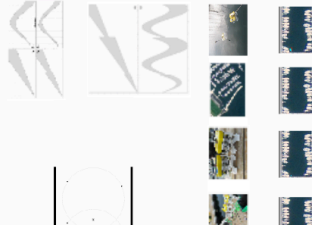


IV - COOPERATIVE
CALIZATION AS A CONSTRAINT
SATISFACTION PROBLEM



V - TESTCASES



VI - CONCLUSION



ENSTA
Bretagne

SWIM 2013

**Cooperative localization of underwater
robots with unsynchronized clocks.**



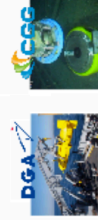
Aymeric BETHENCOURT

June 6th, 2013



I - INTRODUCTION

Autonomous Underwater Vehicle



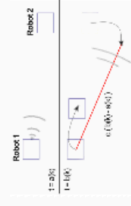
- Swarms: Robustness / Redundancy / Diversity / Scalability / Cooperation

- Problem: Localization specific to Underwater Robots:
 - On the surface: GPS
 - Underwater: No specific infrastructures for localization, Acoustic localization

- Objective: Interval based approach for localizing underwater robots in a moving swarm using sonar communication.

- Approach: **Distributed**

- Challenge:
 1. Consider that the displacement of the robot while the sonar measurement is ongoing is unknown to the position of the localizations.



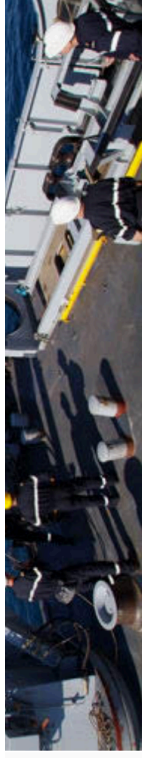
2. Consider the clocks of the robots to be unsynchronized.

- Problem: Solve an inter-temporal system of equations with uncertain times.

- Principle: Given the linear and non-linear state equations, solve a constant localization problem for which interval propagation is particularly powerful.

Autonomous Underwater Vehicle





- Swarm : Robustness (Redundancy / Diversity)
Expandability / Scalability
Cooperation

- Problem : Localization. Specific to Underwater Robots :
On the surface : CDE

Cooperation

- Problem : Localization. Specific to Underwater Robots :
On the surface : GPS.
Underwater : No specific infrastructure for localization.
 - Dead Reckoning
 - Acoustical localization

- Objective : Interval based approach for localizing underwater

Underwater : No specific infrastructure for localization.

- Dead Reckoning
- Acoustical localization

- Objective : Interval based approach for localizing underwater robots in a moving swarm using sonar communication.

- Approach : **Distributive**

- Objective : Interval based approach for localizing underwater robots in a moving swarm using sonar communication.

@

- Approach : **Distributive**

- Originality :

1. Consider that the displacement of the robot while the sonar wave is moving is superior to the precision of the localization.

- Approach : **Distributive**

or

- Originality :

1. Consider that the displacement of the robot while the sonar wave is moving is superior to the precision of the localization.

Robot 1



Robot 2

Robot 1



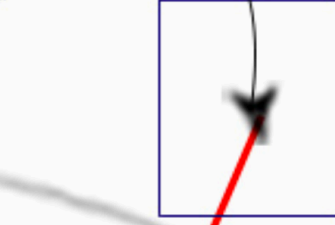
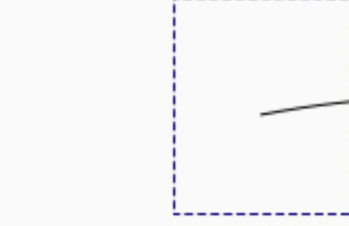
$a(k)$

Robot 2

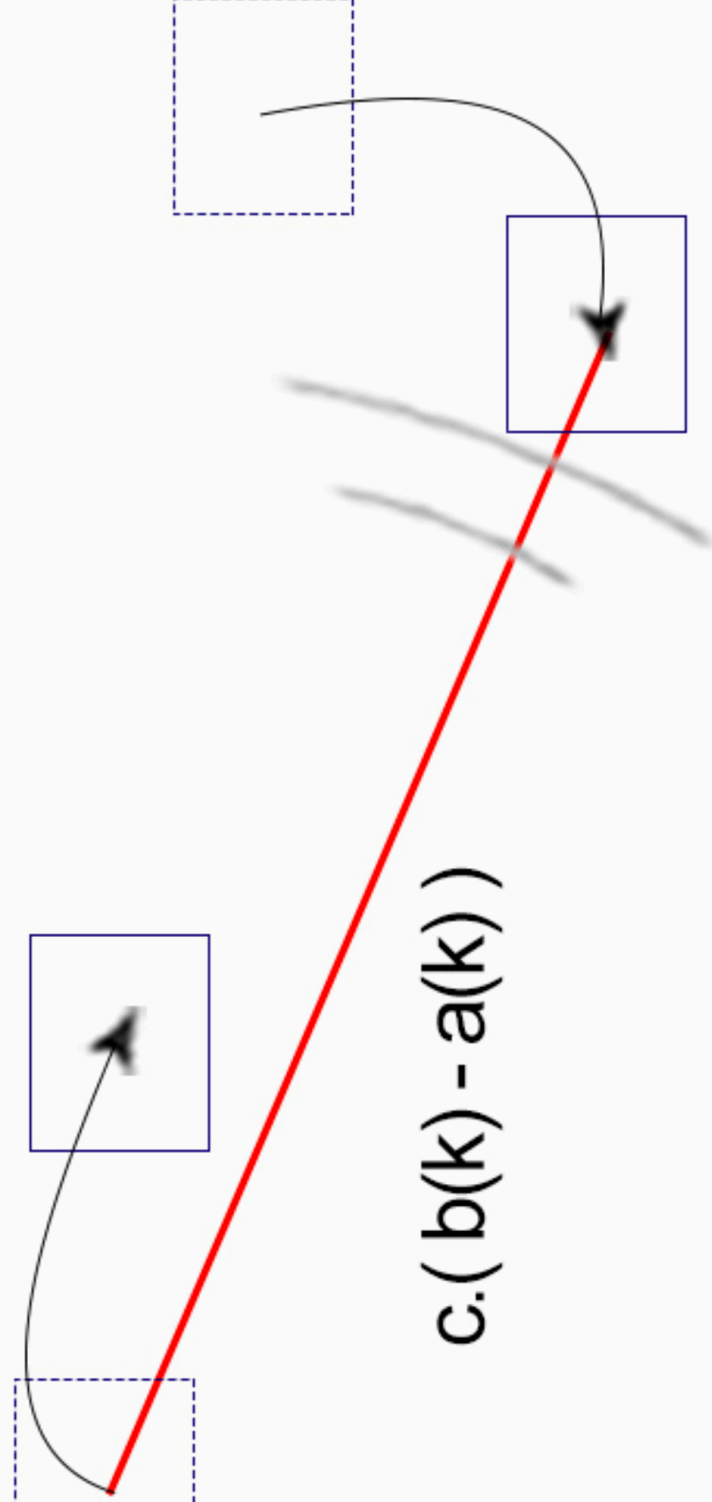


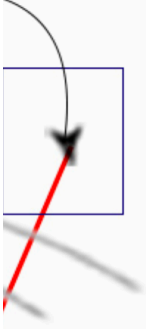
$t = a(k)$

$t = b(k)$



$c.(b(k) - a(k))$





2. Consider the clocks of the robots to be unsynchronized.

- Problem : Solve an inter-temporal system of equations with uncertain time.

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II - PROBLEM STATEMENT

State equations

For all robot i :

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) \quad (\text{evolution equation})$$

$$\mathbf{y}_i = \mathbf{g}(\mathbf{x}_i) \quad (\text{observation equation})$$

Dead Reckoning

GPS, Acousical measurements

$$\tau = h_i(t) \quad (\text{Clock function})$$

Formalization

- (i) $\dot{\mathbf{x}}_i(t) = \mathbf{f}(\mathbf{x}_i(t), \mathbf{u}_i(t)) + \mathbf{n}_x(t)$
- (ii) $g(\mathbf{x}_{i(k)}, a(k), \mathbf{x}_{j(k)}, b(k), a(k), b(k)) = 0$
- (iii) $\hat{a}(k) = h_{i(k)}(a(k))$
- (iv) $\hat{b}(k) = h_{j(k)}(b(k))$
- (v) $\hat{h}_i(t) = 1 + n_h(t)$

k is the number of the ping.

(ii) can be

$$g(\mathbf{x}_i, \mathbf{x}_j, a, b) = \|\mathbf{x}_i - \mathbf{x}_j\| - c(b - a)$$

Formalization

For robot j receiving a ping k from i ,
We know :

$$\hat{a}(k), \hat{b}(k), i(k), j(k), \mathbf{x}_{i(k)}, \mathbf{x}_{j(k)}, \{a(k)\}$$

We want :

$$\{a(k)\}, \{b(k)\}, \mathbf{x}_{i(k)}, \mathbf{x}_{j(k)}, \{b(k)\}$$

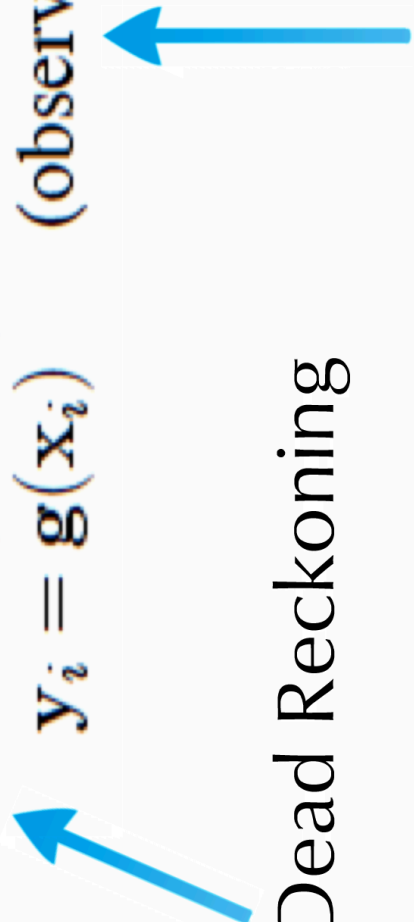
In our context, the trajectories $\mathbf{x}$ and $\mathbf{h}$ belong to a lattice and their domains will be represented by tubes (interval of trajectories).

Formalization

Each ping contains the estimated position box of the emitting robot along with the interval around the emitting time in the robot's clock.

State equations

For all robot i:

$$\begin{array}{ll} \dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) & \text{(evolution equation)} \\ \mathbf{y}_i = \mathbf{g}(\mathbf{x}_i) & \text{(observation equation)} \end{array}$$


Dead Reckoning

GPS, Acoustical measurements

$$\tau = h_i(t) \quad \text{(Clock function)}$$

Formalization

- (i) $\dot{\mathbf{x}}_i(t) = \mathbf{f}(\mathbf{x}_i(t), \mathbf{u}_i(t)) + \mathbf{n}_x(t)$
- (ii) $g(\mathbf{x}_{i(k)}(a(k)), \mathbf{x}_{j(k)}(b(k)), a(k), b(k)) = 0$
- (iii) $\tilde{a}(k) = h_{i(k)}(a(k))$
- (iv) $\tilde{b}(k) = h_{j(k)}(b(k))$
- (v) $\dot{h}_i(t) = 1 + n_h(t)$

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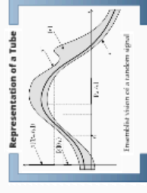
$$\tilde{a}(k), \tilde{b}(k), i(k), j(k), [\mathbf{x}_{i(k)}] ([a(k)])$$

We want :

$$[a(k)], [b(k)], [\mathbf{x}_{j(k)}] ([b(k)])$$

In our context, the trajectories x and h belong to a lattice and their domains will be represented by tubes (interval of trajectories).

III - CONSTRAINT PROPAGATION



Tubes

The tube $\mathbf{X}^*$ is an interval $\mathbf{X}^* = [\mathbf{X}^*_L, \mathbf{X}^*_U]$ for $\mathbf{X}^*$ (represented)

$$\mathbf{X}^*(t) \subseteq \mathbf{X}^*_L(t) \leq \mathbf{X}^*_U(t) \leq \mathbf{X}^*(t)$$

The set of all tubes $\mathcal{T}^*$

Tube arithmetic

We can extend some classical operations we have on functions of $\mathbf{X}^*$, such as sums, multiplications, using to a function in tubes. It suffices to use the classical notion of pointwise arithmetic and inclusion functions for all.

An arithmetic on tubes is thus a direct extension of interval arithmetic.

Operations

Example :

$$f(t) = \int_0^t (1-s) e^{-s} ds = 1 - e^{-t}$$

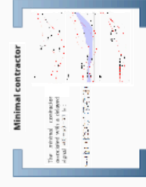
$$f(t) = \int_0^t (1-s) e^{-s} ds = 1 - e^{-t}$$

Tube Contractor

Contractant $\mathcal{Z}(t)$

Associated contractor $\mathcal{C}_t(\mathcal{Z}, t)$

Contracting Completeness $\left(\mathcal{Z}_t(t) \right)_{t \in [0, T]} \rightarrow x \in [a, b]$



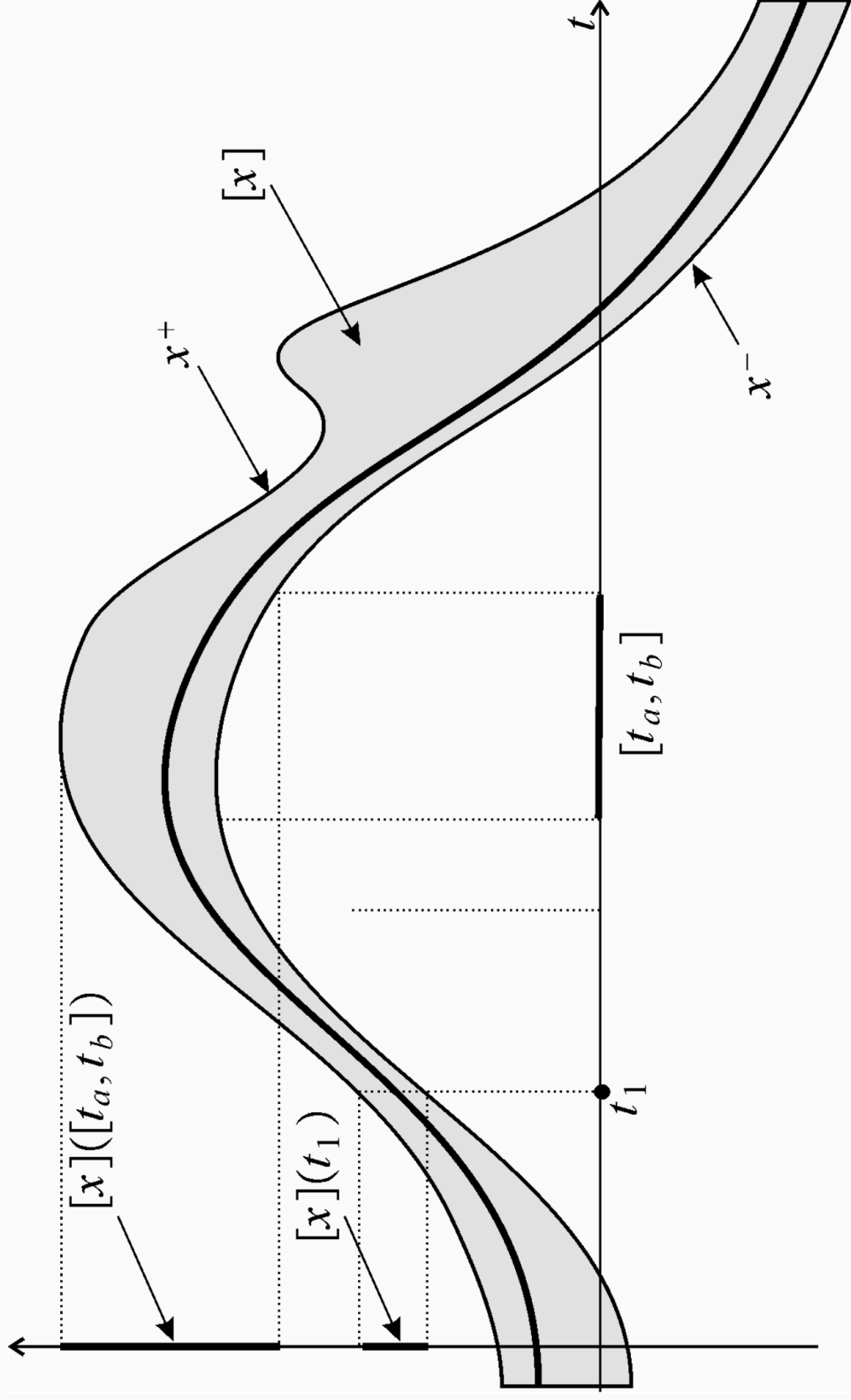
Tubes

The tube $[\mathbf{x}]$ is an interval $[\mathbf{x}^-, \mathbf{x}^+]$ of $\mathcal{F}^n$ (trajectories)

$$\mathbf{x}^-(t) \leq \mathbf{x}(t) \leq \mathbf{x}^+(t)$$

The set of all tubes is $\mathbb{I}\mathcal{F}^n$

Representation of a Tube



Ensemblist vision of a random signal

Tube arithmetic

We can extend some classical operations we have on functions of $\mathcal{F}^n$, such as sums, multiplication, image by a function, to tubes.

It suffices to use the classical notion of interval arithmetic and inclusion functions for all t .

An arithmetic on tubes is thus a direct extension of interval arithmetic.

Operations

Example :

$$[x] + [y] = [x^- + y^-, x^+ + y^+]$$

$$\int_{t_1}^{t_2} [\mathbf{x}] (\tau) d\tau = \left[\int_{t_1}^{t_2} \mathbf{x}^- (\tau) d\tau, \int_{t_1}^{t_2} \mathbf{x}^+ (\tau) d\tau \right]$$

Tube Contractor

Constraint

$$\mathcal{L}(x).$$

Associated
contractor

$$[x] \xrightarrow{C_{\mathcal{L}}} [y]$$

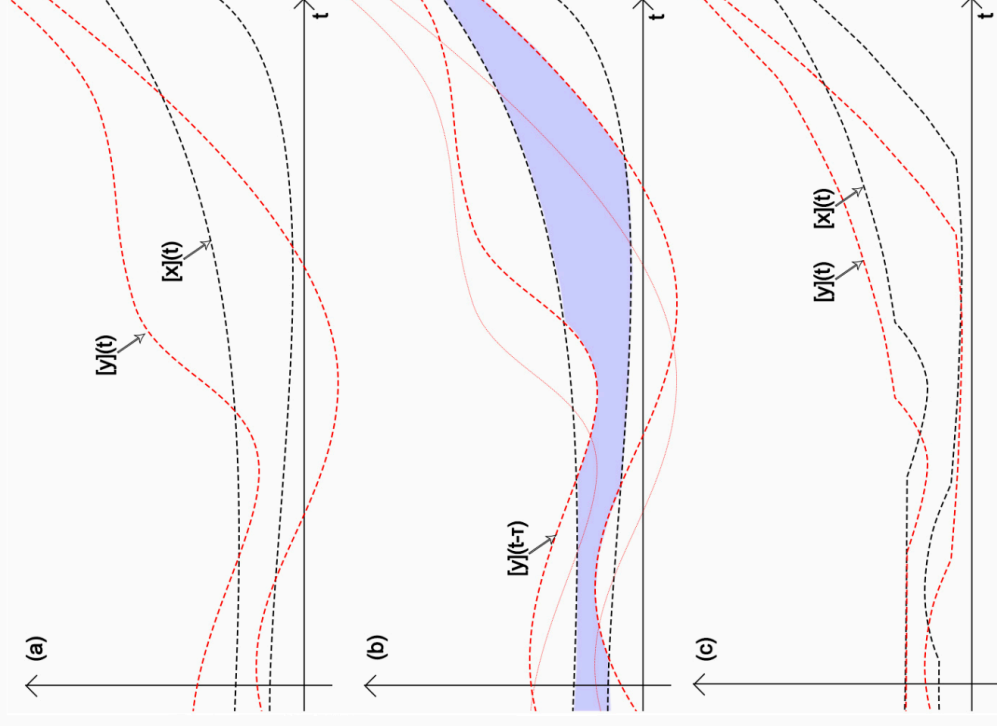
Contracting
Completeness

$$\left(\begin{array}{c} \forall t, [x](t) \subset [y](t) \\ \mathcal{L}(x) \\ x \in [x] \end{array} \right) \Rightarrow x \in [y]$$

Minimal contractor

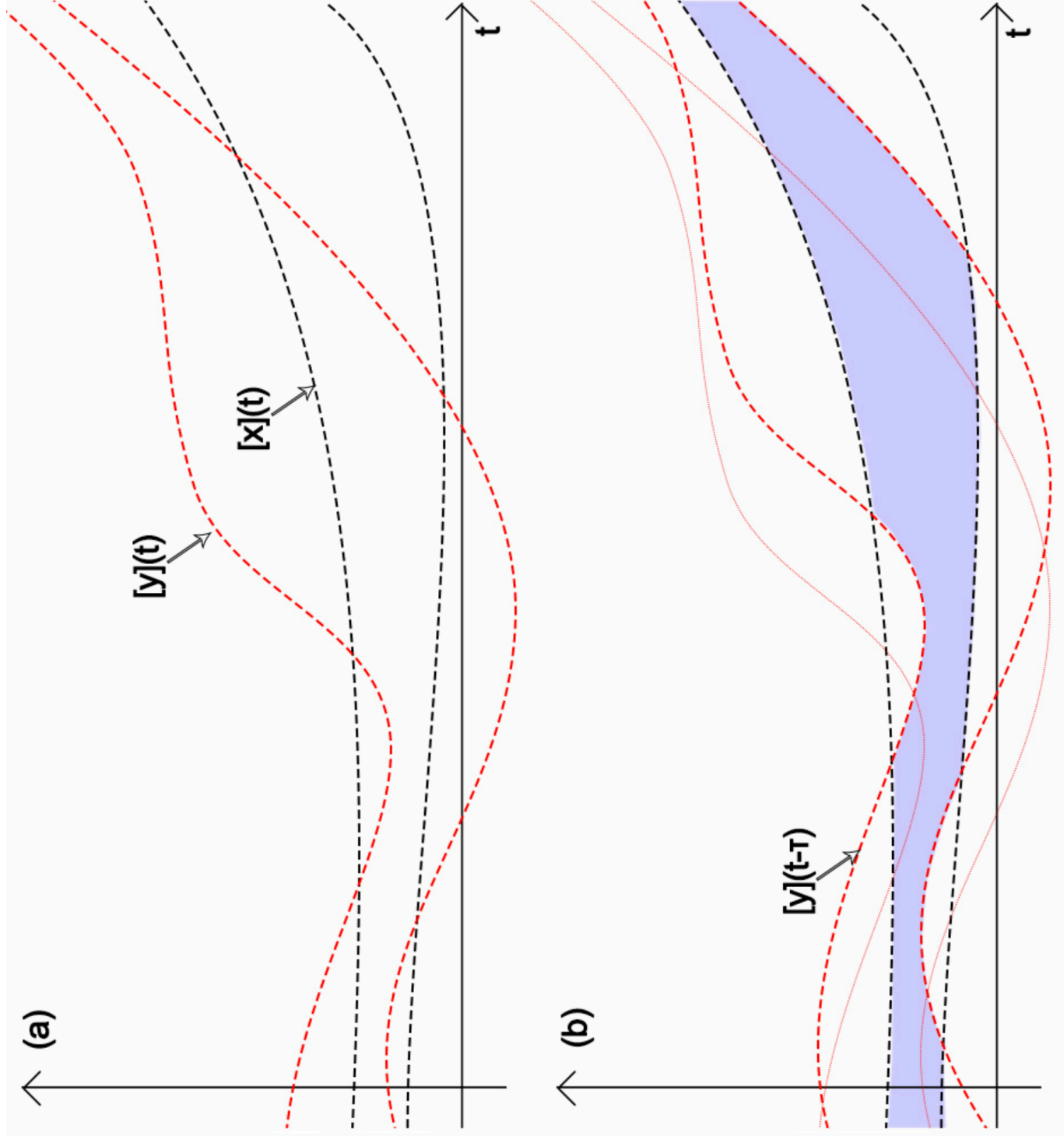
The minimal contractor associated with a delayed signal $x(t) = y(t - T)$ is :

$$C_{delay} \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \left(\begin{array}{l} [x^-(t) \wedge y^-(t - T), x^+(t) \vee y^+(t - T)] \\ [x^-(t + T) \wedge y^-(t), x^+(t + T) \vee y^+(t)] \end{array} \right)$$

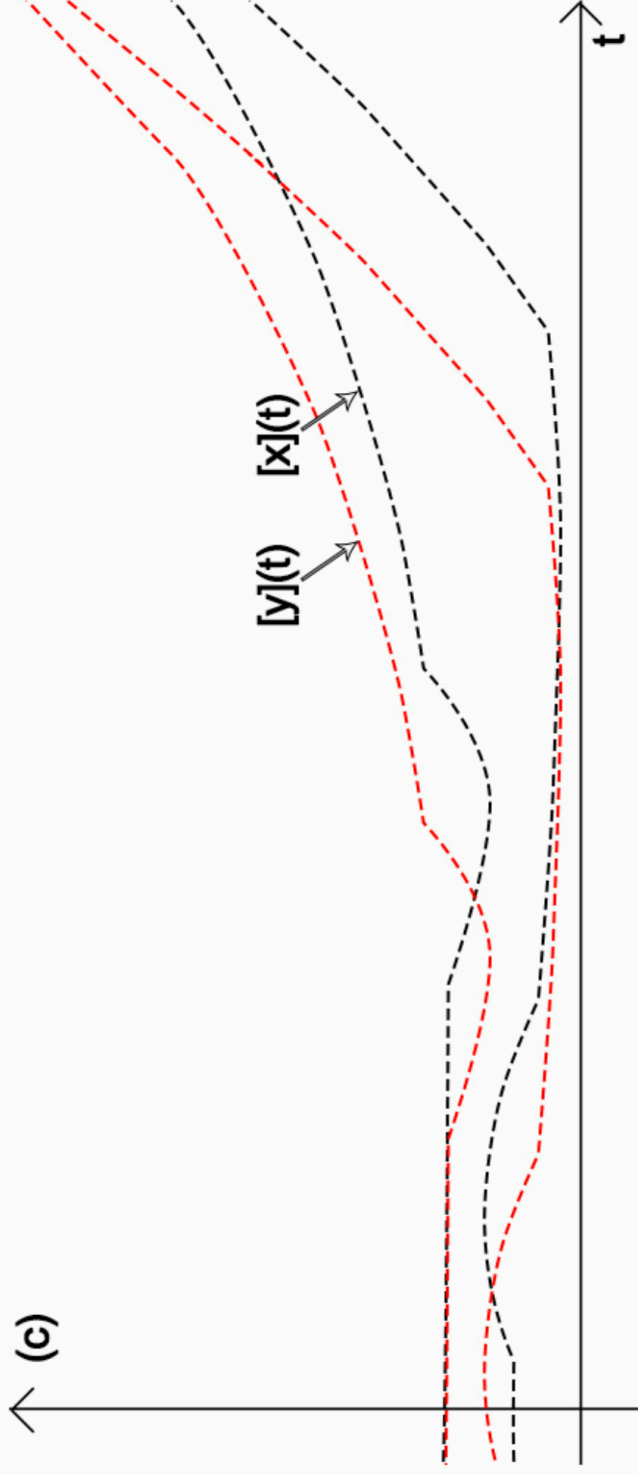
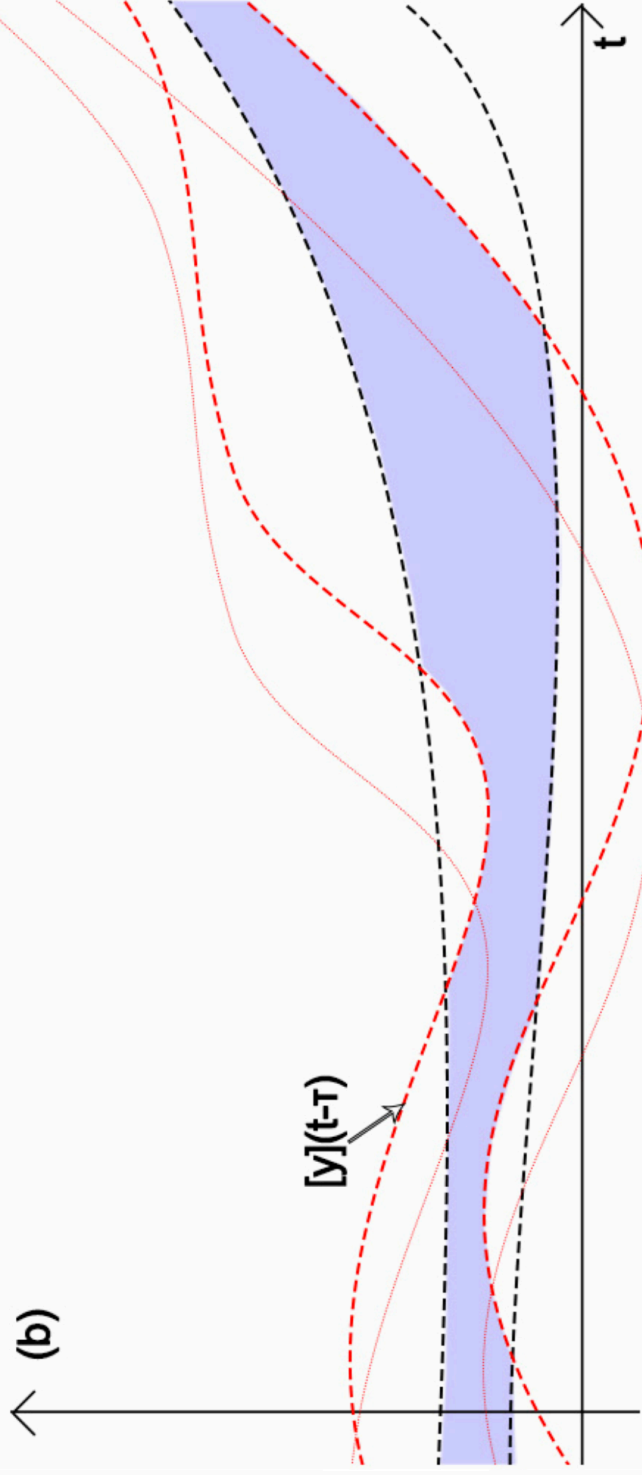


or d

$$\begin{pmatrix} t - T \\ y^+(t) \end{pmatrix}$$



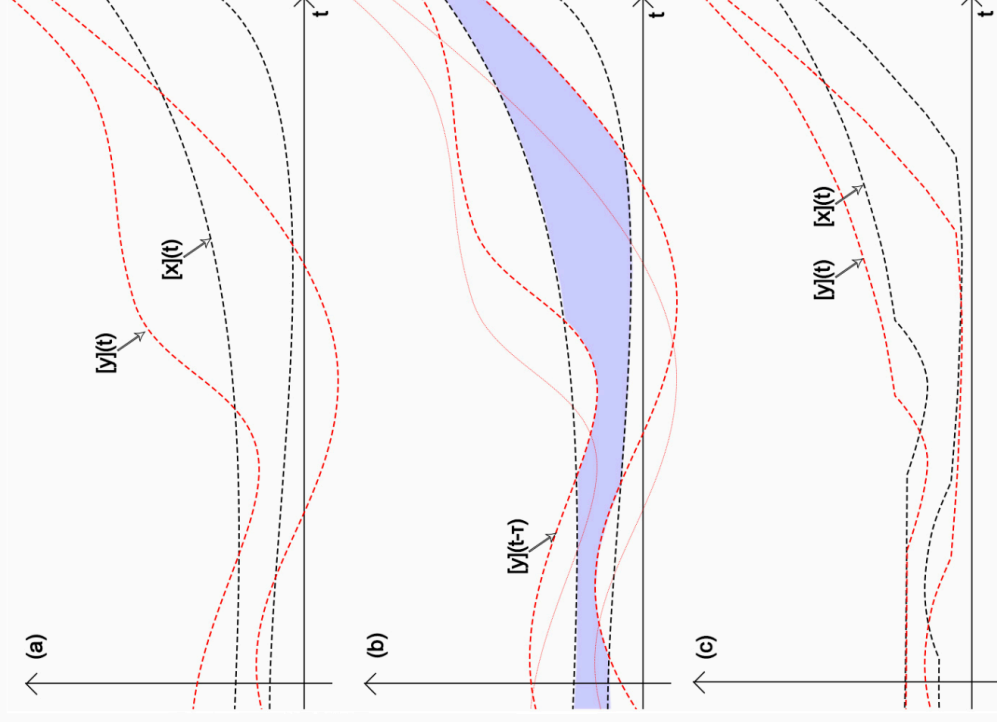
$$\begin{pmatrix} y^+(t) \\ y^-(t-T) \end{pmatrix}$$



Minimal contractor

The minimal contractor associated with a delayed signal $x(t) = y(t - T)$ is :

$$C_{delay} \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \left(\begin{array}{c} [x^-(t) \wedge y^-(t - T), x^+(t) \vee y^+(t - T)] \\ [x^-(t + T) \wedge y^-(t), x^+(t + T) \vee y^+(t)] \end{array} \right)$$



[illegible][illegible]
$$\begin{aligned} \text{For all } t: \\ [\mathbf{x}_u](t) &= [\mathbf{x}_u](t) \int_0^t ([\mathbf{f}](t), [\mathbf{x}_u(t)]) + [\mathbf{u}_x(t)] dr \\ [\mathbf{h}_u](t) &= [\mathbf{h}_u](t) \int_0^t (1 + [\mathbf{n}_u](t)) dr \end{aligned}$$

$$\begin{aligned} \text{For all } t: \\ [\underline{x}_u](t) &= [\underline{x}_u](t) \int_0^t ([\mathbf{F}]([\underline{u}_s(t)], [\underline{x}_s(t)]) + [\underline{n}_x(t)]) dt \\ [\underline{h}_u](t) &= [\underline{h}_u](t) \int_0^t (1 + [\underline{n}_h](t)) dt \end{aligned}$$

$$[h_s](t) - [h_t](t) \cap \int_0^t (1 + [n_h](t)) d\tau$$

[illegible]

$$| \Psi \rangle = | \Psi \rangle_{\text{in}} \otimes | \Psi \rangle_{\text{out}}$$

[illegible]

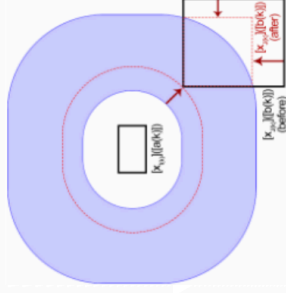
[illegible]

$$\begin{aligned} \frac{\partial \ln \mathcal{L}}{\partial \theta} &= \frac{1}{p(\mathbf{y}|\mathbf{x};\theta)} \frac{\partial p(\mathbf{y}|\mathbf{x};\theta)}{\partial \theta} \\ \frac{\partial \ln \mathcal{L}}{\partial \theta} &= \frac{1}{\int p(\mathbf{y}|\mathbf{x};\theta) d\mathbf{y}} \int \frac{\partial p(\mathbf{y}|\mathbf{x};\theta)}{\partial \theta} d\mathbf{y} = \int \frac{\partial p(\mathbf{y}|\mathbf{x};\theta)}{\partial \theta} p(\mathbf{y}|\mathbf{x};\theta) d\mathbf{y} \end{aligned}$$

$$\begin{aligned} \mathcal{H}^1(\mathbb{R}^n) &= \mathcal{H}^1(\mathbb{R}^n) \cap \mathcal{H}^1(\mathbb{R}^n) \\ &= \mathcal{H}^1(\mathbb{R}^n) \cap \mathcal{H}^1(\mathbb{R}^n) \end{aligned}$$

$$\hat{\mu}^{\pm}(k, \ell) = \{\mu_1^{\pm}(k, \ell), \mu_2^{\pm}(k, \ell), \mu_3^{\pm}(k, \ell)\} \in \mathbb{R}^3(\mathbb{C})$$

10



Contractors associated with the constraints

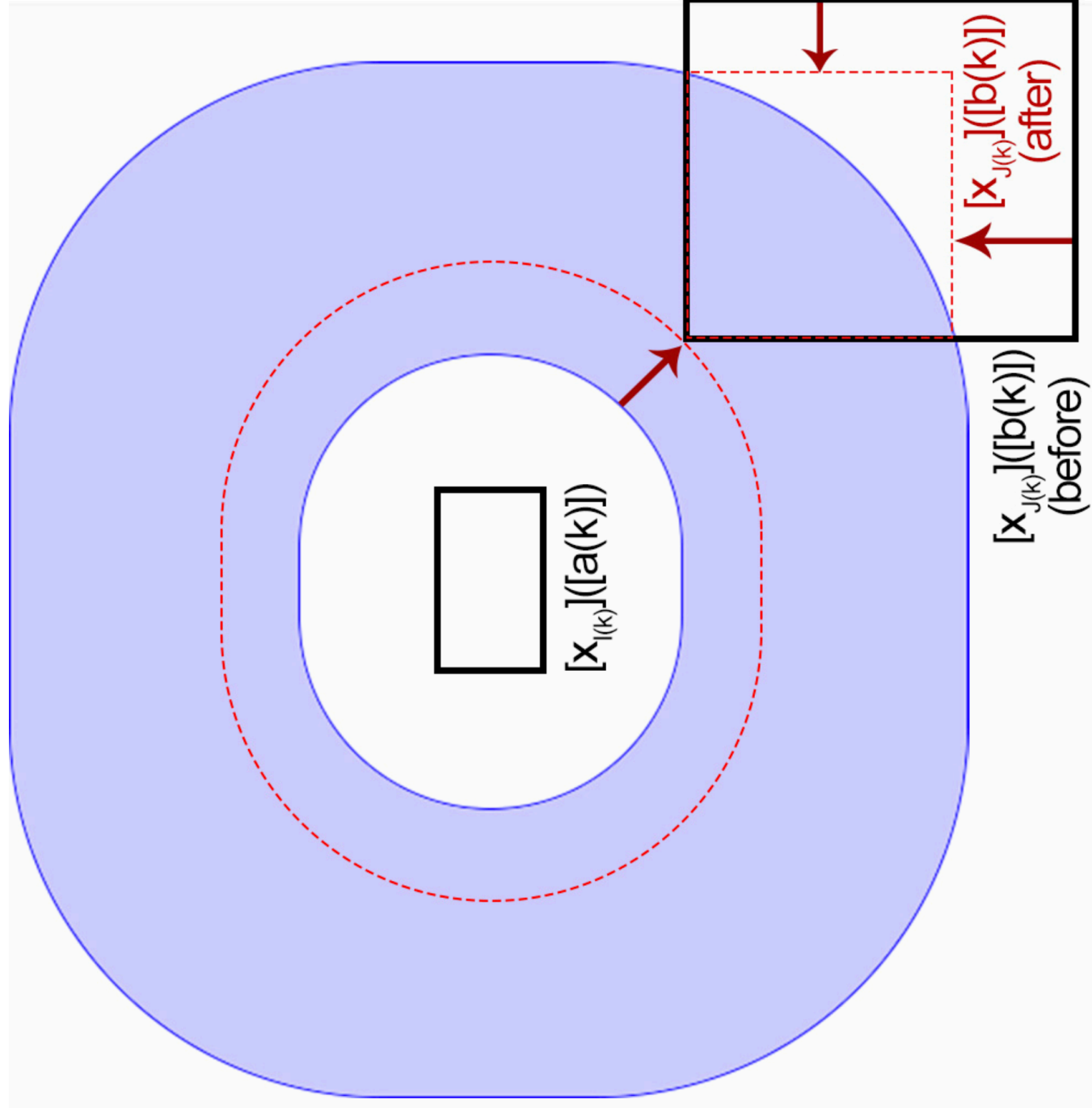
For all t :

$$[\mathbf{x}_i](t) = [\mathbf{x}_i](t) \cap \int_0^t ([\mathbf{f}]([\mathbf{u}_i(t)], [\mathbf{x}_i(t)]) + [\mathbf{n}_x(t)]) d\tau$$

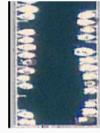
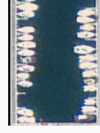
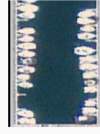
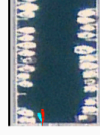
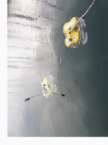
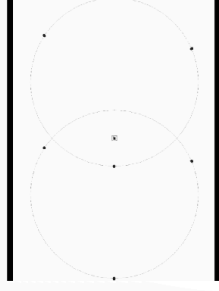
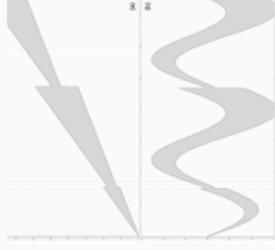
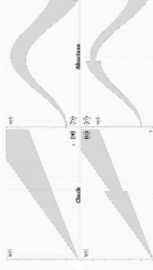
$$[h_i](t) = [h_i](t) \cap \int_0^t (1 + [n_h](t)) d\tau$$

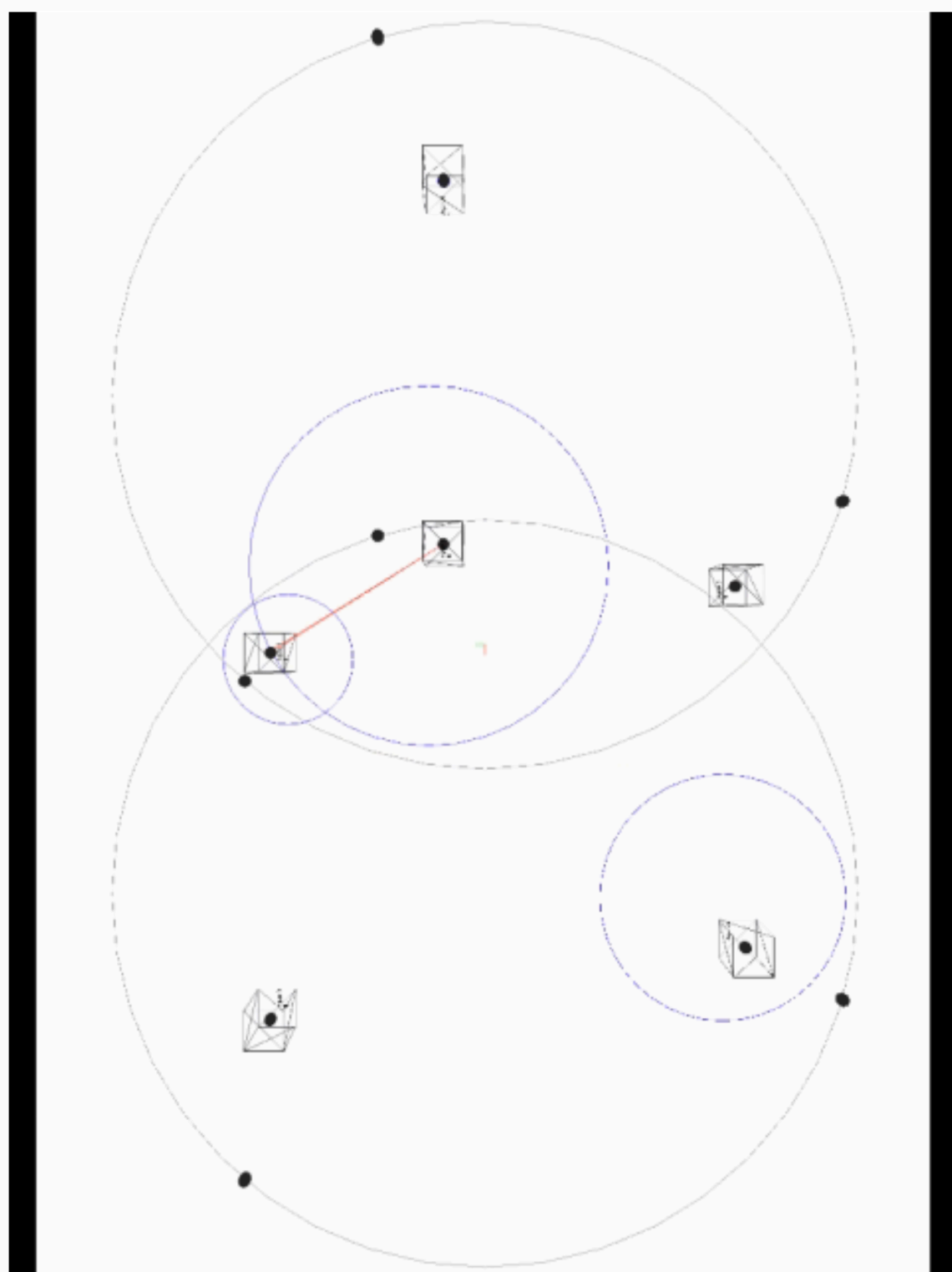
For each k :

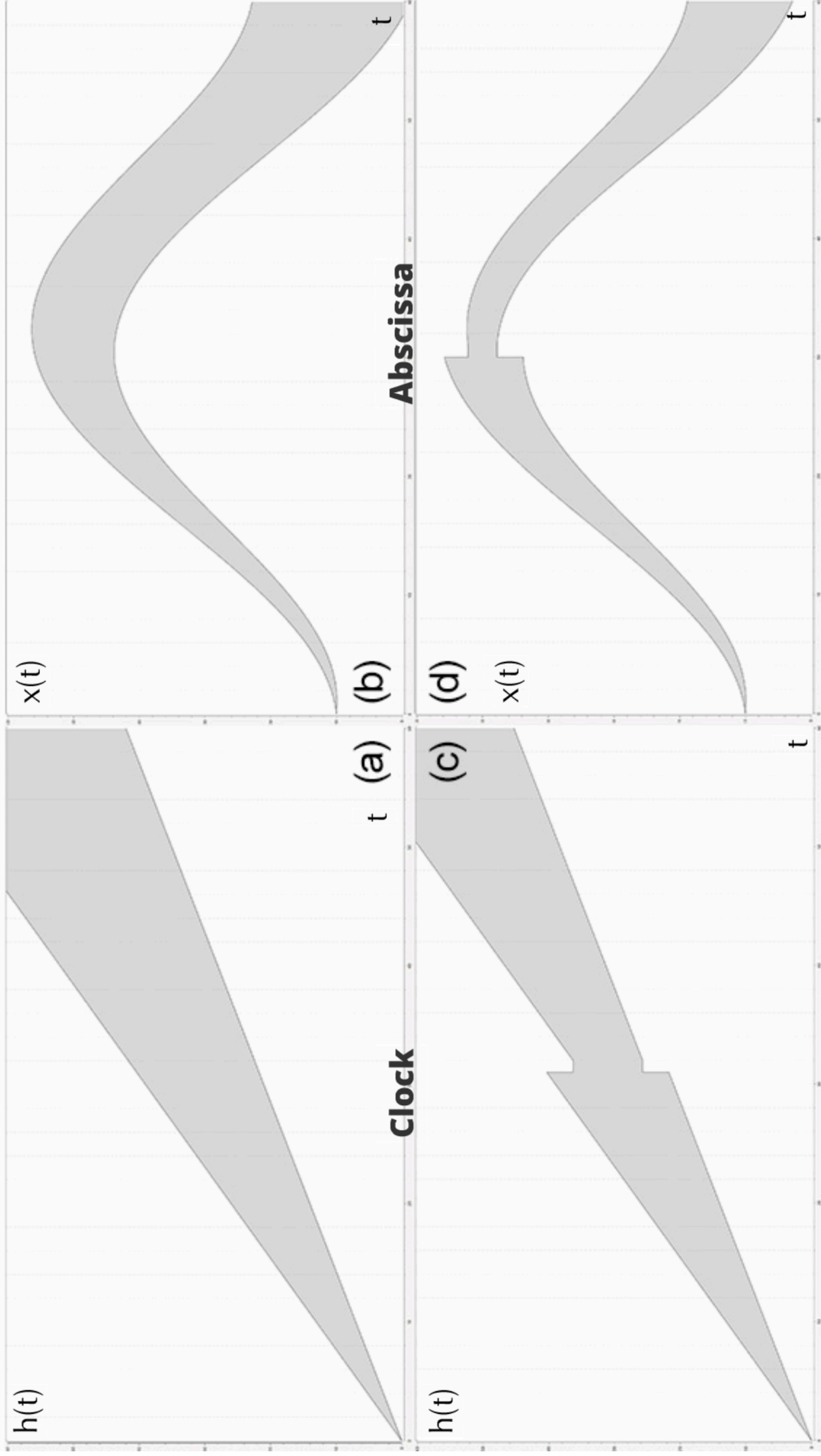
$$\begin{aligned} [a(k)] &= [a(k)] \cap [h_{i(k)}^{-1}([\tilde{a}(k)])] \\ [b(k)] &= [b(k)] \cap [h_{j(k)}^{-1}([\tilde{b}(k)])] \\ [x_{i(k)}]([b(k)]) &= [x_{i(k)}]([a(k)]) \cap (-\sqrt{c^2 * ([b(k)] - [a(k)]^2 - [y_{i(k)}]([a(k)] - [y_{j(k)}]([b(k)])) + [x_{j(k)}]([a(k)]))} \\ [y_{i(k)}]([b(k)]) &= [y_{i(k)}]([a(k)]) \cap (-\sqrt{c^2 * ([b(k)] - [a(k)]^2 - ([x_{i(k)}]([a(k)] - [x_{j(k)}]([b(k)])) + [y_{j(k)}]([a(k)]))} \\ [a(k)] &= [a(k)] \cap (-\sqrt{\frac{[x_{i(k)}]([a(k)] - [x_{j(k)}]([b(k)]))^2 + ([y_{i(k)}]([a(k)] - [y_{j(k)}]([b(k)]))^2}{c^2}} + [b(k)]) \\ [b(k)] &= [b(k)] \cap (\sqrt{\frac{[x_{i(k)}]([a(k)] - [x_{j(k)}]([b(k)]))^2 + ([y_{i(k)}]([a(k)] - [y_{j(k)}]([b(k)]))^2}{c^2}} + [a(k)]) \\ [\tilde{a}(k)] &= [\tilde{a}(k)] \cap [h_{i(k)}]([a(k)]) \\ [\tilde{b}(k)] &= [\tilde{b}(k)] \cap [h_{j(k)}]([b(k)]) \end{aligned}$$

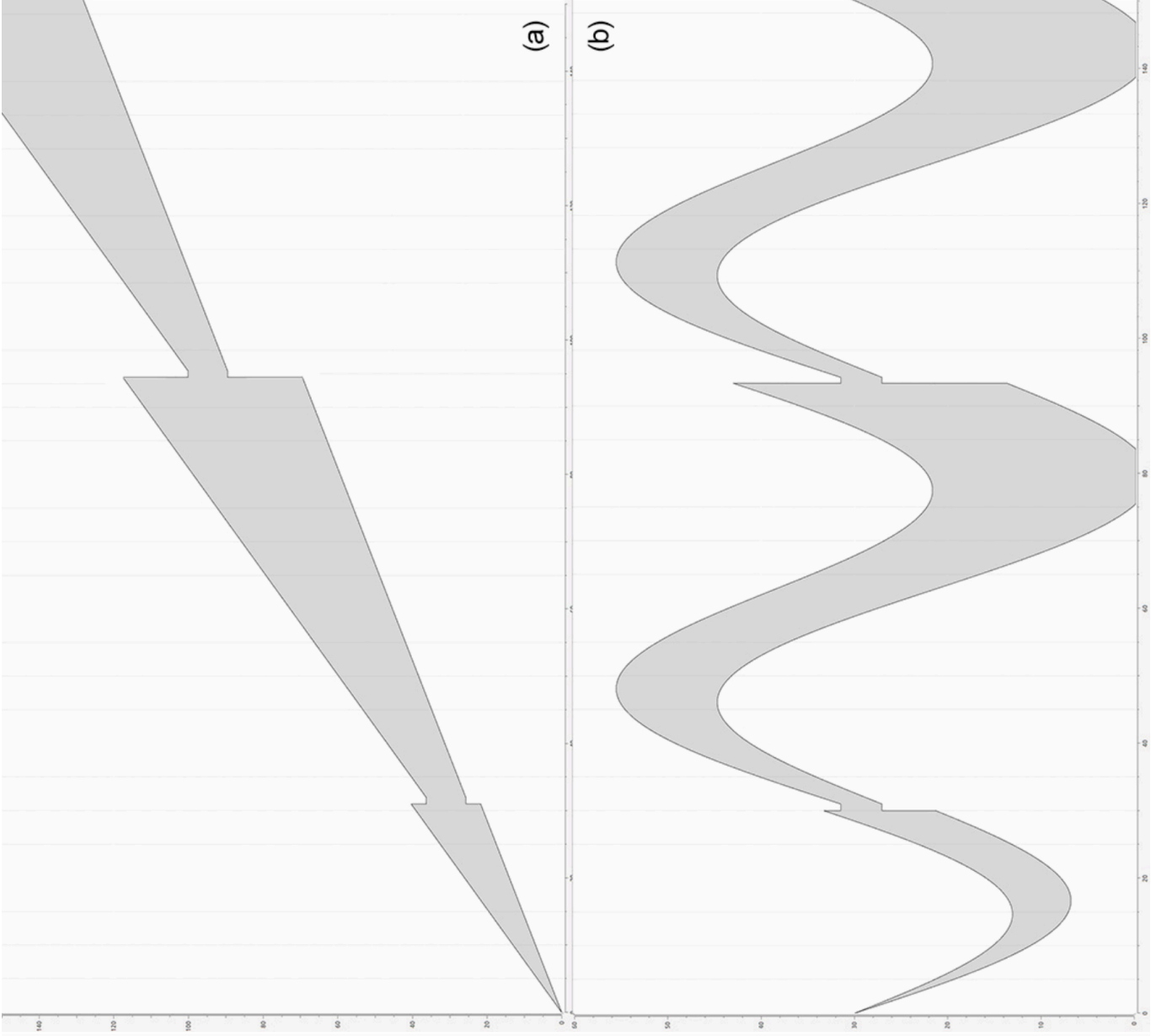


V - TESTCASES







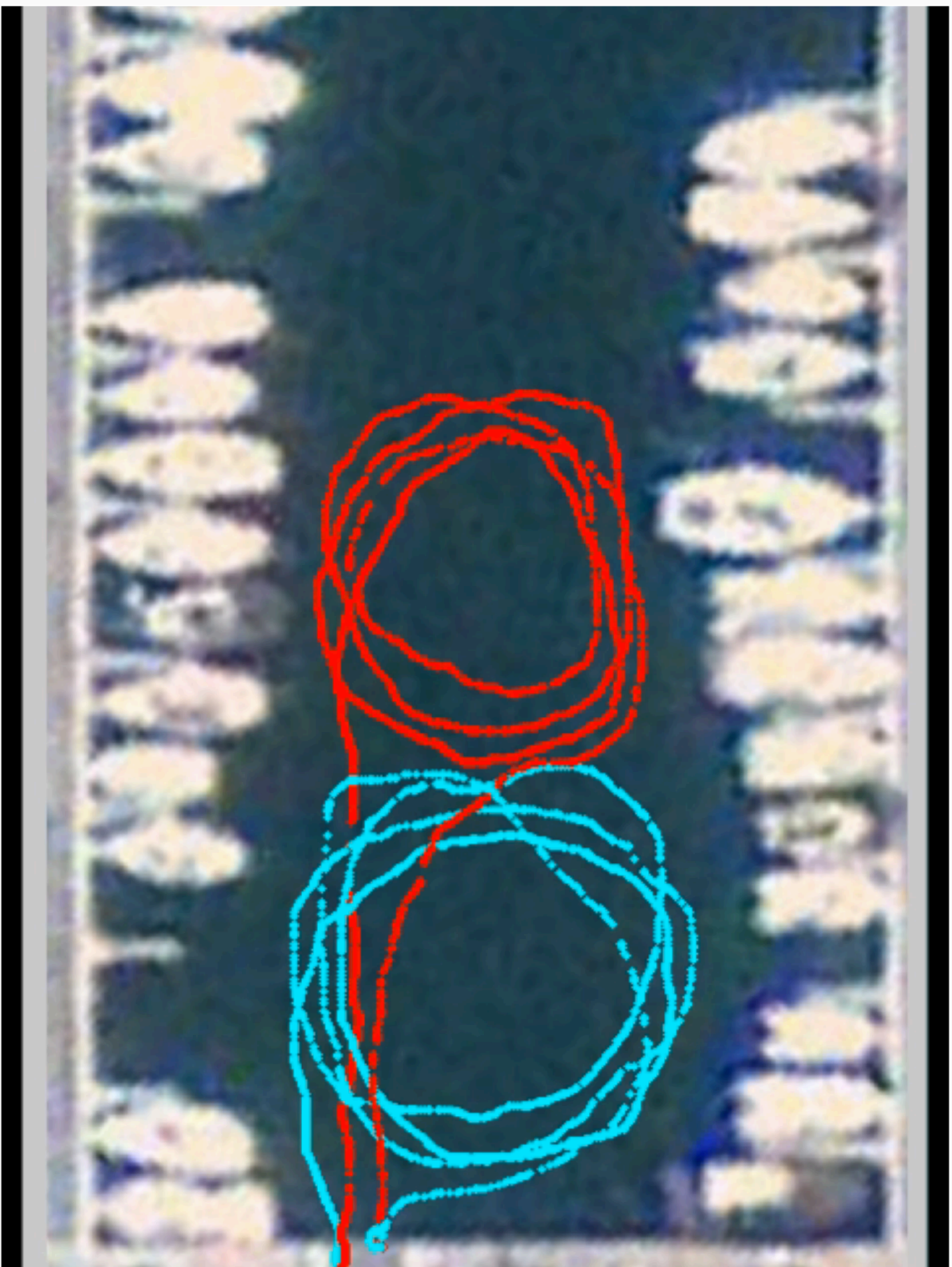


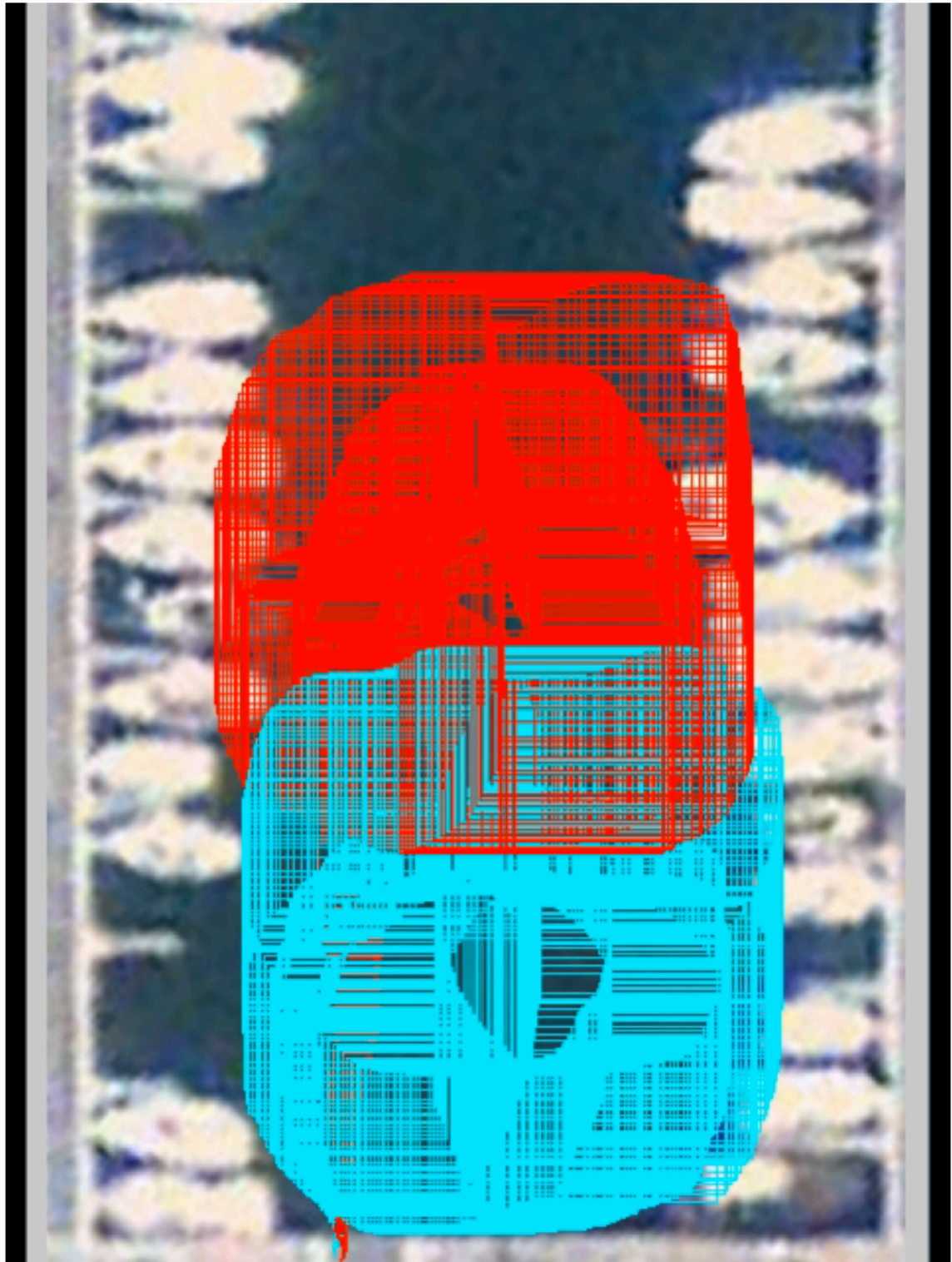


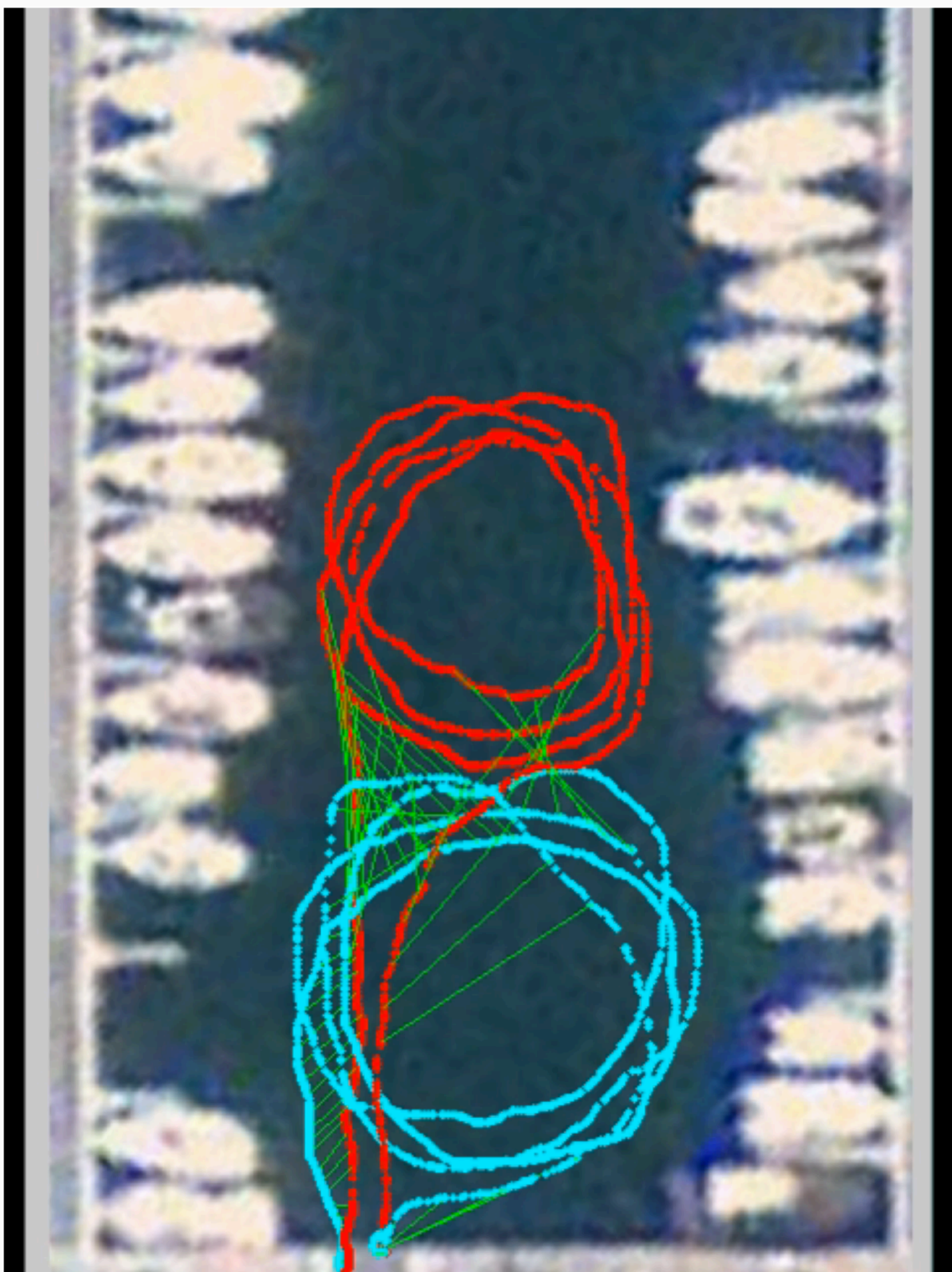


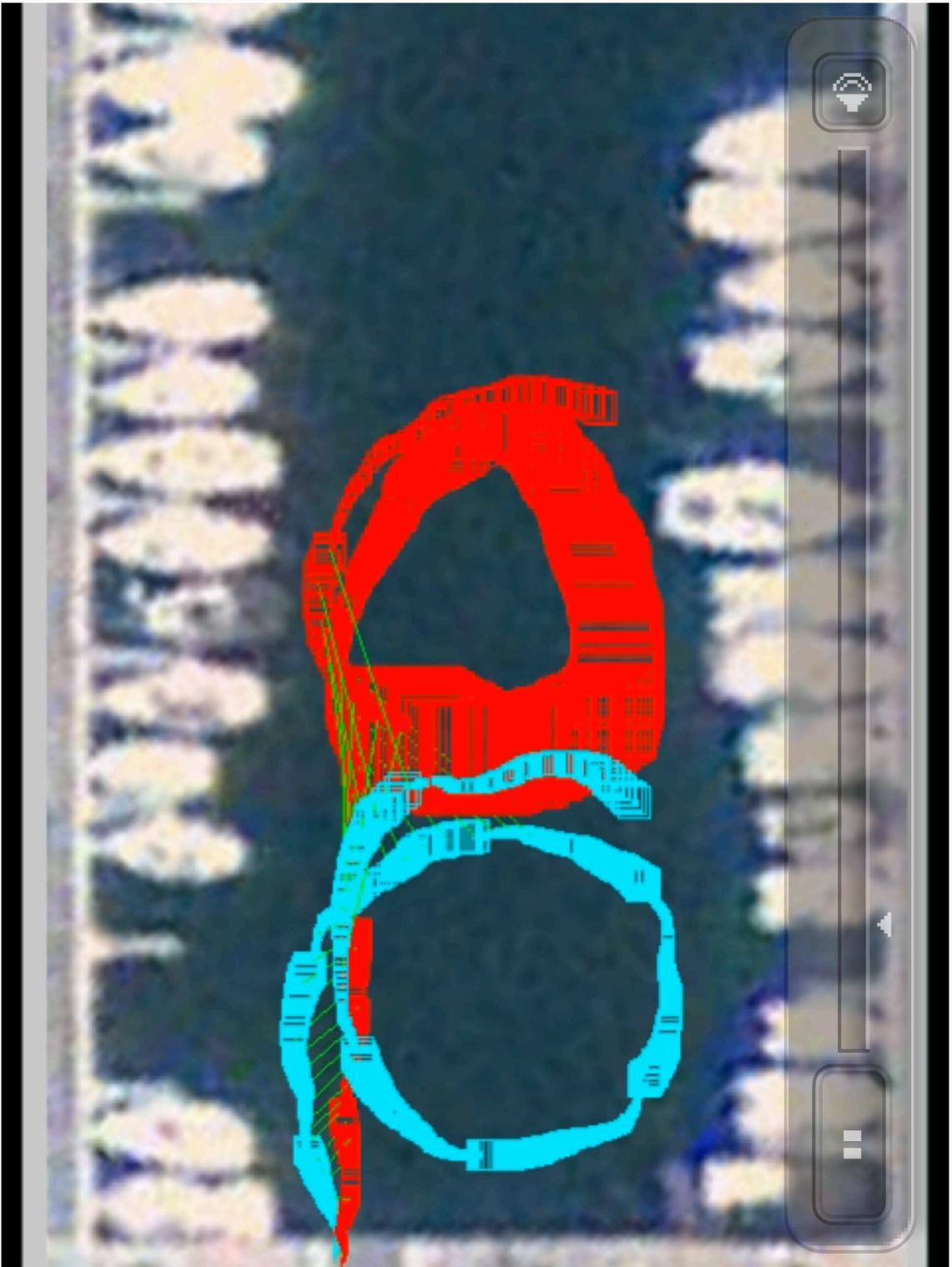












VI - CONCLUSION

Conclusion

- Proposed a solution to solve an inter-temporal constraints satisfaction problem with uncertain time.
- Future :
 - More real experiments have to be made.
 - Make it robust to outliers
 - Absolute time > Matrix of relative times

Article & References



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CODEN PHIL

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Article & References



Cooperative localization of underwater robots with unsynchronized clocks

Ayméric Belhencourt, Luc Jaulin ENSTA-Bretagne, IARSTIC, 2 Rue François Verny, 29200 Brest
 {aymeric.belhencourt,luc.jaulin}@ensta-bretagne.fr

Abstract—This paper proposes an interval based approach for localizing ununderwriter robots in a moving swarm using sonar measurements. The localization problem is solved by computing the displacement of the robot while the same way as moving is supervised to the precision of the localization. Therefore we cannot assume that we measure a true distance between robots at a constant time, but links of the robots at different times. Moreover we'll consider the latency times of the wave are uncertain. Therefore transmitting and receiving times of the wave are uncertain. Therefore we will use an interval arithmetic to propagate the uncertainty in our uncertain times. The principle will be to cast the linear and nonlinear state equations into a constraint satisfaction problem where each interval propagation is particularly serial. The

Index Terms—Tubes, Interval analysis, Localization, AUV, Underwater robots, Swarm, Constraint Propagation, Constraint satisfaction, Contractors.

1. INTRODUCTION

THIS paper proposes a new approach to localize Autonomous Underwater Vehicles (AUV) in a moving environment. Localizing AUV can be particularly difficult underwater because there is no installed infrastructure, and AUV can only access the GPS on the surface. The only available tools for underwater are dead reckoning and acoustical localization system. To solve this problem, we usually consider the robots close to each other and moving slowly enough so that the displacement of the sound wave can be considered negligible compared to the precision of the localization. The distance between the robots is measured and the triangular inequalities are solved to localize the robots [1] [2].

In our approach, we consider fast and well spaced AUVs so that the displacement of the sonar wave can no longer be considered negligible. The measured distance is therefore between the positions of the robots at different times. Fig. 1 illustrates the principle. Moreover we consider the clocks of the robots to be unsynchronized so that the emitting and receiving times are uncertain. We consider our system to be decentralized. Each robot will localize itself using only the data received from other robots.

The first one is the probabilistic approach. If the system is linear, the problem can be solved using for instance the Extended Kalman filter (EKF) [3], and if the problem is nonlinear, a Sequential Monte Carlo (SMC) such as a particle filter can be considered. The second approach is ensemble-based. When the problem is linear, Ellipsoids and Potatoes [4] Data received from other robots.

can be proved more efficient than intervals. However, for non linear systems, only intervals and sub-provings have been proved efficient [5]. The principle is to wrap the solution in an interval, then successively contract this interval around the solution. For this, we use contractors that are associated to the equations of our system. Interval analysis has been showed to be efficient in many localization applications [6] [7].

The originality of this paper is to deal with intertemporal constraints on uncertain times. We consider the position and check of the robots as intervals of trajectories (functions of $t \in \mathbb{R}^n$) and cast the equations as a constraint satisfaction

This paper is organized as follows. Section II proposes a simple and new formalization for cooperative localization with intertemporal constraints on uncertain timesteps. Section III presents the basic notions of constraint propagation. It also introduces intervals of trajectories (called tubes) that are used to solve our problem. The resolution method is then presented in Section IV, and section V provides an illustration of the approach on simple examples. Section VI concludes the paper.

II. PROBLEM STATEMENT

We shall describe our cooperative localization problem. For a robot i in our swarm, consider the state equations of the robot as follows:

$$\begin{aligned} \hat{s}_i &= f(x_i, u_i) && \text{(evolution equation)} \\ y_i &= g(x_i) && \text{(observation equation)} \end{aligned} \quad (1)$$

where $\mathbf{x}_i$ is the state vector of the robot i , $\mathbf{u}_i$ its inputs $\mathbf{u}$ commands, $\mathbf{y}_i$ its outputs or measures, and $\mathbf{f}$ and $\mathbf{g}$ respectively the evolution and observation functions. Notice that the observation function depends on the state of the robot at its current time. In our approach, we consider a sonar communication (or ping) between the robots, which will lead to intertemporal constraints. We consider that each ping encloses the estimated position box of the emitting robot, along with the interval around the emitting time in the robot's clock.

Formalization. An *interleaving relation* (or *ping for short*) is a binary relation $\bowtie$ between two sets of events. It corresponds to a tuple $p = \langle \alpha, \beta, \gamma, \delta \rangle$ where α corresponds to the emission time, $\beta \in \mathbb{R}$ to the reception time, $\gamma \in \{1, \dots, m\}$ the emitting robot, and $\delta \in \{1, \dots, m\}$ the receiver. Due to the causality, we have $\beta \geq \alpha$, or equivalently $\alpha \leq \beta$. An element of the set $\mathcal{P}$ is denoted by p ; the i th element of the tuple p is denoted by $p[i]$. The i th emission and the j th reception are denoted by e_i and r_j , respectively, and by t the true time. We denote by $\tau = h_i(t)$ the clock reading, which for absolute time t matches the i th time function.

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Cooperative localization of underwater robots with unsynchronized clocks

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Abstract—This paper proposes an interval based approach for localizing underwater robots in a moving swarm using sonar communication. The originality of this paper is to consider that the displacement of the robot while the sonar wave is moving is superior to the precision of the localization. Therefore we cannot suppose that we measure a true distance between robots at the same time, but between robots at different times. Moreover we'll consider the clocks of the robots to be unsynchronized so the emitting and receiving times of the wave are uncertain. Therefore the object of this paper is to deal with intertemporal constraints on uncertain times. The principle will be to cast the linear and nonlinear state equations into a constraint satisfaction problem for which interval propagation is particularly powerful. The resulting propagation method will be illustrated in a simulation.

Index Terms—Tubes, Interval analysis, Localization, AUV, Underwater robots, Swarm, Constraint Propagation, Constraint satisfaction, Contractors.

I. INTRODUCTION

THIS paper proposes a new approach to localize Autonomous Underwater Vehicles (AUV) in a moving swarm. Localizing AUV can be particularly difficult underwater as there is no installed infrastructure, and AUV can only access the GPS on the surface. The only available tools underwater are dead reckoning and acoustical localization system. To solve this problem, we usually consider the robots close to each other and moving slowly enough so that the displacement of the sonar wave can be consider negligible compared to the precision of the localization. The distance between the robots is measured and the triangular inequalities are solved to localize the robots [1] [2].

In our approach, we consider fast and well spaced AUV so that the displacement of the sonar wave can no longer be considered negligible. The measured distance is therefore between the positions of the robots at different times. Fig.1 illustrates the principle. Moreover we consider the clocks of the robots to be unsynchronized so that the emitting and receiving times are uncertain. We consider our system to be decentralized. Each robot will localize itself using only the data received from other robots.

Two approaches can be considered to solve this problem. The first one is the probabilistic approach. If the system is linear, the problem can be solved using for instance the Extended Kalman Filter (EKF) [3], and if the problem is nonlinear, a Sequential Monte Carlo (SMC) such as a particle filter [2] can be considered. The second approach is ensemble. When the problem is linear, Ellipsoids and Polytopes [4]

can be proved more efficient than Intervals. However, for non-linear systems, only intervals and sub-pavings have been proved efficient [5]. The principle is to wrap the solution in an interval, then successively contract this interval around the solution. For this, we use contractors that are associated to the equations of our system. Interval analysis has been showed to be efficient in many localization applications [6][7].

The originality of this paper is to deal with intertemporal constraints on uncertain times. We consider the position and clock of the robots as intervals of trajectories (functions of $\mathbb{R} \rightarrow \mathbb{R}^n$) and cast the equations as a constraint satisfaction problem.

This paper is organized as follows. Section II proposes a simple and new formulation for cooperative localization with intertemporal constraints on uncertain times. Section III presents the basic notions of constraint propagation. It also introduces intervals of trajectories (called tubes) that is used to solve our problem. The resolution method is then presented in Section IV, and section V provides an illustration of the approach on simple examples. Section VI concludes the paper.

II. PROBLEM STATEMENT

We shall describe our cooperative localization problem. For a robot i in our swarm, consider the state equations of the robot as follows:

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) & (\text{evolution equation}) \\ \mathbf{y}_i &= \mathbf{g}(\mathbf{x}_i) & (\text{observation equation})\end{aligned}\quad (1)$$

where $\mathbf{x}_i$ is the state vector of the robot i , $\mathbf{u}_i$ its inputs or commands, $\mathbf{y}_i$ its outputs or measures and $\mathbf{f}$ and $\mathbf{g}$ respectively the evolution and observation functions. Notice that the observation function depends on the state of the robot at the current time. In our approach, we consider a sonar communication (or ping) between the robots, which will lead to intertemporal constraints. We consider that each ping encloses the estimated position box of the emitting robot, along with the interval around the emitting time in the robot's clock.

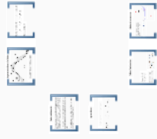
Formalization. An *intertemporal relation (or ping for short)* corresponds to a 4-uplet $\mathbf{p} = (a, b, i, j)$ where $a \in \mathbb{R}$ corresponds to the emission time, $b \in \mathbb{R}$ to the reception time, $i \in \{1, \dots, m\}$ the emitting robot, and $j \in \{1, \dots, m\}$ the receiver. Due to the causality, we have $b > a$, or equivalently (a, b) is an element of the t -plane [8]. Denote by $\mathbf{p}(k)$ the k th ping, and by t the true time. We denote by $\tau = h_i(t)$ the clock function which for an absolute time t matches the inner time

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II - PROBLEM STATEMENT



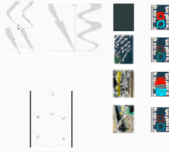
III - CONSTRAINT PROPAGATION



IV - COOPERATIVE LOCALIZATION AS A CONSTRAINT SATISFACTION PROBLEM



V - TESTCASES



VI - CONCLUSION



I - INTRODUCTION



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