Influence of uncertainties on ultrasonic localization systems

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Joint work with ETH Zurich

Problem: localizing a robot which has a sound-emitter (single ping)

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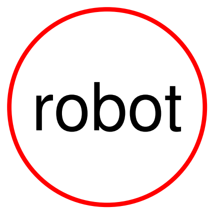

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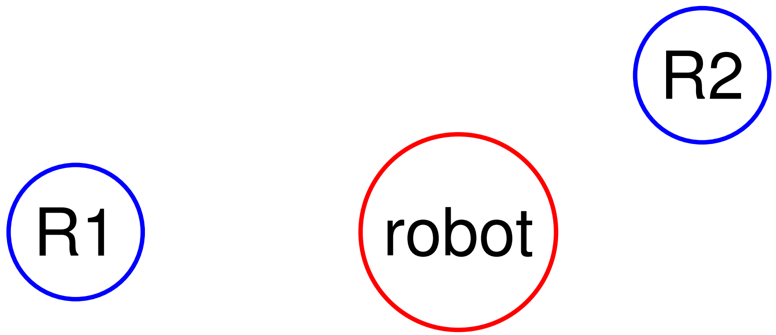

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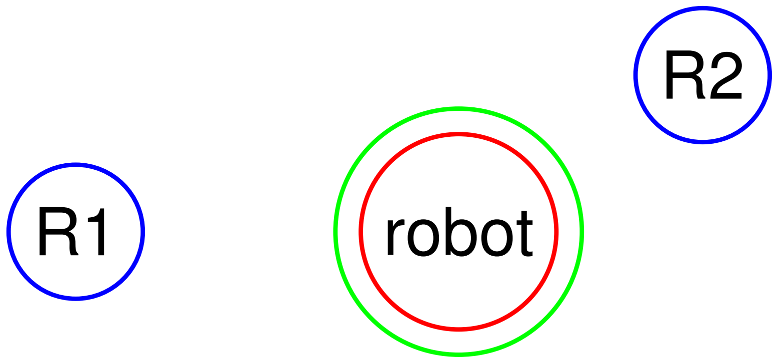

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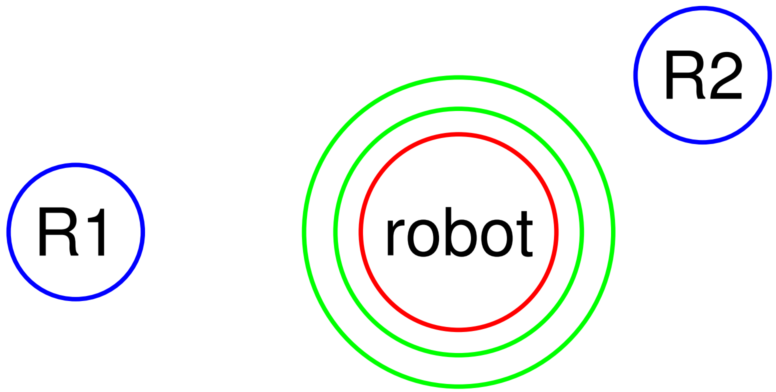

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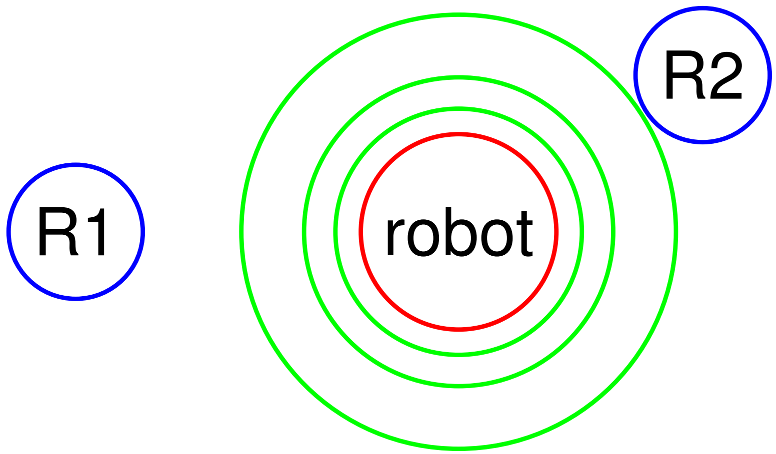

R2 receives the sound signal at time t_2

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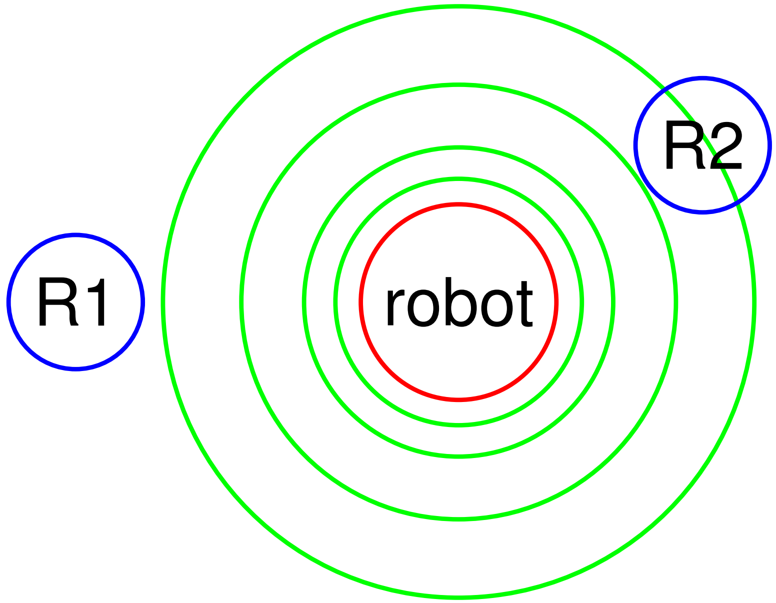

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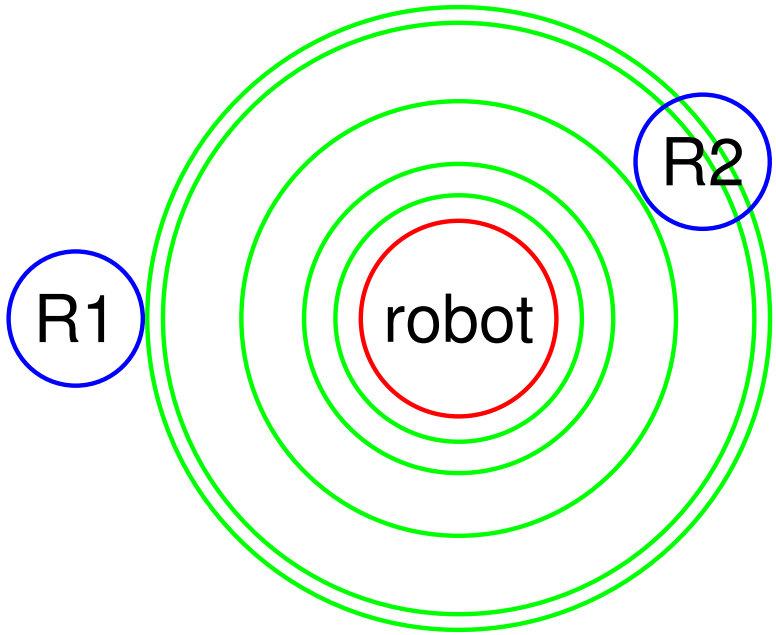

R1 receives the sound signal at time t_1

Time Difference Of Arrival (TDOA): $t_1 - t_2$

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- d_i: distance between robot and receiver R_i
- c: sound velocity

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TDOA is measured, robot location such that $d_1 - d_2$ has a fixed value

$\Rightarrow$ robot is on a hyperbola having R_1, R_2 as focal points

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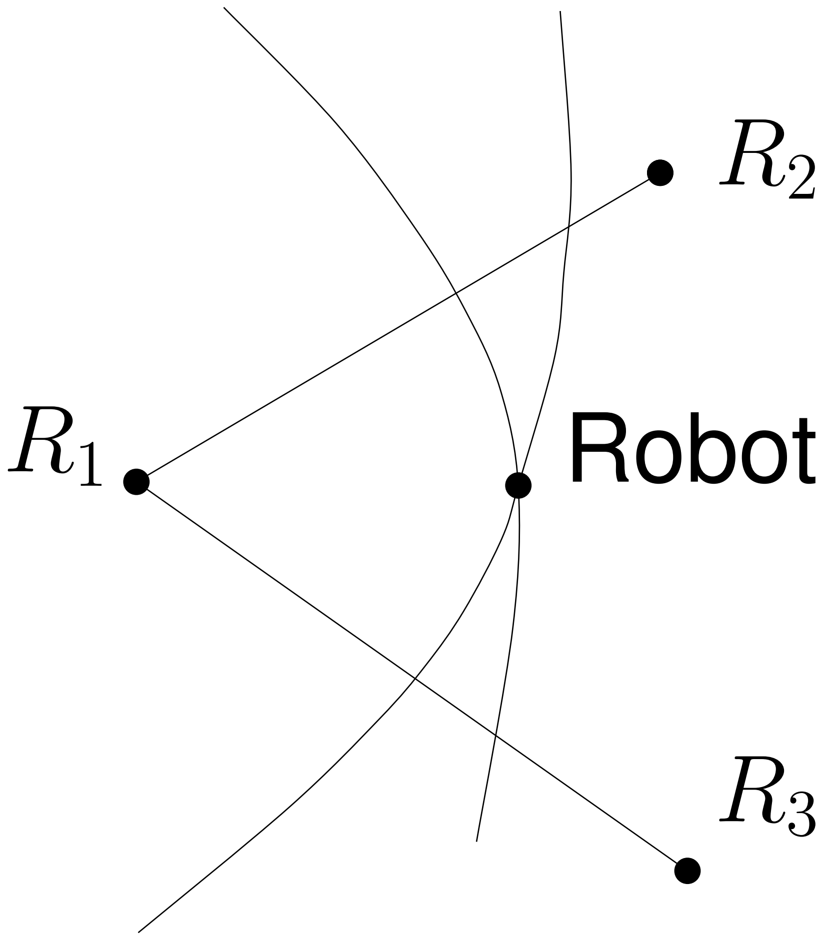
3 receivers: robot is at the intersection of 2 hyperbola

$\Downarrow$

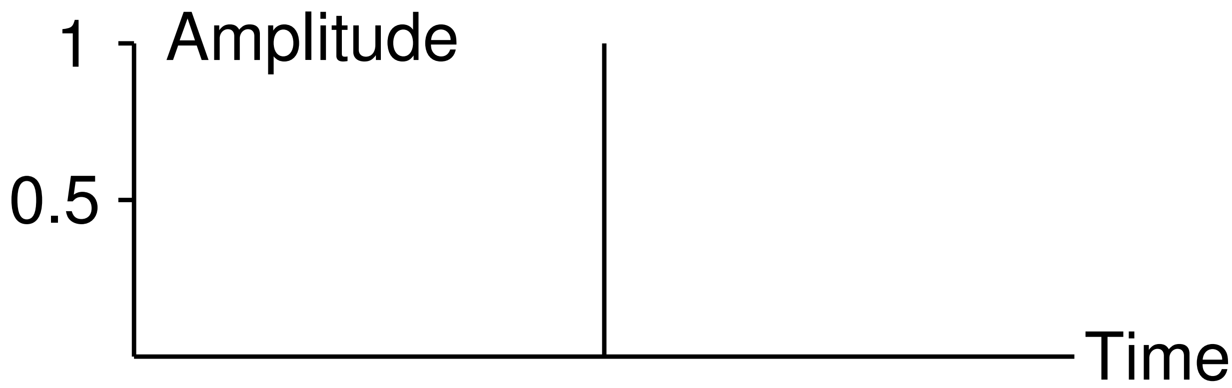
location is, in general, unique



The practice

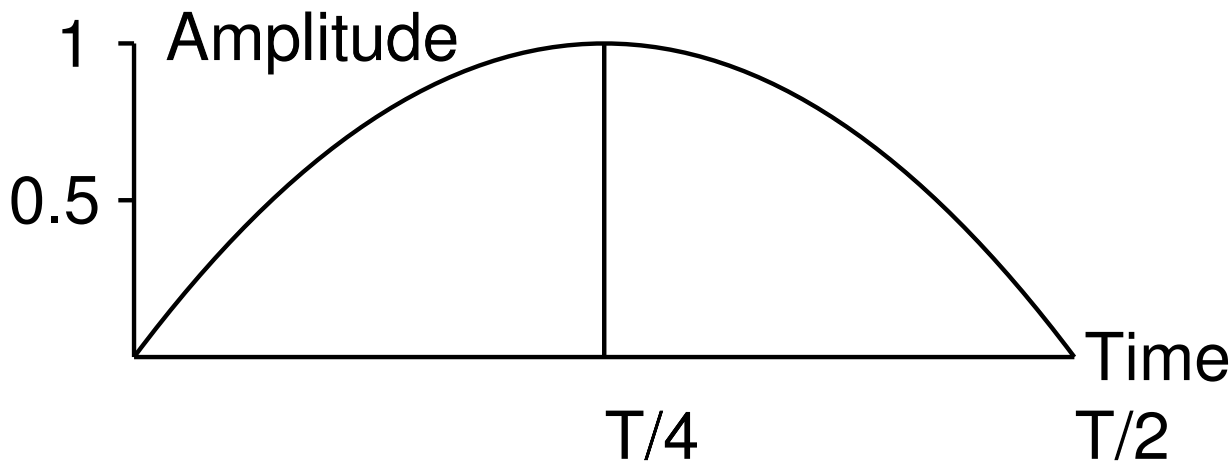
The practice

- the sound signal is not a ping



The practice

- the sound signal is a distribution with frequency f



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The probability distribution

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larger signal

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larger signal

- detection if signal is larger than a given threshold

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Computing the signal

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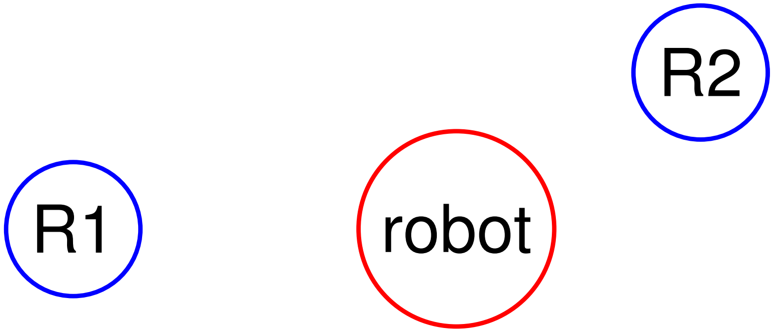

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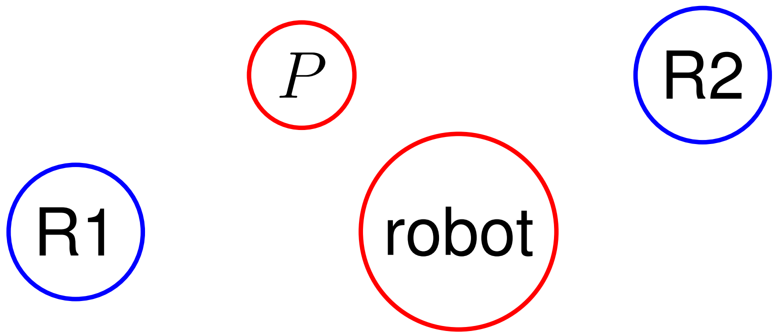

d'_j: distance between P and R_j

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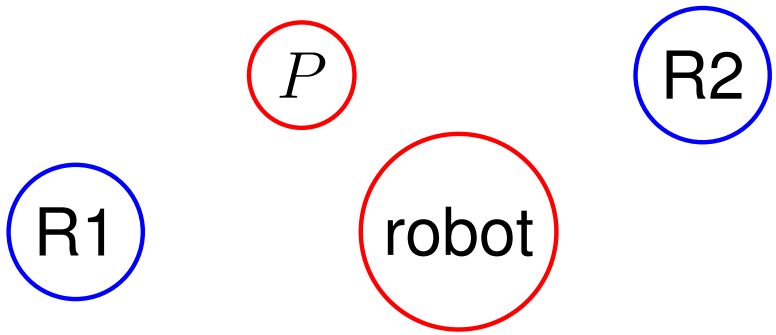

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signal amplitude at P for receivers R_i, R_j: A_{ij}

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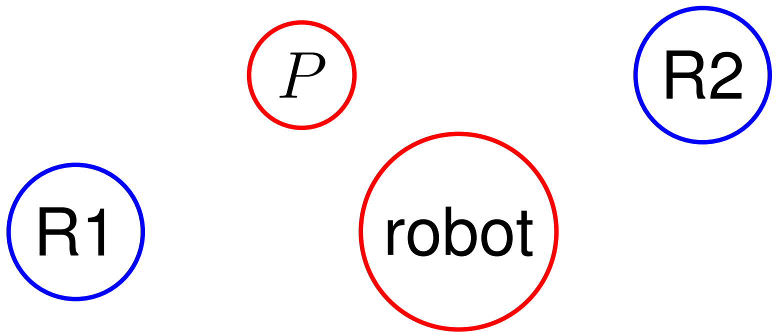

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$$A_{ij} = \sqrt{2\cos(2\pi \frac{f}{c}(d_i - d'_i - d_j + d'_j)) + 2}$$

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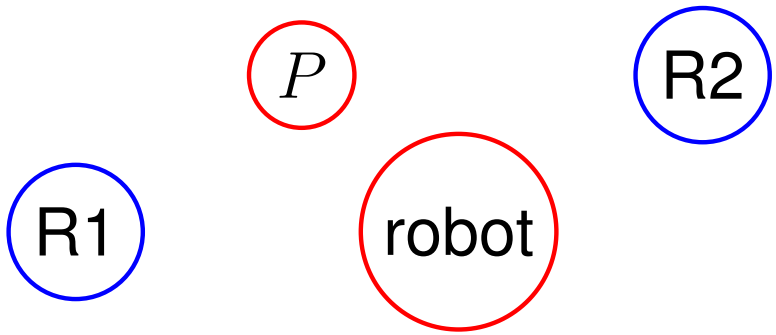

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maximal signal: 2

If for all pairs of receiver $A_{ij} >$ threshold: detection

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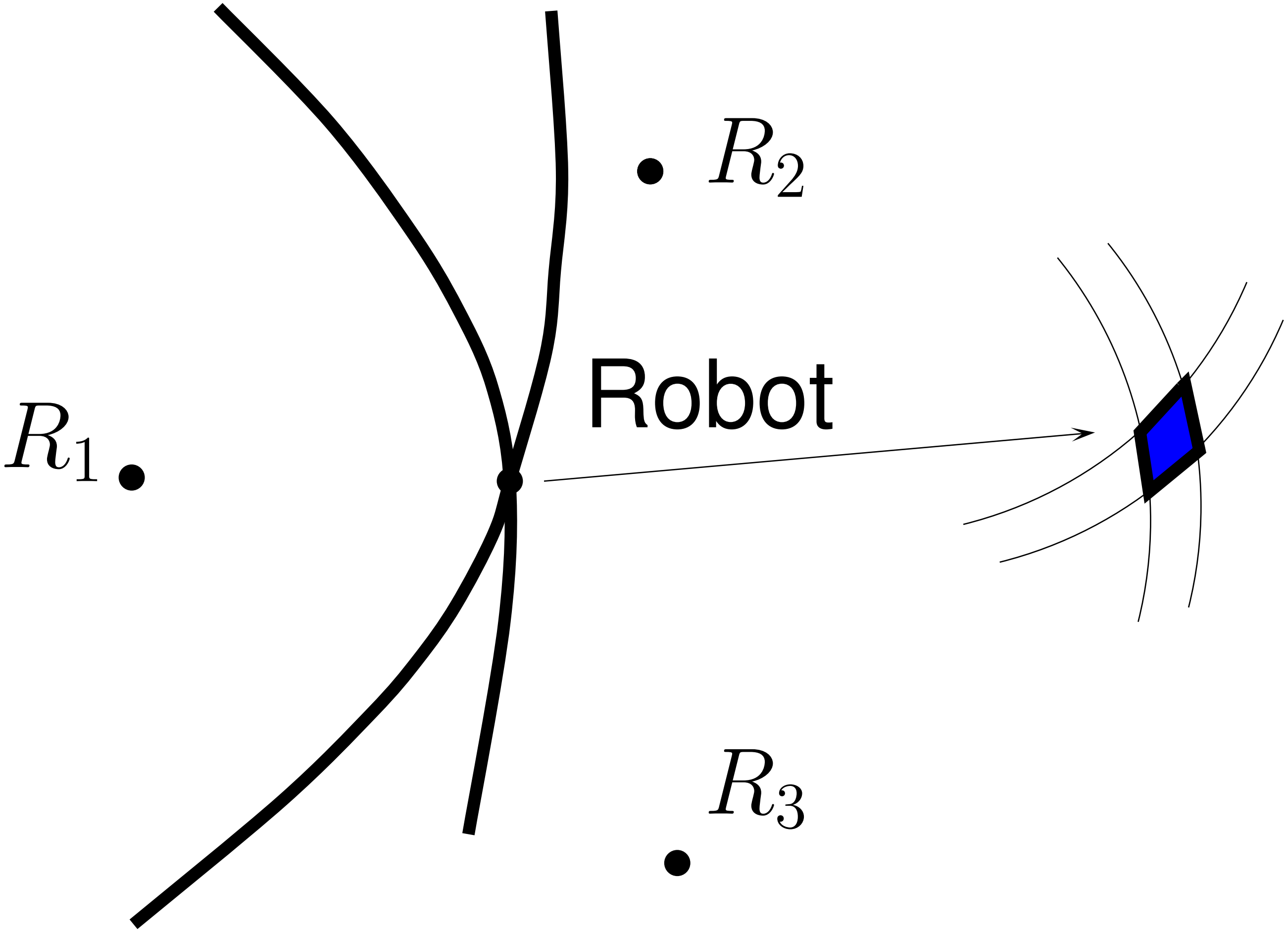

Managing uncertainties

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effect on the localization ?

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- $\mathcal{M}$: list of m 4D-boxes, initially $\mathcal{M} = \{\mathcal{S}_1\}$

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Computing the region

Algorithm $\texttt{Find}(\mathcal{B}, \mathcal{L}, n)$

for $k = 1$ to n **do**

end for

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compute all $F_{ij} = A_{ij}(\mathcal{B}_k)$

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Algorithm `Find`$(\mathcal{B}, \mathcal{L}, n)$

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if $\exists\ i,\ j$ such that $\overline{F_{ij}} < \epsilon$ **then**

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if $\forall\ i, j\ \underline{F} > \epsilon$ **then**

store $\mathcal{B}_k$ as solution, **next**

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end for

`Loop`($\mathcal{S}, \mathcal{M}, l$): same algorithm than `Find` but

- maximum of l bisection
- returns 1 if all $\mathcal{S}$ boxes have been processed, 0 otherwise

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Computing the region

Algorithm `Find`($\mathcal{B}, \mathcal{L}, n$)

for $k = 1$ to n **do**

compute all $F_{ij} = A_{ij}(\mathcal{B}_k)$

if $\exists\ i,\ j$ such that $\overline{F_{ij}} < \epsilon$ **then**

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end if

if $\forall\ i, j\ \underline{F} > \epsilon$ **then**

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if `Loop`($\mathcal{S}, \mathcal{M}, 100$)=1 **then**

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end if

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end for

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Result

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- c in [1465,1496] m/s ($\pm$ 5 degrees variation)

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- c in [1465,1496] m/s ($\pm$ 5 degrees variation)
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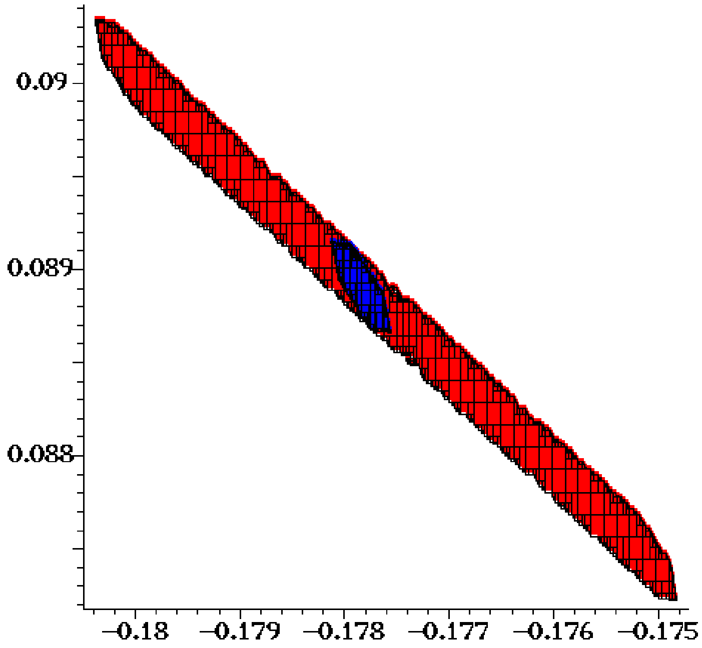

No uncertainty, with uncertainties

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Design problem

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- a given workspace $\mathcal{W}$ for the robot

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Find possible location of R_3 so that:

- for all robot location in $\mathcal{W}$ localization error is $< \alpha$

Design problem

- two receivers in an exactly known location
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Find possible location of R_3 so that:

- for all robot location in $\mathcal{W}$ localization error is $< \alpha$
- allowing to manage uncertainties on the location of R_3

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Approach

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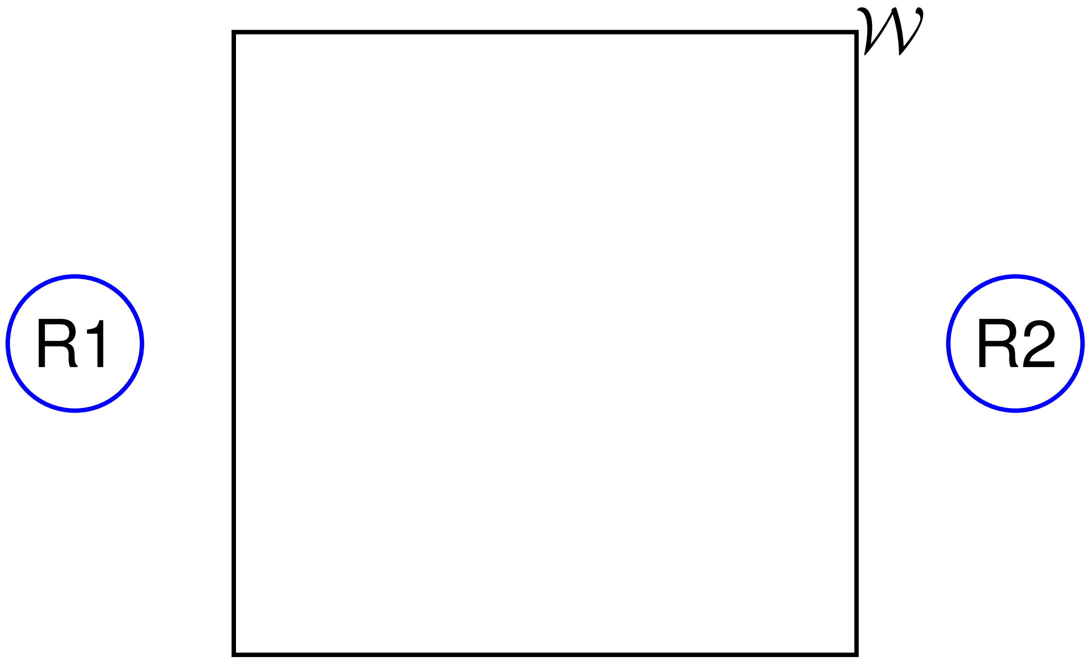

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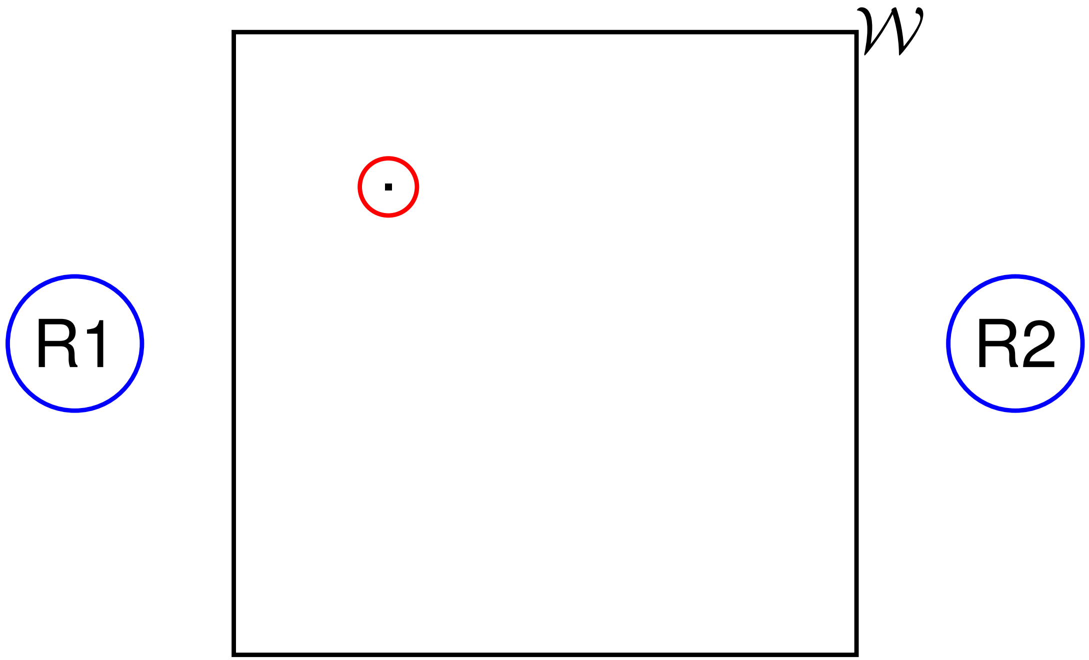

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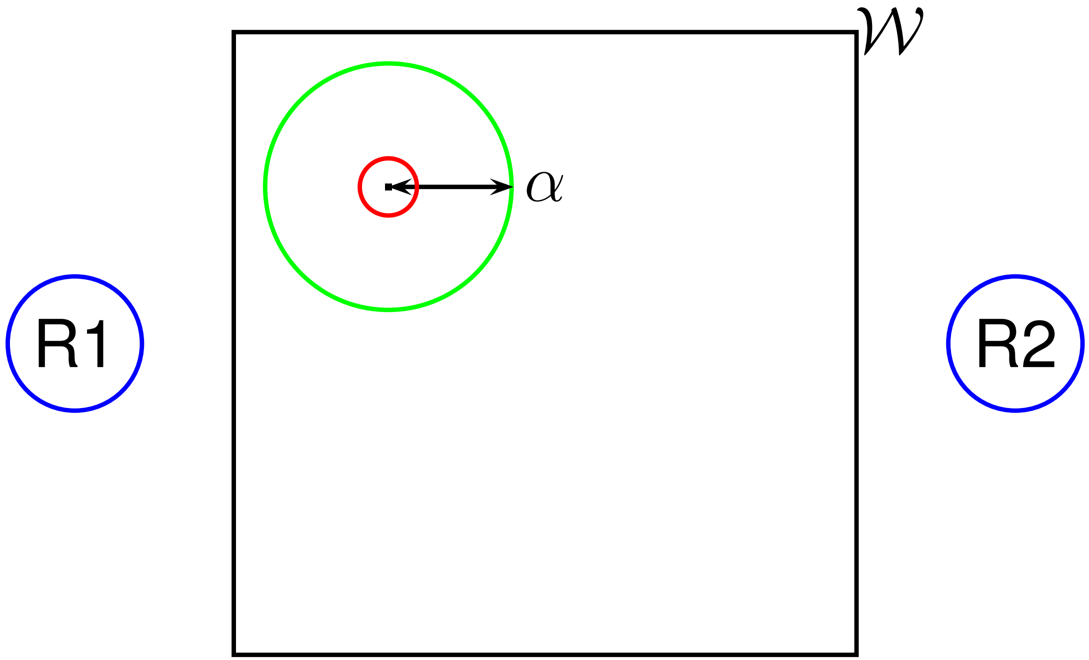

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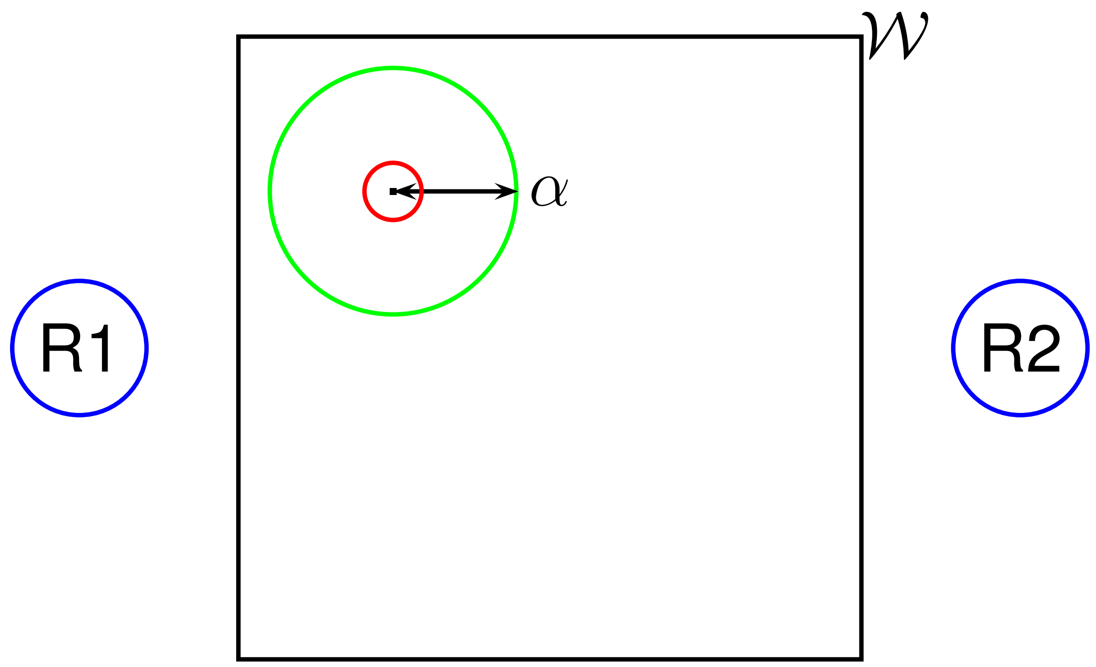

If circle centered at P, radius α has no detectable points

$\Rightarrow$ localization error at P is $< \alpha$

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Algorithm

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- Upper loop: box with x, y (coordinates of R_3)

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computationally expensive

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$$\Downarrow$$

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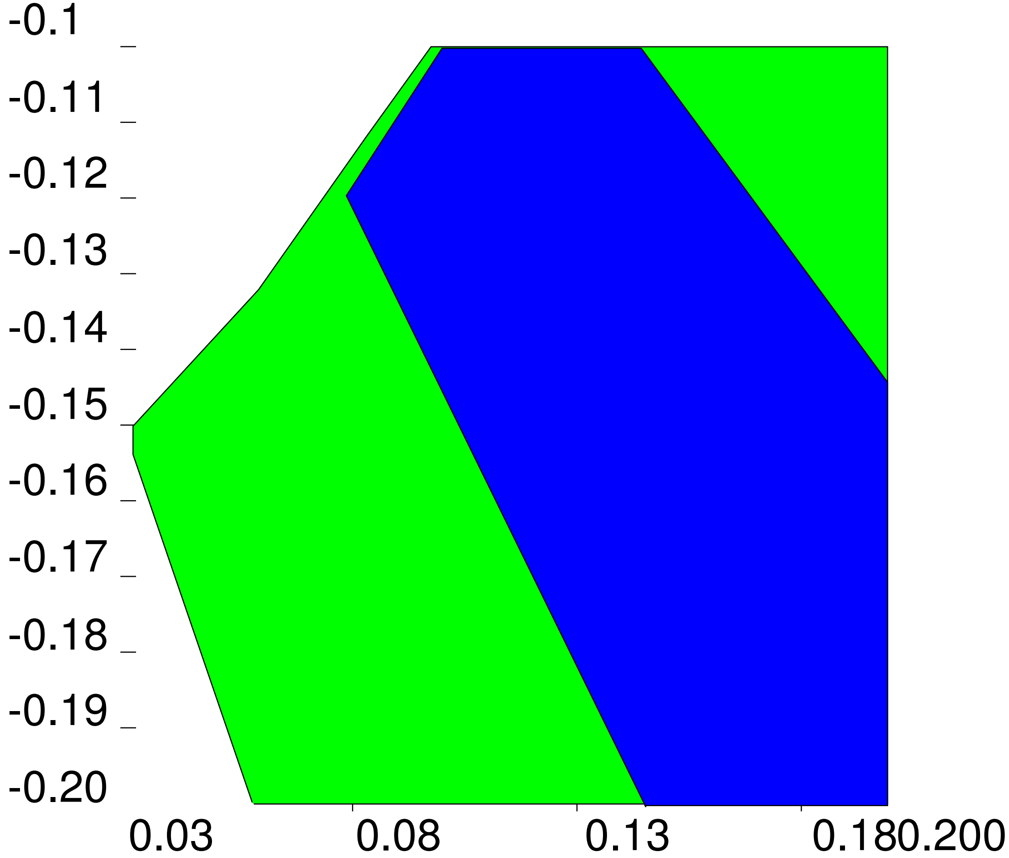

Algorithms

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computationaly expensive: 20 hours on 17 computers

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Algorithm 2:

- select a possible location for R_3 within the result of Algorithm 1
- choose a positioning accuracy for R_3
- check the whole workspace for accuracy

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location 0.123$\pm$ 0.0005, -0.123 $\pm$ 0.005 is valid

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- more realistic localization (inner and outer approximation)

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- design for given performances of the system

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- better signal model (reflection, . . .)

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Conclusion

Analysis

- more realistic localization (inner and outer approximation)

Synthesis

- design for given performances of the system

Prospective

- better signal model (reflection, . . .)
- inaccuracies on the location of R_1, R_2

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