Set-Theoretic Estimation of Hybrid Systems: Motivations and Consequences.

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Outline

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2. State Estimation of Hybrid Systems
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4. Summary

Complex Systems

Complex artifacts such as space probes or process automation systems characterized by:

- High complexity,
- Demand for high performance, availability and safety.

Today's needs

- Added complexity makes these systems vulnerable to unanticipated failures.
- Essential to be able to track the system's operations in order to react appropriately.

Complex Systems as Hybrid Systems

Most controlled/automated systems exhibit continuous dynamics with abrupt switches in their dynamics:

- Exhibit a certain number of functional *modes* of behavior.
- Every mode corresponds to an expected continuous behavior.

Hybrid Model

Such systems can be accurately modeled as *hybrid systems*:

- Model containts a mixture of discrete and continuous variables.
- Discrete dynamics through transitions among *mode* variables.
- Continuous dynamics through sets of discrete-time equations within each mode.

Hybrid System

Defined by:

- A discrete state vector of modes $\mathbf{x}_m$ with domain $X_m = \{\mathbf{m}_1, \cdots, \mathbf{m}_l\}$.
- A continuous state vector $\mathbf{x}_c$.
- A dynamic model of the evolution of the system:
  - Use a discrete timeline, with sampled time index k.
  - At continuous level:

$$\begin{aligned} \mathbf{x}_{c,k} &= f(\mathbf{x}_{c,k-1}, \mathbf{u}_{c,k-1}, \mathbf{w}_{c,k-1}, \mathbf{x}_{m,k}) && (1) \\ \mathbf{y}_{c,k} &= h(\mathbf{x}_{c,k}, \mathbf{v}_{c,k}, \mathbf{x}_{m,k}) && (2) \end{aligned}$$

  - At discrete level, a set of *transitions* among modes: $\mathbf{x}_{m_i,k} \longrightarrow \mathbf{x}_{m_j,k+1}$.

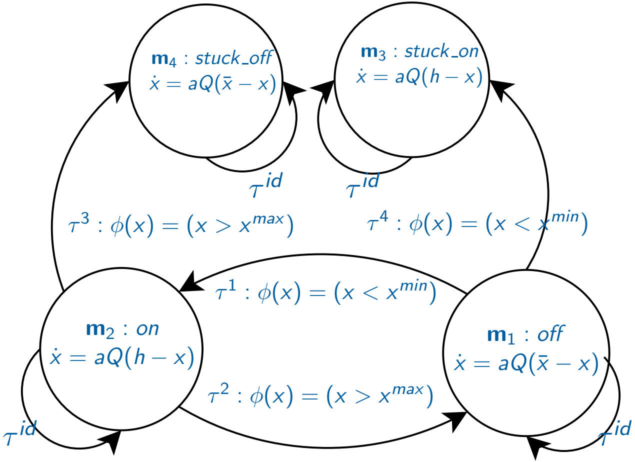


Figure: Thermostat system modeled as a hybrid system.

Uncertainty

- Most plants operate in uncertain environments (e.g. uncertain conditions & disturbances).
- Process is in general uncertain.
- Complex systems are subjected to malfunctions: there's a non zero chance of a fault occurence at every instant.

Observability

- Most often the hybrid system state remains partially observable only.
- Sensors are noisy (a little bit).
- Modeling imposes a level of abstraction: certain switches are not directly observable.

Filtering

Filtering, or *state estimation* is the operation that reconstructs the whole hybrid state based on a stream of measurements and the model of the system.
The *estimated* hybrid state at time-step k is noted $\hat{\mathbf{x}}_k$.

Filtering under uncertainty

- Modern algorithms must cope with uncertainty.
- Two main representations of uncertainty:
  - Probabilities, through Bayesian update.
  - Bounded sets, through novel methods.

State Estimation of hybrid system

The dominant hybrid filtering scheme employs a stochastic representation, for different sets of methods:

- Multi-model filtering: IMM, ...
- Particle filtering: RBPF, ...

Basically: apply a Bayesian belief update. Estimated state: $\hat{\mathbf{x}}_k = (\hat{\mathbf{x}}_{m,k}, \mathbf{p}_{c,k})$ where $\mathbf{p}_{c,k}$ a multivariate distribution.

Why is it dominant ?

- Stochastic estimation converges !
- Easy to implement. Many algorithms for exact and approximated filtering.
- Social factor: dominates because... it dominates !

Drawbacks of the dominant filtering scheme #1

A blowup in the number of state estimates, also called hypotheses, is inevitable.

- Need to track every possible mode sequence.
- Exponential blowup is taken for granted. Treated as a *natural* problem !!
- Has appeared as such in the literature for over 20 years !
- In consequence, most stochastic hybrid filters yield an approximation of the true (modeled) state of the system.

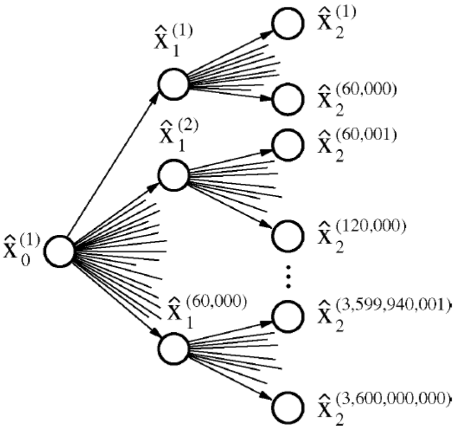


Figure: Blowup in the number of estimates: some states have identical discrete estimate but cannot be merged without statistical loss !

Drawbacks of the dominant filtering scheme #2

- State estimates as a probability distribution with infinite tail over the continuous state (e.g. Gaussian & Kalman filtering):
  - Real information about a system in general yield bounded values...
  - The computational properties of the functional description of the a priori knowledge obscure the real a priori information !
- Most algorithms truncate distributions with precise thresholds on low precision values:
  - Bayesian update of a truncated Gaussian does yield a (truncated) Gaussian...
  - In consequence, the reliability of the produced results can be questionned !

Systems with Faults

- Stochastic modeling of faults often relies on an a priori knowledge about occurences of faults that have never been observed !
  - The literature has produced a plethora of algorithms that apply a rigorous Bayesian update to a priori values !
- The blowup is particularly intractable when the hybrid system represents faults as discrete switches that may occur at anytime.
- Infinite tails add up to the exponential blowup !

Suggests that probability distributions are not a proper representation of uncertainty for our application to hybrid models with faults.

Bounded representation of uncertainty

Mitigating the ambiguity over the system state that plagues the stochastic filters recommends a bounded representation of uncertainty as adopted in set-theoretic approaches.

Advantages of bounded uncertainty

- Guaranteed results: avoid false positive, popular in application fault detection and monitoring.
- Most importantly: allows to circumvent the blowup in estimates !
  - Estimates with identical discrete state can be merged with no loss of information.

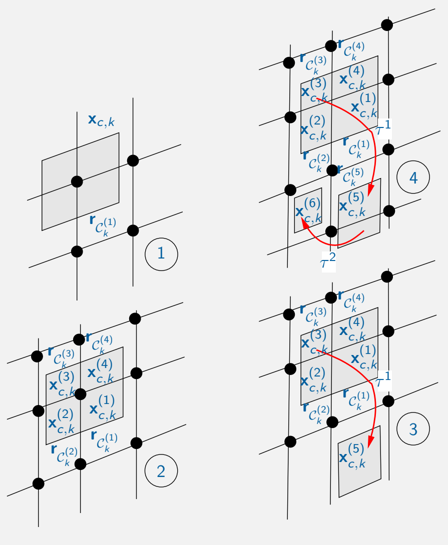
At what cost ?

- Recursive computation of convex bounds suffers from the well-known *wrapping-effect*: requires the costly computation of tight bounds.
- Multiple incident parameters: necessitates estimating over a sliding window.

Set-Theoretic Estimation

A six steps process:

1. Continuous state prediction. Can be approximated by a variety of geometrical shapes (ellipsoids, rectangles, polytopes).
2. Partition of the continuous state estimates according to the switches in the discrete dynamics.
3. Discrete state prediction from each of the partition cells.
4. Transfer of the continuous state.
5. Merging of estimates with the same predicted discrete state.
6. Using observations for pruning the impossible estimates.



$\mathbf{x}_{c,k}$
$\mathbf{r}_{\mathcal{C}_k^{(1)}}$
1
$\mathbf{r}_{\mathcal{C}_k^{(3)}}$
$\mathbf{r}_{\mathcal{C}_k^{(4)}}$
$\mathbf{x}_{c,k}^{(3)}$
$\mathbf{x}_{c,k}^{(4)}$
$\mathbf{x}_{c,k}^{(1)}$
$\mathbf{x}_{c,k}^{(2)}$
$\mathbf{r}_{\mathcal{C}_k^{(1)}}$
$\mathbf{r}_{\mathcal{C}_k^{(2)}}$
$\mathbf{r}_{\mathcal{C}_k^{(5)}}$
$\mathbf{x}_{c,k}^{(5)}$
$\mathbf{x}_{c,k}^{(6)}$
τ^1
τ^2
4
2
3

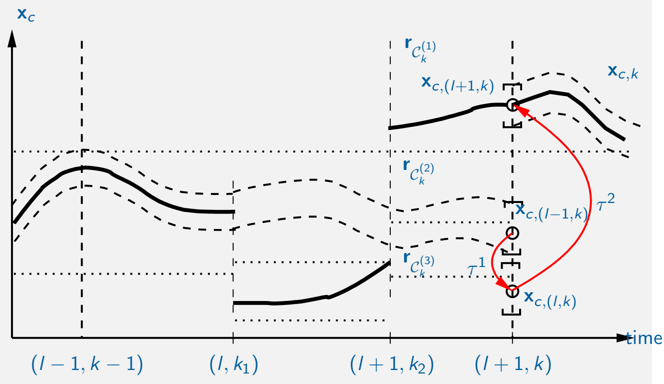


Figure: Multiple switches lead up to time step $(l+2,k)$, making up for fast switches at (l,k_1), $(l+1,k_2)$ between two physical time steps: fast discrete dynamics is reconstructed. Required assumption: continuous behavior is piecewise monotonous.

Experimental Results
Thermostat Example

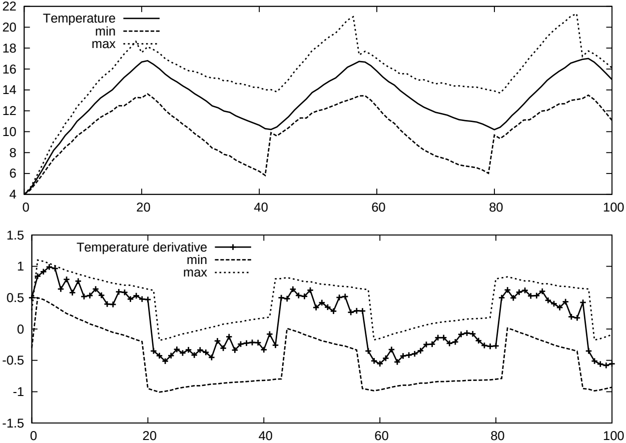


Figure: Thermostat example. Temperature change is observed, temperature is estimated.

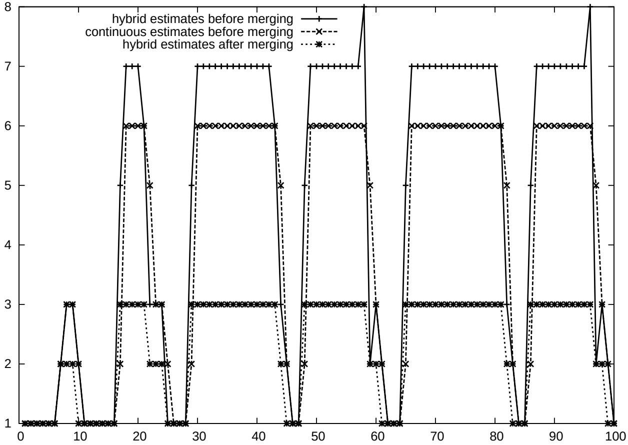


Figure: Number of estimates before and after the merging step.

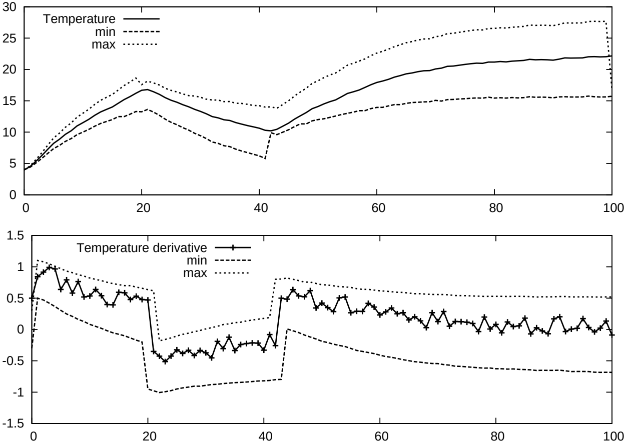


Figure: Thermostat gets stuck on around step 40.

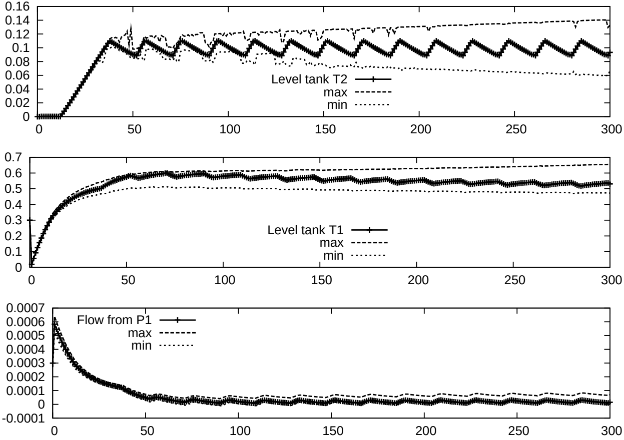
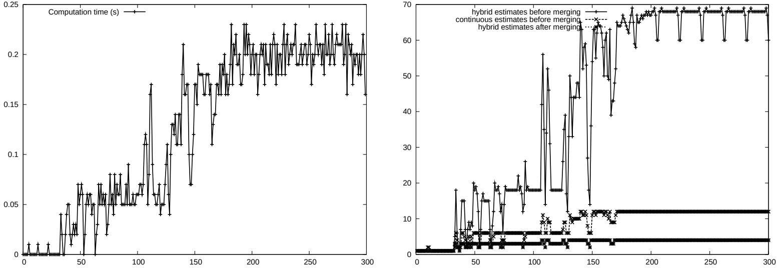


Figure: 2 reservoirs: nominal behavior.

Experimental Results

General Performances



(a) Computation time per step.

(b) Number of estimates before and after the merging step.

Figure: 2 reservoirs: general performances. System has a total of 1350 possible states but tracks 5% of them.

In Summary #1

- Computational burden of stochastic hybrid filters comes from the need for tracking an elevated number of estimates, whereas that of set-theoretic estimation lies in the computation of tight bounds.

In Summary #2

- Set-theoretic estimation of hybrid systems is more complicated than its stochastic counterpart:
  - Framework is less compact,
  - Geometrical shapes require more effort than functional probability distributions.
- Set-theoretic estimation of hybrid system is powerful:
  - Prevent the blowup in the number of estimates,
  - Uses and processes real a priori information.
  - Strictly bounds variable values for fault detection.
- The future lies in mixed set-based/probabilistic representations and the building of dedicated inference techniques.