Feedback-induced attractors in controlled aeroelastic wing and their detection via interval analysis

Max Demenkov

Institute of Control Sciences
Russia, Moscow

Partly joint work with M. Goman, De Montfort University, UK

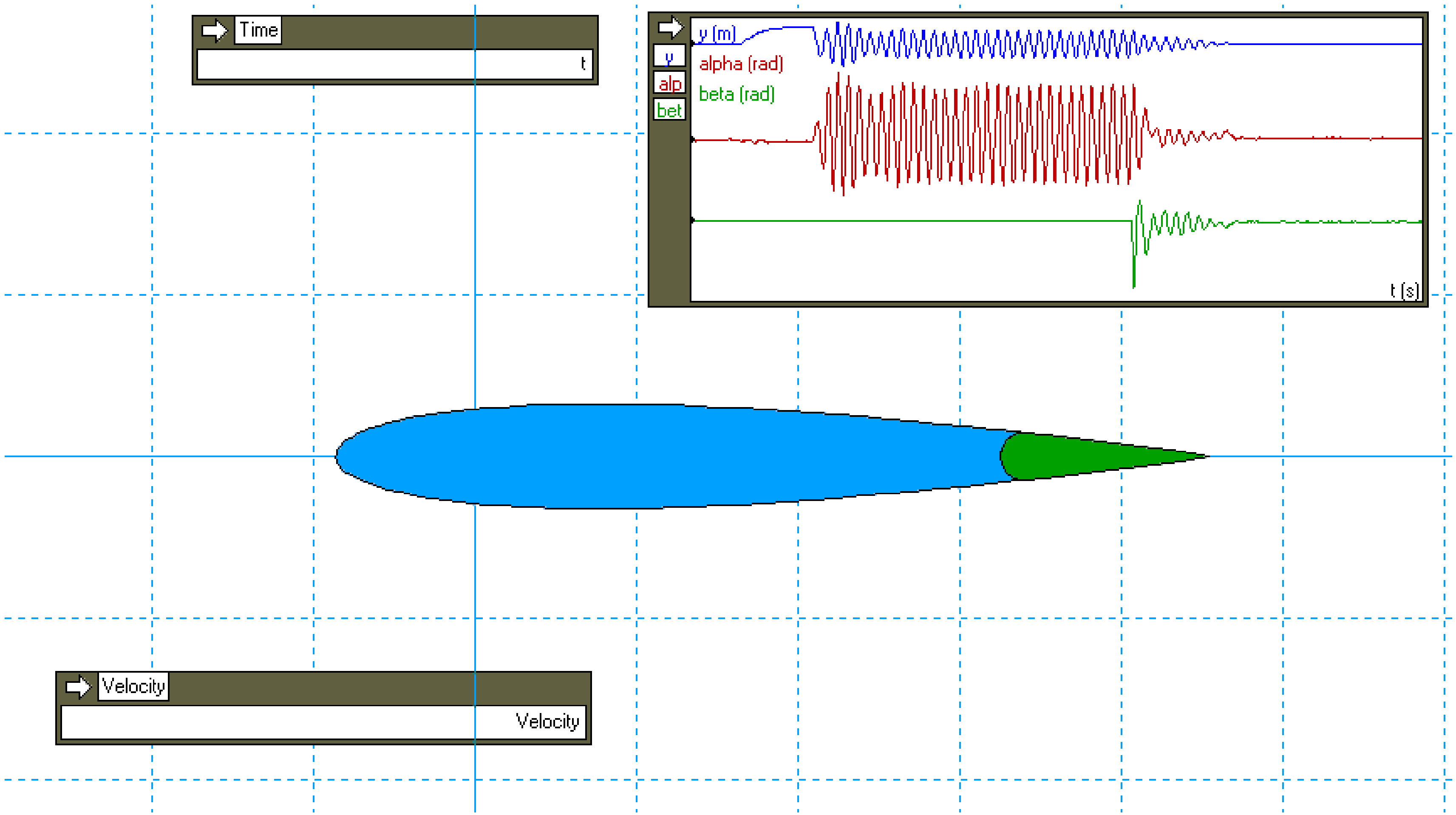
Aeroelastic problem: aircraft wing oscillations

•*Aeroelastic problems* represent a mutual interaction between the aerodynamics and structure of an aerospace vehicle.

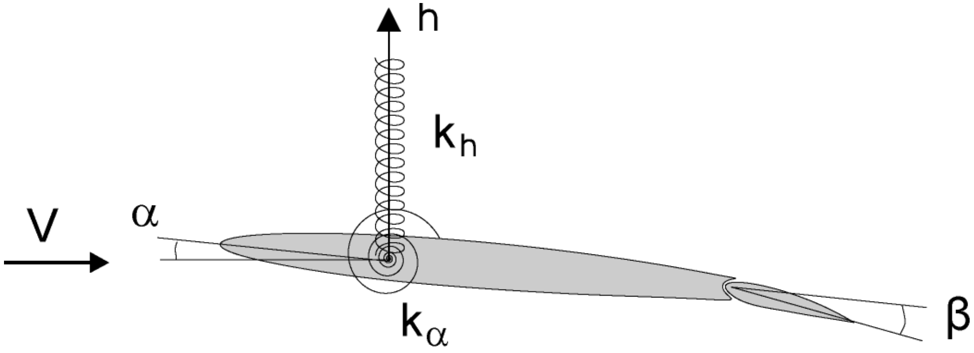
•Aeroelastic systems are inherently nonlinear and these nonlinearities can lead to pathologies such as limit-cycle oscillations

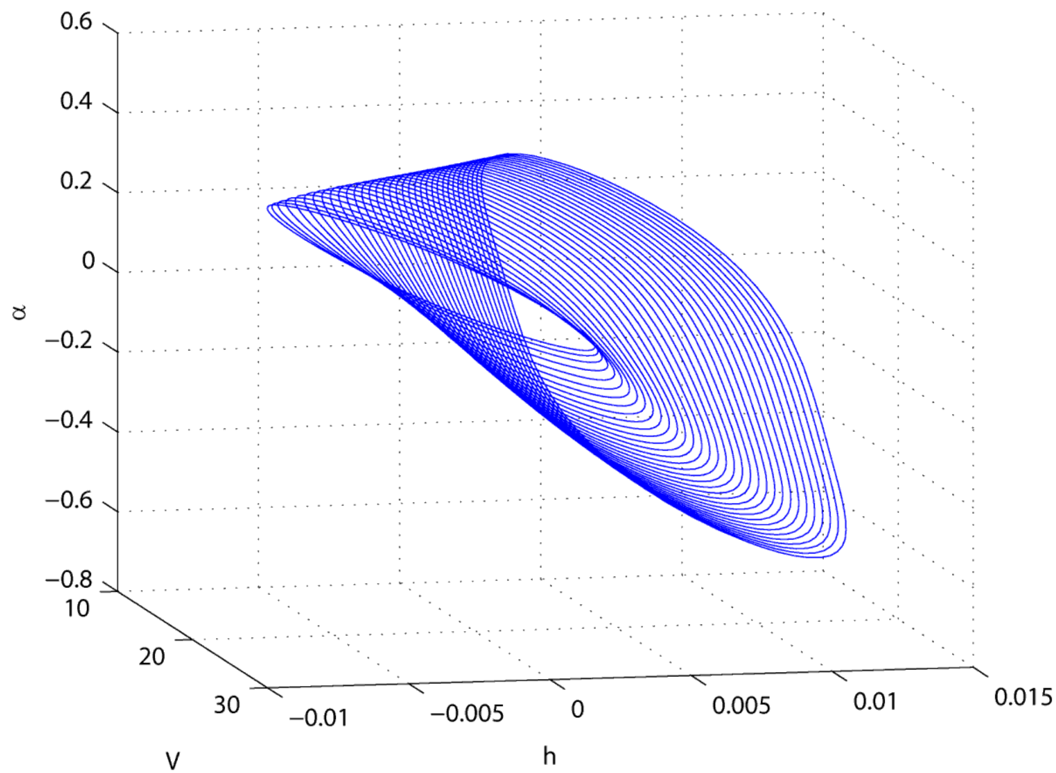
•Excessive vibrations (flutter) in aeroelastic systems can lead to catastrophic structural failures

Flutter suppression via feedback control



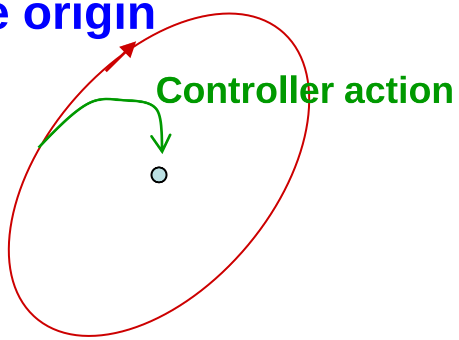
Geometric view on the vibration suppression problem



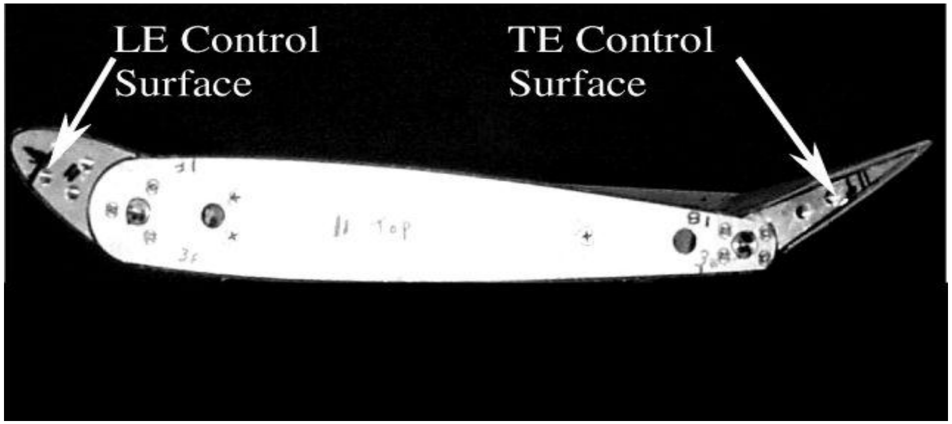


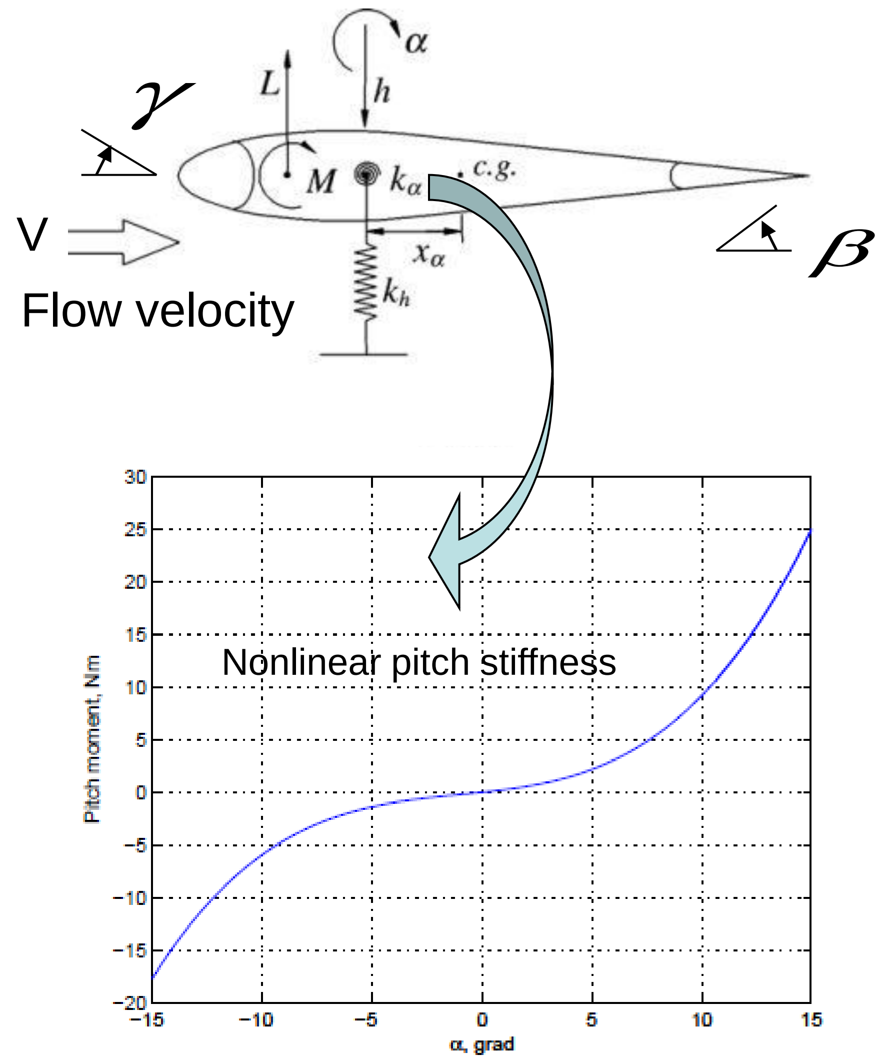
- **During vibrations, the system goes along a closed curve in the system state-space – it is a *limit cycle***

- **Our goal is to destroy all limit cycles in the system state-space and force all trajectories to be attracted by the origin**



Experimental apparatus at Texas A&M University





$$A\ddot{x} + B\dot{x} + C(\alpha)x = Du, \quad x = \begin{bmatrix} h \\ \alpha \end{bmatrix}, \quad u = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 \quad 0 & 1 \quad 0 \\ 0 \quad 0 & 0 \quad 1 \\ -A^{-1}C(\alpha) & -A^{-1}B \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \quad 0 \\ 0 \quad 0 \\ A^{-1}D \end{bmatrix} u$$

$$-u_{\max} \le \beta, \gamma \le u_{\max}$$

Model described in Strganac et al., AIAA Journ. Of Guidance Control Dyn. 2004

Controller based on *nonlinear dynamic inversion:* unconstrained case

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix} \begin{matrix}0 & 0\end{matrix} & \begin{matrix}1 & 0\end{matrix}\\ \begin{matrix}0 & 0\end{matrix} & \begin{matrix}0 & 1\end{matrix}\\ -A^{-1}C(\alpha) & -A^{-1}B \end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}\begin{matrix}0 & 0\end{matrix}\\ \begin{matrix}0 & 0\end{matrix}\\ A^{-1}D\end{bmatrix}u$$

$$\Downarrow$$

$$u(x,\dot{x}) = D^{-1}[C(\alpha) + B\dot{x} + A(K_x x + K_{\dot{x}}\dot{x})]$$

$$\Downarrow$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \multicolumn{2}{c}{K_x} & \multicolumn{2}{c}{K_{\dot{x}}} \end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}$$

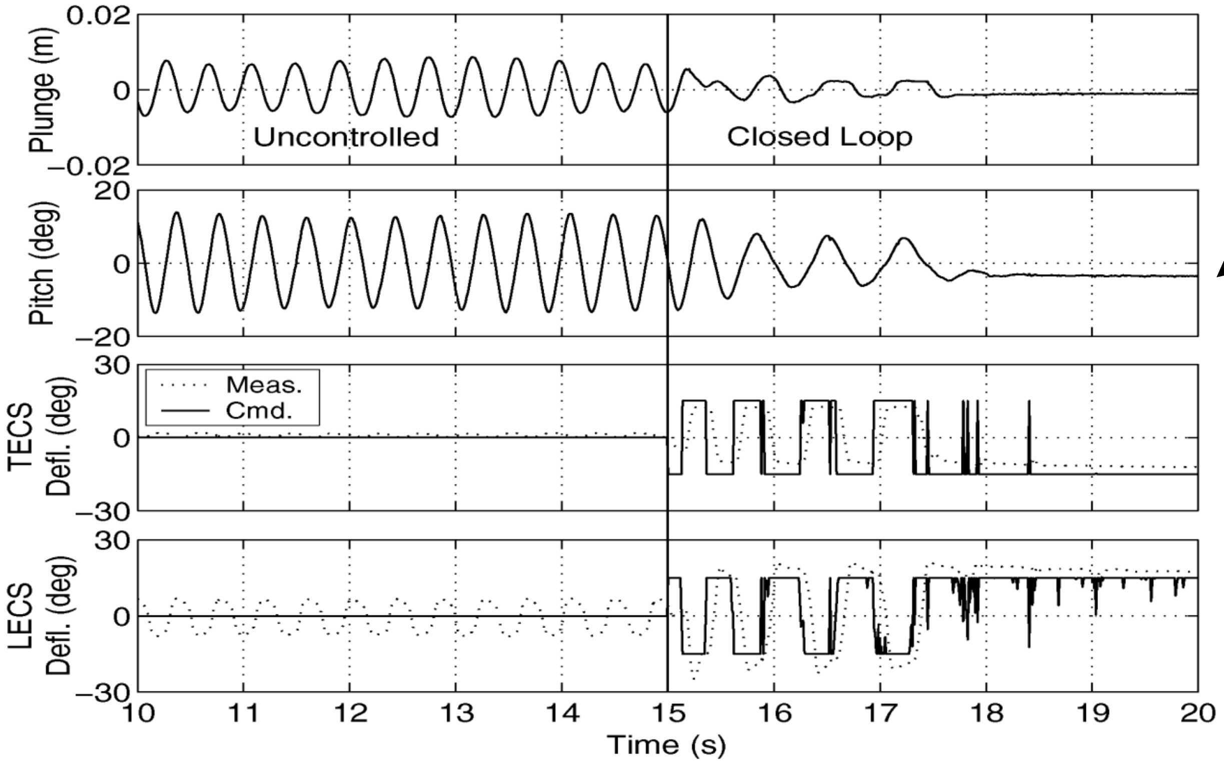
- Very popular method in the aerospace industry

- Nice linear system with predefined stability and performance characteristics

- Cancellation of nonlinearity cannot be exact, therefore control law should incorporate some robustness properties

Experimental facts: sometimes system is stabilized in non-zero trim conditions

α



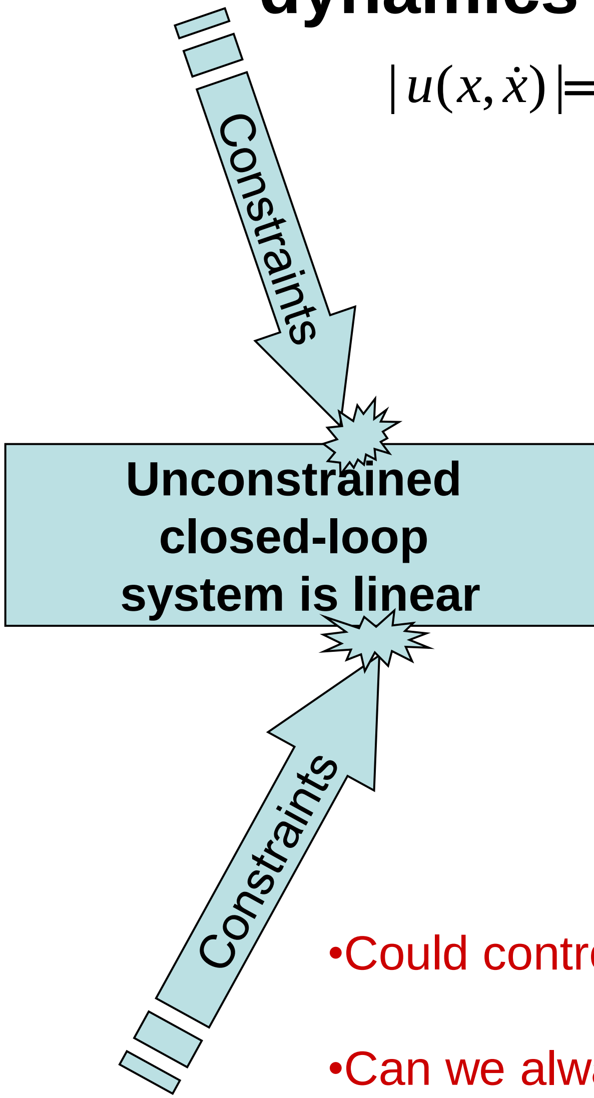
Stable equilibrium:
-5 deg in pitch angle !

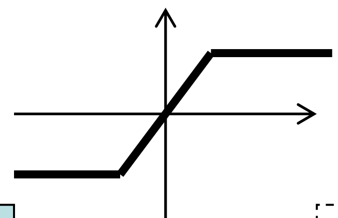
- **The equilibria cannot be found by extensive numerical simulation on a computer**

- **The particular source of the problem was not clear (attributed to dry friction)**

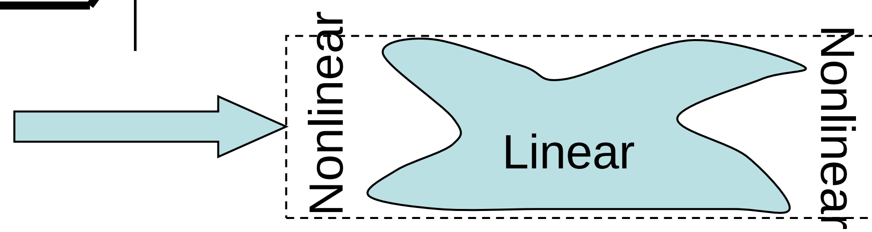
Control constraints lead to unexpected nonlinear dynamics in the closed-loop system

$$|u(x,\dot{x})| = \left| D^{-1}[C(\alpha) + B\dot{x} + A(K_x x + K_{\dot{x}}\dot{x})] \right| \le u_{\max}$$





$$\mathbf{sat}(u) = \begin{cases} u_{\max}, & u > u_{\max} \\ u, & |u| \le u_{\max} \\ -u_{\max}, & u < u_{\max} \end{cases}$$



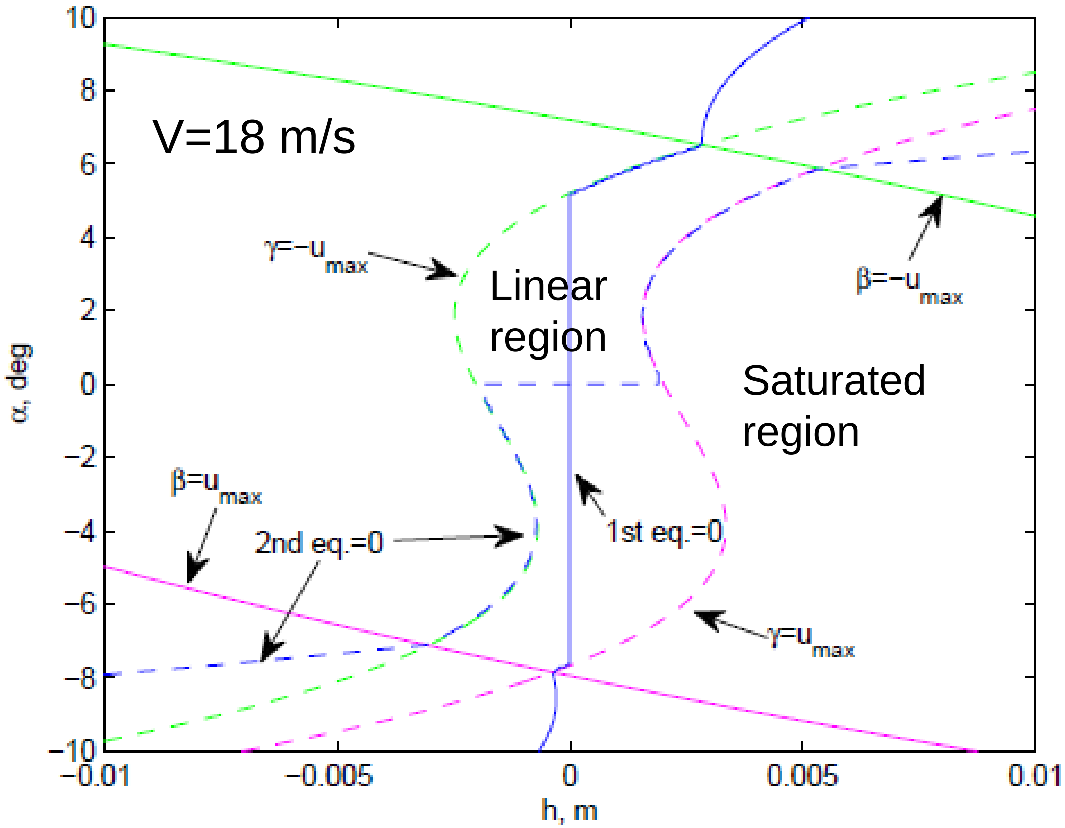
Linear system is not valid in the whole state space, but only in some part of it !

- Could control constraints lead to unzero equilibria in the system ?
- Can we always find these equilibria in the system model ?

Equilibria analysis in a plane: two-dimensional system of equations

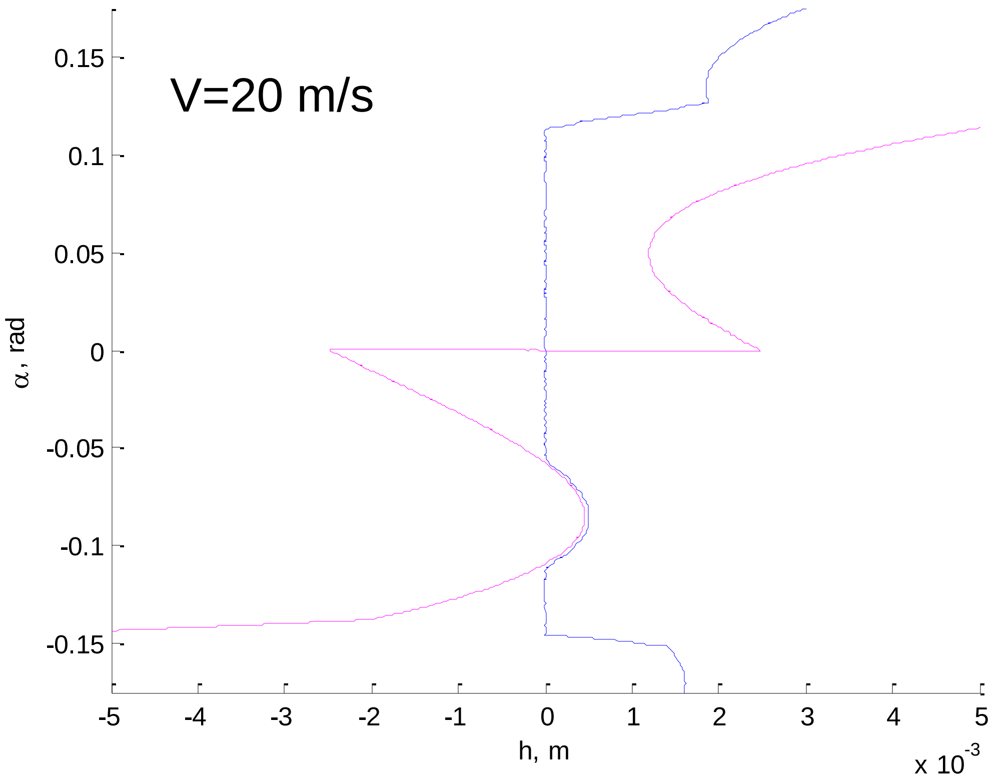
$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = 0 \Rightarrow f(x) = \ddot{x} = A^{-1}(D\,\mathbf{sat}(u(x,0)) - C(\alpha)x) = 0$$

$$\mathbf{sat}(u(x,0)) = \mathbf{sat}(D^{-1}[C(\alpha)x + AK_x x]),\ x = \begin{bmatrix} h \\ \alpha \end{bmatrix},\ u = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$



- Treat each of two equations as a two-dimensional function
- Draw zero level contours
- Their intersection gives us an equilibrium

Suspicious behaviour of the contour curves



- Zero curves for both equations are very close to each other
- There is no actual intersection between them
- Could a small perturbation in the system provoke their intersection ?

Structural stability

•The notion of structural stability was first introduced by Andronov and Pontryagin in 1937

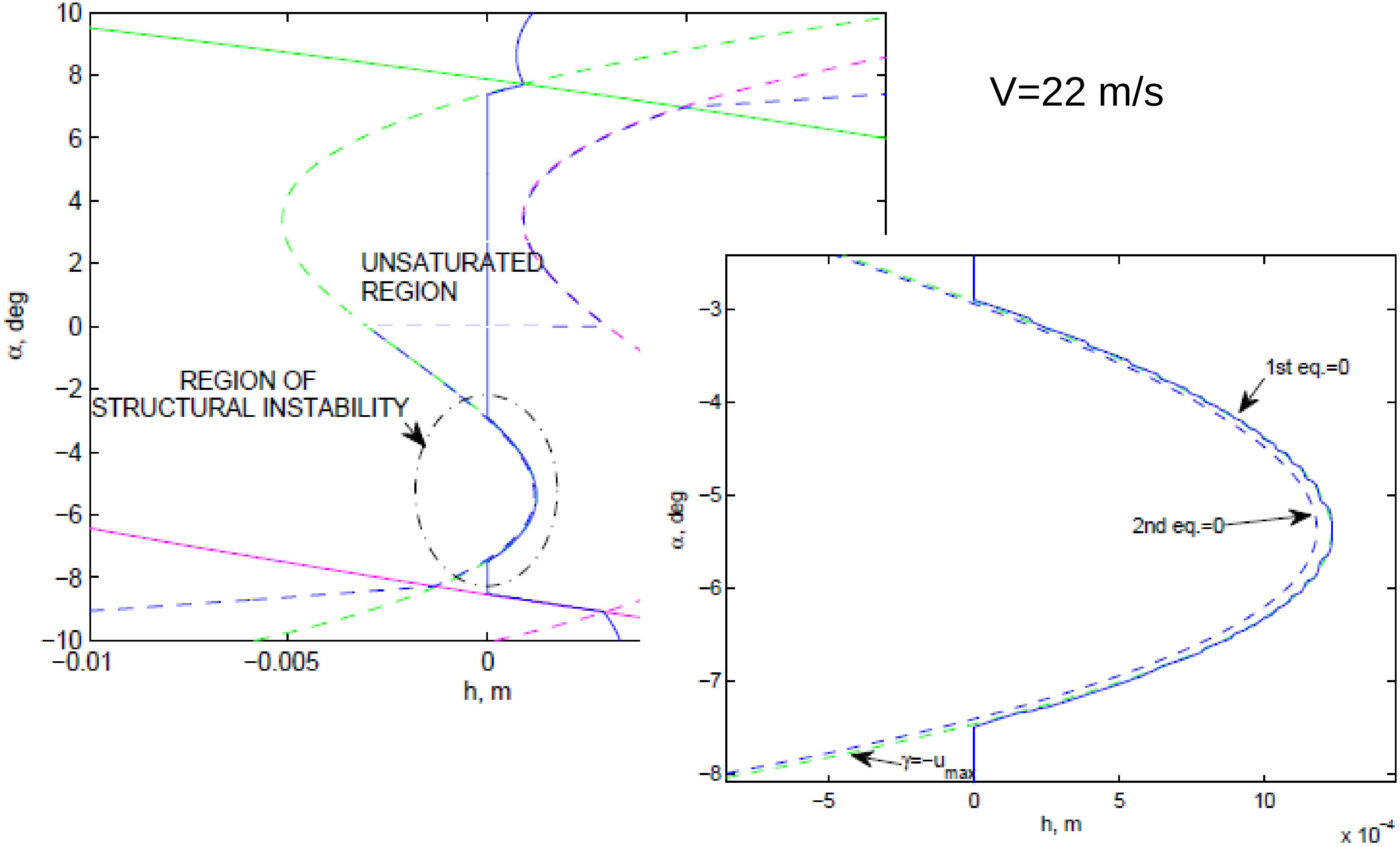
•Informally, we say that a system is structurally stable if small variations in the model does not change qualitatively the set of trajectories originating from all initial conditions in the state space

•It is possible to define bifurcations in both smooth and non-smooth systems via the structural stability concept

•Real-life applications of the concept are rare: Pai *et al.*, 1995 (power systems); Kaslik and Balint, 2007 (aerospace reentry vehicle); Sumida *et al.*, 2007 (clinical study). The paper of Sumida noted that even any numerical degree of the structural stability was not defined previously.

Indication of structural instability in our system

V=22 m/s



How to automatize structural instability detection ?

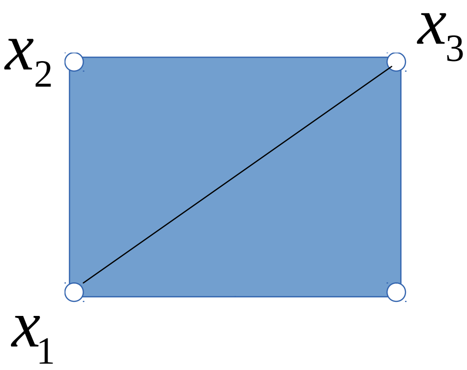
- We tried to apply MATLAB-INTLAB-based methods but with no success – because of heavy overestimation every small interval extension of the system contains zero !

- Instead, we switched to different approach – first, represent system as piecewise-linear and then apply interval methods

- We do not want to find point solutions – instead, we are looking for a collection of boxes which indicate regions of structural instability

- Somehow, the drawback of interval analysis is its advantage in our case

Piecewise-linearization



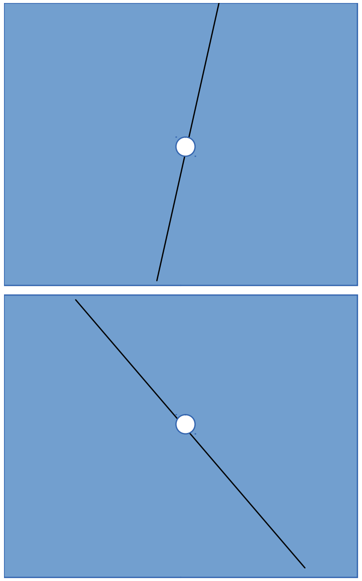
$$x \in R^2$$

- State-space is divided into cubes
- Every cube is splitted into simplices
- For every simplex, derive linear functions as a solution of linear equations:

$$d_k{}^T x_1 + c_k = f(x_1)$$
$$d_k{}^T x_2 + c_k = f(x_2)$$
$$d_k{}^T x_3 + c_k = f(x_3)$$

$$d_k = \begin{bmatrix} d_{k1} \\ d_{k2} \end{bmatrix}$$

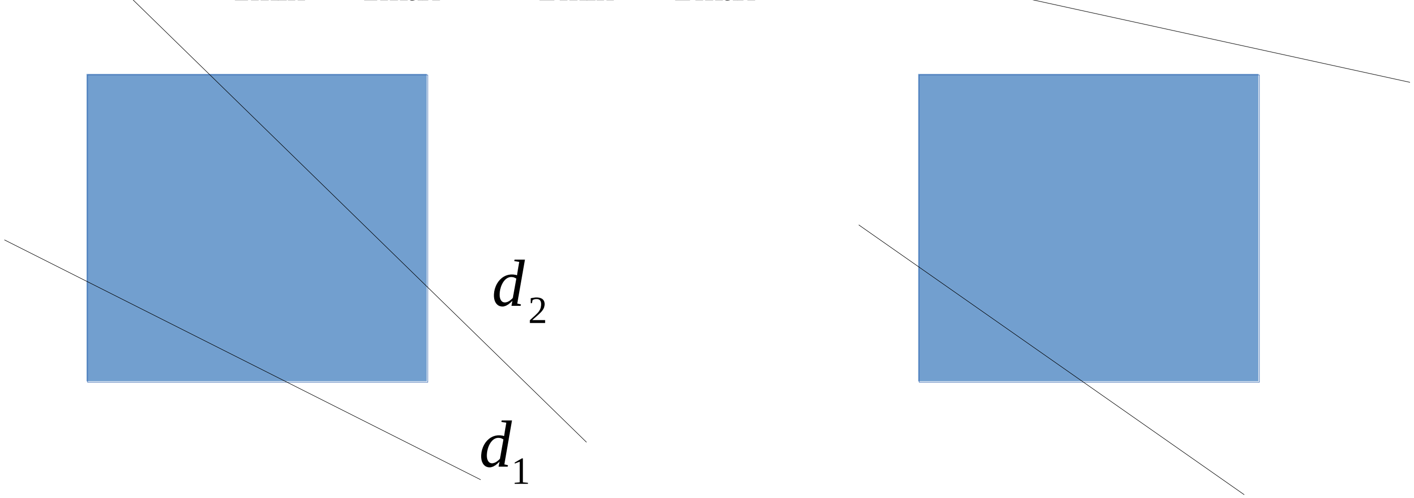
Piecewise-linearization: trust region method



- State-space is divided into cubes
- For every cube, we linearize around center point
- We assume that linearization “works well” inside the cube

Linear functions over a cube

$$X = [[x_{1\min}, x_{1\max}], [x_{2\min}, x_{2\max}], \ldots]$$



Intersection is possible

No intersection

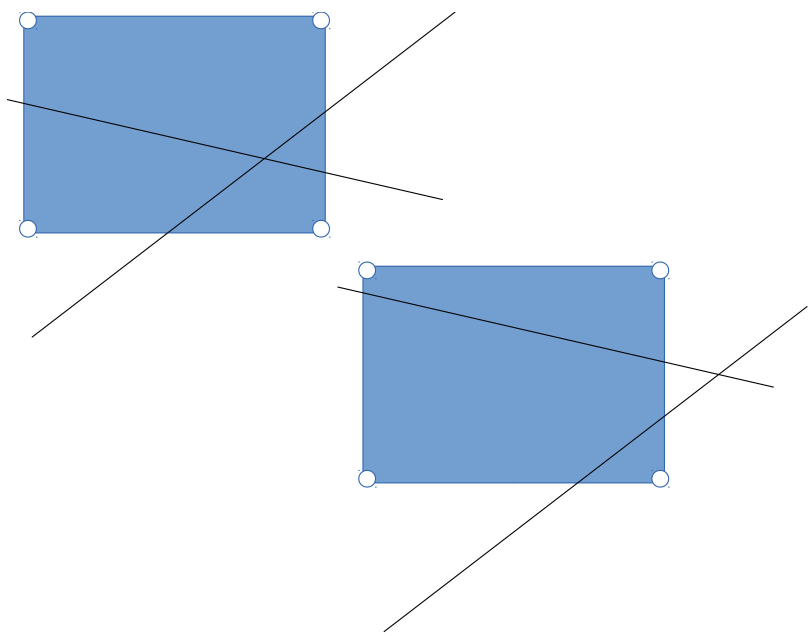
$$\max_{x \in X} \; d_{1,2}{}^T x + c_{1,2} > 0$$

$$\max_{x \in X} \; -d_{1,2}{}^T x - c_{1,2} < 0$$

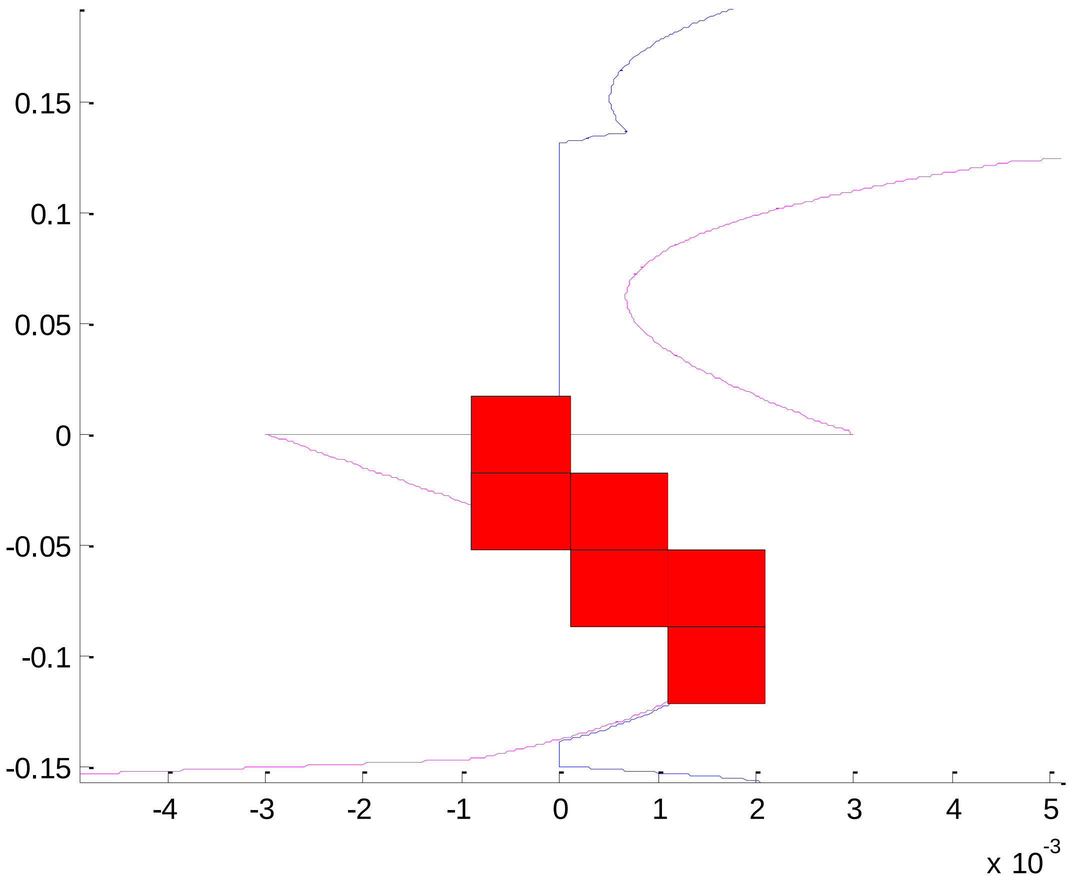
$$\max_{x \in X} d_k{}^T x = \sum_i d_{ki} \max(d_{ki} x_{i\min}, d_{ki} x_{i\max})$$

Piecewise-linear algorithm

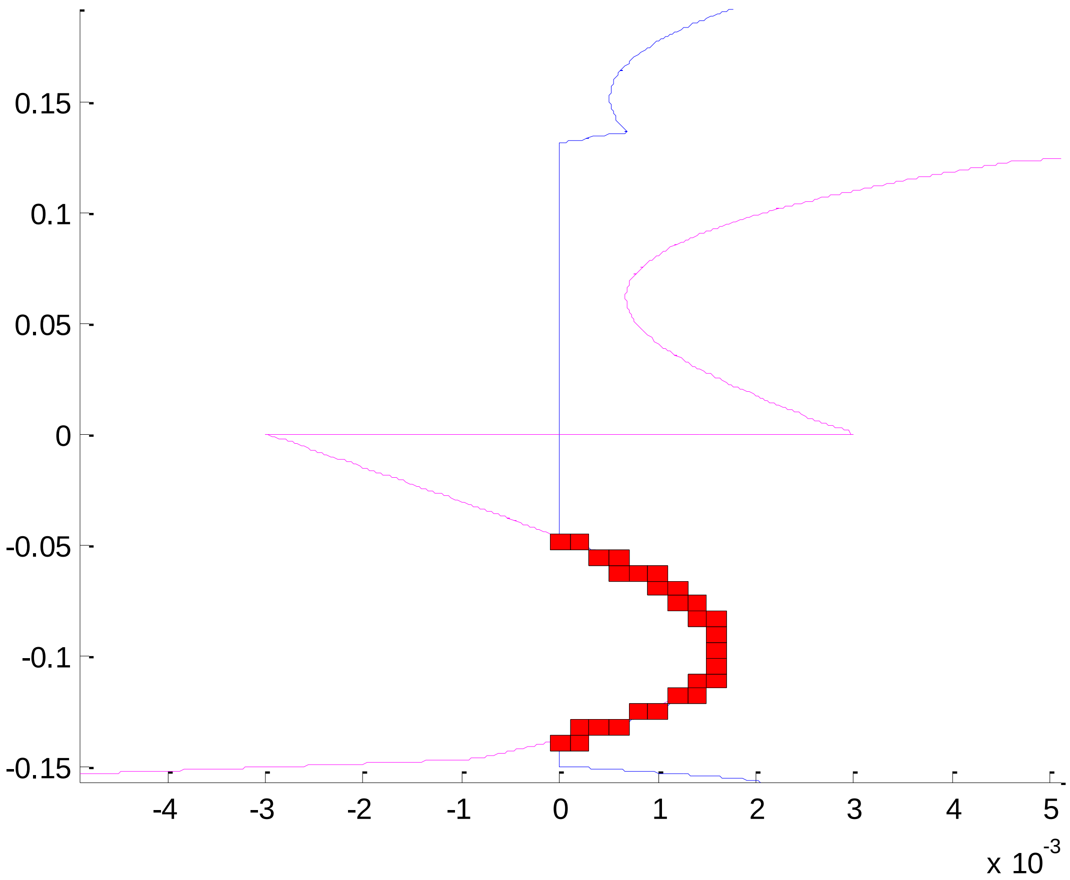
- Check if cube contains all linear functions
- Try to find their intersection as a solution of linear system
- Check if this solution is inside the cube
- If not, or there is no solution, mark the cube as “suspicious”



Piecewise-linear identification of structural instability: large grid



Piecewise-linear identification of structural instability: smaller grid



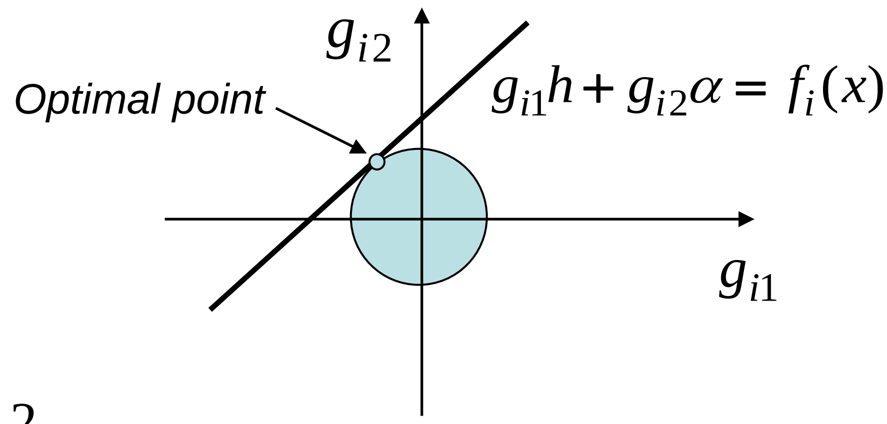
Artificial perturbation of the system

Let us introduce some hypothetical (linear) unmodeled dynamics:

$$f_{\mathrm{mod}}(x) = f(x) + g(x), \quad \|g(x)\| \le \varepsilon_1, \ \left\|\frac{dg(x)}{dx}\right\| \le \varepsilon_2$$

$$A\ddot{x} + B\dot{x} + C(\alpha)x - Ag(x) = Du$$

We use the formula for the minimum distance from the origin in (g_{i1}, g_{i2}) -space:



$$g(x) = Gx = \begin{bmatrix} g_{11}h & g_{12}\alpha \\ g_{21}h & g_{22}\alpha \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$$

$$g_{i1} = \frac{-f_i h}{h^2 + \alpha^2}, \quad g_{i2} = \frac{-f_i \alpha}{h^2 + \alpha^2}, \ i = 1,2$$

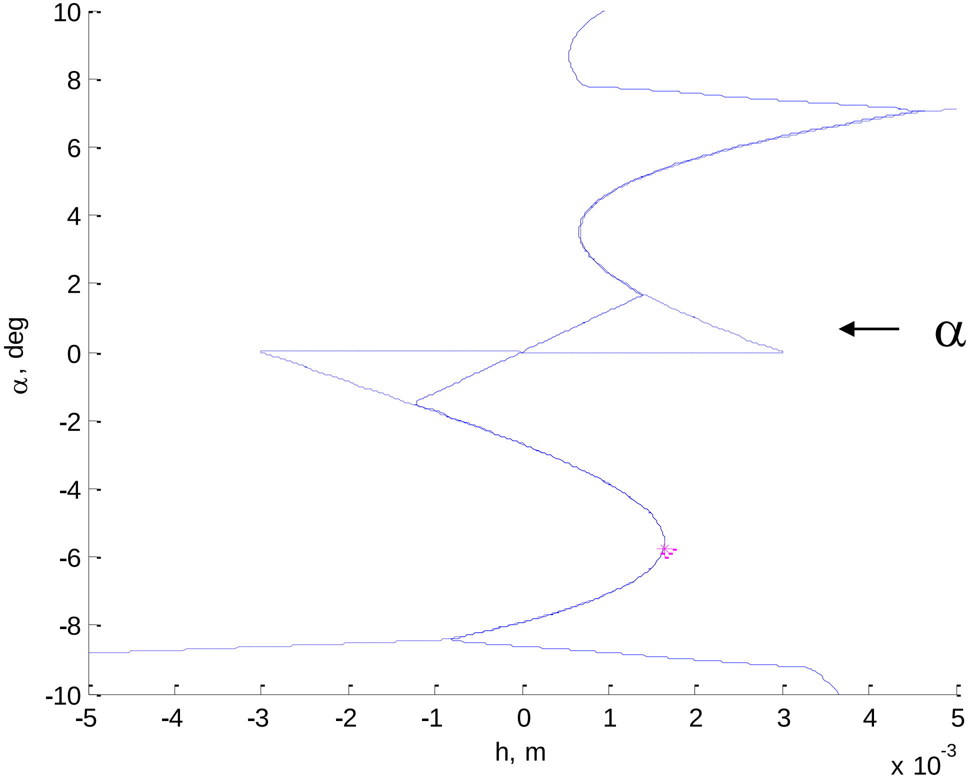
Numerical example: artificial stable equilibrium

$$V = 22\,m/s,\; h = 0.001636157\,m,\;\; \alpha = -0.1000768\; rad$$

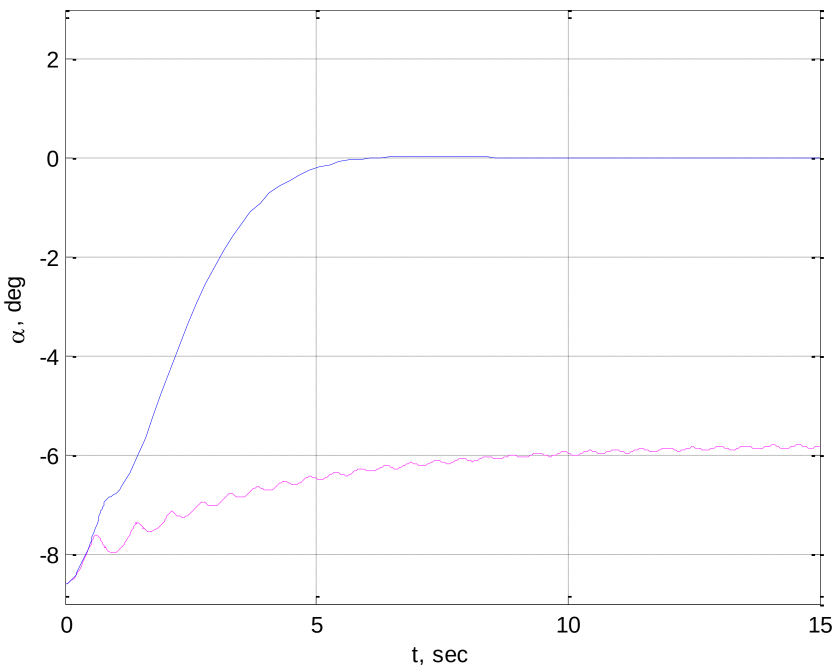
$$Gx = \begin{bmatrix} -0.0008h & 0.0479\alpha \\ -0.0009h & 0.0538\alpha \end{bmatrix}$$

Open-loop “nominal” dynamics:
min 600 times difference !

$$-A^{-1}C(\alpha)x = \begin{bmatrix} -214.1395h & -24.3894\alpha \\ 859.9288h & -129.7758\alpha \end{bmatrix}$$



← α →



Numerical bifurcation analysis of the closed-loop *nonlinear* system

- **We use MATLAB-based toolbox MATCONT for the bifurcation analysis of the closed-loop limit cycles**

- **A bifurcation of a *nonlinear* system is a qualitative change in its dynamics produced by varying parameters – for example, change of number of cycles and equlilibria in the system**

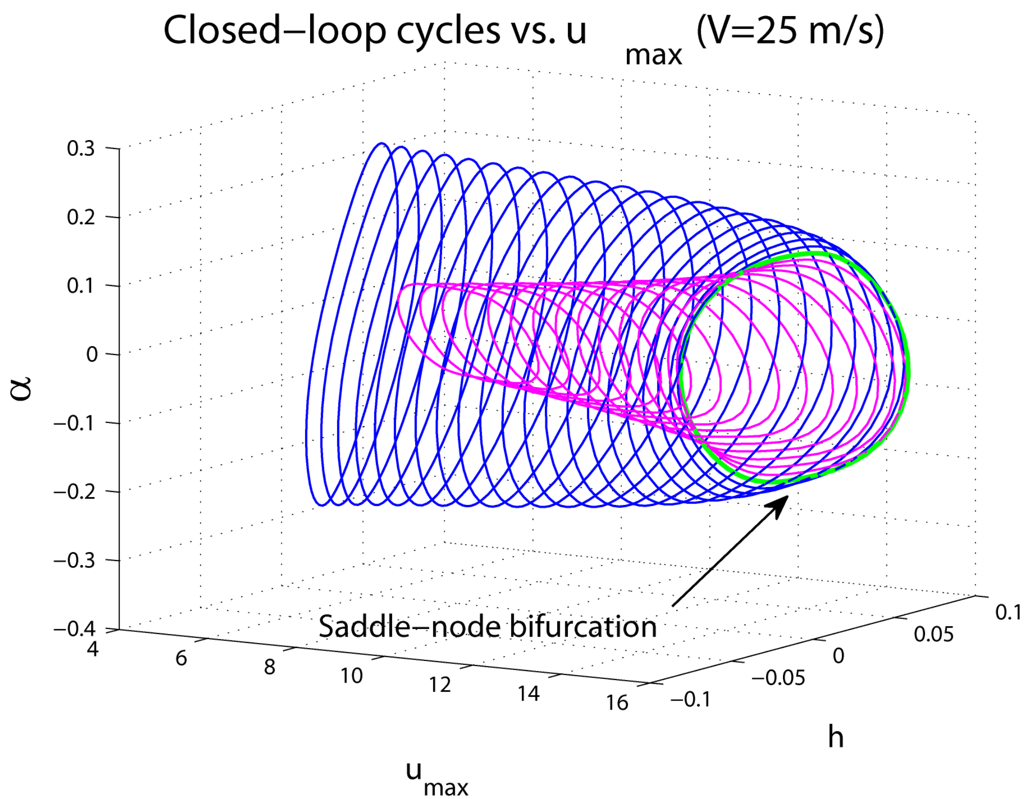
- **The controller is designed using linear-quadratic criterion and ONE ACTUATOR, saturation is considered**

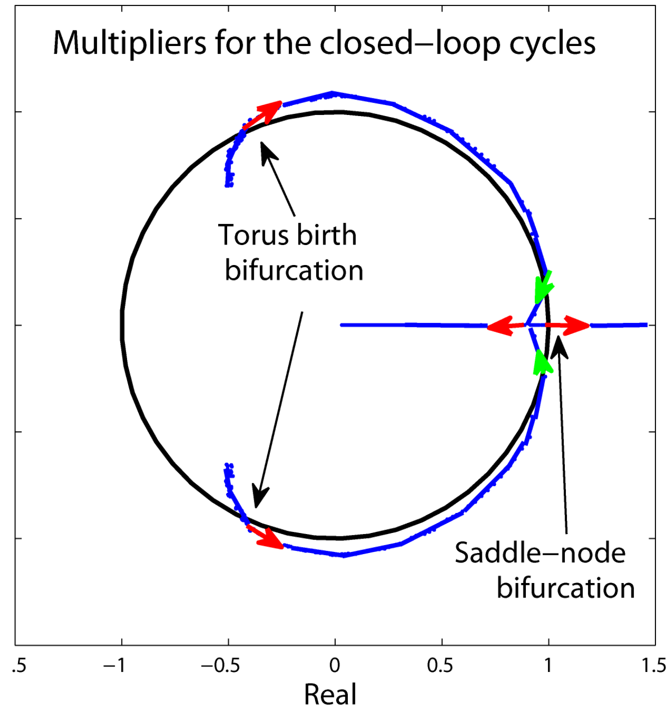
- **During bifurcation analysis, we change only one parameter – *maximum control amplitude***

- **MATCONT works only with smooth systems – therefore we smoothed saturation function**

See Yu.A. Kuznetsov, Elements of applied bifurcation theory, Springer 1995-2004

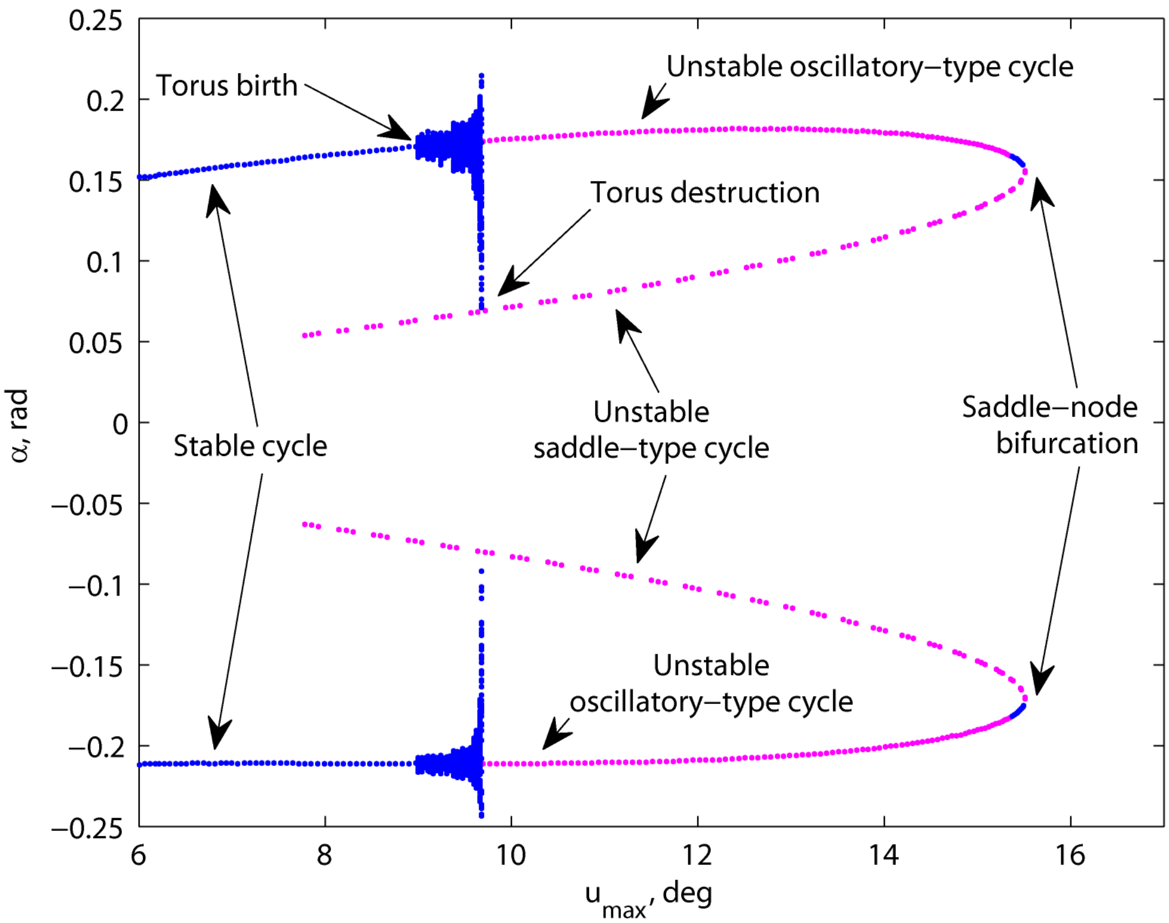
Numerical results





The numerical continuation with various V from 11 to 30 m/s reveals that the *saddle-node (or fold) bifurcation* occurs at some u_{max} between 15 and 16 degrees. The bifurcation scenario implies the collision and disappearance of two limit cycles, the stable one and the unstable one.

Bifurcation diagram for V= 25 m/s



Being initially stable at small u_{max}, the closed-loop limit cycle first loses its stability after the *torus birth* via the Neimark-Sacker bifurcation and then, after the second Neimark-Sacker bifurcation, regains stability in a very short range of u_{max} before its final disappearance

Conclusions

- **Random attractors appearance in experiments is explained from the theoretical viewpoint**
- **Simple numerical method for structural instability detection is proposed based on piecewise-linearization**
- **The proposed method can help in feedback design for flutter suppression**
- **See paper of my colleagues with application of piecewise-linear analysis to more realistic model:**

Kolesnikov, E. and Goman, M., Analysis of Aircraft Nonlinear Dynamics Using Non-Gradient Based Numerical Methods and Attainable Equilibrium Sets, AIAA Atmospheric Flight Mechanics Conference, No. AIAA-2012-4406, August 2012