Improvements on flow/guard intersection for nonlinear hybrid reachability*

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Computing the reachable set of hybrid dynamical systems in a reliable and verified way is an important step when addressing verification or synthesis tasks. This issue is still a challenge for uncertain nonlinear hybrid dynamical systems. Recently, [1] proposed a method for solving the flow/guard intersection problem. [1] derives an analytical expression for the boundaries of continuous flows using interval Taylor methods and techniques for controlling the wrapping effect, then the event detection and localization problem underlying flow/guard intersection is expressed as a constraint-satisfaction problem (CSP).

In this study, we improve the efficiency of the latter method by introducing several modifications [2]. First, we use QR-factorization based geometrical transformations in our own implementation of the ODE solver, in order to control wrapping effect as suggested by Lohner [3]. Then, we rely on the standard contractor HC4Revise as implemented in IBEX toolbox [4] to solve the CSP underlying the detection and localization problems. Finally, we optimize our bisection strategy by partitioning mainly time variable while using the contractor to prune and reduce the domain for the state variables.

The performance of the new method is illustrated on several examples involving typical hybrid systems, with continuous dimensions up to 6 : a non-holonomic vehicule model, a glycemic model, and a ball bouncing on a sinusoidal surface. Our evaluations show very promising performance.

References

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