Application of Interval Analysis : safe path planning

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DEMAR / LIRMM / INRIA

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Laboratoire d'Informatique de Robotique et de Microélectronique de Montpellier

LIRMM

INRIA

Plan

1. Context

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- Safety
- few sensors $\Rightarrow$ No feedback , No control loop.
- Validation on humanoid robots.

Modeling

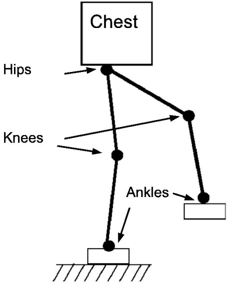


Fig.: 2D model of paraplegian patient under FES

We modelize the patient as a serial chain with 6 degrees of freedom in the saittal plane.

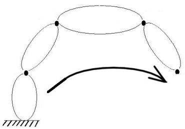
modeling

Forward recurrency (i =1 to N) Given $q, \dot{q}, \ddot{q}, X_0, \dot{X}_0, \ddot{X}_0$

$$\begin{aligned} X_i &= f_1(X_{i-1}, q_i) \\ \dot{X}_i &= f_2(\dot{X}_{i-1}, q_i, \dot{q}_i) \\ \ddot{X}_i &= f_3(\ddot{X}_{i-1}, q_i, \dot{q}_i, \ddot{q}_i) \end{aligned} \tag{1}$$

With

$$X_i = [x_i, y_i, \theta_i]^T \tag{2}$$



modeling

Backward recuurency : (i = N to 0)

$$F_i = g(F_{i+1}, q_i, \dot{q}_i, \ddot{q}_i, X_i, \dot{X}_i, \ddot{X}_i) \tag{1}$$

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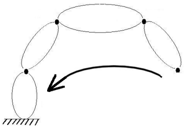
$$F_i = [Fx_i, Fy_i, \Gamma_i]^T \tag{2}$$



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Semi-Infinite Programming [Hettich and Kortanek(1993)]

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We present a new method to deal with the inequality constraints.

Usual Discretization $\Rightarrow$ Time-point Discretization

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[Reemtsen(1998)] presents how to compute and adapt $\{t_0, t_1, ..., t_{N-1}, t_N\}$

Usual Discretization : Illustration of constraint violation

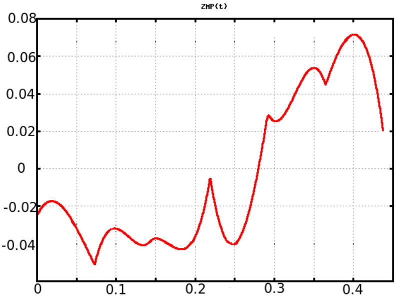

Fig.: Representation of ZMP(t)

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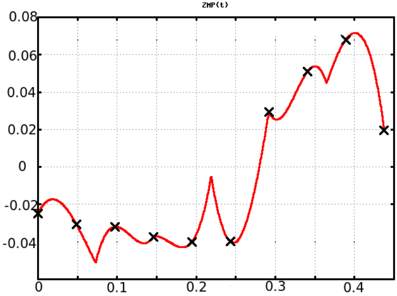

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We take 10 points to evaluate the constraint function

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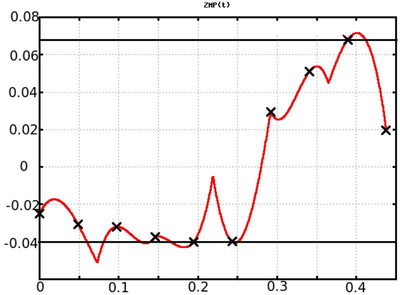

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The points satisfy the constraint whereas the continuous function violate it

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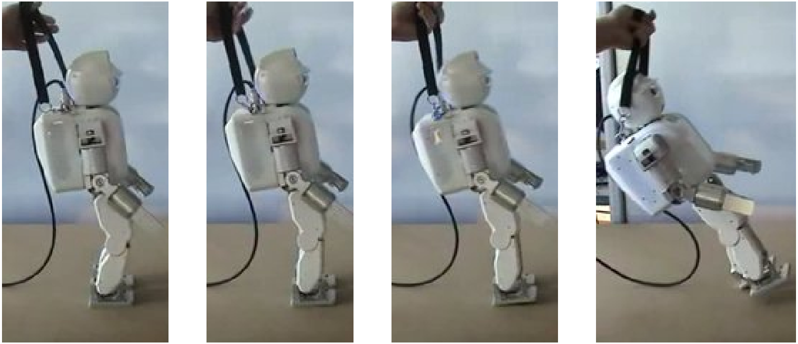

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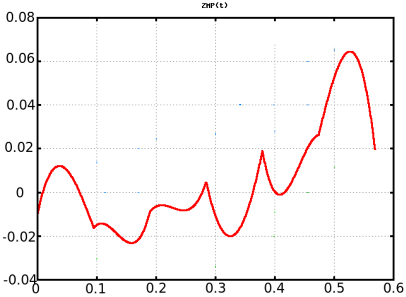
The safe discretization is done through Interval Analysis.

Safe discretization : illustration



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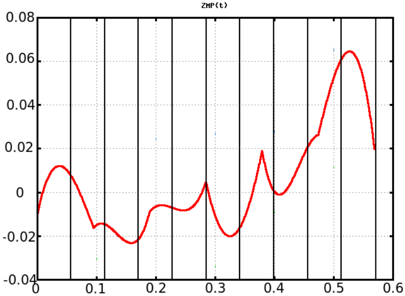

Fig.: Representation of ZMP(t)

Split into 10 time-intervals

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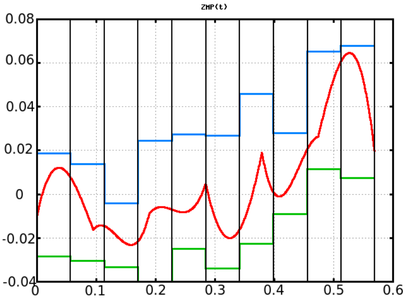

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Compute the extrema through interval analysis

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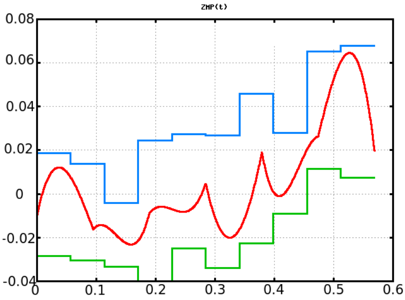
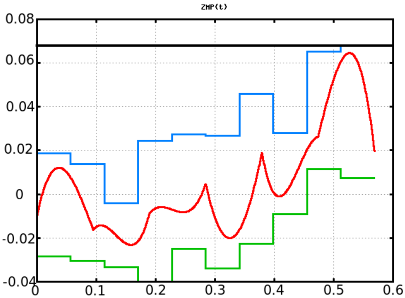


Fig.: Representation of ZMP(t)

Return the values to the algorithm

Safe discretization : illustration



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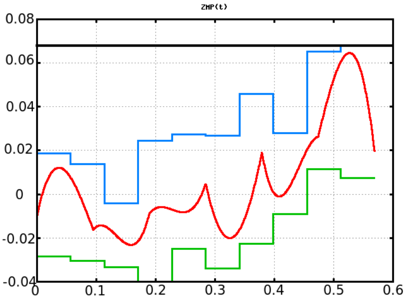
The ZMP constraint is never violates ⇒ the robot keeps its balance

Safe discretization : illustration



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Some Methods from Interval Analysis allow to get better computation [Hansen and Walster(2004)]

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	usual path planning	safe path planning
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Tab.: Comparison of usual and safe path planning.

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  - to more complex systems : 3D.

Thank you for your attention !

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