CSPs and Quantified CSPs

G. Verger

LIRMM - CNRS/University of Montpellier II, France

June 19th 2008 / SWIM 2008

What is as CSP?

- Consist of:
  - A set of variables $X = \{x_1, \ldots, x_n\}$
  - variables' domains (finite sets of possible values)
  - A set of constraints $C = \{c_1, \ldots, c_k\}$

Example

$x \in \{1,2\}, y \in \{1,2\}, z \in \{1,2\}, x = y, x \neq z, y > z$

- Solutions of a CSP:
  - Assignment of value from its domain to every variable satisfying all the constraints.

Example

$(x,y,z) = (2,2,1)$ is a solution of above problem

Systematic Search Methods

- Characteristics
  - Exploring the search space
  - Complete and sound
  - Efficiency issues
- Two algorithms
  - Generate-and-Test (GT)
  - Backtracking(BT)

Generate-and-Test

- Probably the most general problem solving method
- Algorithm :
  1. Instantiate all variables
  2. Test its satisfaction
  3. If not a solution, go back to 1
- Drawbacks :
  - Blind generator
- Possible improvements
  - Guiding the generator ($\Rightarrow$ Local Search)
  - Checking consistency during generation ($\Rightarrow$ BT)

Backtracking

- Incrementally extends a partial solution towards a complete solution.
- Algorithm :
  1. Instantiate a variable
  2. Check the consistency
  3. If there is a variable not instantiated, go to 1
- Drawbacks :
  - Trashing, redundant work
  - Late detection of inconsistencies
- Possible improvements
  - Loop-back methods
  - Look-ahead methods

Consistency Techniques

- CSP as a graph :
  - Nodes are variables
  - Edges are constraints
- Aim : Removing inconsistent values from domains
- Different kinds of techniques :
  - Node Consistency (NC)
  - Arc Consistency (AC)
  - Path Consistency (PC)
  - k-consistency
- Not Complete

Constraint Propagation

- Systematic search = not efficient
- Consistency checking = not complete
- $\Rightarrow$ Combine search and check!
- Two methods :
  - Look-ahead : prevent conflicts (FC, AC, SAC...)
  - Look-back : intelligent restoring after conflict (Backjumping, Dynamic BT)
- Current Solvers use BT + AC.

Heuristics

- CSP is NP-Complete
- Choices are made during BT
- $\Rightarrow$ are there better ways to explore the tree search?
- Two kinds of heuristics :
  - Variable Ordering Heuristics = Fail-First (MinDom, MaxDeg, Dom/Deg, Dom/wDeg)
  - Value Ordering Heuristics = Succeed-First (MinConflict, Geelen's Promise)
- Current Solvers use Dom/wDeg + MinConflict

What is a QCSP

- Consists of:
  - A sequence of variables $X = \{x_1, \ldots, x_n\}$
  - variables' quantifiers (existential or universal)
  - variables' domains (finite sets of possible values)
  - A set of constraints $C = \{c_1, \ldots, c_k\}$

Example

$\exists x_1 \in \{1,2,3\} \forall y_1 \in \{1,2\} \exists x_2 \in \{1,2\}\ x_1 \neq x_2, y_1 = x_2$

Definition of a QCSP (2)

Example

$\exists x_1 \in \{1,2,3\} \forall y_1 \in \{1,2\} \exists x_2 \in \{1,2\}\ x_1 \neq x_2, y_1 = x_2$

- A Scenario is :
  - an assignment of value from its domain to every variable satisfying all the constraints.
  - the solution of corresponding CSP

Example

$(x_1, y_1, x_2) = (1,2,2)$ is a scenario of the previous QCSP

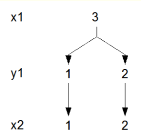
Definition of a QCSP (3)

Example

$$\exists x_1 \in \{1,2,3\} \forall y_1 \in \{1,2\} \exists x_2 \in \{1,2\}\; x_1 \neq x_2, y_1 = x_2$$

- A Strategy is :
  - the tree of winning scenarii corresponding to the QCSP

Strategy



Solving a QCSP

- Transforming into CSP : exponential space!
- what techniques can we bring from classical CSP?
  - Backtracking?
  - Constraint Checking?
  - Heuristics?

Backtracking

- Backtracking seems a reasonable start
- Algorithm :
  - Instantiate a variable
  - Check the consistency
  - If there is a variable not instantiated, go to 1
  - if all are instantiated, go to last unchecked universal variable

Backtracking

- Backtracking seems a reasonable start
- Algorithm :
  - Instantiate a variable (Order of variables!)
  - Check the consistency
  - If there is a variable not instantiated, go to 1
  - if all are instantiated, go to last unchecked universal variable

Backtracking

- Backtracking seems a reasonable start
- Algorithm :
  - Instantiate a variable (Order of variables!)
  - Check the consistency (Adapt Consistency techniques)
  - If there is a variable not instantiated, go to 1
  - if all are instantiated, go to last unchecked universal variable

Backtracking

- Backtracking seems a reasonable start
- Algorithm :
  - Instantiate a variable (Order of variables!)
  - Check the consistency (Adapt Consistency techniques)
  - If there is a variable not instantiated, go to 1
  - if all are instantiated, go to last unchecked universal variable

Consistency Techniques

- As in CSP for existential variables
- for universal variables : removing one value = Fail

The example of Quantified AC

$x \in \{1,2,3\}, y \in \{1,2\}$

- $\exists x \exists y, x \neq y$ AC detects nothing
- $\forall x \exists y, x \neq y$ AC detects nothing
- $\forall x \forall y, x \neq y$ AC detects inconsistency!
- $\exists x \forall y, x \neq y$ AC removes 1 and 2 from x's domain!

$\forall x \forall y$ and $\exists x \forall y$ can be checked during preprocessing!

Heuristics

- QCSP is PSPACE-Complete
- Choices are made during BT
- $\Rightarrow$ are there better ways to explore the tree search?
- Two kinds of heuristics :
  - Variable Ordering Heuristics
    - Fail-First (MinDom, MaxDeg, Dom/Deg, Dom/wDeg)
      Inside a block!
  - Value Ordering Heuristics
    - Succeed-First (MinConflict) $\Rightarrow$ Existential variables
    - Fail-First (MaxConflict) $\Rightarrow$ Universal variables

Techniques from SAT and QBF

Some techniques from boolean world.

- The Pure-Value Rule
  - From Pure-Literal in QBF
- Solution-Directed BackJumping
  - Cube learning in QBF

More...

- Constraint Satisfaction Problem:
  - Since 1974 (Montanari), 1977 (Mackworth)
  - Reference: Handbook of Constraint Programming (2006)
  - Solvers: ILOG Solver, JChoco, Minion...
  - Benchmarks: CSP-lib on the web
- Quantified CSP:
  - Since 2002 (Bordeaux, Monfroy)
  - No reference
  - Solvers: Qcsp-solve, QeCode, BlockSolve
  - No benchmarks!