Improving the Ray Tracing of Implicit Surfaces using Interval Arithmetic

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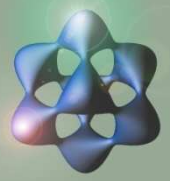
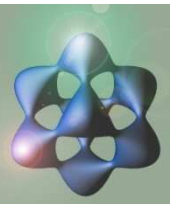


Section 1

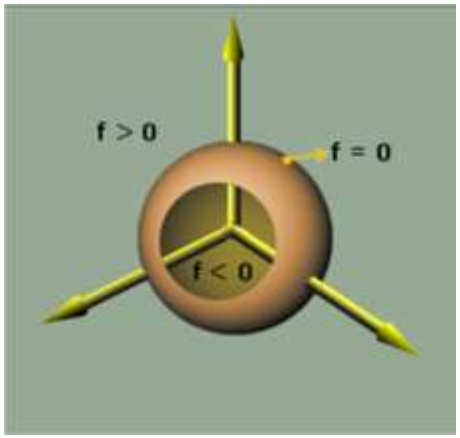
Introduction

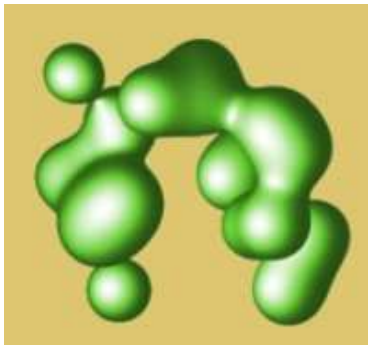


Implicit Surfaces (IS)

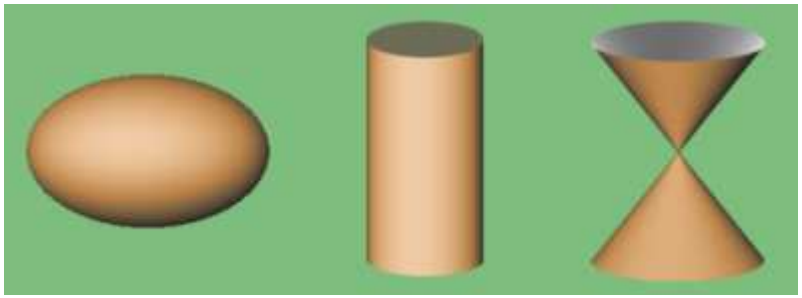
$$f: \Omega \in \mathbf{R}^3 \longrightarrow \mathbf{R}$$

f(x, y, z) = 0 : Surface
f(x, y, z) < 0 : Points Inside (volume)
f(x, y, z) > 0 : Points Outside

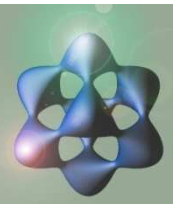




Blobs

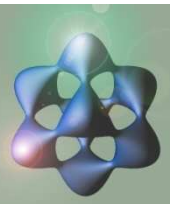


Quadrics



Application of IS

Modeling using Constructive Solid Geometry operations

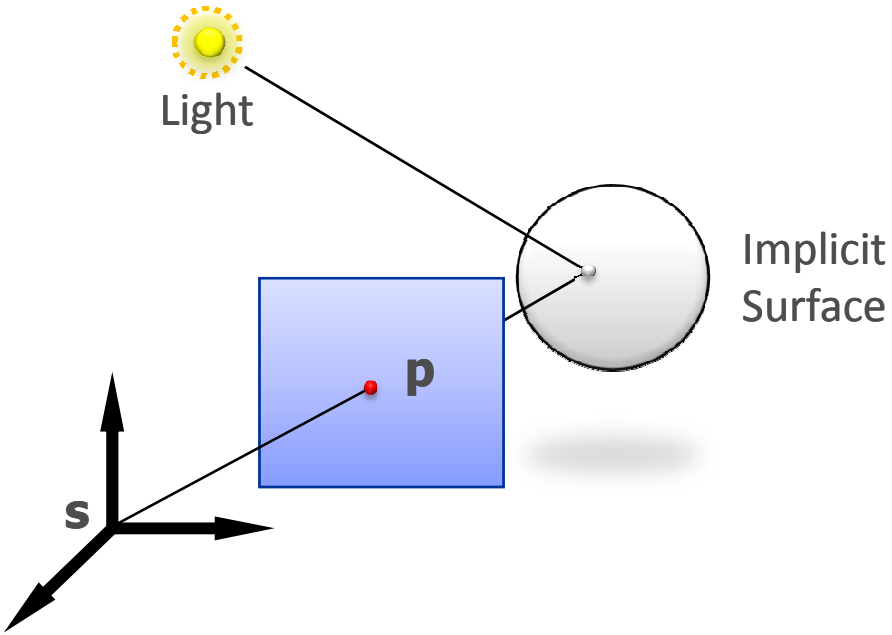


Ray Tracing IS

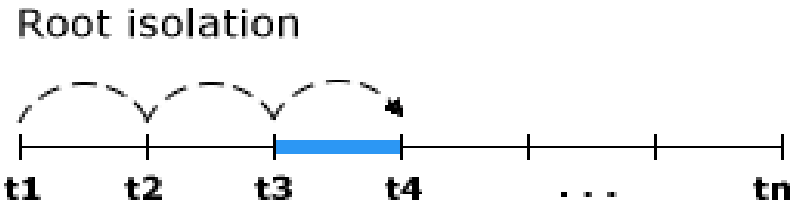
Intersection test

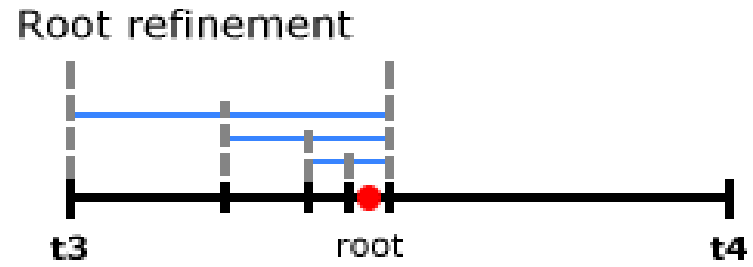
$$x = s_x + t(x_p - s_x) \qquad y = s_y + t(y_p - s_y) \qquad z = s_z + t(z_p - s_z)$$

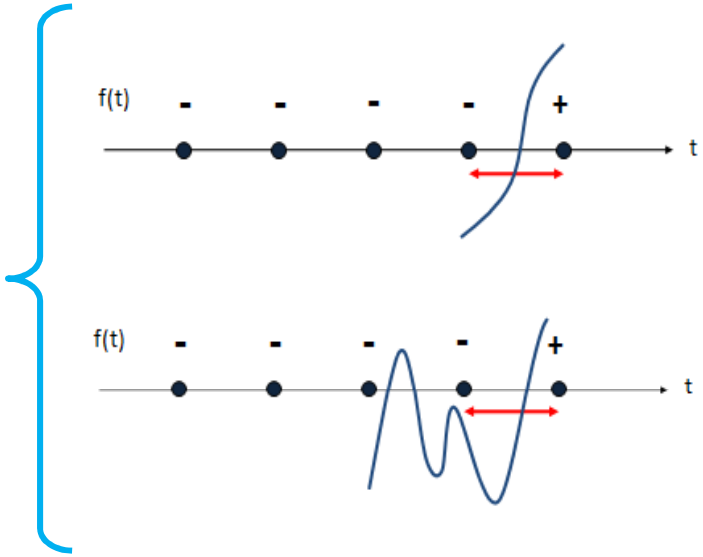
$$F(t) = F(s_x + t(x_p - s_x), s_y + t(y_p - s_y), s_z + t(z_p - s_z))$$



Using point sampling

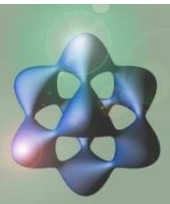






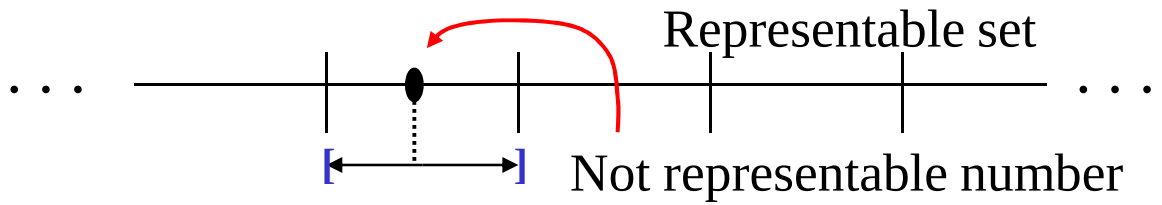
Solution approaches

Lipschitz constants, Interval Arithmetic, Affine arithmetic

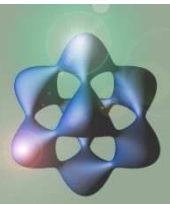


Interval Arithmetic

- Computers can not represent all the set of real numbers: rounding problems.
- Interval Arithmetic:

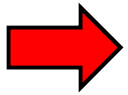


- Successfully applied in different areas: structures, control, supervision, medicine.



Interval Ray Tracing IS (ii)

$$F(T) = F(s_x + T(x_p - s_x), s_y + T(y_p - s_y), s_z + T(z_p - s_z))$$



$0 \notin F(T)$: The interval T does not contain roots.

Mitchell

- Solve root isolation using IA (Bisection, Interval Newton)

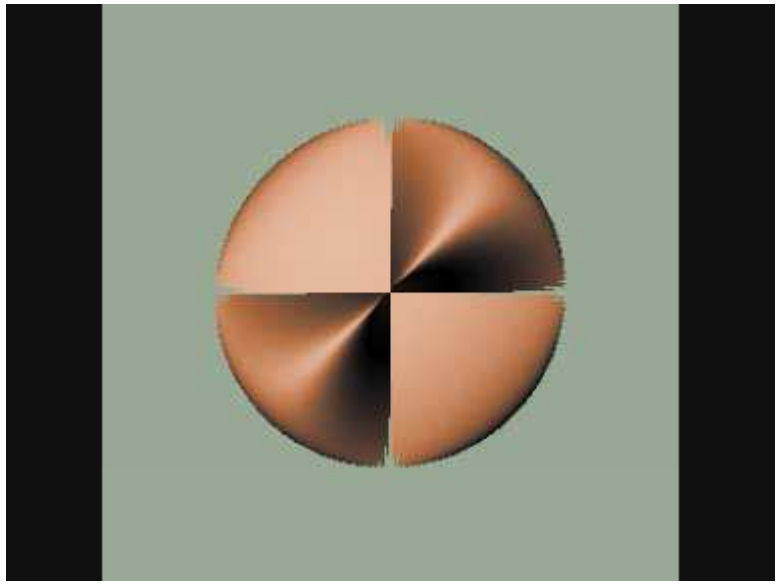
Capriani et al.

- Alefeld-Hansen operator combined as a complement of Interval Newton Method

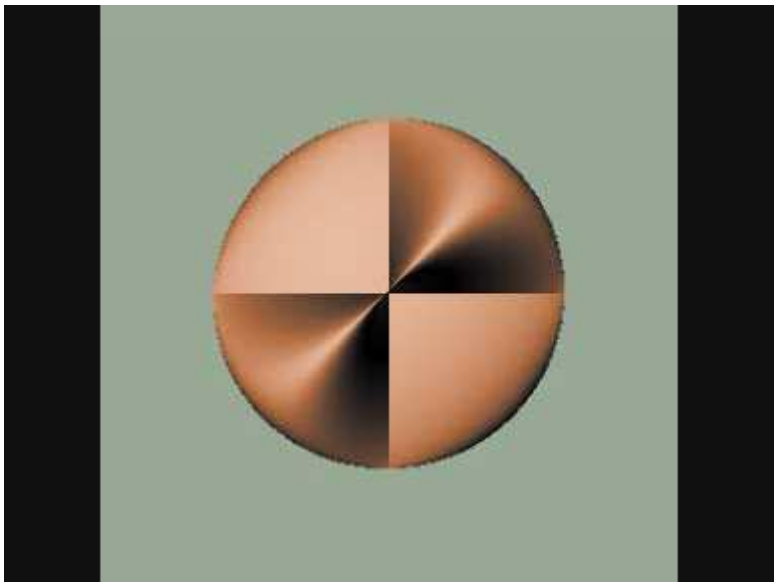
San Juan et al. (MRFro)

- Improved bisection using conditionals

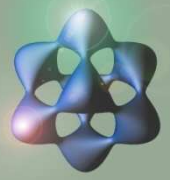
Point sampling vs. IA



Point sampling



Interval Arithmetic

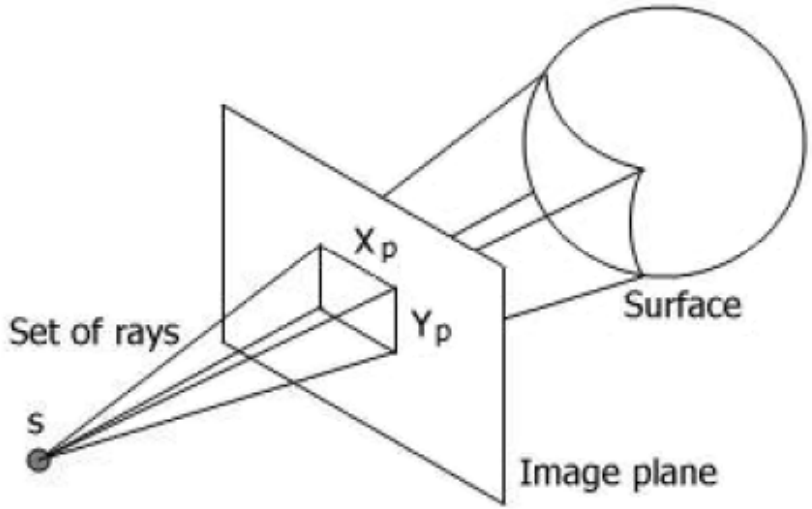


Section 2

Efficiency improvement

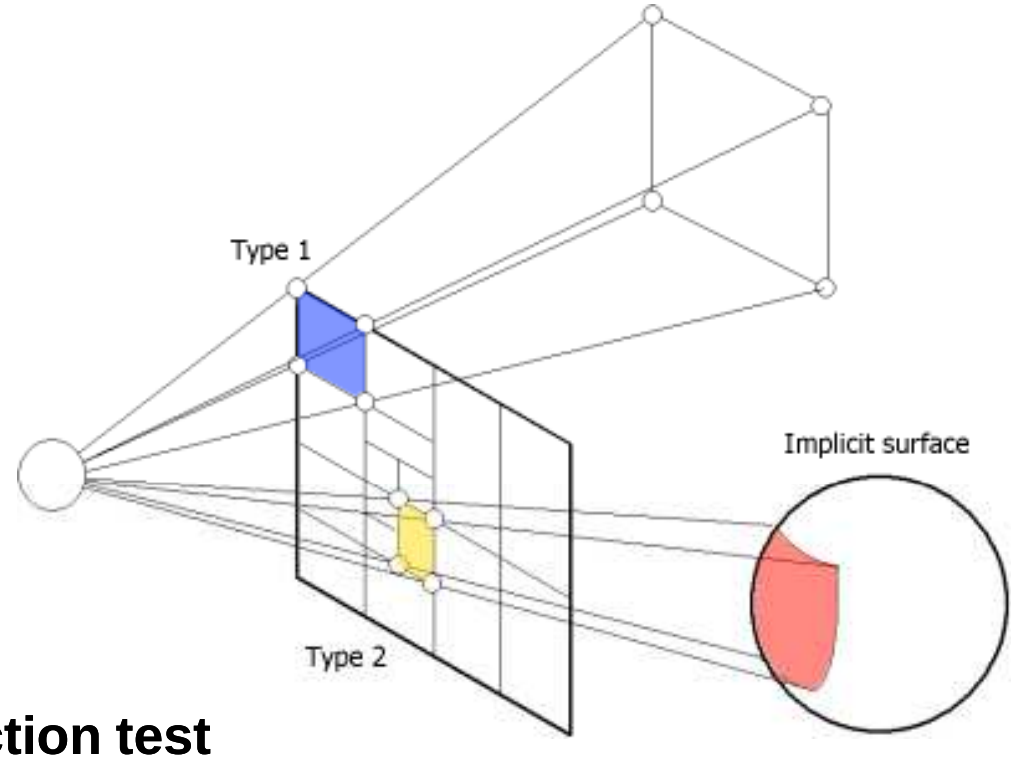
Evaluating a Set of Rays

$$F(T,X_p,Y_p) = F(s_x + T(X_p - s_x), s_y + T(Y_p - s_y), s_z + T(z_p - s_z))$$



Evaluation of a set of rays instead of a unique ray

Evaluating areas instead of points

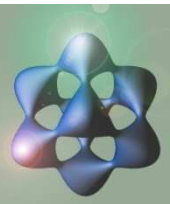


Rejection test

- Identify areas without rays intersecting the surface

Inclusion test

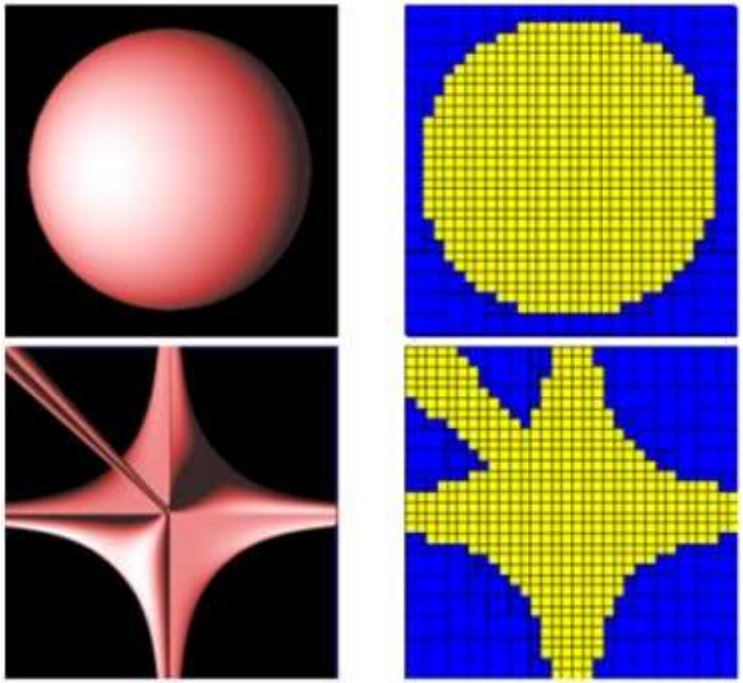
- Identify areas in which all the rays intersect the surface

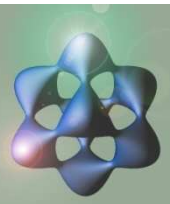


Rejection Test (i)

Semantic:

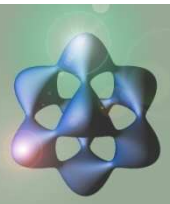
- "For all values of X, and for all values of Y, for all the values of T, $0 \notin fR(X,Y,T)$ "





Rejection Test (ii)

```
Function Reject(Intervals X_s,Y_s,Z_s)
    T=Interval(0,t)
    add T to ListT
    For every T in ListT
      If Width(T) < ε_T
        Exit For
      EndIf
      If 0 ∉ fR(X_s, Y_s, T)
        Drop T from ListT
      Else
        Bisect T into T_1,T_2
        Add T_1,T_2 to ListT
      EndIf
    EndFor
    If empty(ListT)
      Box does not have Intersections
    Else
      Box can have Intersections
    EndIf
```



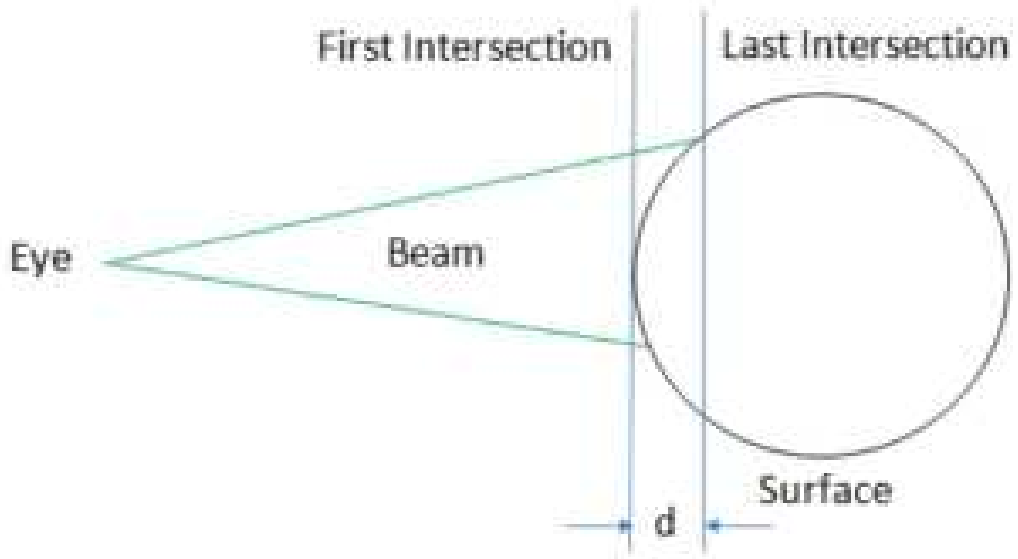
Inclusion Test (i)

"For all values of X, and for all values of Y, there are values of T, $0 \in$ fR(X,Y,T)
"

```
Function Include(Intervals X_s,Y_s)
    Meet=[-∞,∞]
    T=Interval(0,∞)
    add T to ListT
    For every T in ListT
      If Width(T) < ε_T
        Exit For
      EndIf
      If 0 ∈ fR(X_p,Y_p,T)
        R_a = fR(X_p,Y_p,T.Inf)
        R_b = fR(X_p,Y_p,T.Sup)
        Meet = Meet ∧ R_a ∧ R_b
        If 0 ∈ Meet
          The Beam Intersects completely the surface
        Else
          Bisect T into T_1,T_2
          Add T_1,T_2 to ListT
        EndIf
      EndIf
    EndFor
```

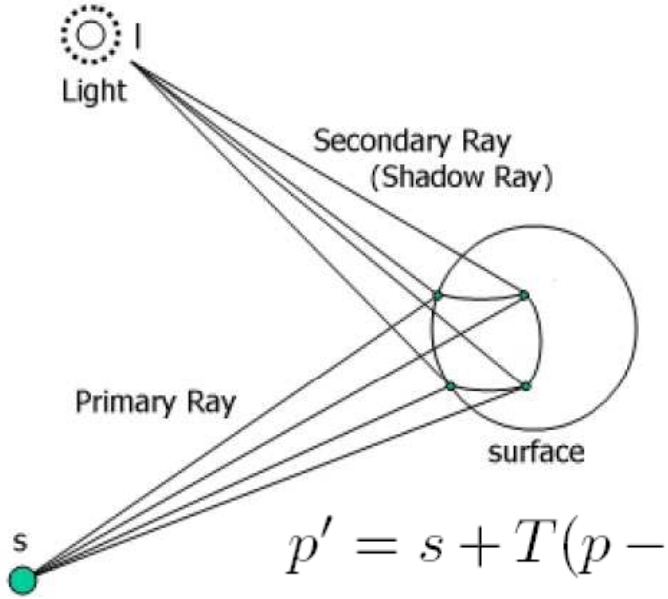
Inclusion Test (ii)

- Also, the distance between the first and last intersection is used to measure the change of the surface inside the area.

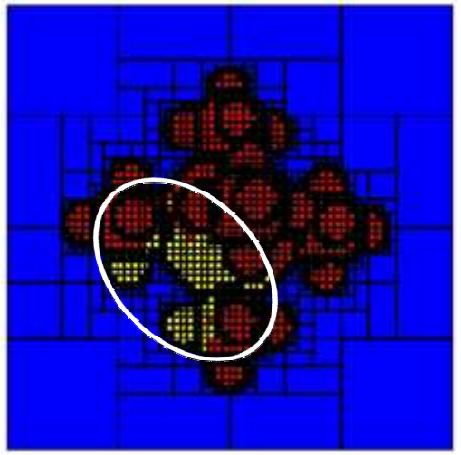


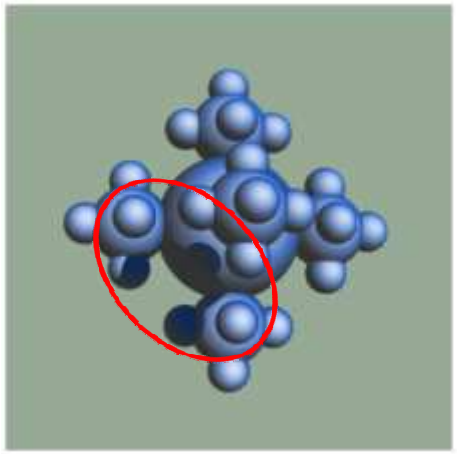
Shadow Regions

- Used only for Inside Regions



$$p' = s + T(p - s)$$



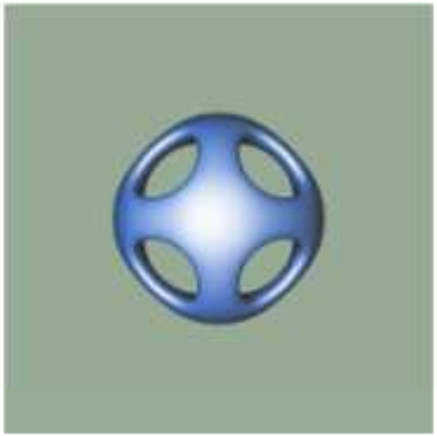
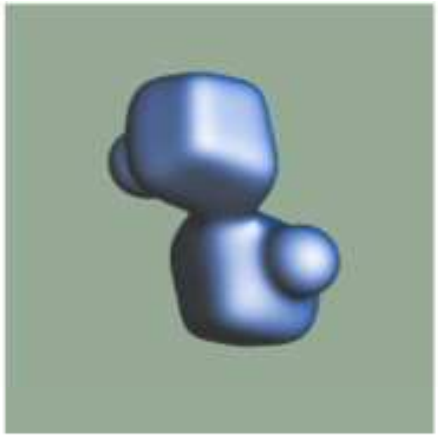
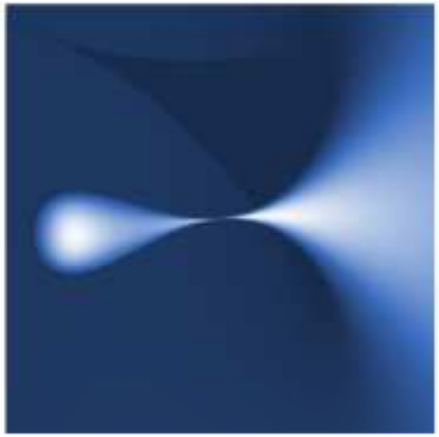


Shadow Ray : $r'(T') = l + T'(p' - l)$

Inclusion Function :

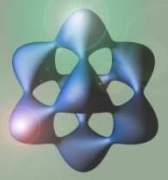
$$F(T') = F(l_x + T'(X'_p - l_x), l_y + T'(Y'_p - l_y), l_z + T'(Z'_p - l_z)) \quad , \quad T' = [0, \infty]$$

Some results

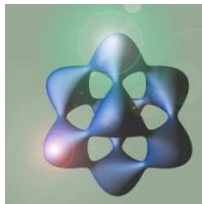




Strategy: Four rays in undefined areas, one ray in other areas

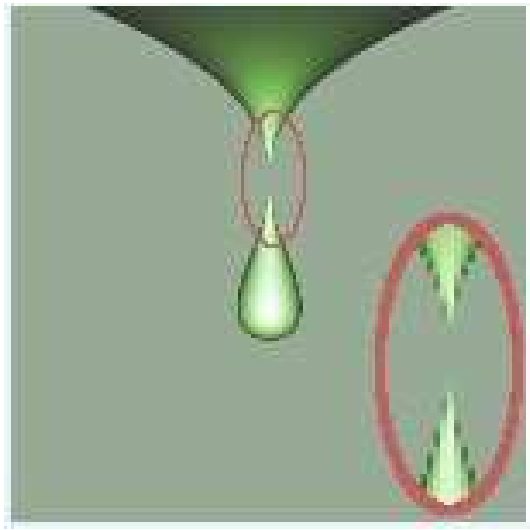
	Presented Method		Previous technique		
Surface	Time	Rays Traced	Time	Rays Traced	% Improvement
Orthocircle	8,8	68631	21,1	375003	139%
Blob	11,6	58273	35,6	414406	205%
Drop	35	460495	106	1620000	202%

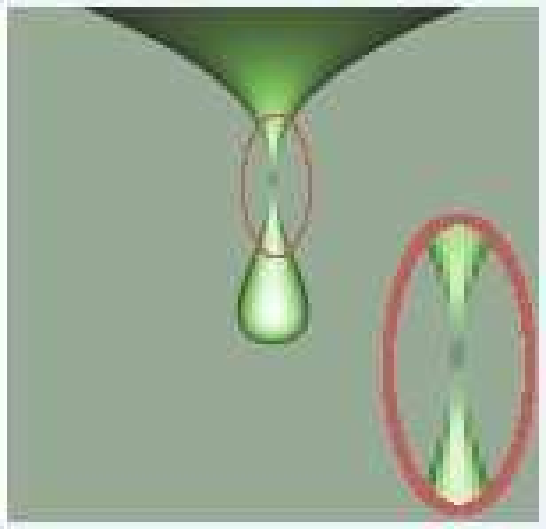


Anti aliasing Algorithm

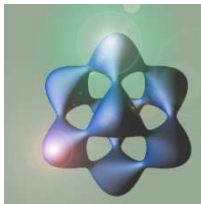


Aliasing in Ray Tracing of IS

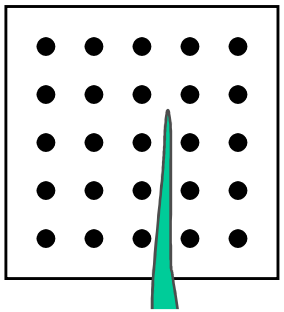




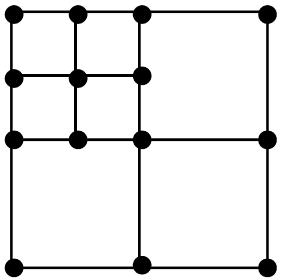
- Aliasing problems appear even when IA is applied in the intersection test.



Anti aliasing Techniques



Regular Sampling

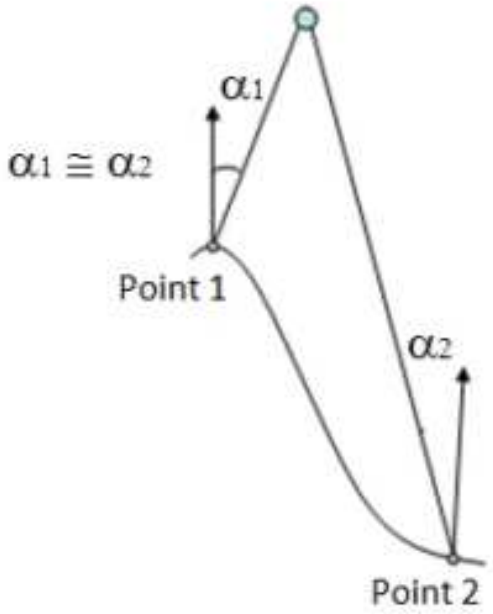


Adaptive Sampling

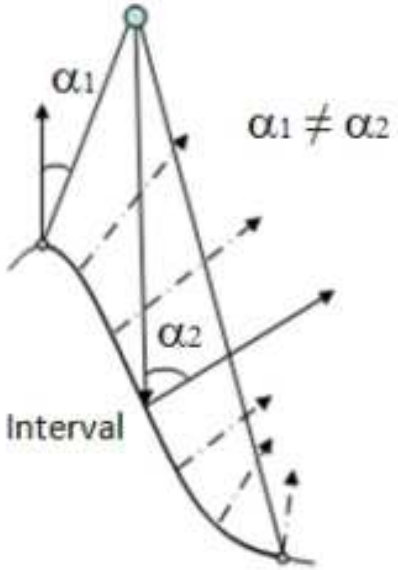
- There are many anti aliasing techniques: adaptive sampling, cone tracing, beam tracing
- Adaptive sampling is still “sampling”
  - Bounding boxes for small objects
  - Problems with long thin objects (J. Genetti et al)
- Neighbor information (beam tracing, cone tracing)
  - Complex intersection test (J. Amanatides)

Interval Adaptive Anti aliasing (IAA)

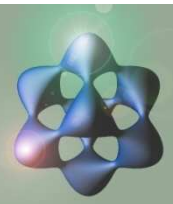
- Use a beam representing a set of rays
- IAA allows to know variation over all the rays intersecting a surface inside an area, not only between points.



Point sampling



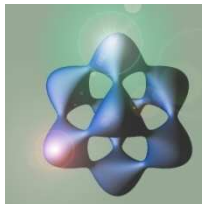
IAA algorithm



Results of IAA

Only IA

Using IAA



Thank you!