Interval and Boolean constraint propagation

for simultaneous localization and map building

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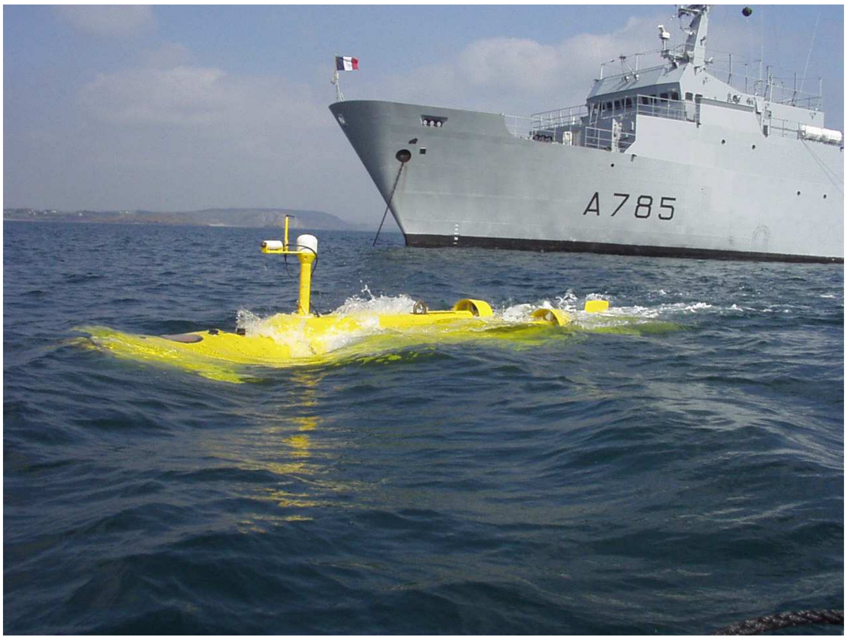
1 The Redermor


A785

2 SLAM

Localization

Given a map, determine the robot's location. The landmark locations are known.

The localization problem is a state estimation problem. The model of the system is

$$\begin{cases} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) \end{cases}$$

where $\mathbf{x} = (x, y, z, \phi, \theta, \psi, v)$. The extended Kalman filter can thus be used.

SLAM (simultaneous localization and mapping)
or **CLM** (concurrent localization and mapping**)**

The landmark locations are unknown.

Determine the locations of the robot as well as the locations of the landmarks.

The SLAM problem is a parameter/state estimation problem. The model of the system is

$$\begin{cases} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}, \mathbf{p}) \end{cases}$$

where $\mathbf{p} = (x_1^\ell, y_1^\ell, z_1^\ell, x_2^\ell, y_2^\ell, z_2^\ell, \dots)$ are the coordinates of the landmarks.

If $\mathbf{z} = (\mathbf{x}, \mathbf{p})$ is the new state vector, the model can be written

$$\begin{cases} \dot{\mathbf{z}} & = & \mathbf{f}_z(\mathbf{z}, \mathbf{u}) \\ \mathbf{y} & = & \mathbf{g}_z(\mathbf{z}) \end{cases}$$

where

$$\mathbf{f}_z(\mathbf{z}, \mathbf{u}) = \begin{pmatrix} \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{0} \end{pmatrix}$$

The SLAM problem becomes a state estimation problem.

Why choosing an interval approach ?

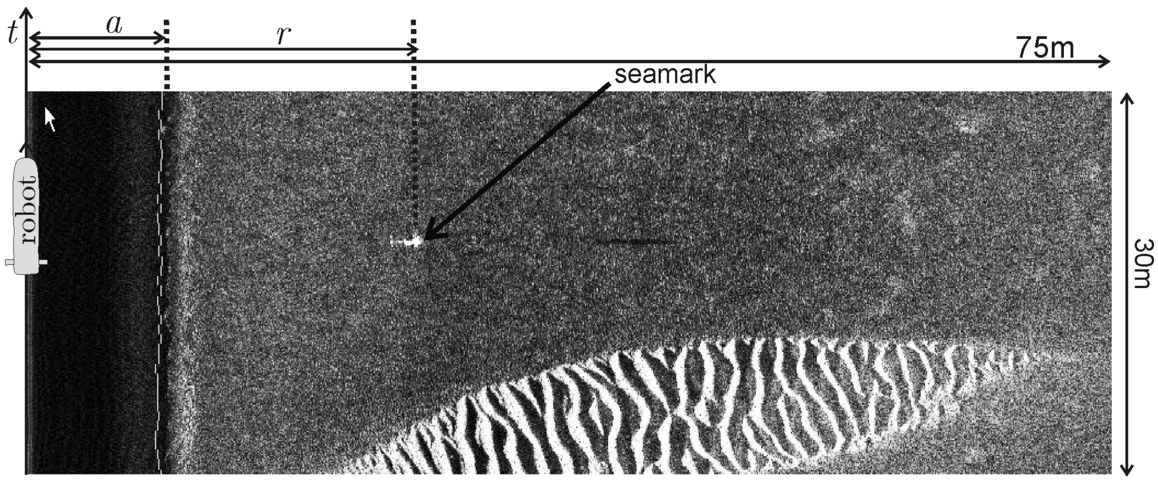
1) A reliable (probabilistic or set membership) method is needed.
2) The model is nonlinear.
3) The noises are non Gaussian and their pdf are unknown.
4) Error bounds are provided by the constructor of all available sensors.
5) The data are clean (no outlier, the bounds are guaranteed).
6) A huge number of redundant data are available.

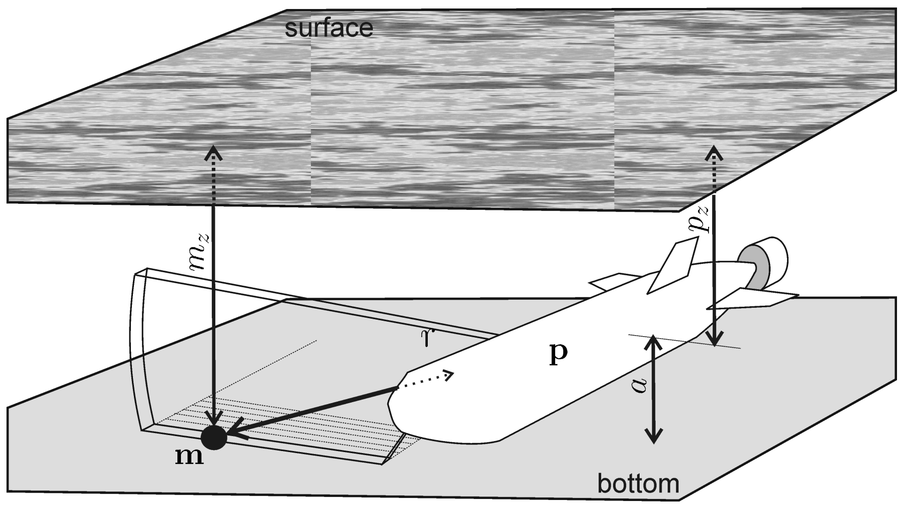
3 Sensors

A GPS (Global positioning system), at the surface only.

$$
\begin{aligned}
t_0 &= 6000, \quad \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\
t_f &= 12000, \quad \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m
\end{aligned}
$$

A sonar (KLEIN 5400 side scan sonar). Makes it possible to compute an estimation $\tilde{r}$ of the distance r from the robot to the detected object.

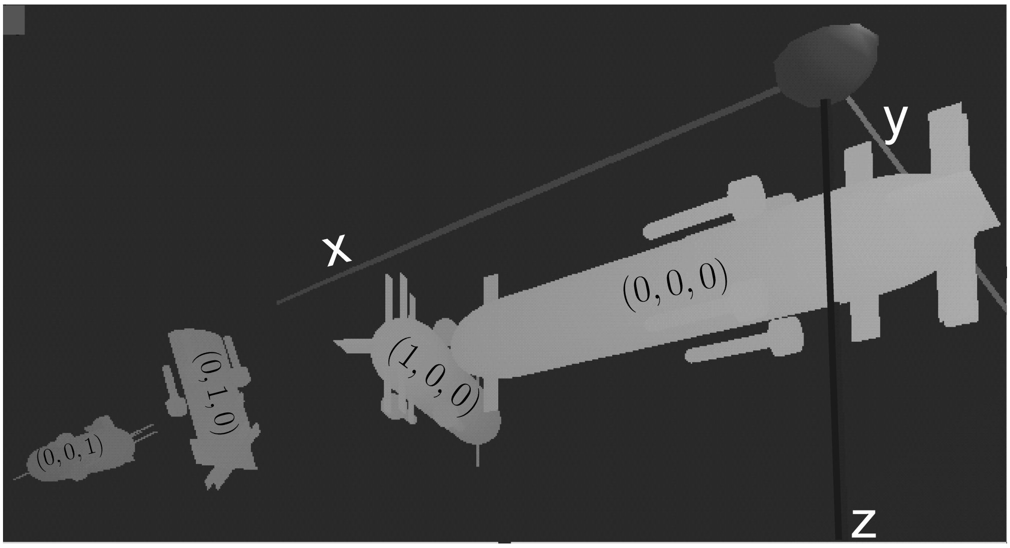



surface
m_z
p_z
r
$\mathbf{p}$
a
$\mathbf{m}$
bottom

A Loch-Doppler. Returns the speed of the robot $\mathbf{v}_r$ expressed in the robot frame. Also returns the altitude a of the robot $\pm$ 10cm.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ, the pitch θ, and the head ψ the robots.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



A barometer computes the depth of the robot

$$p_z(t) \in [-1.5, 1.5] + \tilde{d}.[0.98, 1.02]$$

The interval $[-1.5, 1.5]$ may change depending on the strength of waves and tides.

4 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t) \text{ and } p_z(t).$$

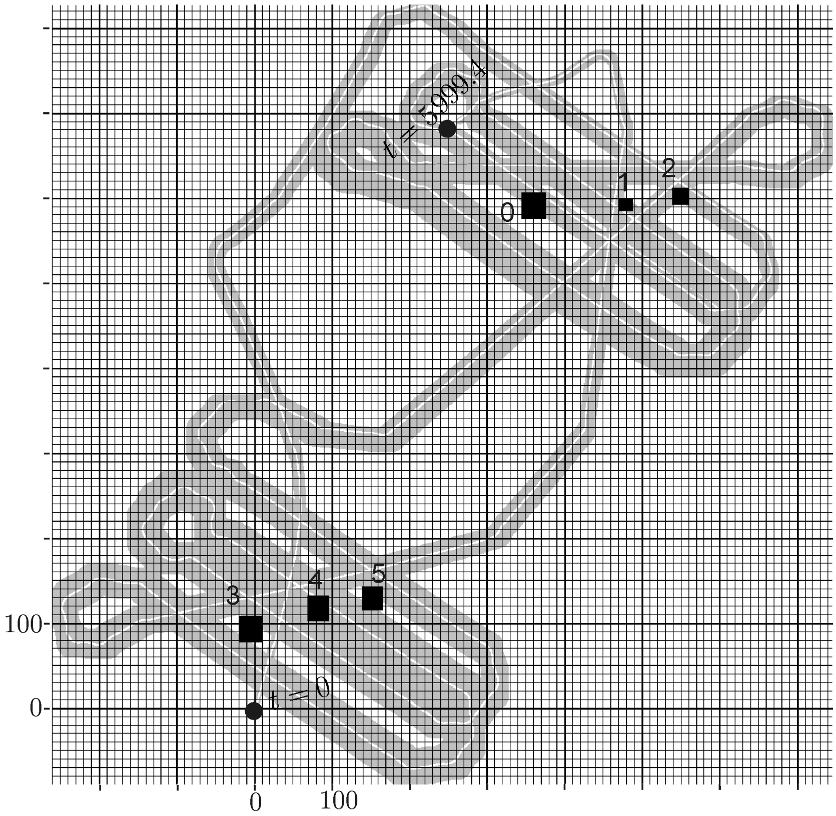
Six objects have been detected by the sonar:

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

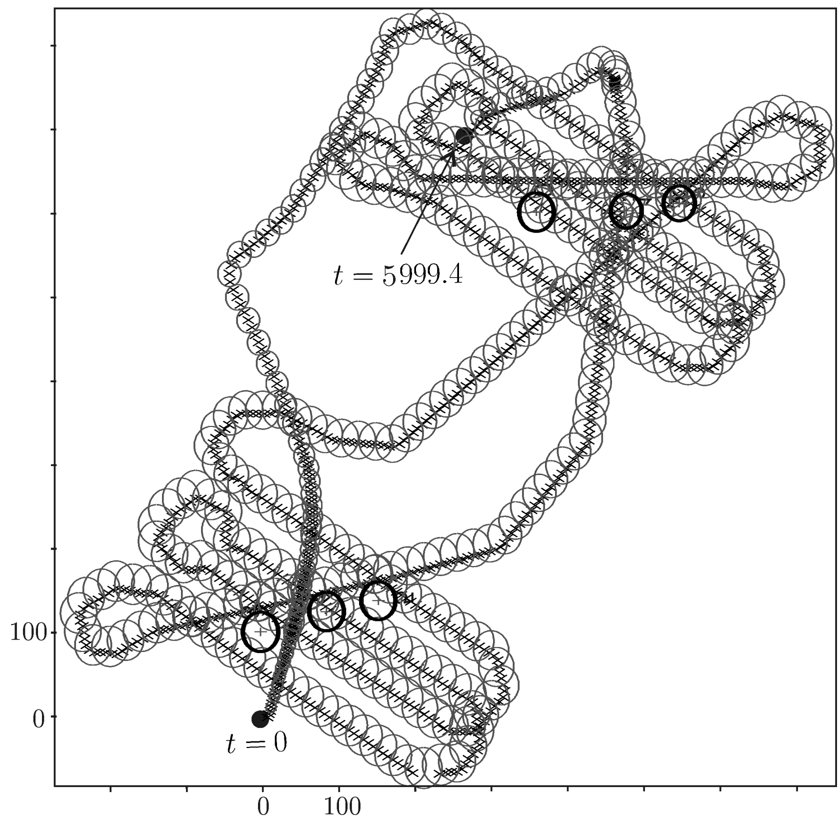
5 Constraints satisfaction problem

$$
\begin{array}{l}
t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}, \\
i \in \{0, 1, \dots, 11\}, \\
\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}, \\
\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)), \\
\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix}, \\
\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix}, \\
\mathbf{R}(t) = \mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t), \\
\mathbf{p}(t + 0.1) = \mathbf{p}(t) + 0.1 * \mathbf{R}(t).\mathbf{v}_r(t), \\
||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i), \\
\mathbf{R}^\mathsf{T}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2}, \\
m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5]
\end{array}
$$

6 Results



Interval constraint propagation approach



Extended Kalman approach

7 Matching problem

For the six objects detected by the sonar, the object number is assumed to be known

i	0	1	2	3	4	...	11
$\tau(i)$	7054	7092	7374	7748	9038	...	11688
$\sigma(i)$	1	2	1	0	1	...	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	...	15.05

Often, in a sonar context, the correspondances are unknown.

i	0	1	2	3	4	...	11
$\tau(i)$	7054	7092	7374	7748	9038	...	11688
$\sigma(i)$	σ_0	σ_1	σ_2	σ_3	σ_4	...	σ_{11}
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	...	15.05

Here, we will assume that we know the total number of objects which is equal to 6.

We need a constraint to perform a contraction!

If $\mathbf{m}_i = \mathbf{m}\left(\sigma\left(i\right)\right)$, then

$$\text{card}\left\{\mathbf{m}_0, \mathbf{m}_2, \ldots, \mathbf{m}_{11}\right\} = 6,$$

which be viewed as a constraint. It can also be written, in a more classical way, by

$$6\_\text{value}\left(\mathbf{m}_0, \mathbf{m}_2, \ldots, \mathbf{m}_{11}\right).$$

Example. If

$$m_1 = 7, m_2 = 8, m_3 = 7, m_4 = 9$$

$$\text{2-value}\,(m_1, m_2, m_3, m_4) \;\; \text{is false}$$
$$\text{3-value}\,(m_1, m_2, m_3, m_4) \;\; \text{is true}$$
$$\text{4-value}\,(m_1, m_2, m_3, m_4) \;\; \text{is false}$$

If

$$\begin{aligned} m_1 &\in [3,6], m_2 \in [5,9], m_3 \in [2,4], m_4 \in [8,14], \\ &\text{2-value}\,(m_1, m_2, m_3, m_4) \end{aligned}$$

then

$$\ldots$$

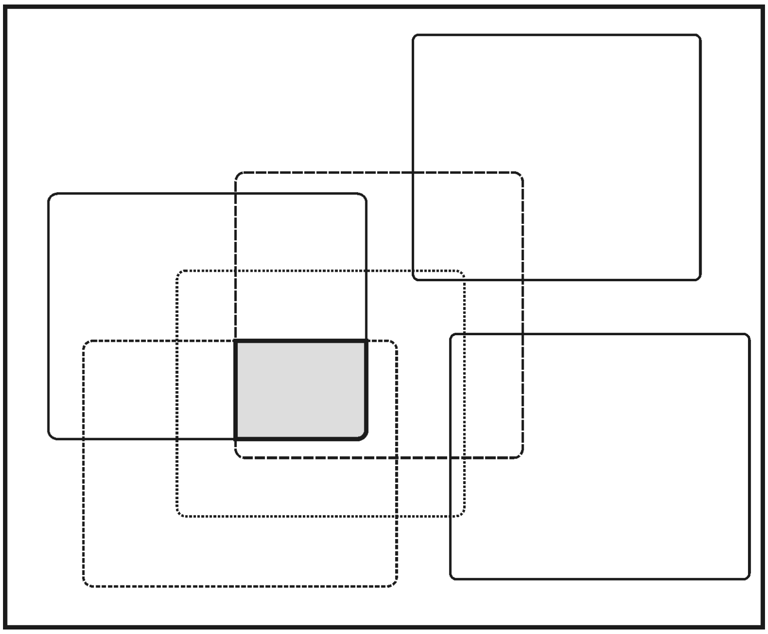
If

$$\begin{aligned} m_1 \in\ & [3,6], m_2 \in [5,9], m_3 \in [2,4], m_4 \in [8,14], \\ & \text{2-value}\,(m_1, m_2, m_3, m_4) \end{aligned}$$

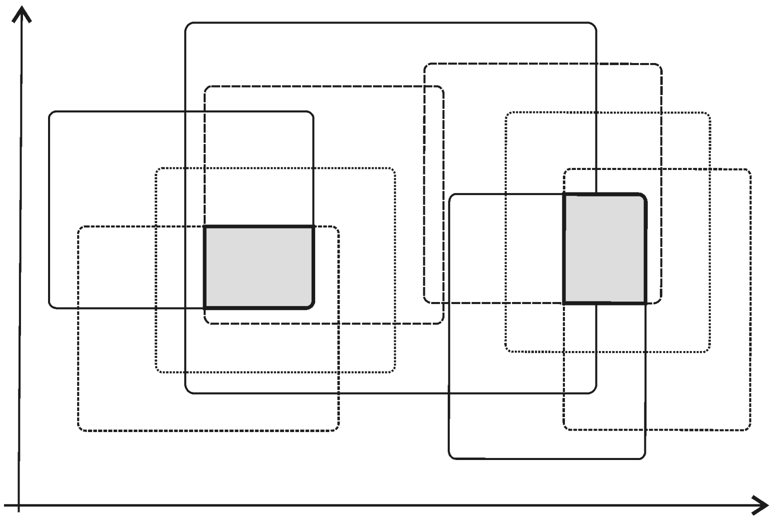
implies

$$m_1 \in [3,4], m_2 \in [8,9], m_3 \in [3,4], m_4 \in [8,9],$$

Recall the "all equal except q" constraint



Contraction with respect to the constraint
All_equal_except_2$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5, \mathbf{x}_6)$



All boxes 9 boxes will be contracted to one the two two grey box.