SAUC'$\mathbb{I}$SSE : our interval underwater robot

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ENSIETA, Brest

Girona, monday, july 30, 2007

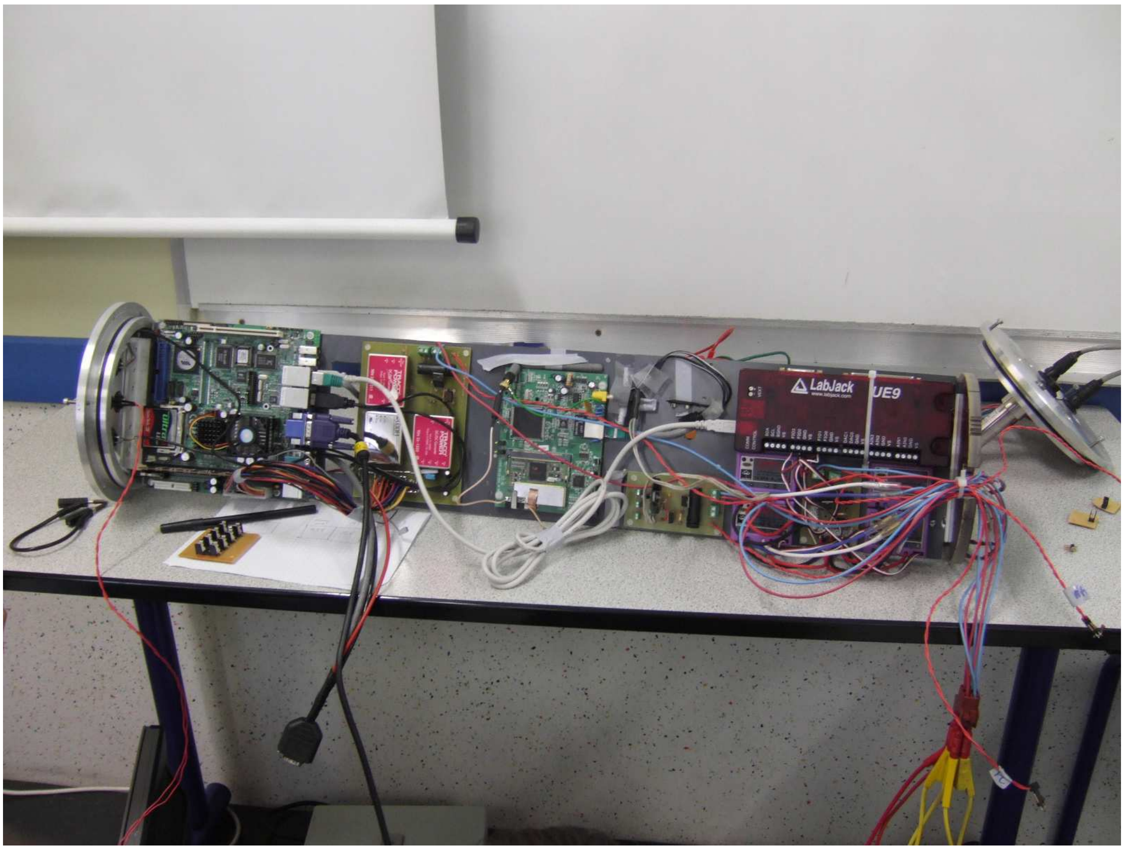
1 SAUC'E competition

In Portsmouth, July 12-15, 2007.

1.1 Principle

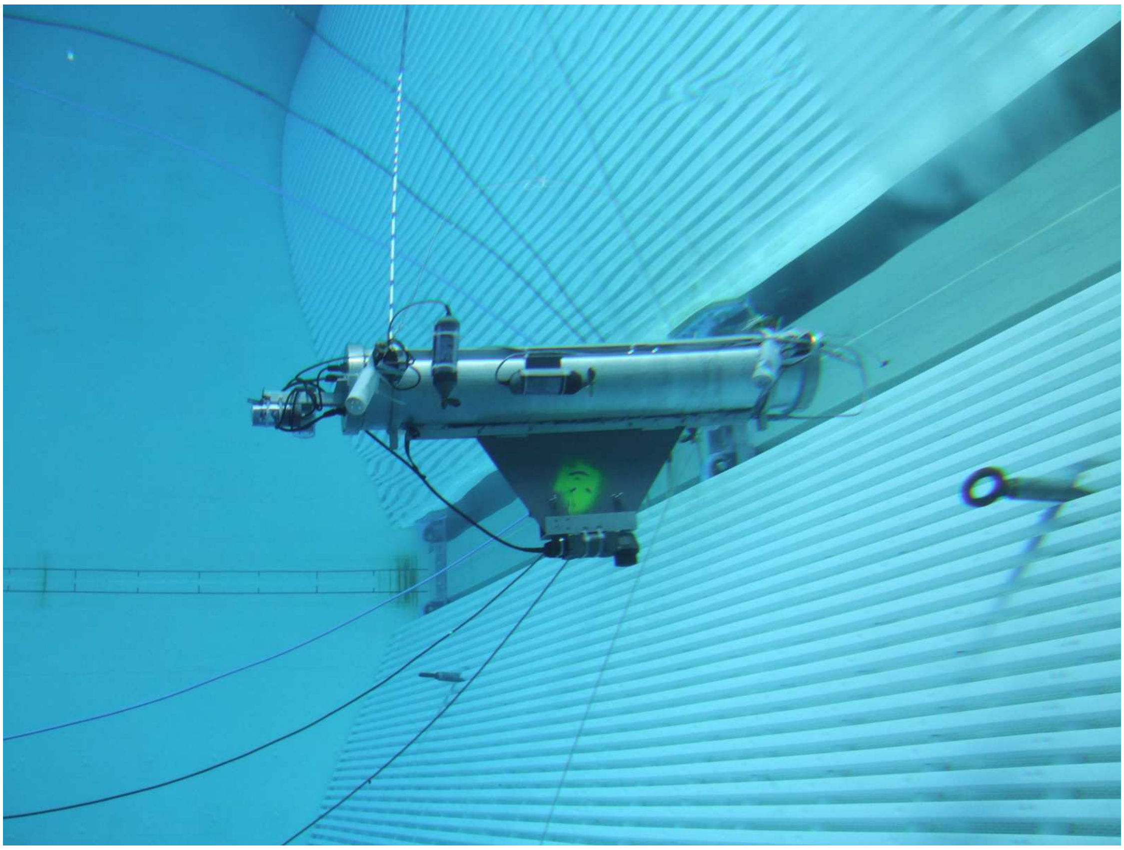
1.2 SAUC'ISSE

(show the movie)

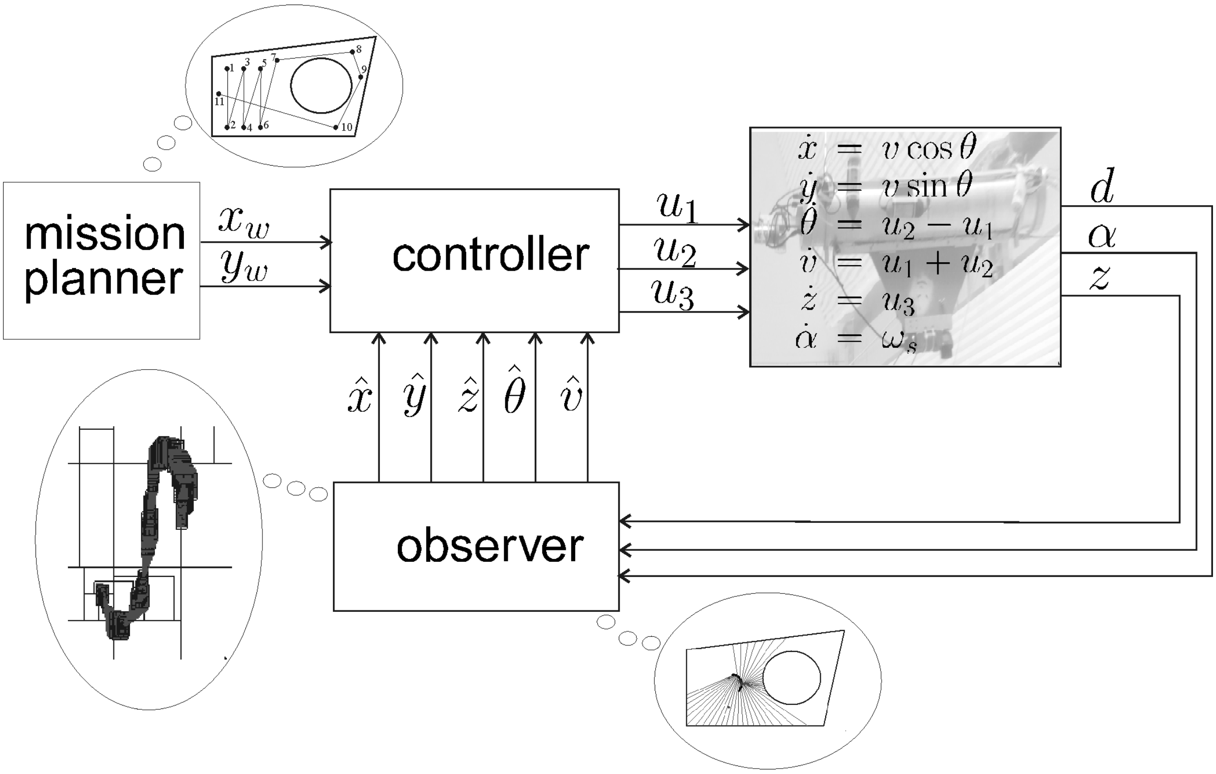


LabJack
UE9


AR
HAUT



1.3 Localization and control


mission planner
x_w
y_w
controller
u_1
u_2
u_3
$\dot{x} = v\cos\theta$
$\dot{y} = v\sin\theta$
$\dot{\theta} = u_2 - u_1$
$\dot{v} = u_1 + u_2$
$\dot{z} = u_3$
$\dot{\alpha} = \omega_s$
d
α
z
$\hat{x}$ $\hat{y}$ $\hat{z}$ $\hat{\theta}$ $\hat{v}$
observer

1.4 Model

State equations

$$\left\{ \begin{array}{lcc} \dot{x} & = & v \cos \theta \\ \dot{y} & = & v \sin \theta \\ \dot{\theta} & = & u_2 - u_1 \\ \dot{v} & = & u_1 + u_2 - v \\ \dot{\alpha} & = & \omega_s \end{array} \right.$$

Observation equation:

input : (x, y, θ, α)

$\tilde{\mathbf{u}} = (\cos(\theta+\alpha);\sin(\theta+\alpha))$; $\ell = \infty$;

$\mathbf{m} = (x\ y)^{\mathsf{T}}$;

for $j = 1$ to n

  if $\det\left(\mathbf{a}_j - \mathbf{m}, \tilde{\mathbf{u}}\right) . \det\left(\mathbf{b}_j - \mathbf{m}, \tilde{\mathbf{u}}\right) \geq 0$ then next j;

  $d := \frac{\det(\mathbf{m}-\mathbf{a}_j - \mathbf{m}, \mathbf{b}_j - \mathbf{a}_j)}{\det(\tilde{\mathbf{u}}, \mathbf{b}_j - \mathbf{a}_j)}$

  if $d < 0$ then next j;

  $\ell := \min(\ell, d)$;

next j

return (ℓ);

2 Set observers

2.1 Principle

Consider the discrete time dynamic system

$$\begin{cases} \mathbf{x}(k+1) & = & \mathbf{f}_k(\mathbf{x}(k)) \\ \mathbf{y}(k) & = & \mathbf{g}(\mathbf{x}(k)) \end{cases}$$

In a bounded-error context, we generally assume that

$$\mathbf{y}(k) \in \mathbb{Y}(k).$$

We can thus define recursively the feasible set $\mathbb{X}(k)$ for $\mathbf{x}(k)$:

(i) $\mathbb{X}(k) = \mathbb{R}^n$, if $k \leq 0$.

(ii) $\mathbb{X}(k+1)$ is the set of all $\mathbf{x}(k+1) \in \mathbb{R}^n$ such that

$$\begin{cases} \exists \mathbf{x}(k) \in \mathbb{X}(k), \\ \exists \mathbf{y}(k) \in \mathbb{Y}(k). \\ \mathbf{x}(k+1) = \mathbf{f}_k(\mathbf{x}(k)) \\ \mathbf{y}(k) = \mathbf{g}(\mathbf{x}(k)) \end{cases}$$

The set $\mathbb{X}(k)$ of all state vectors that are consistent with the past can be computed recursively as follows

$$\mathbb{X}(k+1) = \mathbf{f}_k\left(\mathbb{X}(k)\right) \ \cap \left(\mathbf{f}_k \circ \mathbf{g}^{-1}\right)\left(\mathbb{Y}(k)\right).$$

2.2 Dealing with outliers

To robustify against outliers, we make the following assumption:

Outliers may exist for the outputs but within any time window of length ℓ we never have more than q outliers.

We can thus define recursively the feasible set $\mathbb{X}(k)$ for $\mathbf{x}(k)$:

(i) $\mathbb{X}(k) = \mathbb{R}^n$, if $k \leq 0$.

(ii) and $\mathbb{X}(k+1)$ is the set of all $\mathbf{x}(k+1) \in \mathbb{R}^n$ such that

$$\begin{cases} \exists \mathbf{x}(k-\ell) \in \mathbb{X}(k-\ell), \dots, \exists \mathbf{x}(k) \in \mathbb{X}(k), \\ \exists \mathbf{y}(k-\ell), \dots, \exists \mathbf{y}(k), \\ \bigwedge_{i \in \{0,\dots,\ell\}} \mathbf{x}(k-i+1) = \mathbf{f}_{k-i}(\mathbf{x}(k-i)) \\ \bigwedge_{i \in \{0,\dots,\ell\}} \mathbf{y}(k-i) = \mathbf{g}(\mathbf{x}(k-i)) \\ \bigwedge_{i \in \{0,\dots,\ell\}}^{\{q\}} (\mathbf{y}(k-i) \in \mathbb{Y}(k-i)) . \end{cases}$$

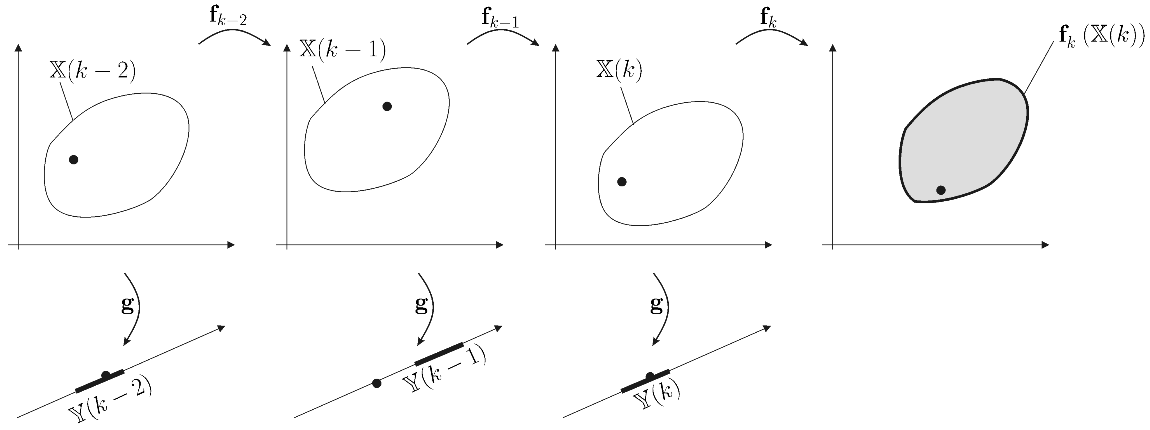
Moreover, we will assume that all $\mathbf{f}_k$ are bijective.

Theorem: The feasible set for the state vector assuming a maximum of q outliers is

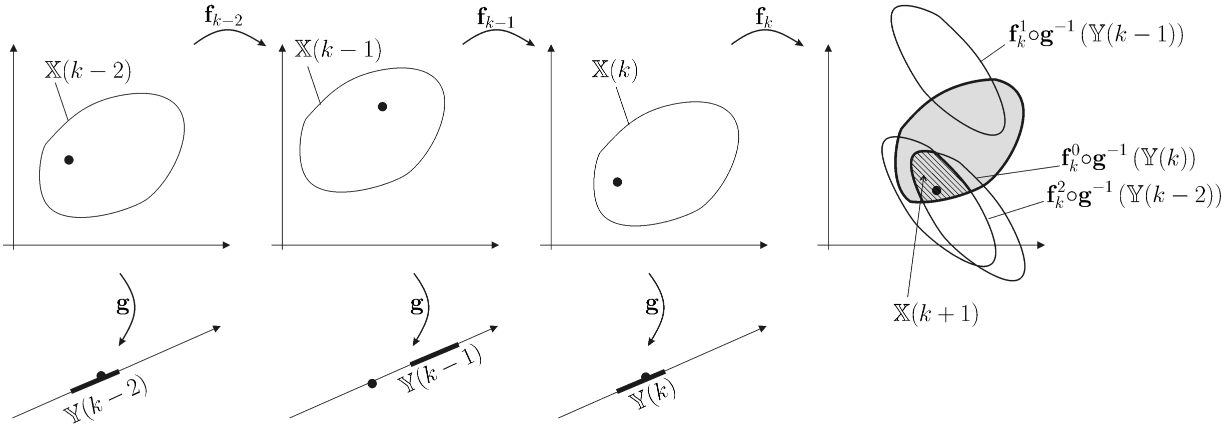
$$\mathbb{X}(k+1) = \mathbf{f}_k\left(\mathbb{X}(k)\right) \ \cap \ \bigcap_{i\in\{0,\ldots,\ell\}}^{\{q\}} \mathbf{f}_k^i \circ \mathbf{g}^{-1}\left(\mathbb{Y}(k-i)\right).$$

where

$$\mathbf{f}_k^i(\mathbf{x}(k-i)) = \mathbf{f}_k \circ \mathbf{f}_{k-1} \circ \ldots \circ \mathbf{f}_{k-i}\left(\mathbf{x}(k-i)\right).$$



$\mathbf{f}_{k-2}$
$\mathbf{f}_{k-1}$
$\mathbf{f}_k$
$\mathbb{X}(k-2)$
$\mathbb{X}(k-1)$
$\mathbb{X}(k)$
$\mathbf{f}_k\ (\mathbb{X}(k))$
$\mathbf{g}$
$\mathbf{g}$
$\mathbf{g}$
$\mathbb{Y}(k-2)$
$\mathbb{Y}(k-1)$
$\mathbb{Y}(k)$



$\mathbf{f}_{k-2}$
$\mathbf{f}_{k-1}$
$\mathbf{f}_k$
$\mathbb{X}(k-2)$
$\mathbb{X}(k-1)$
$\mathbb{X}(k)$
$\mathbf{f}_k^1 \circ \mathbf{g}^{-1}(\mathbb{Y}(k-1))$
$\mathbf{f}_k^0 \circ \mathbf{g}^{-1}(\mathbb{Y}(k))$
$\mathbf{f}_k^2 \circ \mathbf{g}^{-1}(\mathbb{Y}(k-2))$
$\mathbb{X}(k+1)$
$\mathbf{g}$
$\mathbf{g}$
$\mathbf{g}$
$\mathbb{Y}(k-2)$
$\mathbb{Y}(k-1)$
$\mathbb{Y}(k)$

Proof of the theorem: Define

$$\mathbf{f}_k^i(\mathbf{x}(k-i)) = \mathbf{f}_k \circ \mathbf{f}_{k-1} \circ \ldots \circ \mathbf{f}_{k-i}\left(\mathbf{x}(k-i)\right).$$

By applying several times the equivalence

$$(\exists \mathbf{a} \in \mathbb{A}, \mathbf{g}(\mathbf{b}) = \mathbf{a}) \Leftrightarrow \mathbf{g}(\mathbf{b}) \in \mathbb{A}$$

we get that $\mathbf{x}(k+1) \in \mathbb{X}(k+1)$ is equivalent to

$$\left\{ \begin{array}{l} \bigwedge\limits_{i \in \{0,\ldots,\ell\}} \left(\left(\mathbf{f}_k^i\right)^{-1} (\mathbf{x}(k+1)) \in \mathbb{X}(k-i) \right) \\ \{q\} \\ \bigwedge\limits_{i \in \{0,\ldots,\ell\}} \left(\mathbf{g} \circ \left(\mathbf{f}_k^i\right)^{-1} (\mathbf{x}(k+1)) \in \mathbb{Y}(k-i) \right). \end{array} \right.$$

By applying several times the equivalence

$$\mathbf{g}(\mathbf{b}) \in \mathbb{A} \Leftrightarrow \mathbf{b} \in \mathbf{g}^{-1}\left(\mathbb{A}\right),$$

we get that $\mathbf{x}(k+1) \in \mathbb{X}(k+1)$ is equivalent to

$$\left\{ \begin{array}{l} \bigwedge\limits_{i \in \{0,\ldots,\ell\}} \mathbf{x}(k+1) \in \mathbf{f}_k^i\left(\mathbb{X}(k-\ell)\right) \\ \{q\} \\ \bigwedge\limits_{i \in \{0,\ldots,\ell\}} (\mathbf{x}(k+1)) \in \left(\mathbf{f}_k^i\right) \circ \mathbf{g}^{-1}\left(\mathbb{Y}(k-i)\right). \end{array} \right.$$

Thus

$$\mathbb{X}(k+1) = \bigcap_{i \in \{0,\ldots,\ell\}} \mathbf{f}_k^i \left(\mathbb{X}(k-i)\right) \cap \bigcap_{i \in \{0,\ldots,\ell\}}^{\{q\}} \mathbf{f}_k^i \circ \mathbf{g}^{-1} \left(\mathbb{Y}(k-i)\right)$$

Since, this formula is valid for all k, we get

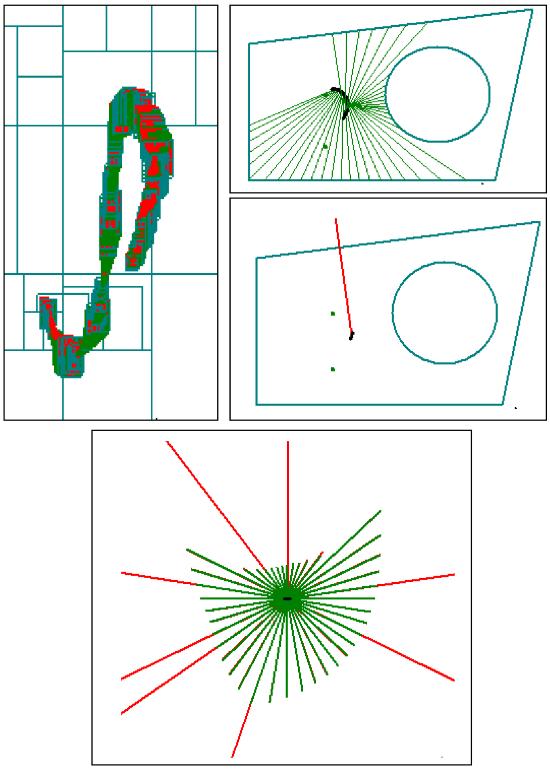
$$\mathbf{f}_k^0 \left(\mathbb{X}(k)\right) \subset \mathbf{f}_k^1 \left(\mathbb{X}(k-1)\right) \subset \mathbf{f}_k^2 \left(\mathbb{X}(k-2)\right) \subset \ldots$$

and thus

$$\bigcap_{i \in \{0,\ldots,\ell\}} \mathbf{f}_k^i \left(\mathbb{X}(k-i)\right) = \mathbf{f}_k^0 \left(\mathbb{X}(k)\right)$$

Finally,

$$\mathbb{X}(k+1) = \mathbf{f}_k^0 \left(\mathbb{X}(k-i)\right) \cap \bigcap_{i \in \{0,\ldots,\ell\}}^{\{q\}} \mathbf{f}_k^i \circ \mathbf{g}^{-1} \left(\mathbb{Y}(k-i)\right).$$



3 Intervals

A random variable x of $\mathbb{R}$ can be represented by an interval $[x]$ such that

$$\mathrm{Supp}\,(p_x) \subset [x].$$

Advantage : The manipulation is much more easy in a nonlinear context.

3.1 Arithmetics

If $\diamond \in \{+, -, ., /\}$, we have

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] \,.$$

For instance,

$$\begin{array}{rl} [-1,3]+[2,5] & = [1,8], \\ [-1,3].[2,5] & = [-5,15], \\ [-1,3]/[2,5] & = [-\frac{1}{2}, \frac{3}{2}], \end{array}$$

3.2 Projection

Consider x, y, z three variables which satisfy

$$\begin{aligned} x &\in [-\infty, 5], \\ y &\in [-\infty, 4], \\ z &\in [6, \infty], \\ z &= x + y. \end{aligned}$$

Values < 2 for x, < 1 for y and > 9 for z are inconsistant.

Since $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ and $z = x + y$, we have

$$\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}$$

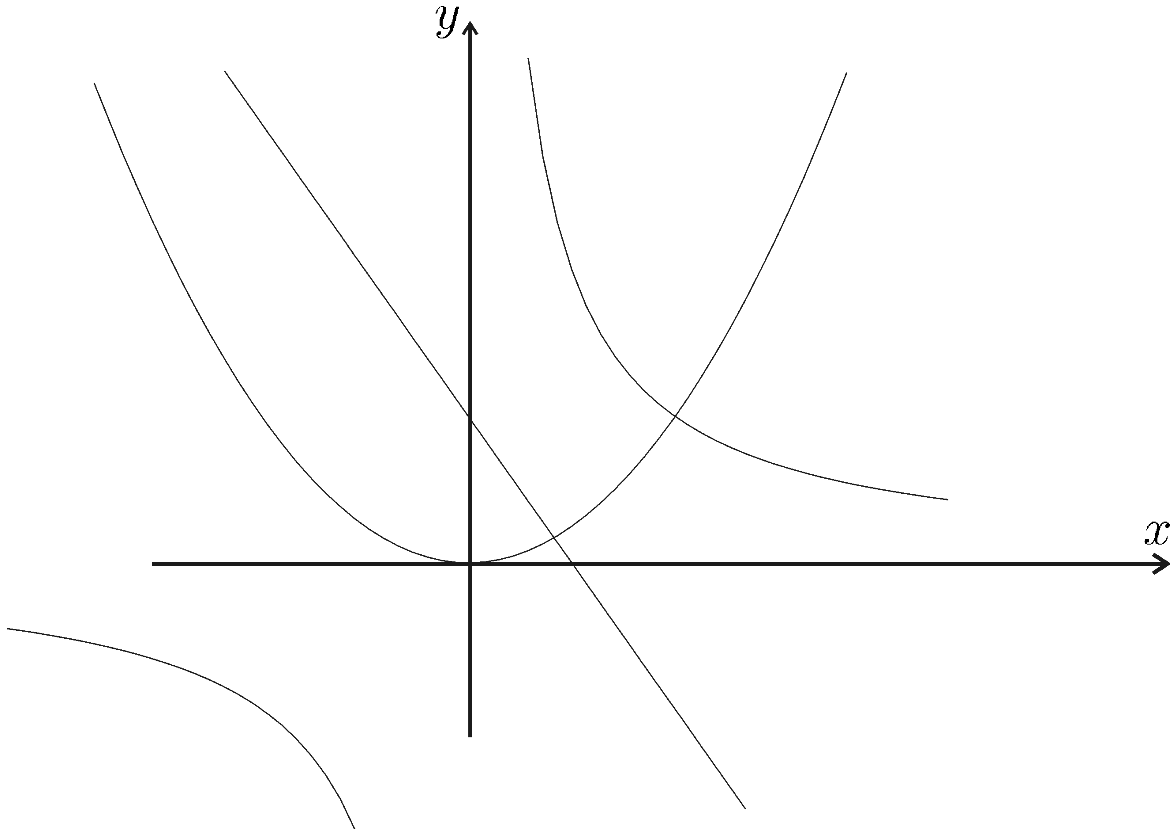
3.3 Propagation

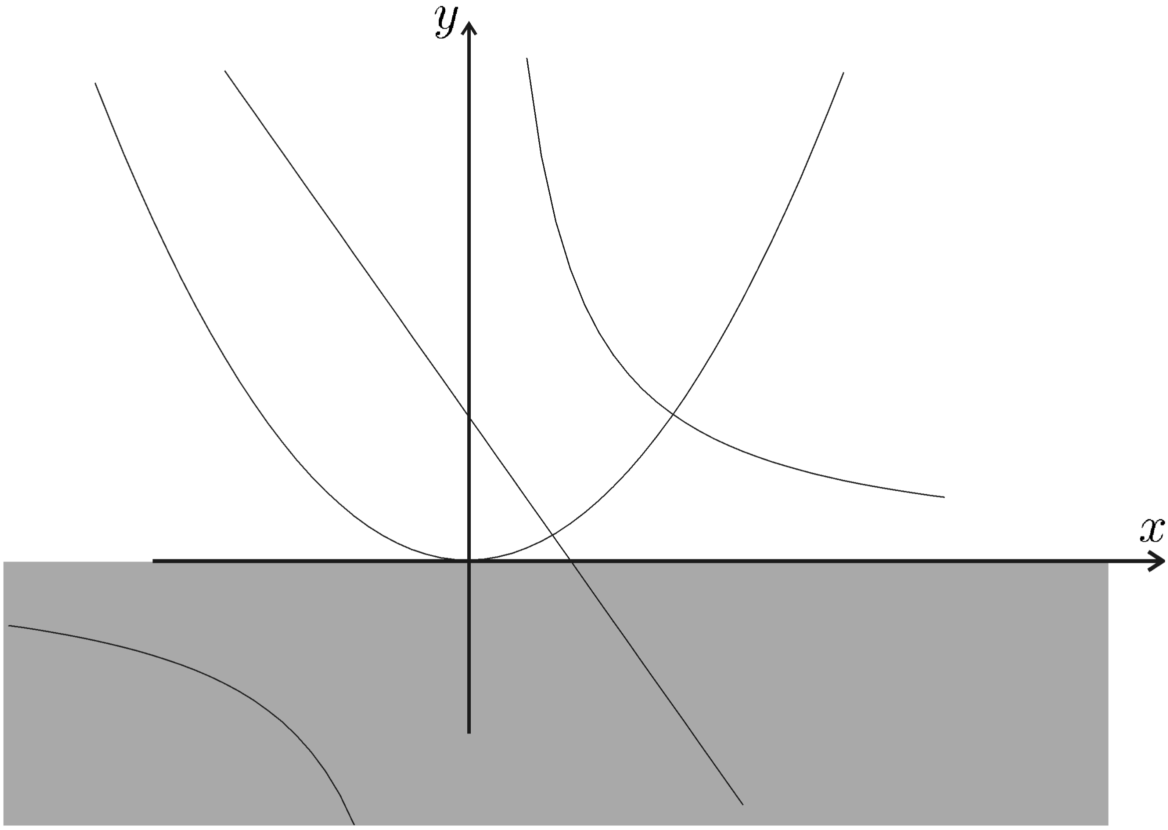
Consider the three constraints

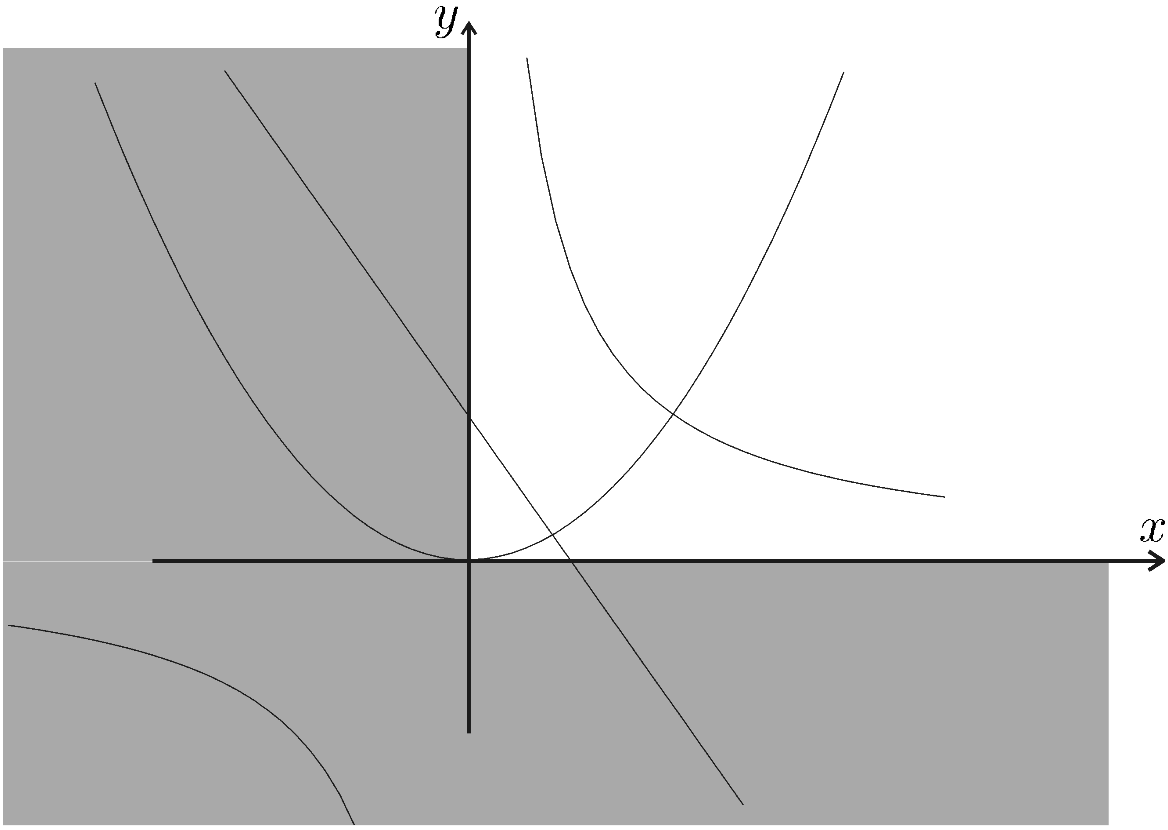
$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$

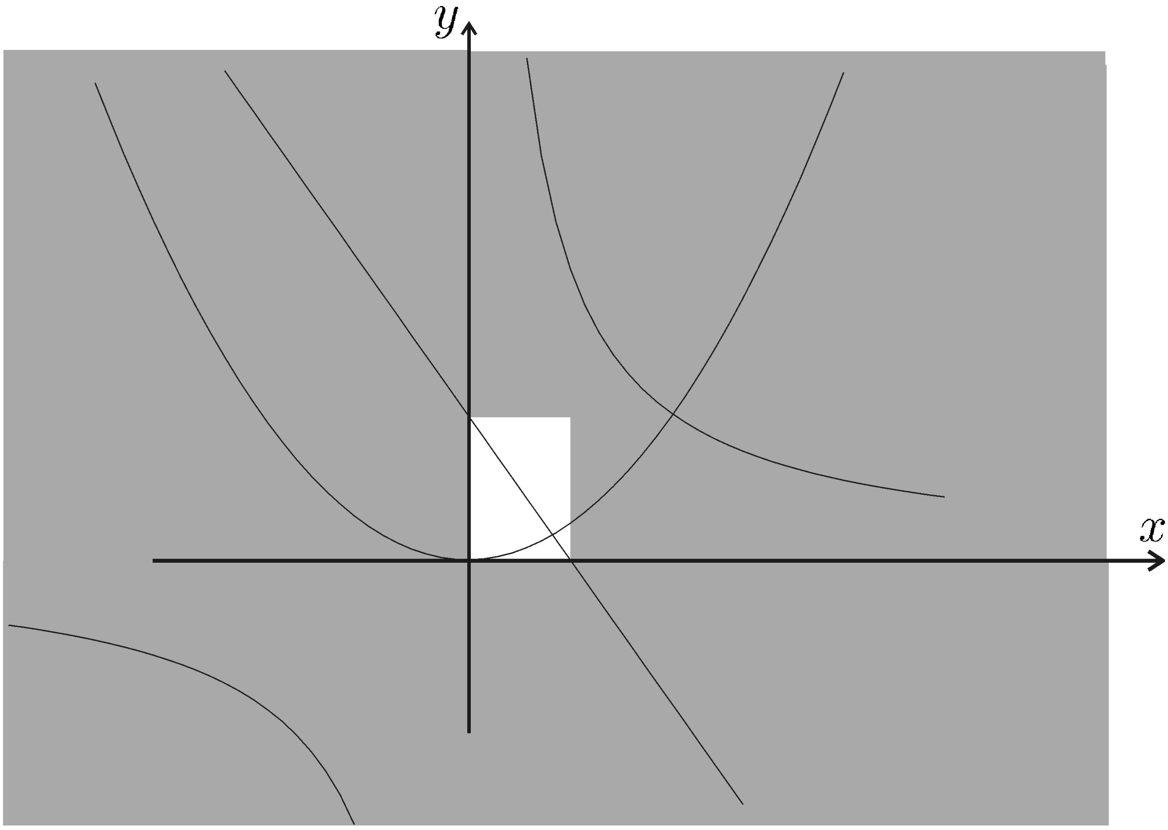
To each variable we assign the domain $[-\infty, \infty]$.

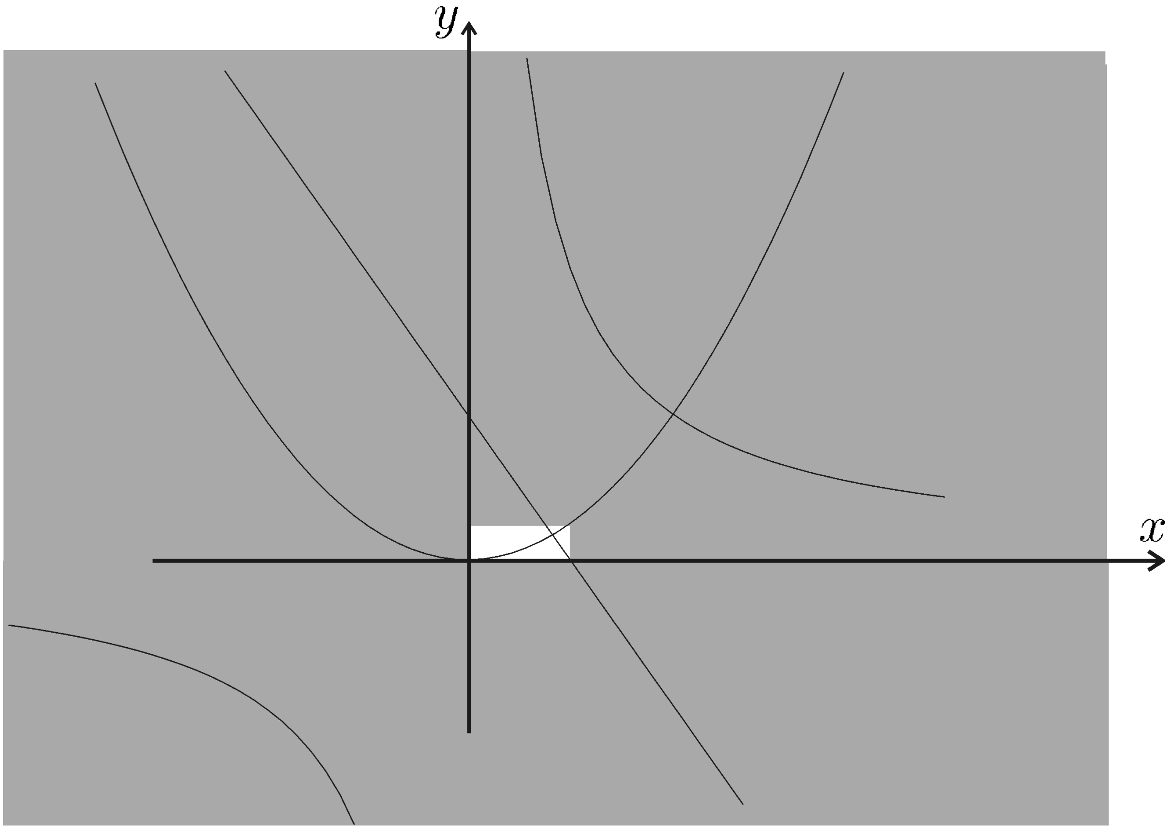
Constraint propagation amounts to project all constraints until equilibrium.


y
x

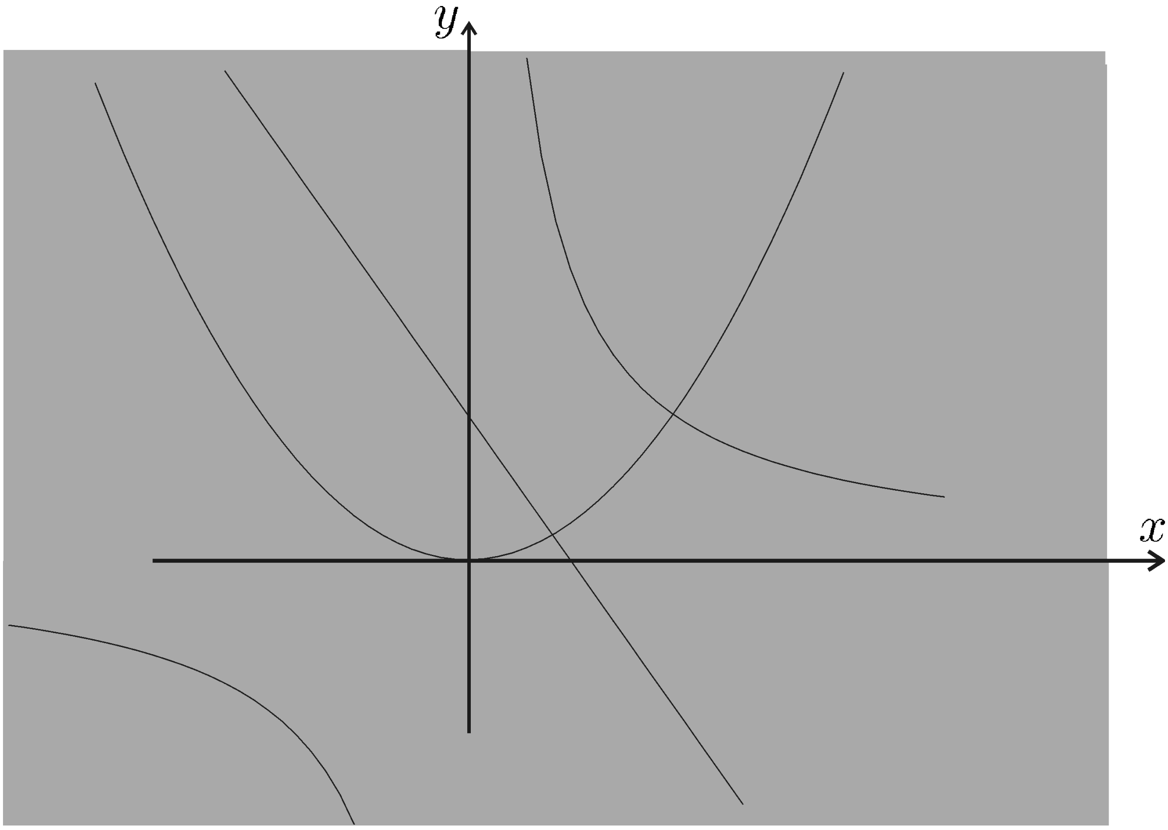

y
x

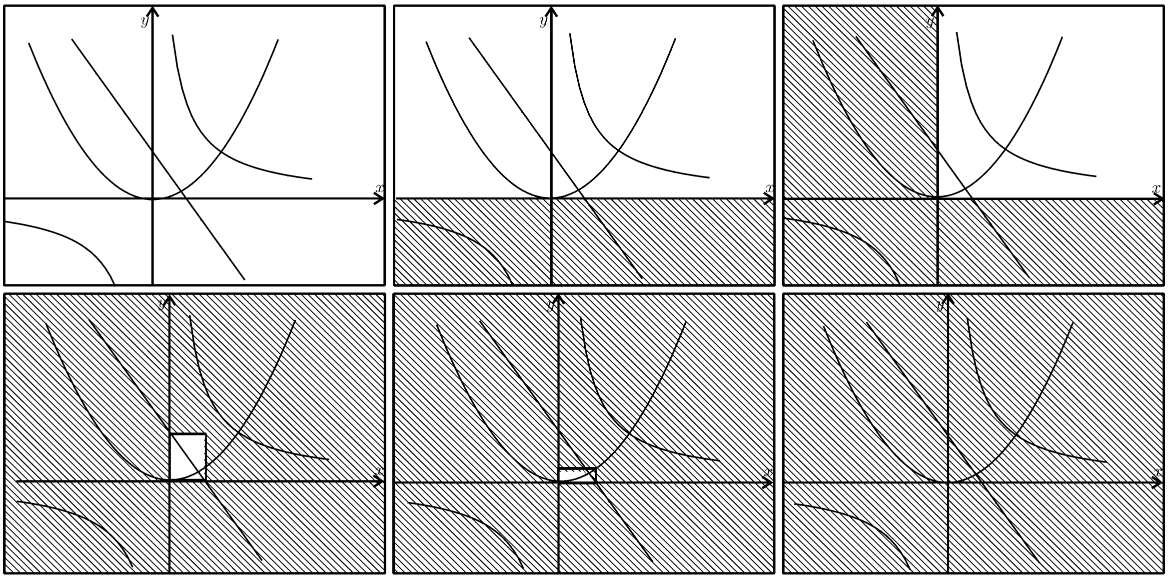

y
x


y
x



y
x


y
x


y
x

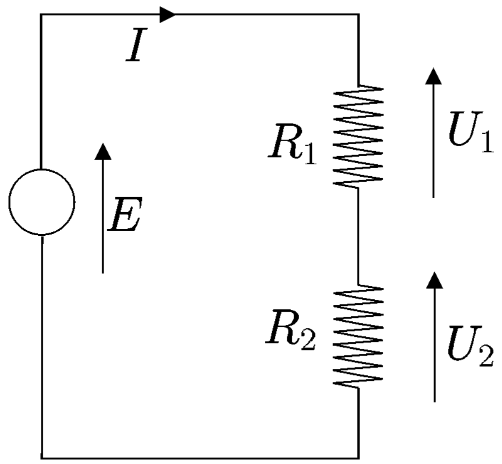
$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty, \infty]^2 \;=\; [0, \infty] \\
(C_2) \Rightarrow & x \in & 1/[0, \infty] \;= [0, \infty] \\
(C_3) \Rightarrow & y \in & [0, \infty] \;\cap\; ((-2)\,.[0, \infty] + 1) \\
 & & = [0, \infty] \;\cap\; ([-\infty, 1]) = [0, 1] \\
 & x \in & [0, \infty] \;\cap\; (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0, 1] \cap [0, 1/2]^2 = [0, 1/4] \\
(C_2) \Rightarrow & x \in & [0, 1/2] \;\cap\; 1/[0, 1/4] = \emptyset \\
 & y \in & [0, 1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

3.4 Bounded-error estimation

One battery and two resistors

Battery : $E = 25V$,

Resistors : $R_1 = 2\Omega, R_2 = 3\Omega$.



Constraints

$$\begin{aligned} P &= EI;\ E = (R_1 + R_2)\, I; \\ U_1 &= R_1 I;\ U_2 = R_2 I;\ E = U_1 + U_2. \end{aligned}$$

Initial domains

$$\begin{array}{ccc} R_1 \in [0, \infty]\Omega & R_2 \in [0, \infty]\Omega & E \in [23, 26]\text{V} \\ P \in [124, 130]\text{W} & I \in [4, 8]\text{A} & \\ U_1 \in [10, 11]\text{V} & U_2 \in [14, 17]\text{V}. & \end{array}$$

Contracted domains

$$\begin{array}{ccc} R_1 \in [\mathbf{1.84}, \mathbf{2.31}]\,\Omega & R_2 \in [\mathbf{2.58}, \mathbf{3.35}]\Omega & E \in [\mathbf{24}, 26]\text{V} \\ P \in [124, 130]\text{W} & I \in [\mathbf{4.769}, \mathbf{5.417}]\,\text{A} & \\ U_1 \in [10, 11]\text{V} & U_2 \in [14, \mathbf{16}]\text{V}. & \end{array}$$

3.5 Contractors

If $\mathbb{X}$ is a subset of $\mathbb{R}^n$, a *contractor* $\mathcal{C}_{\mathbb{X}}$ is an operator such that

$$\begin{array}{cc} \forall[\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(completeness)} \end{array}$$

We shall say that

$\mathcal{C}$ is *monotonic* if	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset \mathcal{C}_{\mathbb{X}}([\mathbf{y}])$
$\mathcal{C}$ is *minimal* if	$\forall[\mathbf{x}] \in \mathbb{IR}^n,\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathbb{X}]$
$\mathcal{C}$ is *idempotent* if	$\forall[\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}(\mathcal{C}_{\mathbb{X}}([\mathbf{x}])) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}])$.
$\mathcal{C}$ is *convergent* if	$\forall \mathbf{x} \notin \mathbb{X}, \exists \varepsilon > 0,$ $(\mathbf{x} \in [\mathbf{x}], w([\mathbf{x}]) < \varepsilon \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = \emptyset)$

If $\mathbb{Z}$ is defined from $\mathbb{X}$ and $\mathbb{Y}$, we can often build $\mathcal{C}_{\mathbb{Z}}$ from $\mathcal{C}_{\mathbb{X}}$ and $\mathcal{C}_{\mathbb{Y}}$:

$\mathbb{Z} = \mathbb{X} \times \mathbb{Y}$	$\mathcal{C}_{\mathbb{Z}}([\mathbf{z}]) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \times \mathcal{C}_{\mathbb{Y}}([\mathbf{y}]),$
$\mathbb{Z} = \mathbb{X} \cap \mathbb{Y}$	$\mathcal{C}_{\mathbb{Z}}([\mathbf{z}]) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathcal{C}_{\mathbb{Y}}([\mathbf{y}]),$
$\mathbb{Z} = \mathbb{X} \cup \mathbb{Y}$	$\mathcal{C}_{\mathbb{Z}}([\mathbf{z}]) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \sqcup \mathcal{C}_{\mathbb{Y}}([\mathbf{y}]).$

3.6 Enclosing sets

The following algorithm generates a subpaving enclosing $[\mathbf{x}] \cap \mathbb{X}$.

Algorithm Enclose(in: $[\mathbf{x}]$, out: $\mathcal{L}$)
$\mathcal{L} := \{[\mathbf{x}]\}$;
while we have time and $\mathcal{L} \neq \emptyset$
    pull $([\mathbf{x}],\mathcal{L})$;
    contract($[\mathbf{x}]$) with respect to $\mathbb{X}$
    if $[\mathbf{x}] \neq \emptyset$, bisect $[\mathbf{x}]$ and push the resulting boxes into $\mathcal{L}$;
end while
return $\mathcal{L}$.

3.7 q-intersection

If

$$\mathbb{X} = ([a] \cap [b]) \cup ([b] \cap [c]) \cup ([a] \cap [c]) = \bigcap^{\{1\}} \{[a], [b], [c]\}.$$

then a minimal contractor for $\mathbb{X}$ can be obtained as follows

Algorithm $\mathcal{C}([x])$
1 $\mathcal{X} = \left\{x^-, x^+, a^-, a^+, b^-, b^+, c^-, c^+\right\}$
2 $\mathcal{V} = \mathcal{X} \cap [x] \cap \mathbb{X}$
3 Return the smallest interval enclosing $\mathcal{V}$.

For instance, if

$$\mathbb{X} = \bigcap^{\{1\}} \{[1,3],[2,7],[6,9]\} \text{ and } [x] = [0,5],$$

we have

$$\mathcal{X} = \{0,5,1,3,2,7,6,9\}$$

and

$$\mathcal{V} = \mathcal{X} \cap [x] \cap \mathbb{X} = \{0,5,1,3,2\} \cap \mathbb{X} = \{3,2\}\,.$$

Thus

$$\mathcal{C}([x]) = [\{3,2\}] = [2,3].$$

3.8 Vector case

We have

$$\begin{aligned}
[\mathbf{x}] \cap [\mathbf{y}] &= ([x_1] \cap [y_1]) \times \cdots \times ([x_n] \cap [y_n])\,, \\
[\mathbf{x}] \cup [\mathbf{y}] &\subset ([x_1] \cup [y_1]) \times \cdots \times ([x_n] \cup [y_n])\,.
\end{aligned}$$

If

$$\mathbb{X} = \bigcap^{\{q\}} \{[\mathbf{a}], [\mathbf{b}], [\mathbf{c}]\} \subset \mathbb{R}^n$$

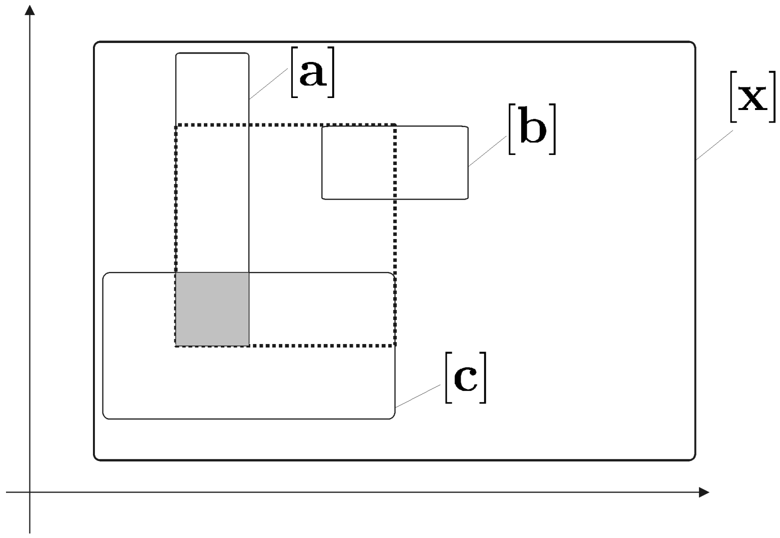
$$\mathbb{X} \subset \underbrace{\left(\bigcap^{\{q\}} \{[a_1], [b_1], [c_1]\} \right)}_{\mathbb{X}_1} \times \cdots \times \underbrace{\left(\bigcap^{\{q\}} \{[a_n], [b_n], [c_n]\} \right)}_{\mathbb{X}_n}$$

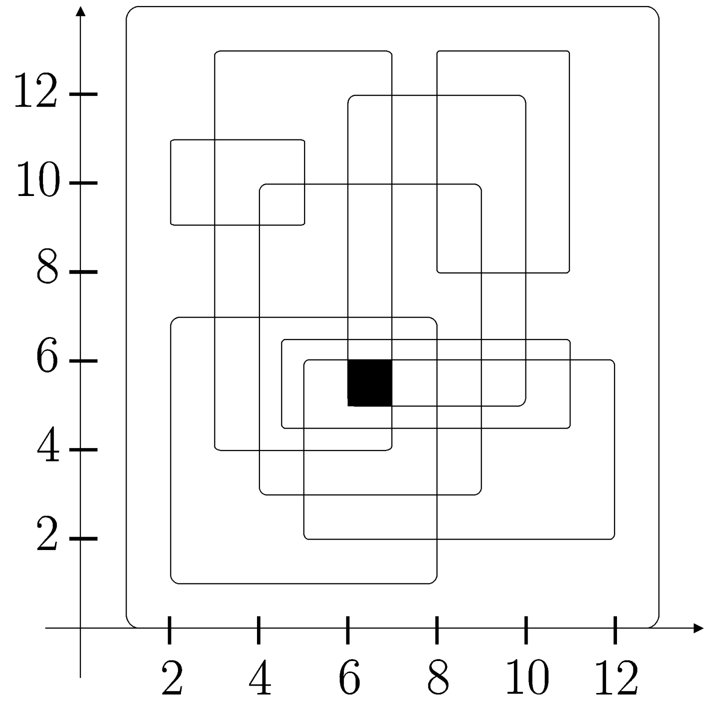
Thus, a contractor for $\mathbb{X}$ is given by

$$\mathcal{C}_{\mathbb{X}}\left([\mathbf{x}]\right) = \mathcal{C}_{\mathbb{X}_1}\left([x_1]\right) \times \cdots \times \mathcal{C}_{\mathbb{X}_n}\left([x_n]\right).$$

Example: Take

$$\mathbb{X} = \bigcap^{\{1\}} \{[\mathbf{a}], [\mathbf{b}], [\mathbf{c}]\}.$$

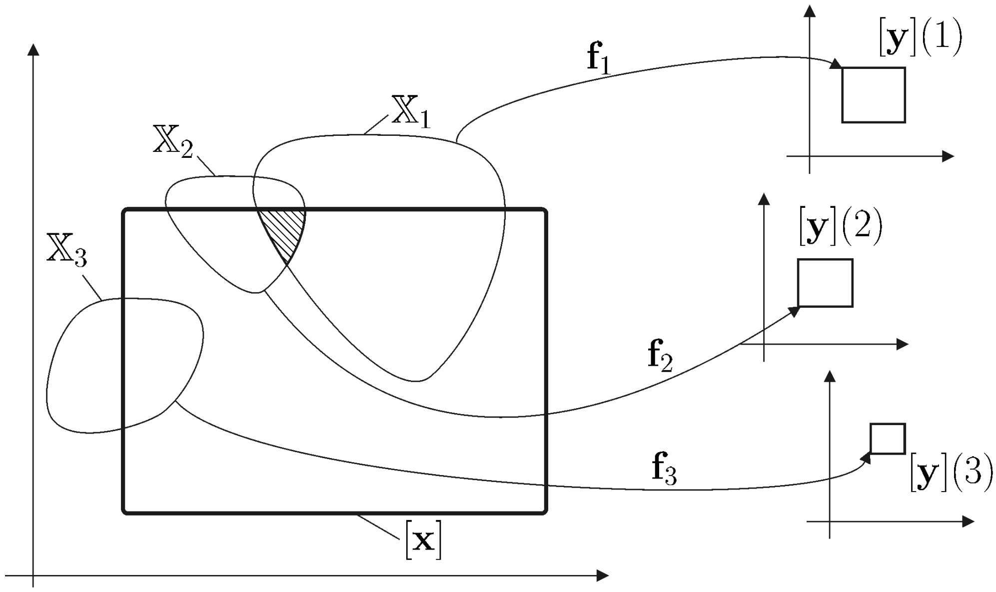




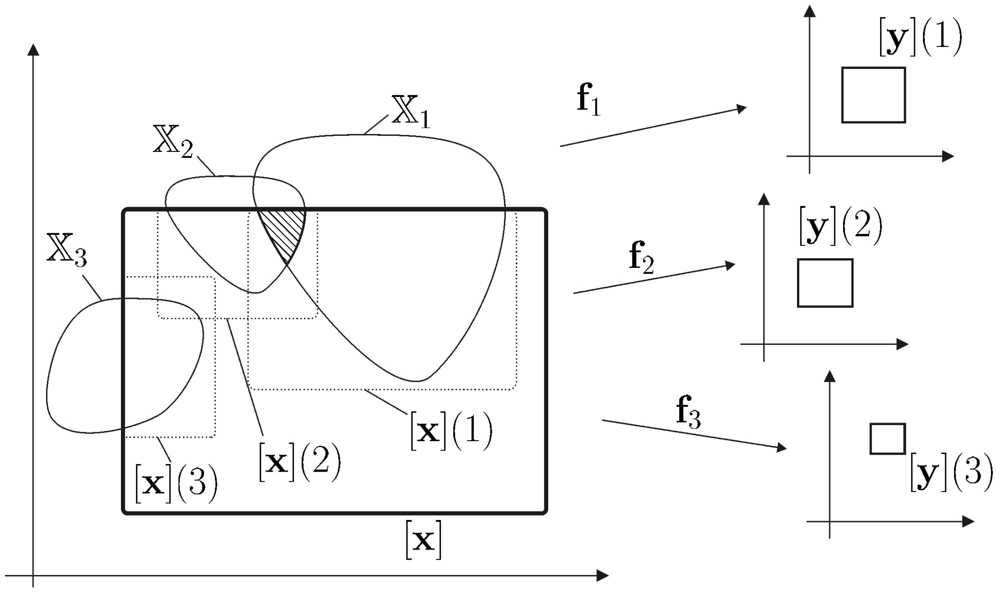
The black box is the 2-intersection of 9 boxes

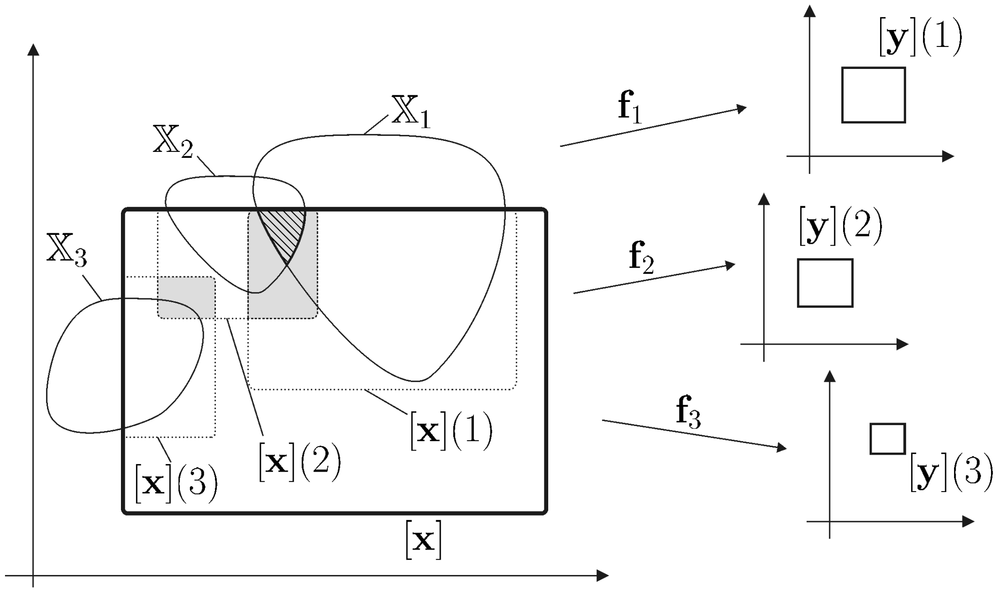
3.9 Relaxed set inversion

$$\mathbb{X} = \bigcap_{i \in \{1,\dots,\ell\}}^{\{q\}} \underbrace{\mathbf{f}_i^{-1}\left([\mathbf{y}](i)\right)}_{\mathbb{X}_i}$$

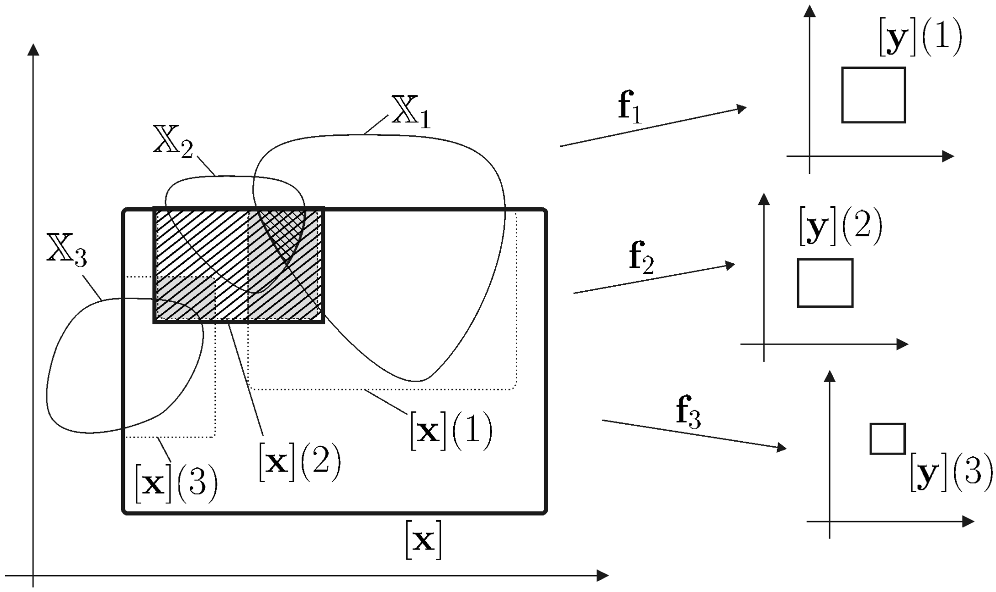


$\mathbf{f}_1$
$[\mathbf{y}](1)$
$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$[\mathbf{y}](2)$
$\mathbf{f}_2$
$\mathbf{f}_3$
$[\mathbf{y}](3)$
$[\mathbf{x}]$


$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$\mathbf{f}_1$
$\mathbf{f}_2$
$\mathbf{f}_3$
$[\mathbf{y}](1)$
$[\mathbf{y}](2)$
$[\mathbf{y}](3)$
$[\mathbf{x}](1)$
$[\mathbf{x}](2)$
$[\mathbf{x}](3)$
$[\mathbf{x}]$



$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$[\mathbf{x}](1)$
$[\mathbf{x}](2)$
$[\mathbf{x}](3)$
$[\mathbf{x}]$
$\mathbf{f}_1$
$\mathbf{f}_2$
$\mathbf{f}_3$
$[\mathbf{y}](1)$
$[\mathbf{y}](2)$
$[\mathbf{y}](3)$


$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$[\mathbf{x}](1)$
$[\mathbf{x}](2)$
$[\mathbf{x}](3)$
$[\mathbf{x}]$
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$\mathbf{f}_3$
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$[\mathbf{y}](2)$
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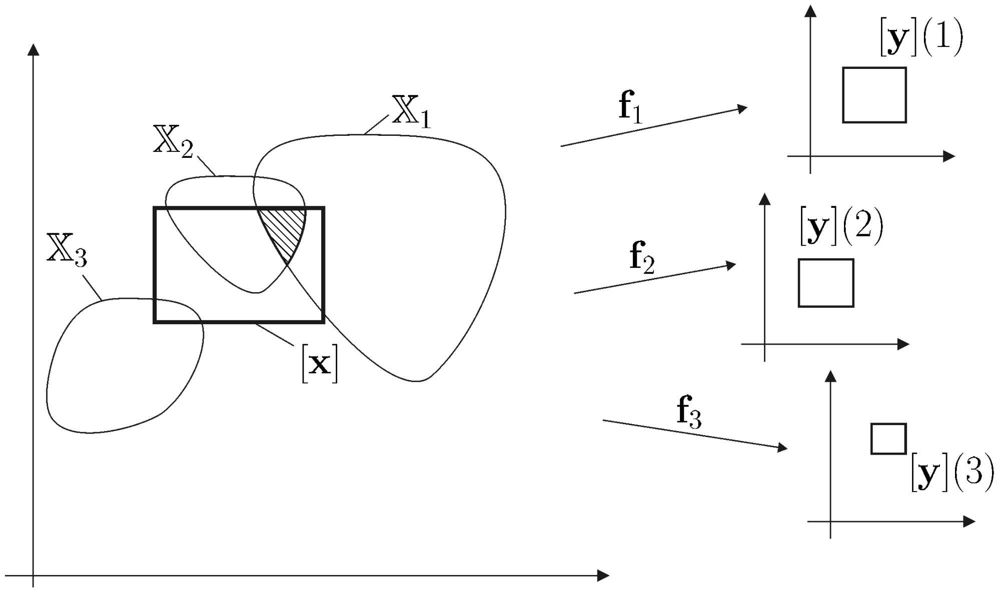

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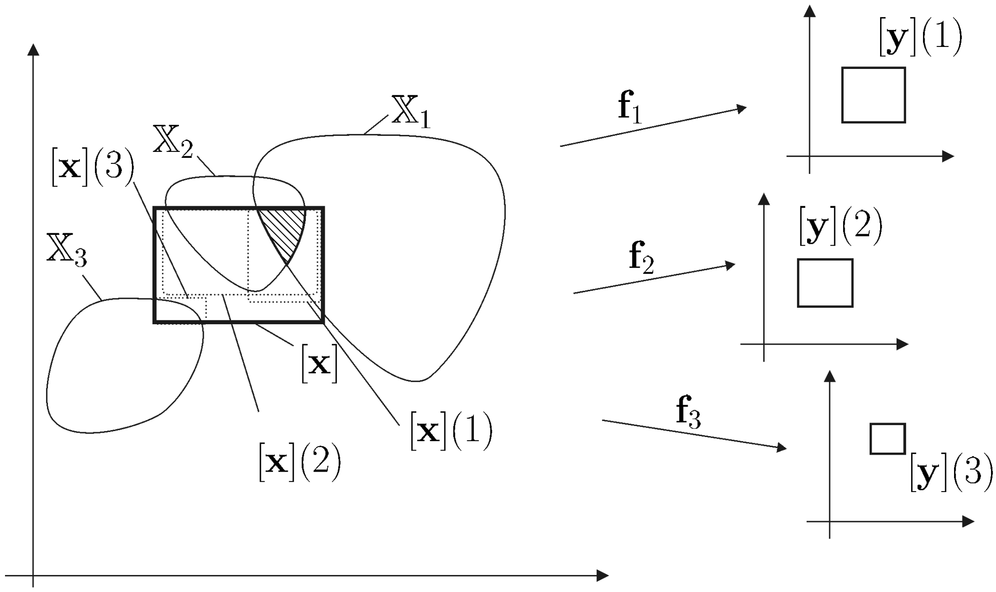
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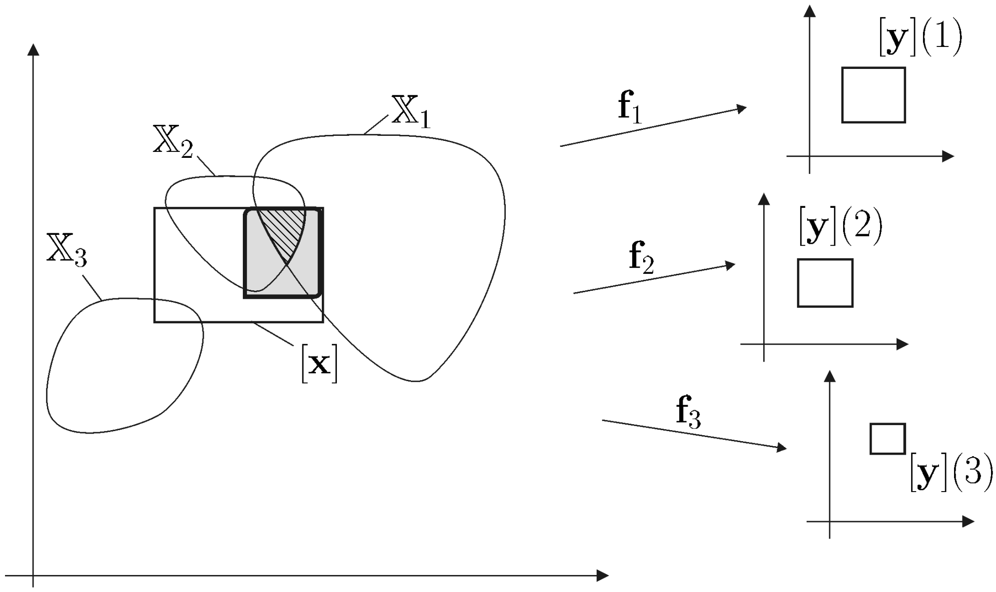

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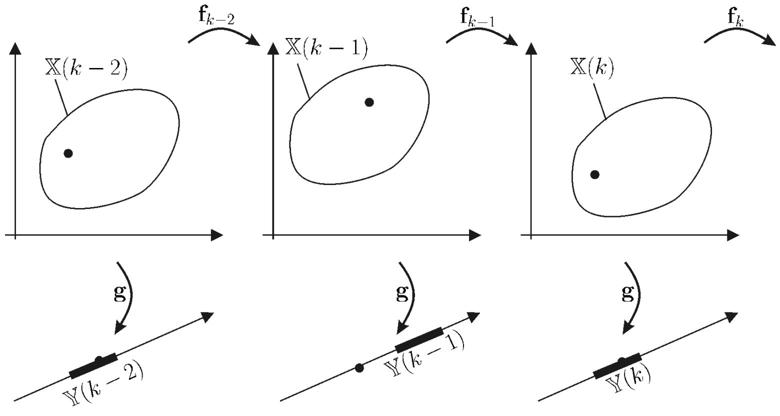
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$\mathbf{f}_{k-1}$
$\mathbf{f}_k$
$\mathbb{X}(k-2)$
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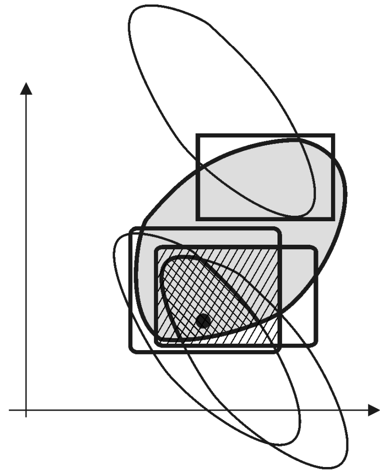
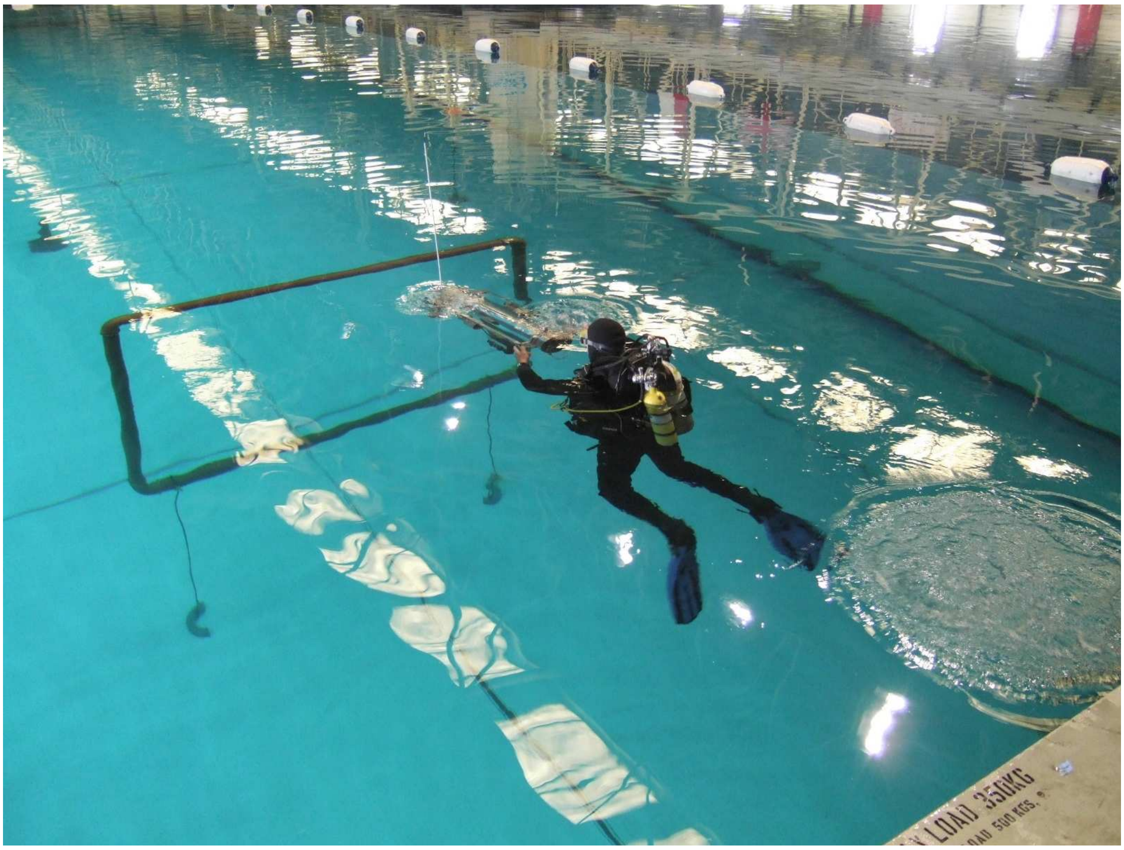


4 Competition


SAUC'ISSE
Ensieta
PILOT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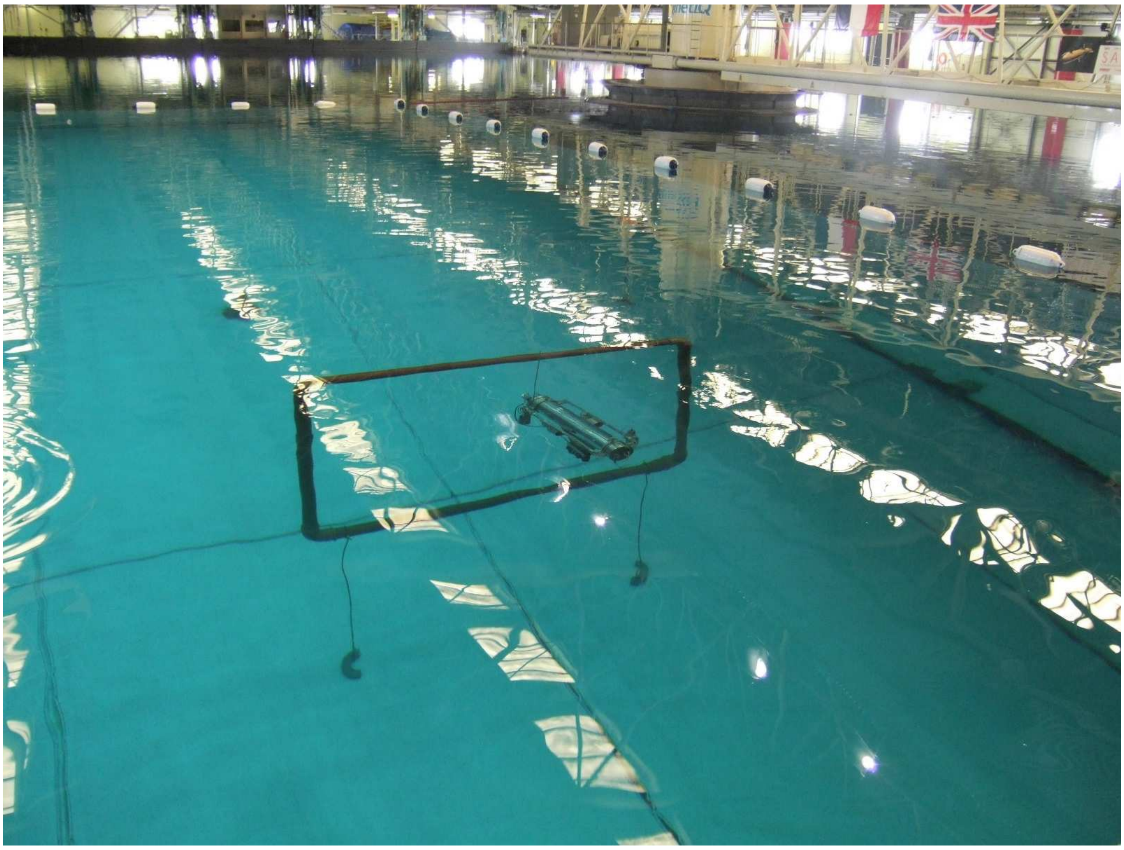
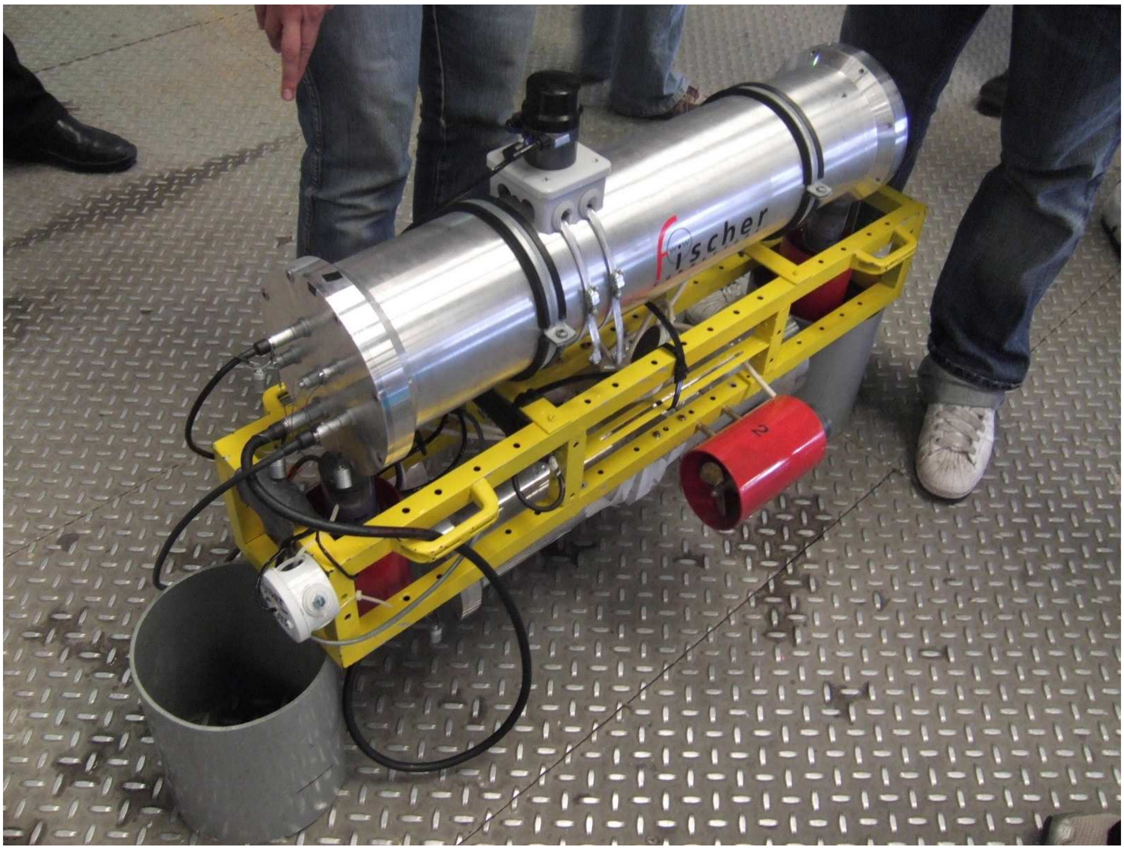
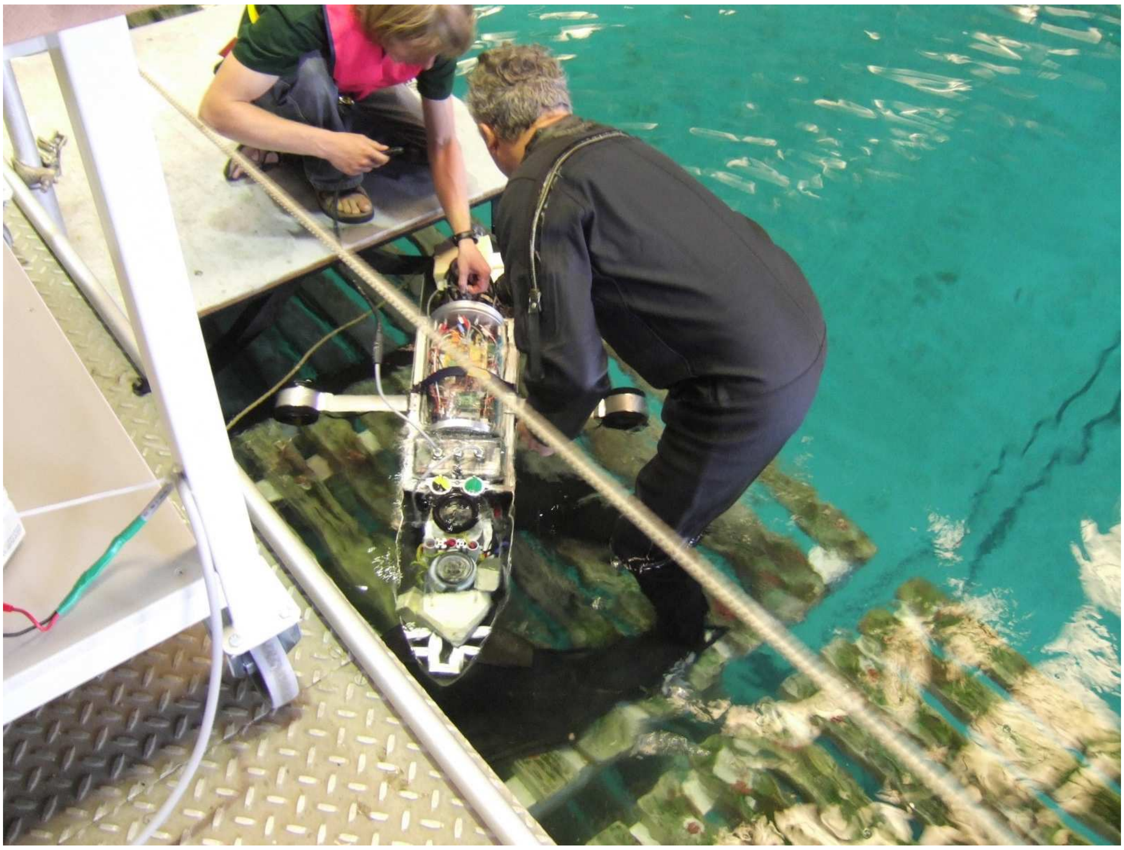
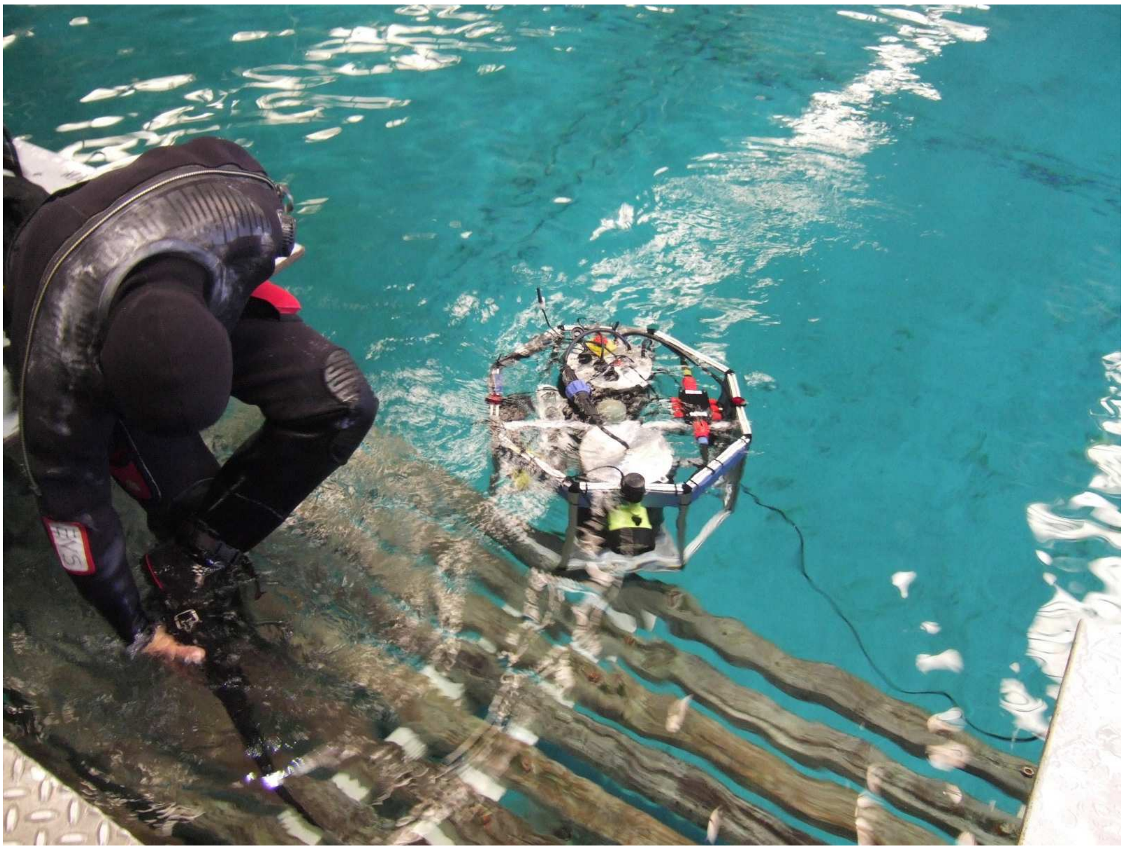



Team Nessie
bp



fischer






REAP