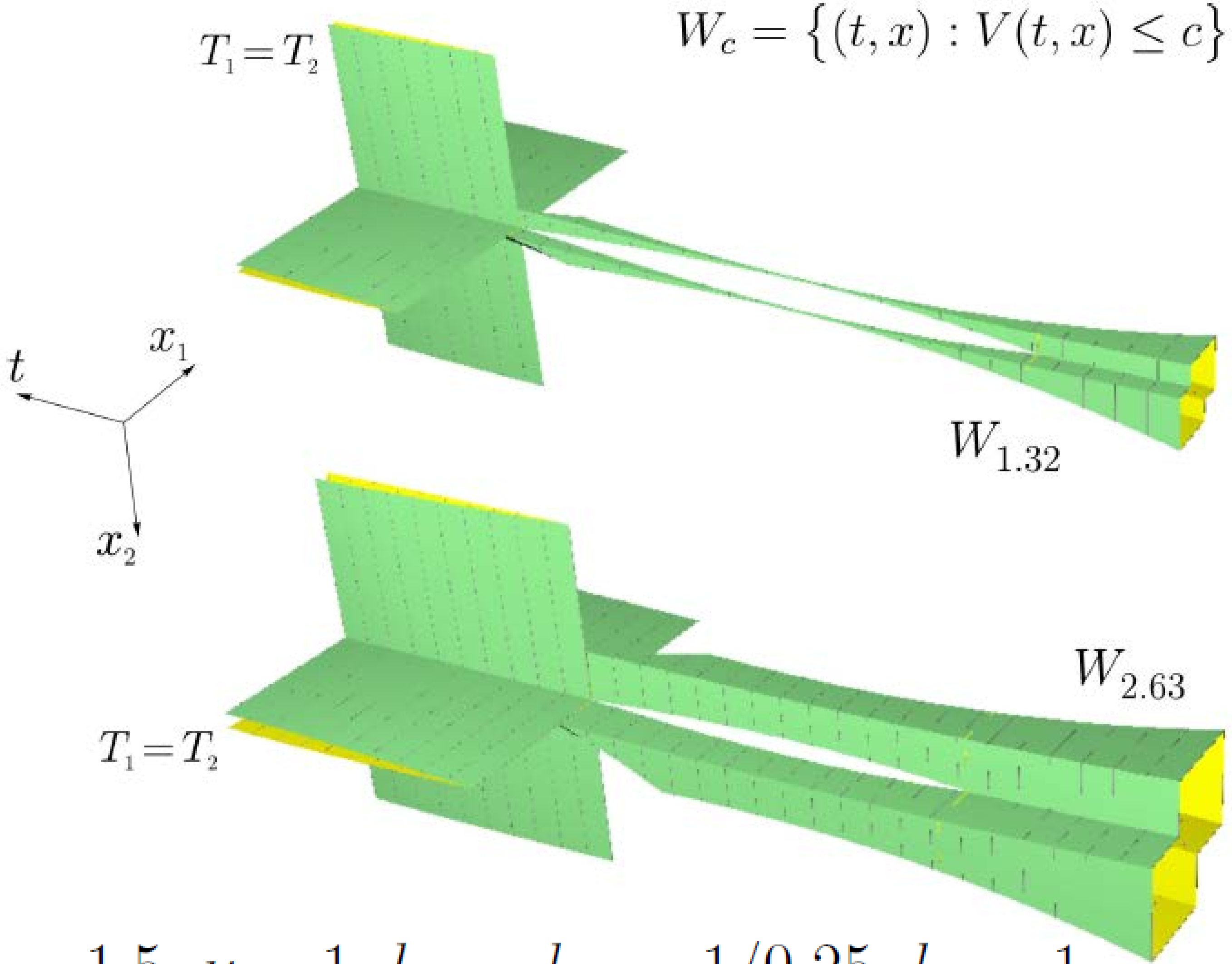
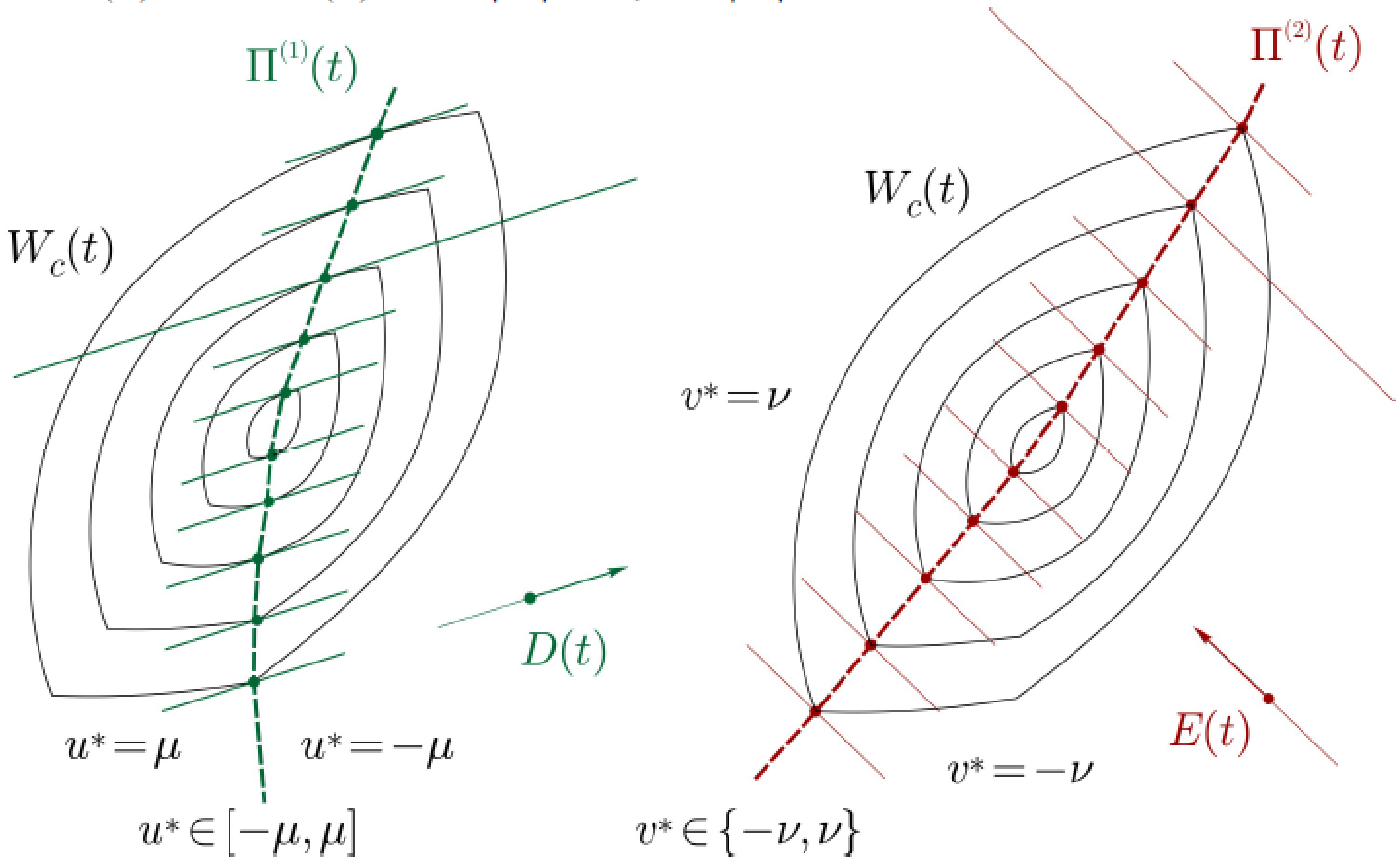
Varying Advantage of Pursuers (1)



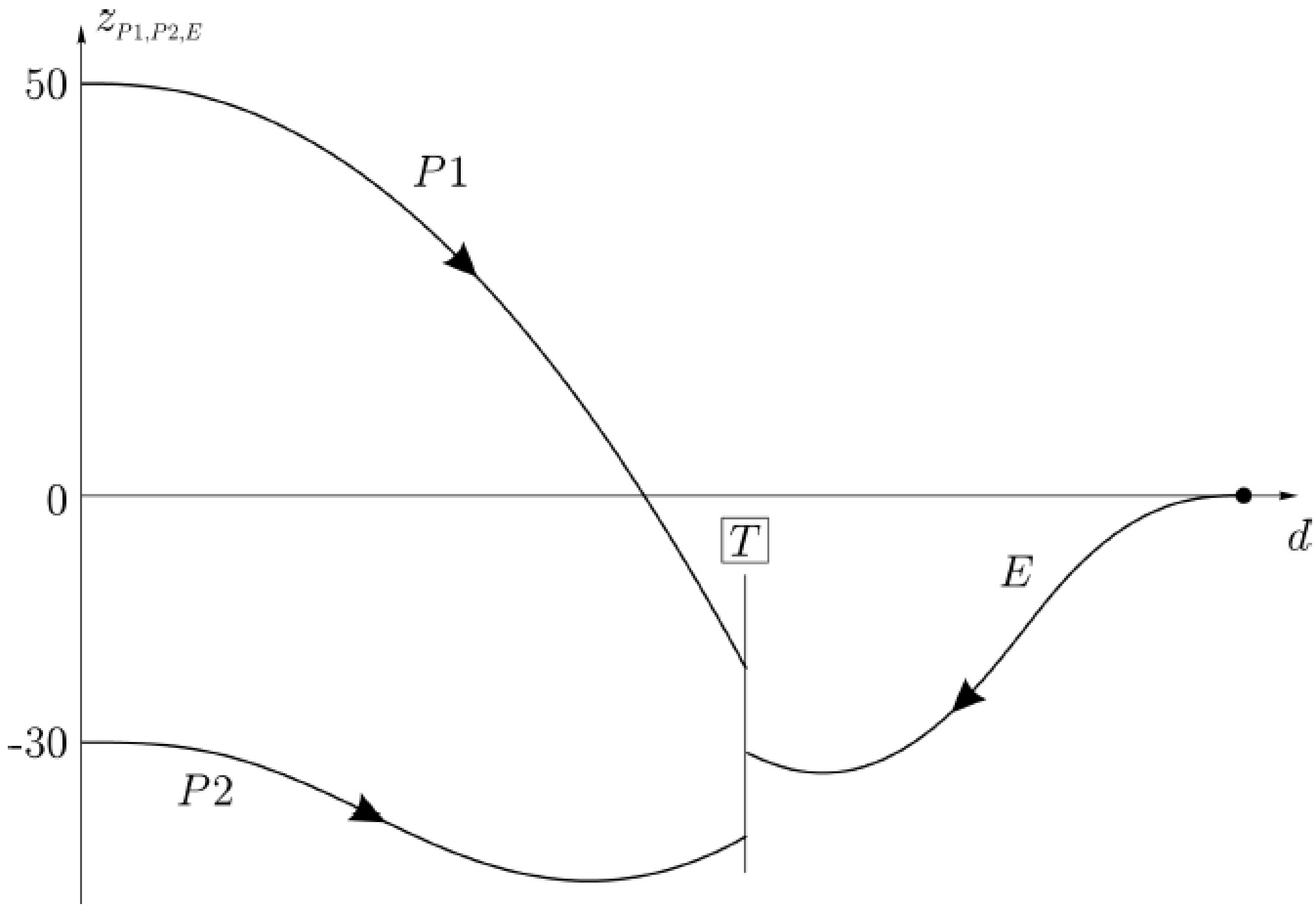
$\mu_1 = \mu_2 = 1.5,\ \nu = 1,\ l_{P_1} = l_{P_2} = 1/0.25,\ l_E = 1,$
$T_1 = T_2 = 15$

$$\dot{x} = D(t)u + E(t)v, \quad |u| \le \mu, \; |v| \le \nu$$



Convex case

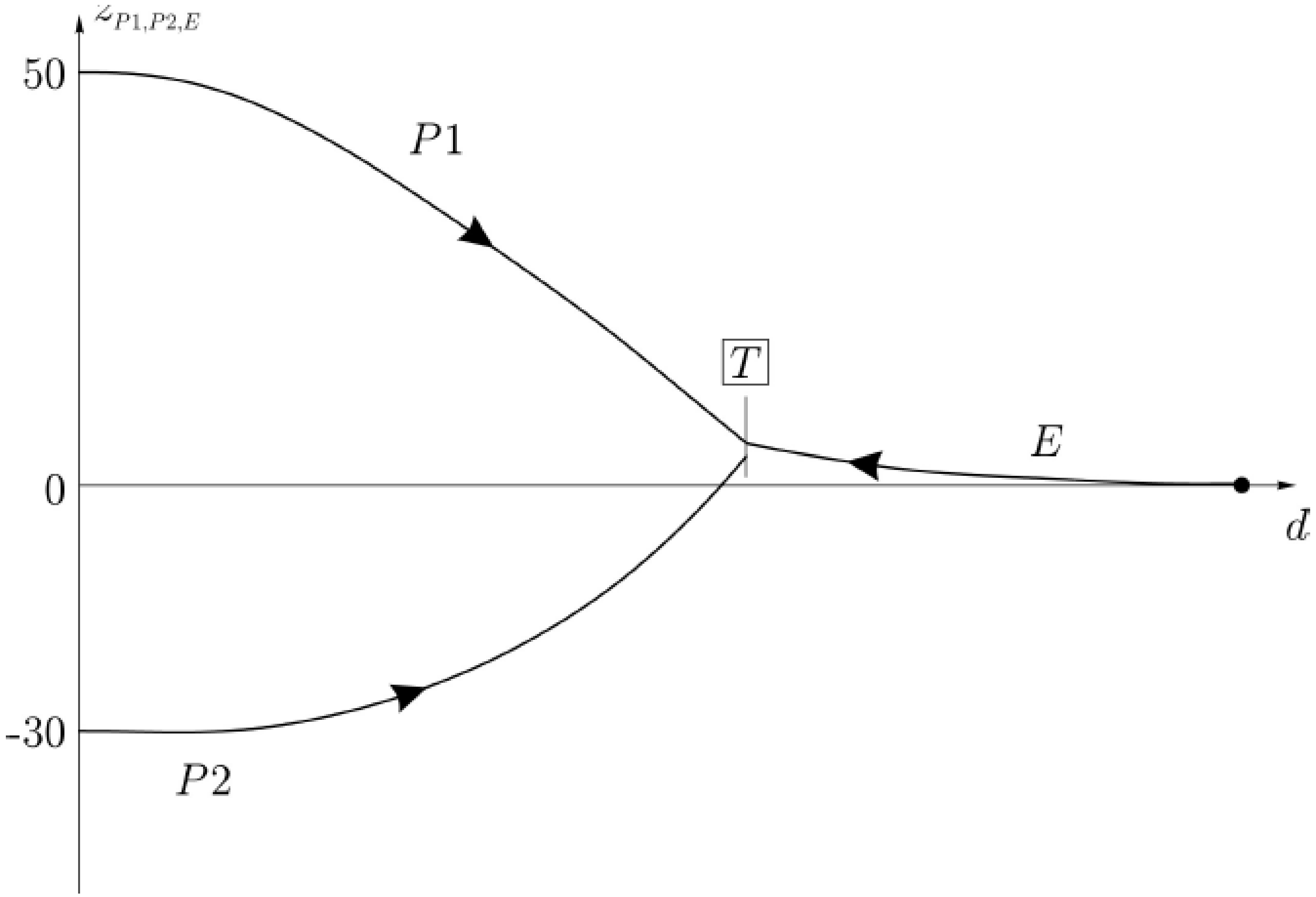
Trajectory Under Quasioptimal Controls of Both Players; Weak Pursuers



$$\mu_1 = 0.7,\ \mu_2 = 0.95,\ \nu = 1,\ l_{P_1} = 1/1.3,\ l_{P_2} = 1/0.4,\ l_E = 1,$$

$$T = T_1 = T_2 = 15$$

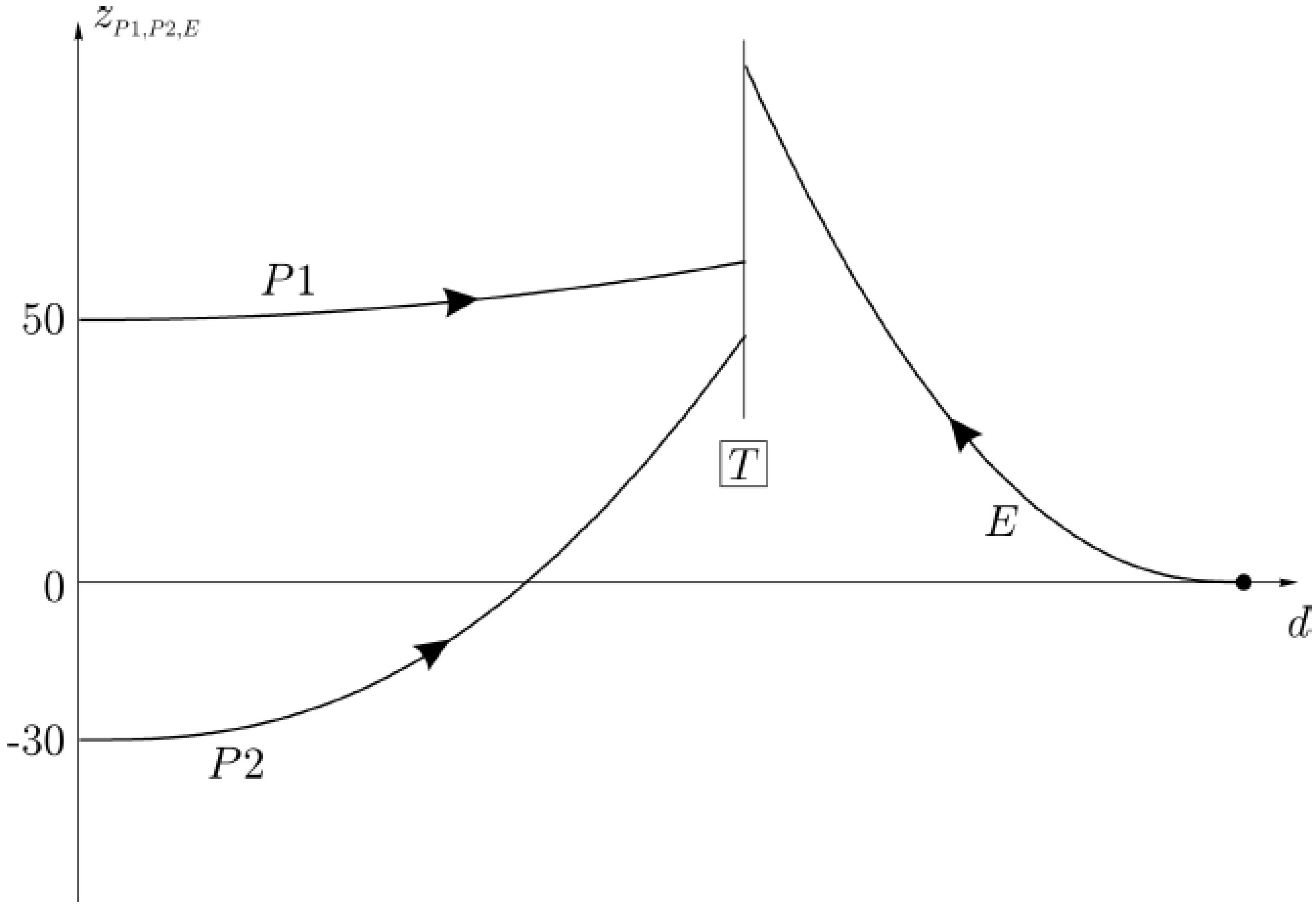
Quasioptimal Control of the First Player and Random Control of the Second One



$$\mu_1 = 0.7,\ \mu_2 = 0.95,\ \nu = 1,\ l_{P_1} = 1/1.3,\ l_{P_2} = 1/0.4,\ l_E = 1,$$

$$T = T_1 = T_2 = 15$$

Quasioptimal Controls
One Pursuer is Very Weak



$$\mu_1 = 0.1,\ \mu_2 = 0.95,\ \nu = 1,\ l_{P_1} = 1/9.0,\ l_{P_2} = 1/0.4,\ l_E = 1,$$
$$T = T_1 = T_2 = 15$$

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Conclusion

- This work is dealing with computing complex capture / reachability sets
- More particularly, we studied a cooperative engagement of two pursuers in front of one evader
- A method for computing quasi-optimal controls from the capture sets has been proposed
- Capture zones (no escape zones) help designers to set up parameters of vehicles in pursuit evasion situations:
  - Vehicule features (manoeuvrability, agility)
  - Spacing between the vehicles when starting the end game
- More complicated acceleration laws (second order control laws; more realistic tranfer functions) could be taken into account
- Extend the Zero Effort Miss computation to 3D simulations

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