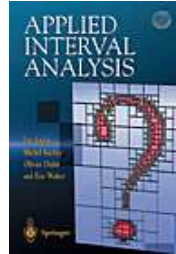


Calcul par intervalles, applications en robotique

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Séminaire au LIP (Lyon)
Lundi 14 mai 2007.

1 Propagation de contraintes sur les intervalles

1.1 Arithmétique sur les intervalles

Si $\diamond \in \{+, -, ., /, \max, \min\}$, on a

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [?, ?], \\ [-1, 3] \cdot [2, 5] &= [?, ?], \\ [-1, 3] / [2, 5] &= [?, ?], \\ [-1, 3] \vee [2, 5] &= [?, ?]. \end{aligned}$$

Si $\diamond \in \{+, -, \cdot, /, \max, \min\}$, on a

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3] \cdot [2, 5] &= [-5, 15], \\ [-1, 3] / [2, 5] &= [-\tfrac{1}{2}, \tfrac{3}{2}], \\ [-1, 3] \vee [2, 5] &= [2, 5]. \end{aligned}$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\begin{aligned}\sin([0, \pi]) &= [\text{?}, \text{?}], \\ \text{sqr}([-1, 3]) &= [\text{?}, \text{?}], \\ \text{abs}([-7, 1]) &= [\text{?}, \text{?}], \\ \text{sqrt}([-10, 4]) &= [\text{?}, \text{?}], \\ \log([-2, -1]) &= [\text{?}, \text{?}].\end{aligned}$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\sin([0, \pi]) = [0, 1],$$

$$\text{sqr}([-1, 3]) = [-1, 3]^2 = [0, 9],$$

$$\text{abs}([-7, 1]) = [0, 7],$$

$$\text{sqrt}([-10, 4]) = \sqrt{[-10, 4]} = [0, 2],$$

$$\log([-2, -1]) = \emptyset.$$

1.2 Projection d'une contrainte

Soient x, y, z trois variables telles que

$$x \in [-\infty, 5],$$

$$y \in [-\infty, 4],$$

$$z \in [6, \infty],$$

$$z = x + y.$$

Les valeurs < 2 pour x , < 1 pour y et > 9 pour z sont inconsistantes.

En effet, puisque $x \in [-\infty, 5]$, $y \in [-\infty, 4]$, $z \in [6, \infty]$ et $z = x + y$, nous avons

$$z = x + y \Rightarrow z \in [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ = [6, \infty] \cap [-\infty, 9] = [6, 9].$$

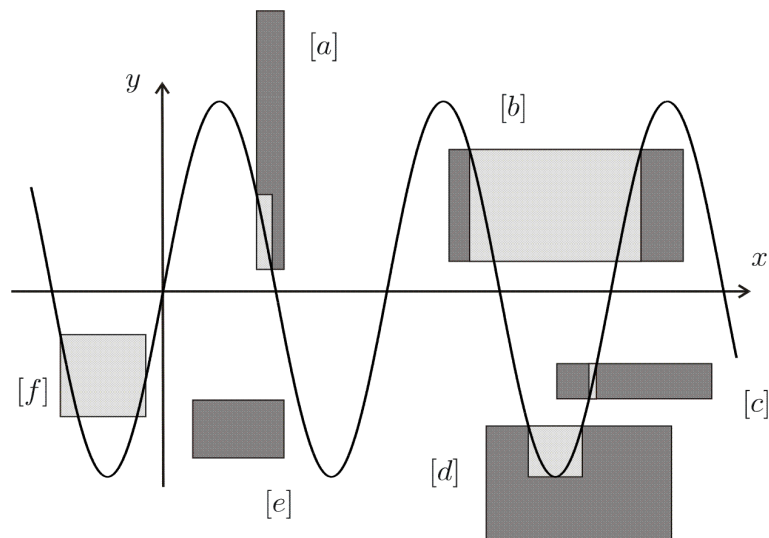
$$x = z - y \Rightarrow x \in [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ = [-\infty, 5] \cap [2, \infty] = [2, 5].$$

$$y = z - x \Rightarrow y \in [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ = [-\infty, 4] \cap [1, \infty] = [1, 4].$$

Pour la contrainte

$$y = \sin x, \quad x \in [x], y \in [y]$$

le problème est un peu plus difficile.



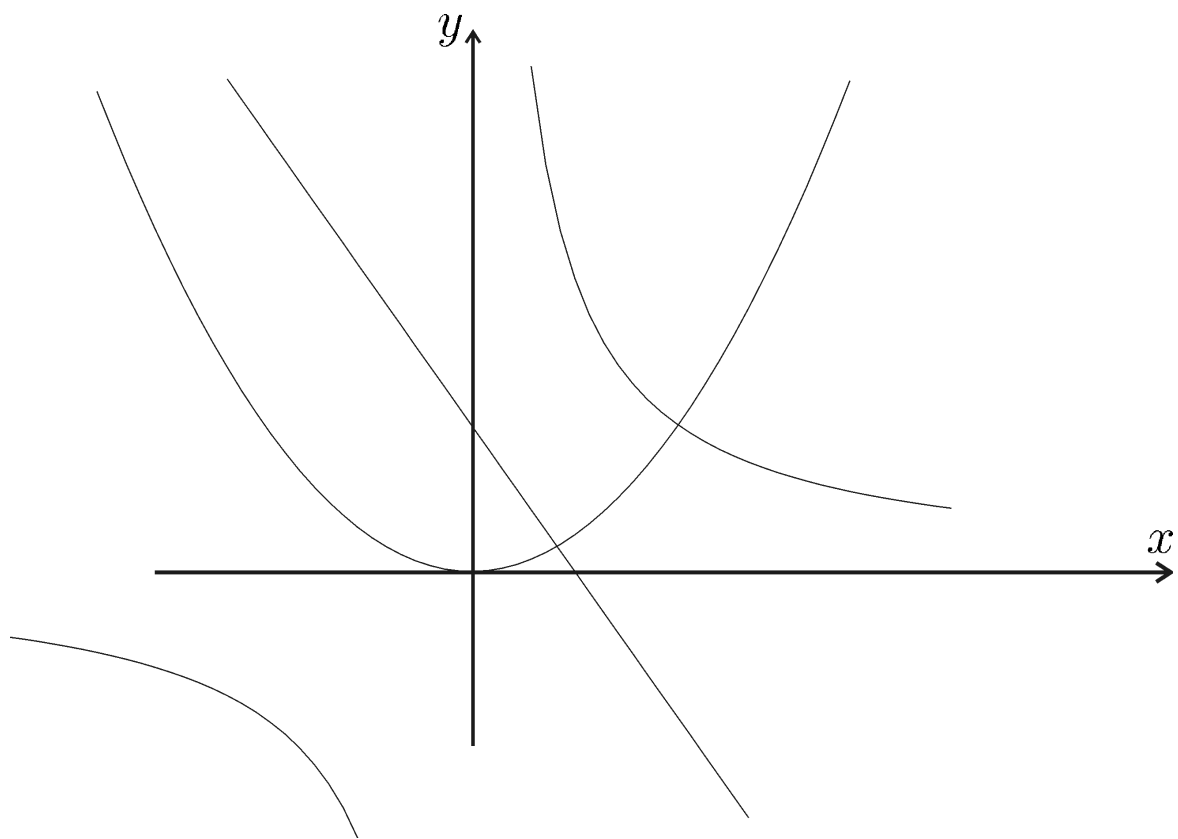
1.3 Propagation

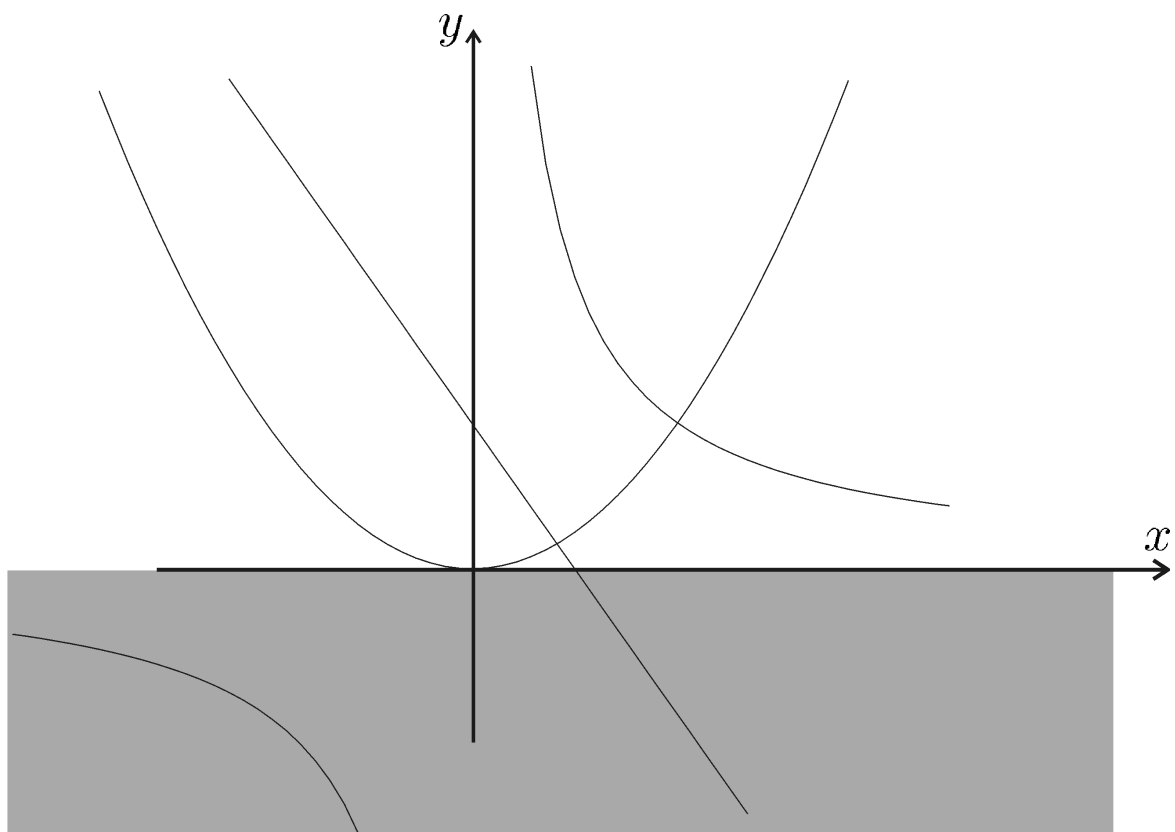
Considérons le problème à trois contraintes

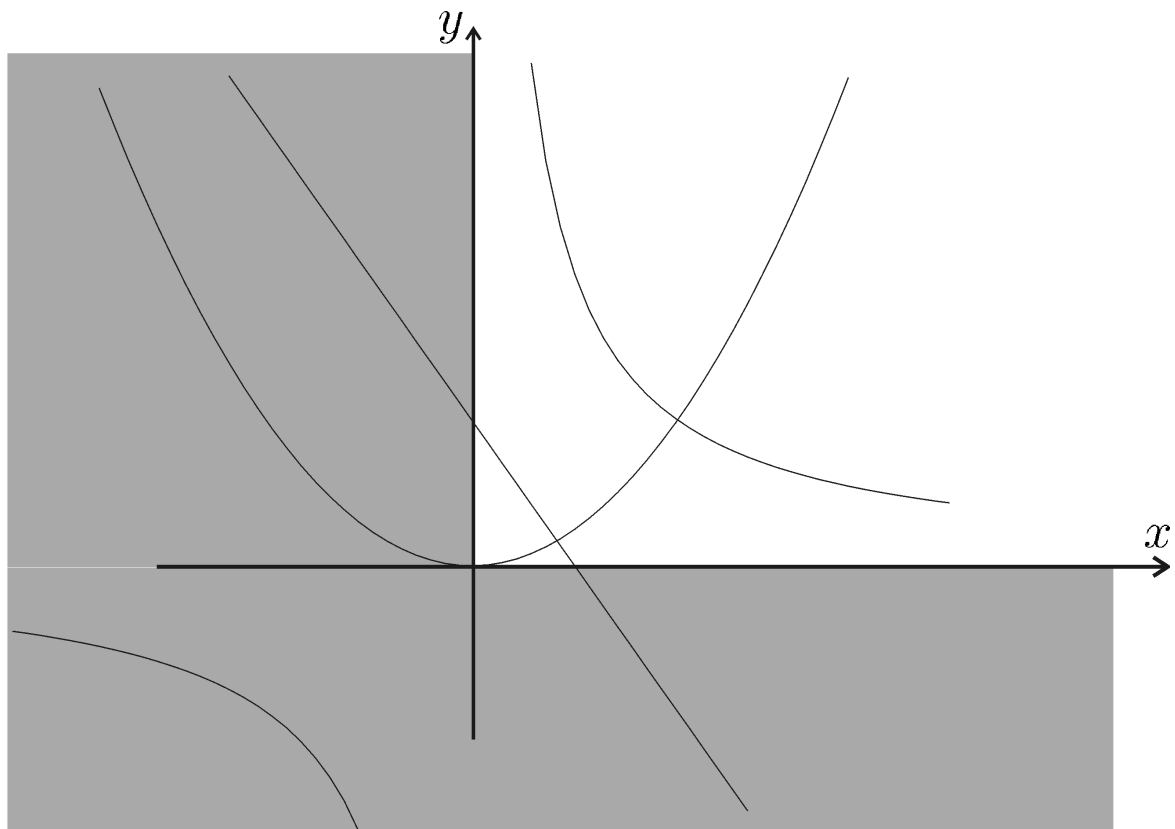
$$\begin{cases} (C_1) : & y = x^2 \\ (C_2) : & xy = 1 \\ (C_3) : & y = -2x + 1 \end{cases}$$

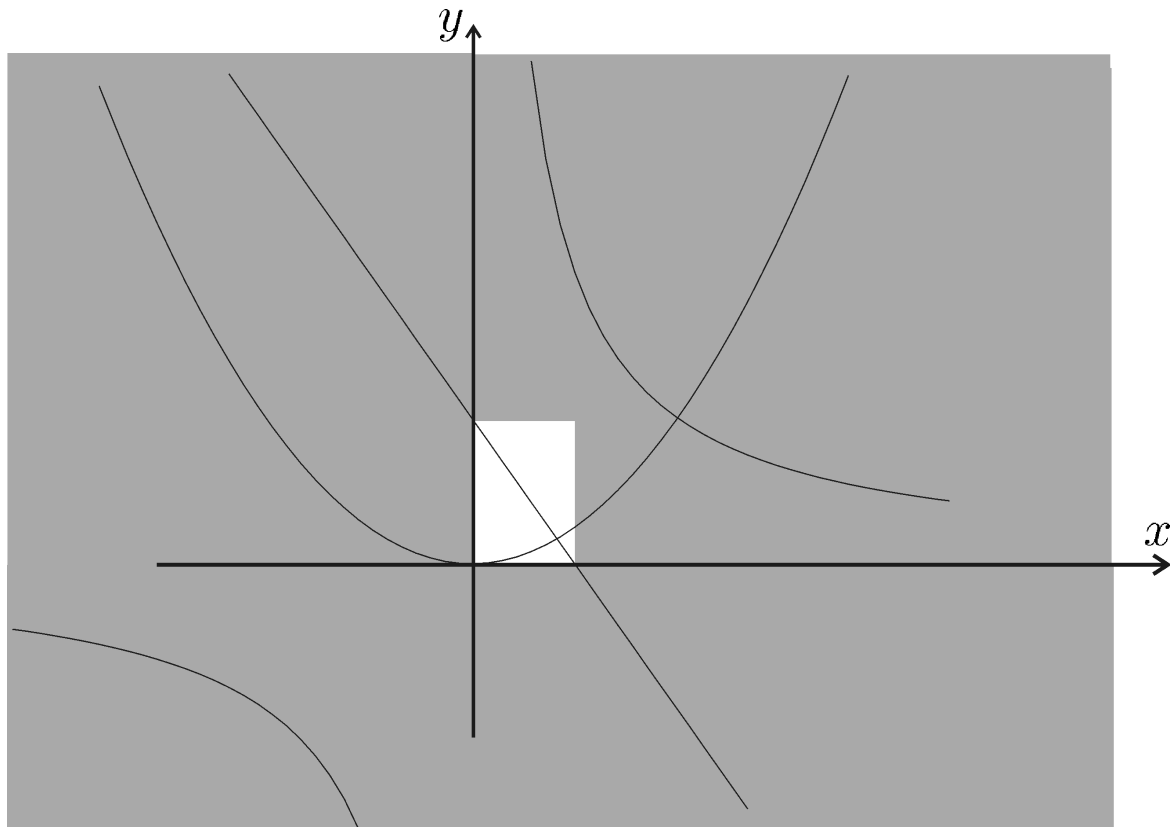
Pour chacune de ces variables, nous affectons le domaine $[-\infty, \infty]$.

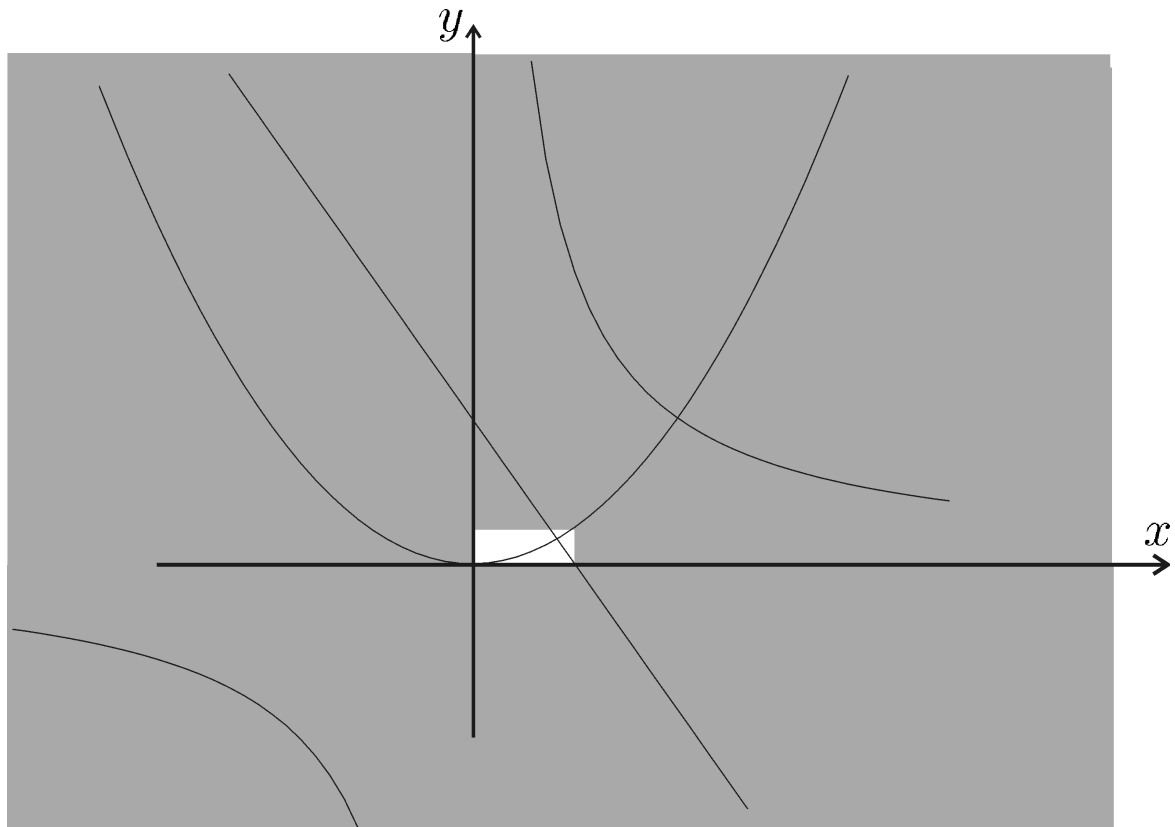
La *propagation* consiste à projeter ces contraintes jusqu'à l'équilibre.

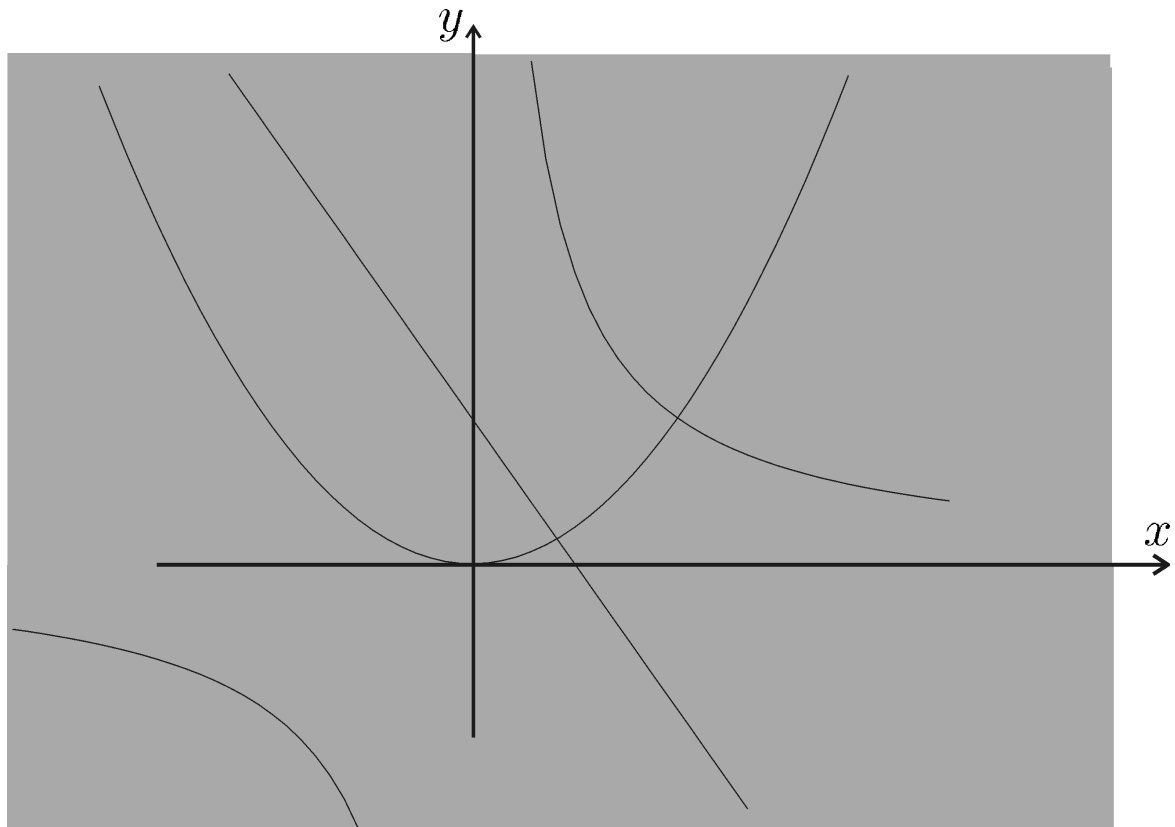


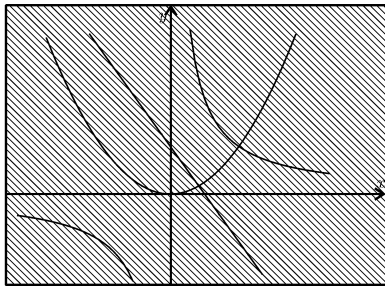
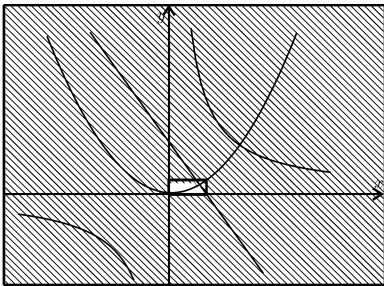
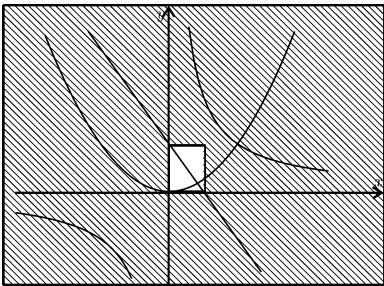
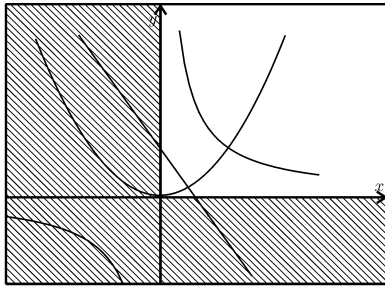
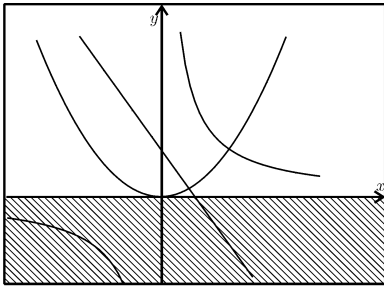
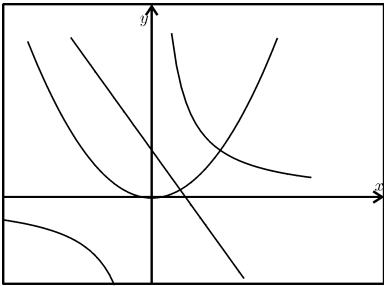












$$(C_1) \Rightarrow y \in [-\infty, \infty]^2 = [0, \infty]$$

$$(C_2) \Rightarrow x \in 1/[0, \infty] = [0, \infty]$$

$$(C_3) \Rightarrow y \in [0, \infty] \cap ((-2) \cdot [0, \infty] + 1) \\ = [0, \infty] \cap ([-\infty, 1]) = [0, 1]$$

$$x \in [0, \infty] \cap (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}]$$

$$(C_1) \Rightarrow y \in [0, 1] \cap [0, 1/2]^2 = [0, 1/4]$$

$$(C_2) \Rightarrow x \in [0, 1/2] \cap 1/[0, 1/4] = \emptyset$$

$$y \in [0, 1/4] \cap 1/\emptyset = \emptyset$$

1.4 Décomposition

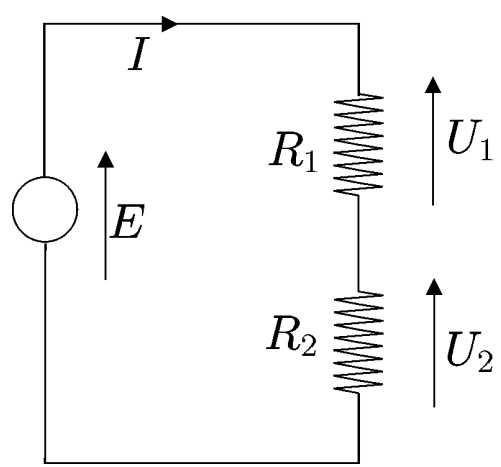
Pour des contraintes plus complexes une décomposition est nécessaire. Par exemple

$$\begin{aligned} x + \sin(y) - xz &\leq 0, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{aligned}$$

se décompose comme suit.

$$\left\{ \begin{array}{lll} a = \sin(y) & x \in [-1, 1] & a \in]-\infty, \infty[\\ b = x + a & y \in [-1, 1] & b \in]-\infty, \infty[\\ c = xz & z \in [-1, 1] & c \in]-\infty, \infty[\\ b - c = d & & d \in]-\infty, 0] \end{array} \right.,$$

1.5 Problème d'estimation



Contraintes

$$P = EI; E = (R_1 + R_2) I;$$

$$U_1 = R_1 I; U_2 = R_2 I; E = U_1 + U_2.$$

Domaines initiaux

$$\begin{array}{ll} R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\ E \in [23, 26]\text{V}, & I \in [4, 8]\text{A}, \\ U_1 \in [10, 11]\text{V}, & U_2 \in [14, 17]\text{V}, \\ P \in [124, 130]\text{W}, & \end{array}$$

Les domaines contractés sont

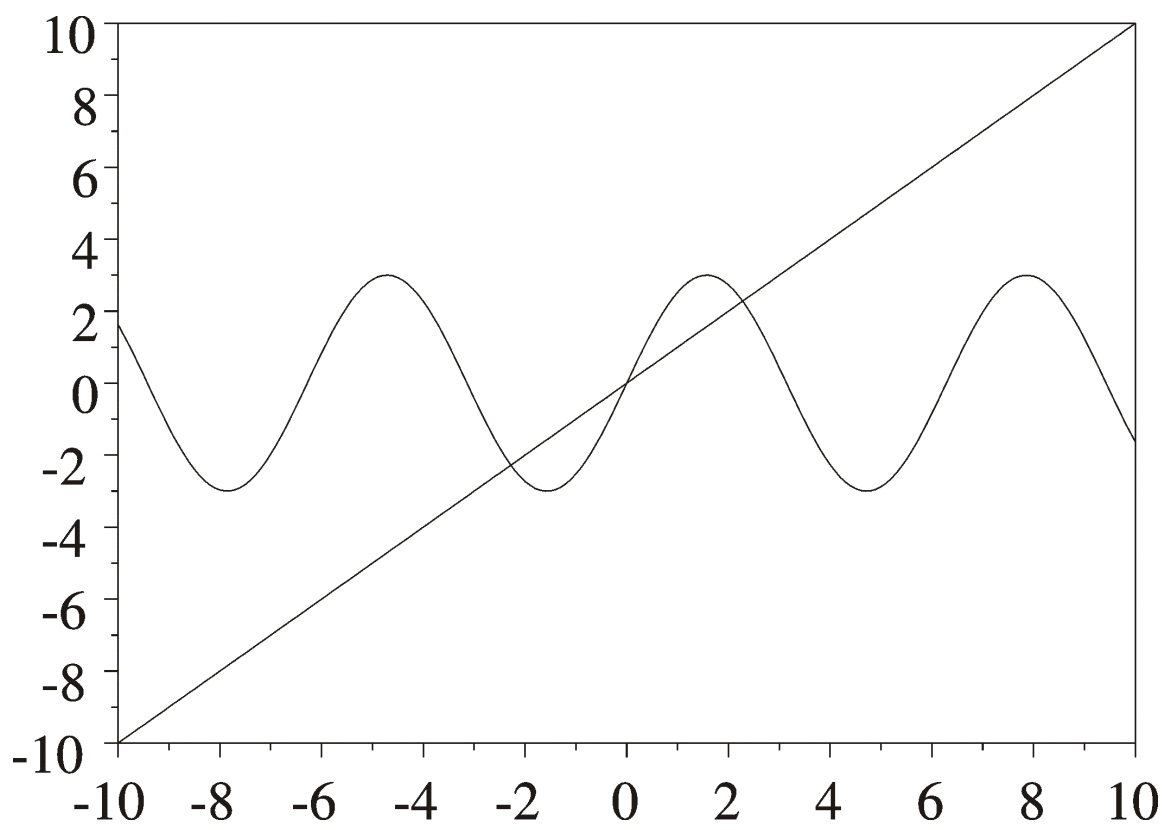
$$\begin{aligned} R_1 &\in [1.84, 2.31] \Omega, & R_2 &\in [2.58, 3.35] \Omega, \\ E &\in [24, 26] \text{V}, & I &\in [4.769, 5.417] \text{A}, \\ U_1 &\in [10, 11] \text{V}, & U_2 &\in [14, 16] \text{V}, \\ P &\in [124, 130] \text{W}, \end{aligned}$$

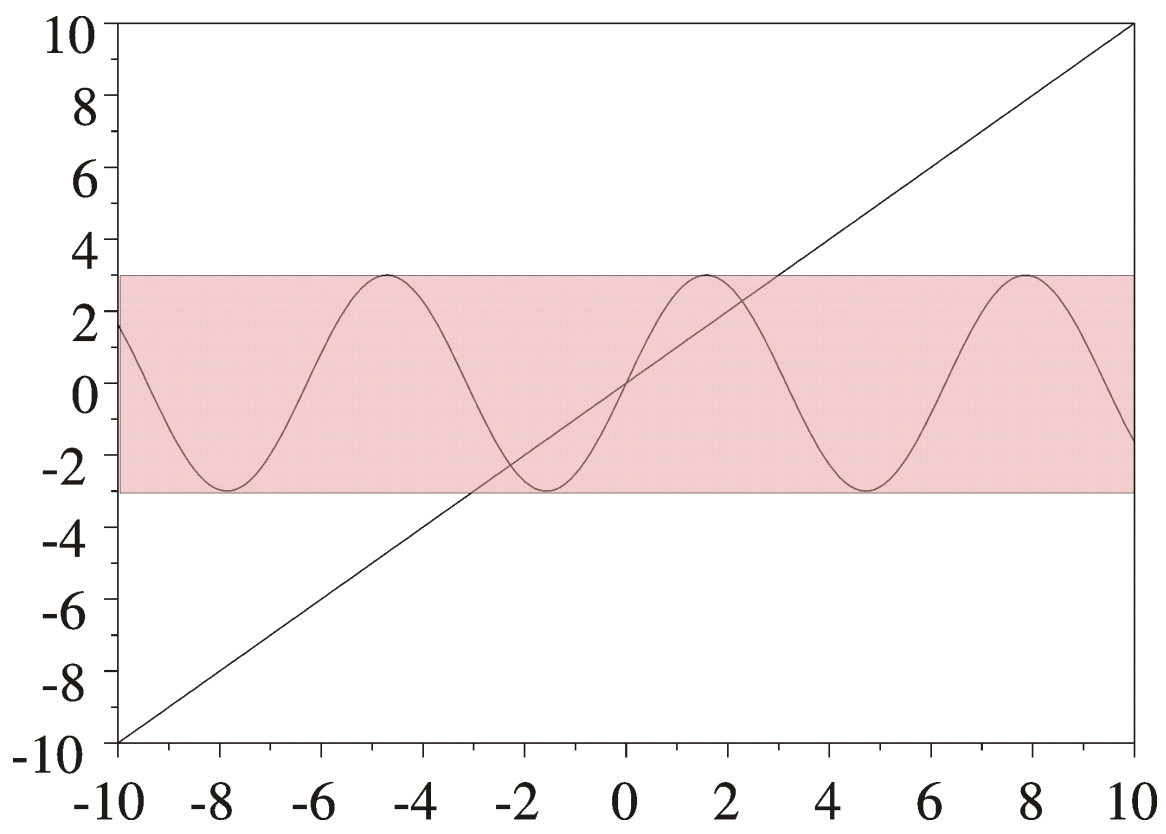
2 Résolution d'équations

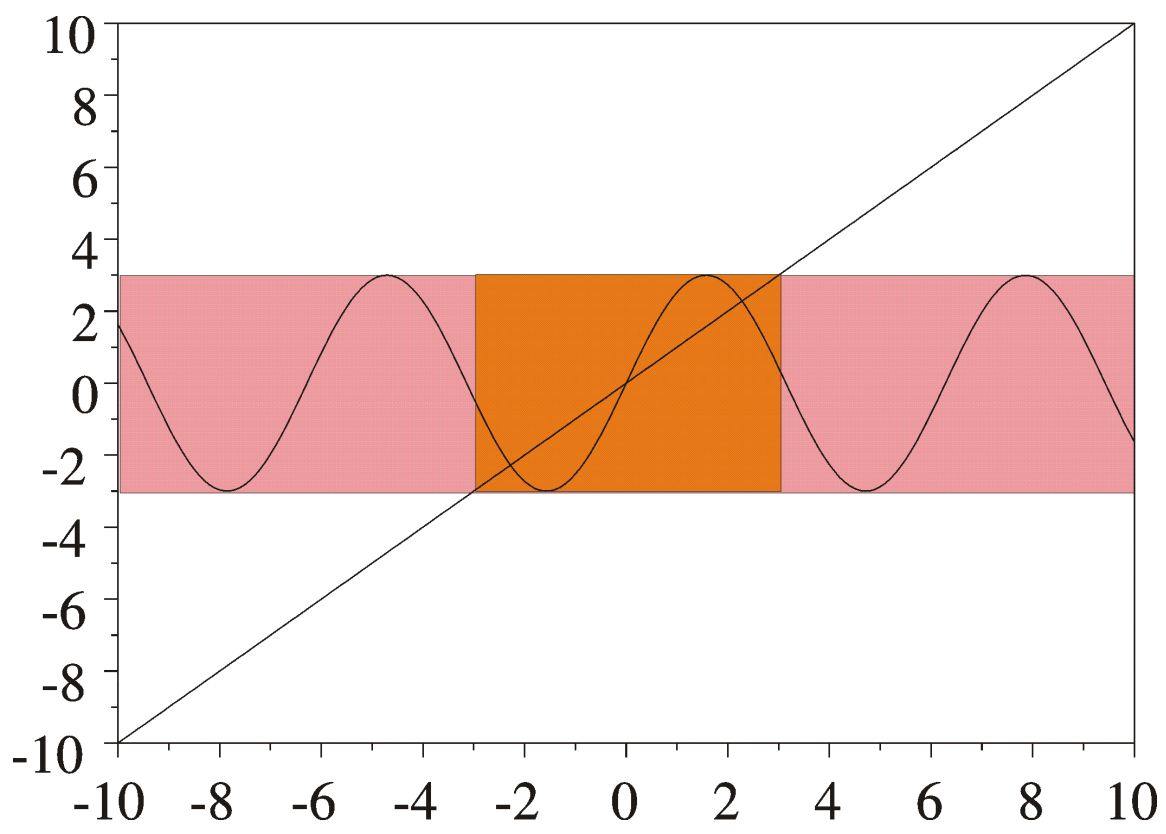
2.1 Principe

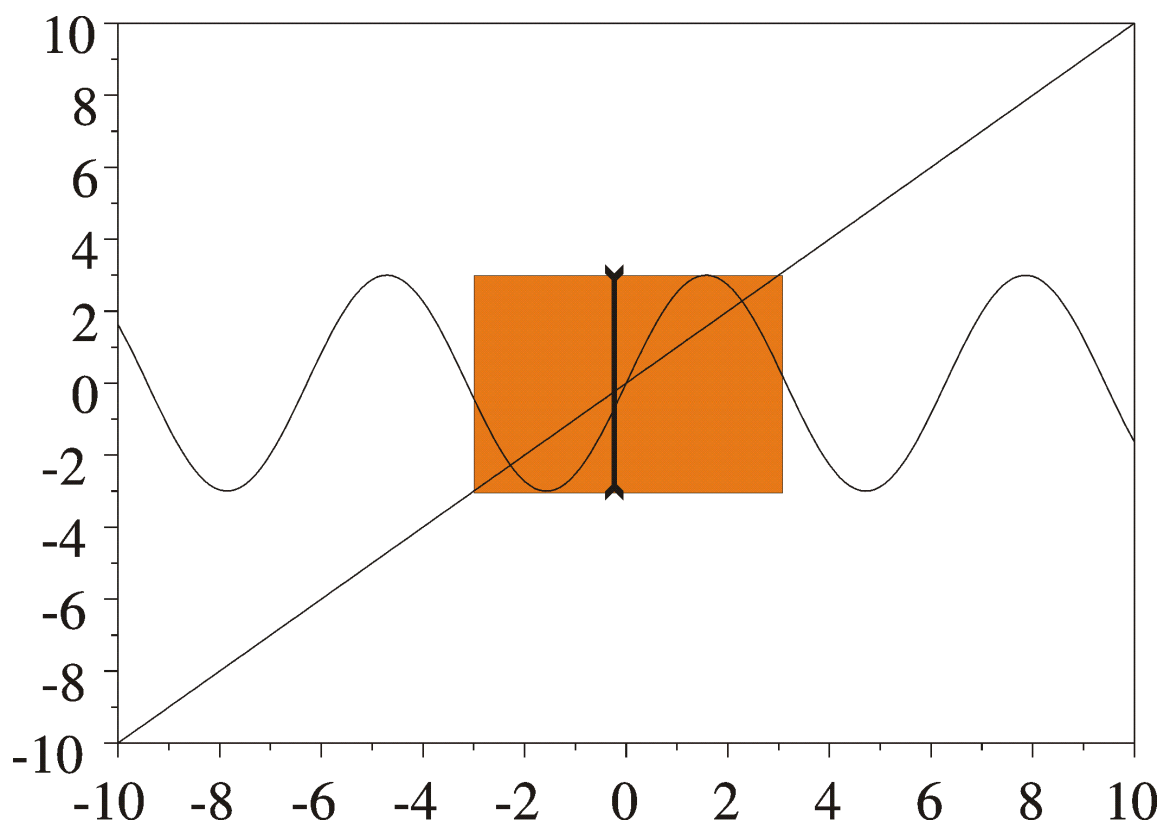
Considérons le système

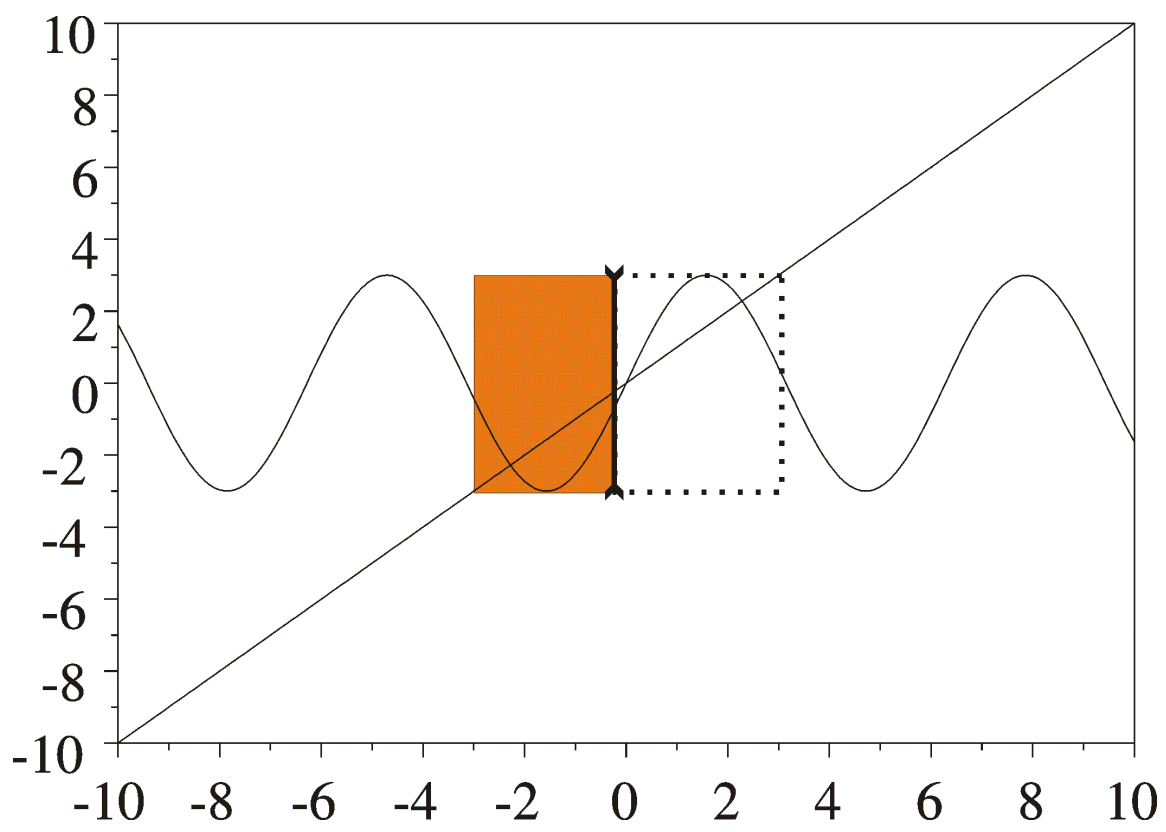
$$\begin{cases} y = 3 \sin(x) \\ y = x \end{cases} \quad x \in \mathbb{R}, y \in \mathbb{R}.$$

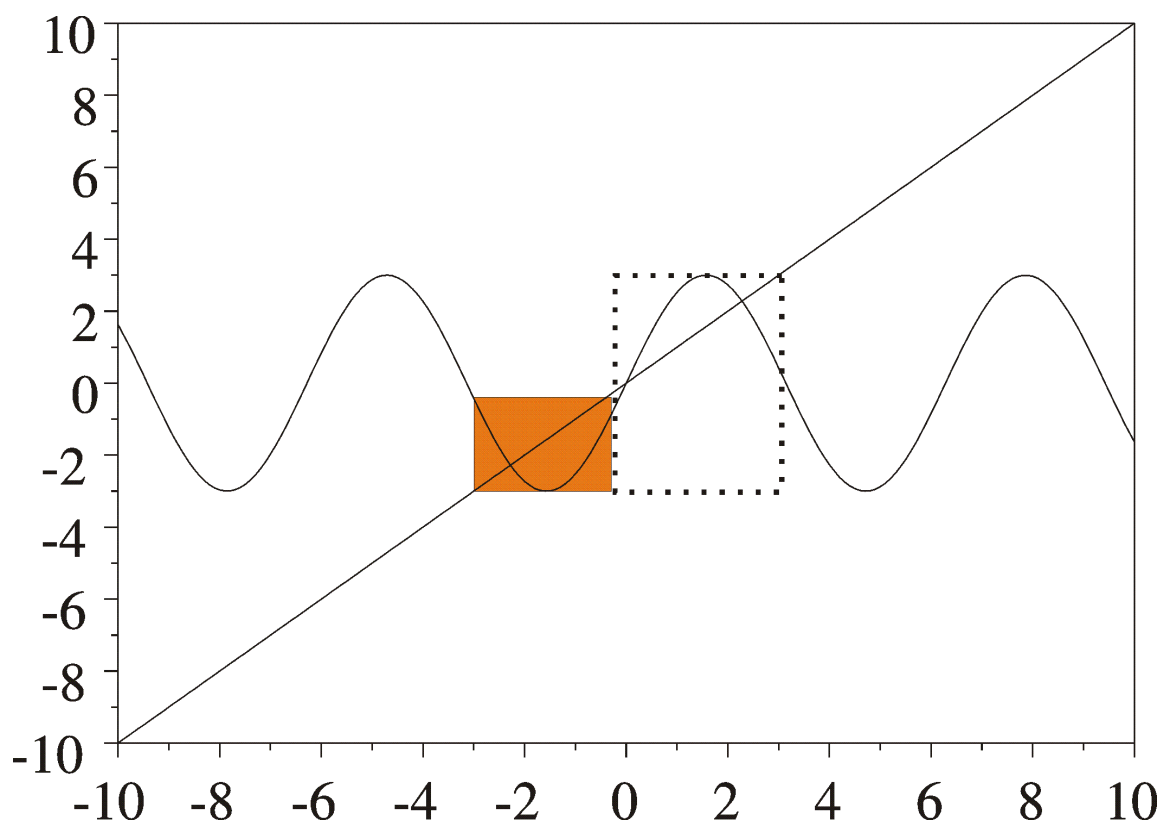


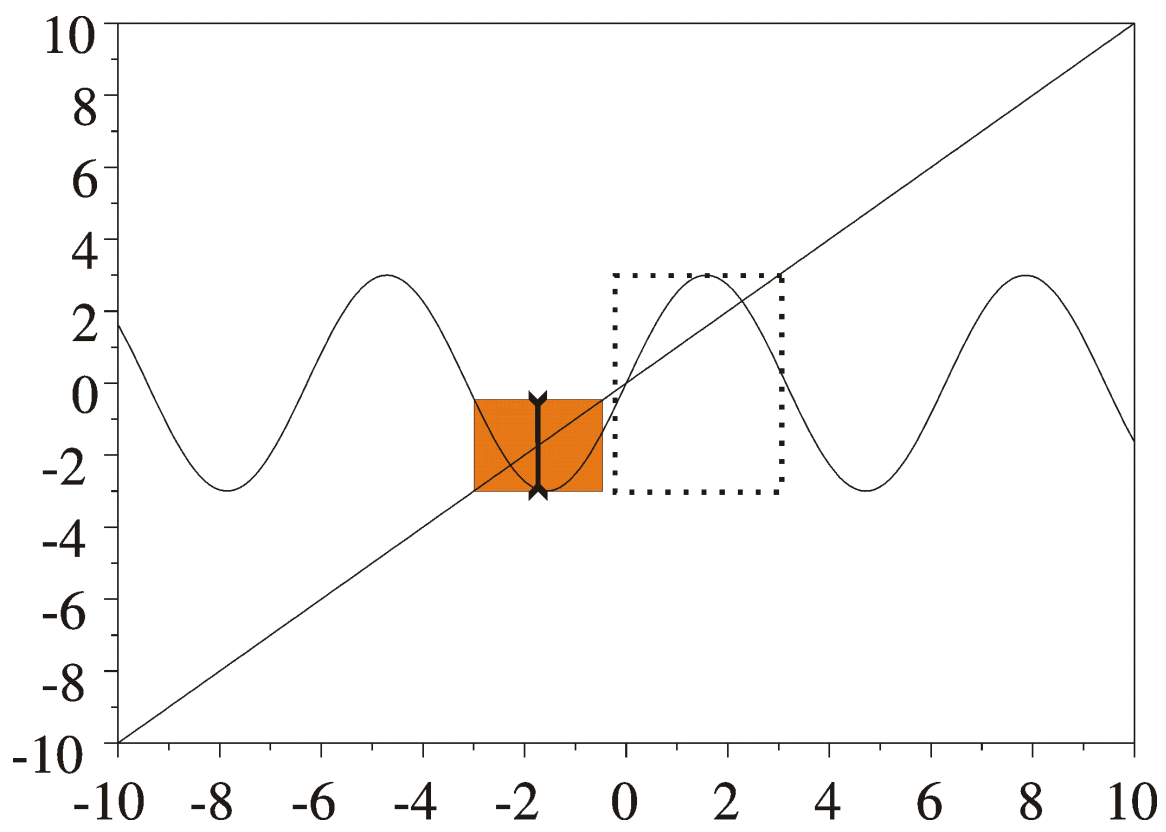


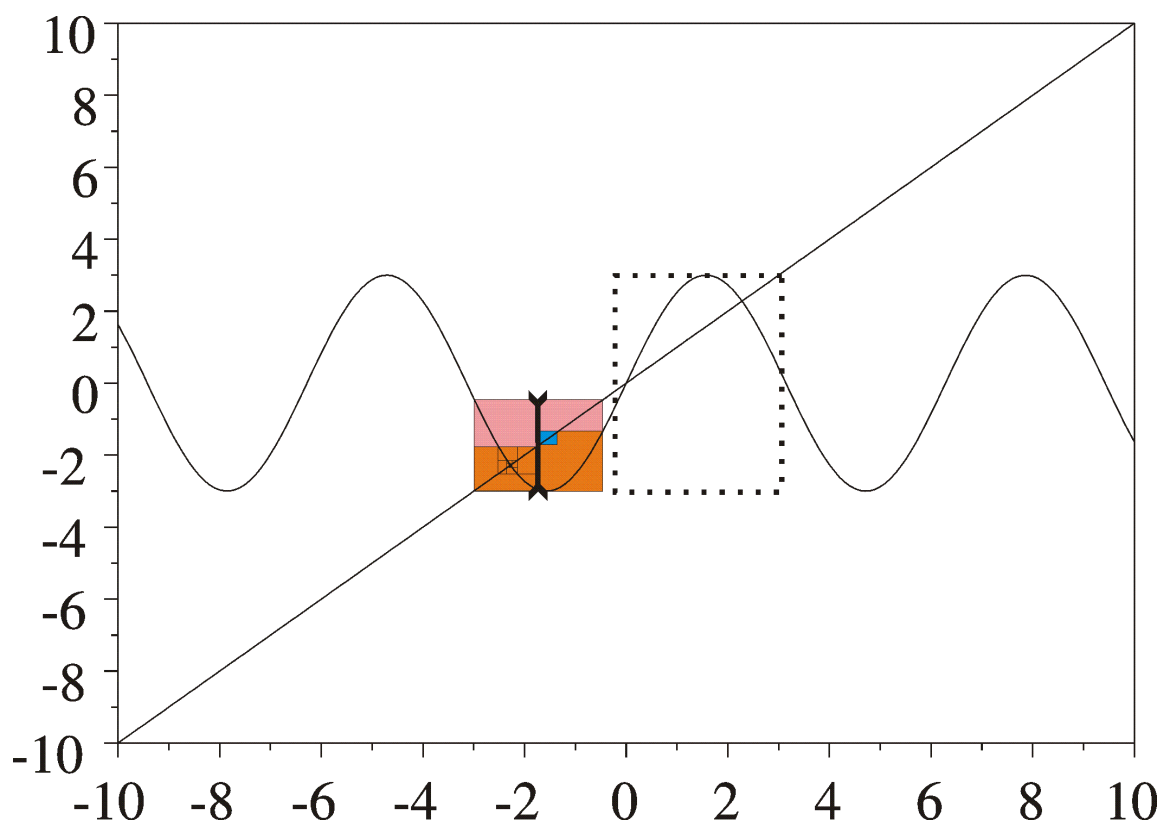


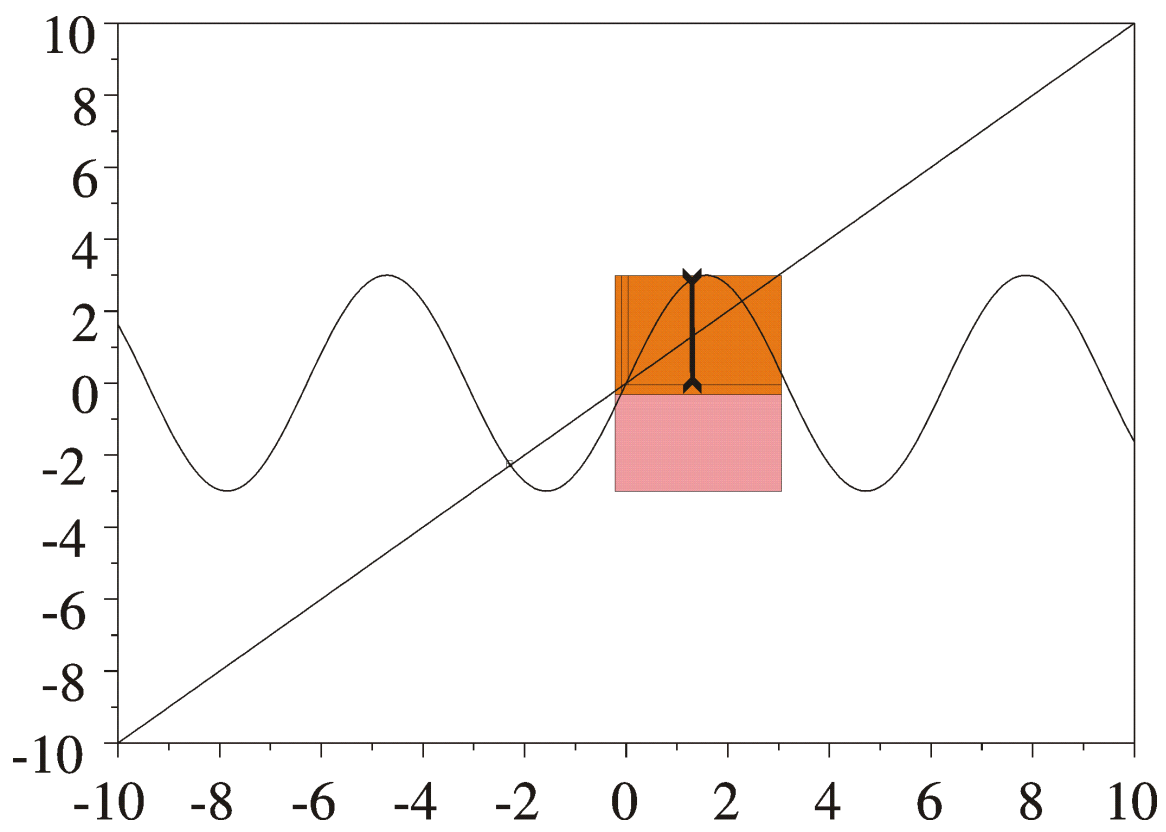


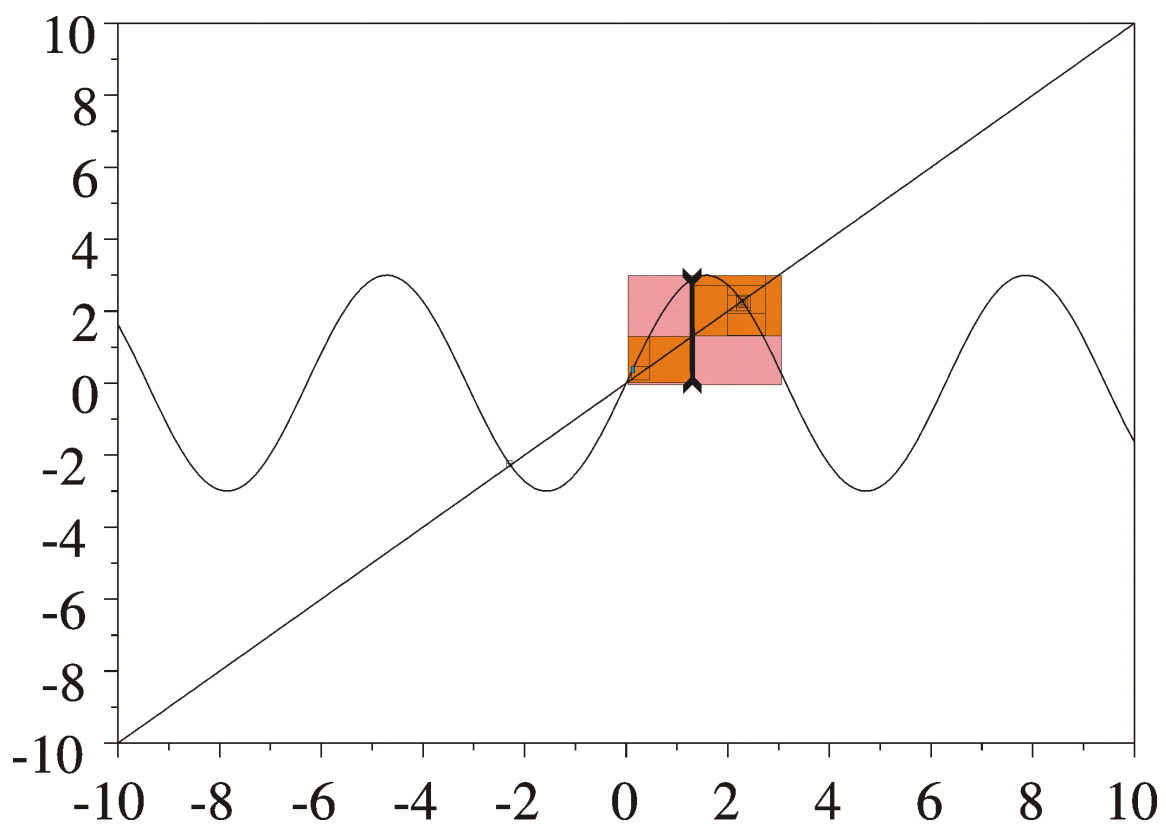




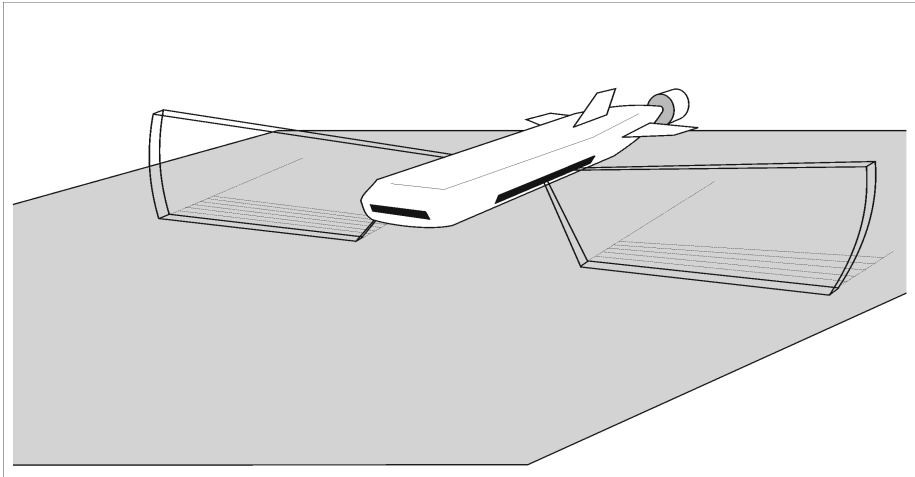






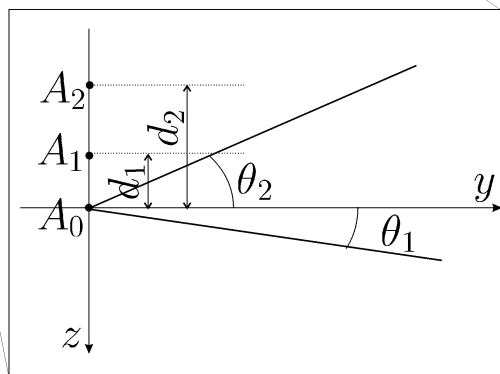
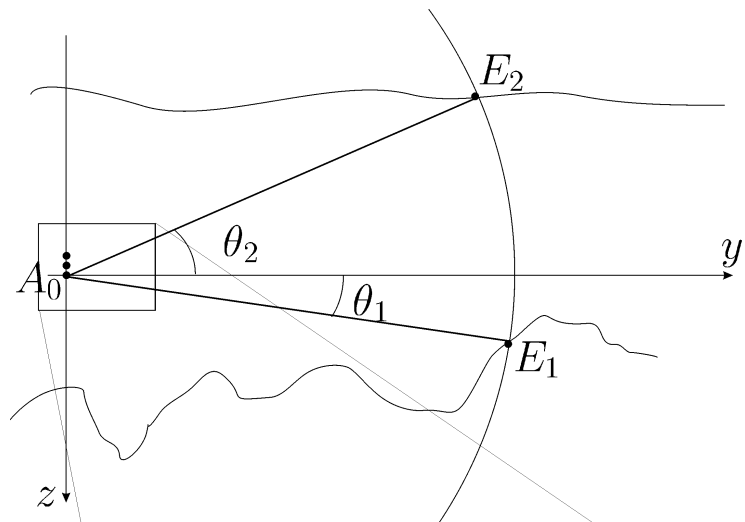


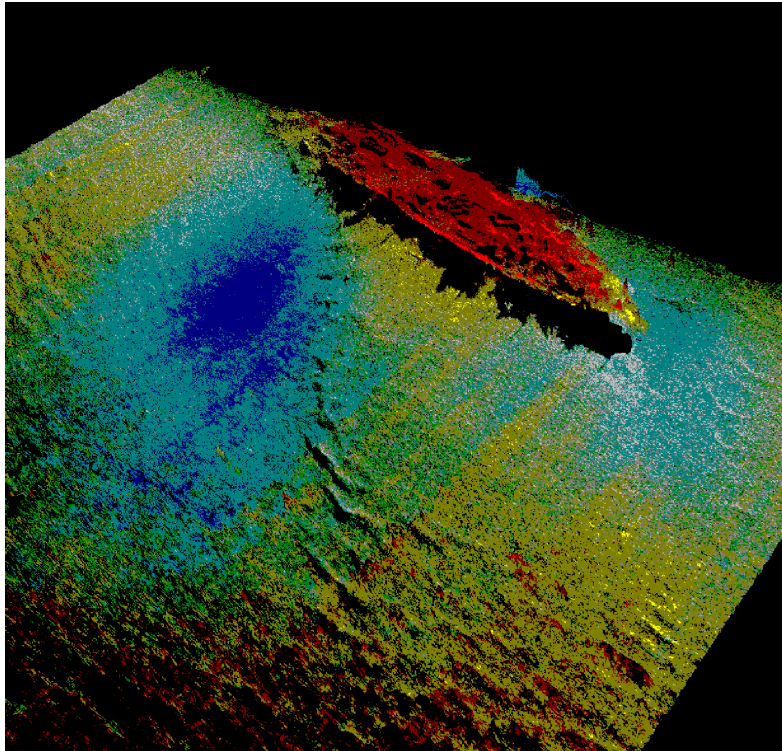
2.2 Imagerie 3d du fond de l'océan



Chaque sonar possède trois antennes A_0, A_1, A_2 . L'onde émise par A_0 est $s(t) = e^{2\pi f_0 t}$.

On a $c = 1500 \text{ ms}^{-1}$, $\lambda = 3 \text{ mm}$, $f_0 = 455 \text{ kHz}$.





3 Approximation intérieure

On passe au complémentaire. Par exemple, si

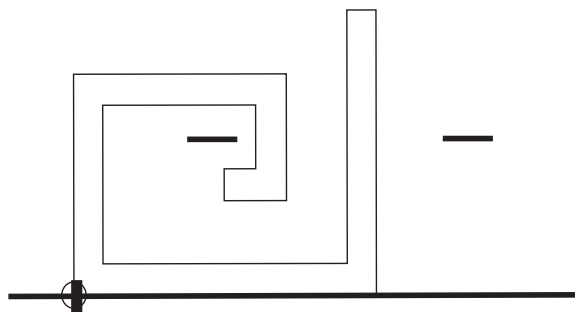
$$\mathbb{S} = \left\{ (x,y) \in \mathbb{R}^2, f(x,y) \leq 0 \text{ et } g(x,y) \leq 0 \right\}.$$

Son complémentaire est

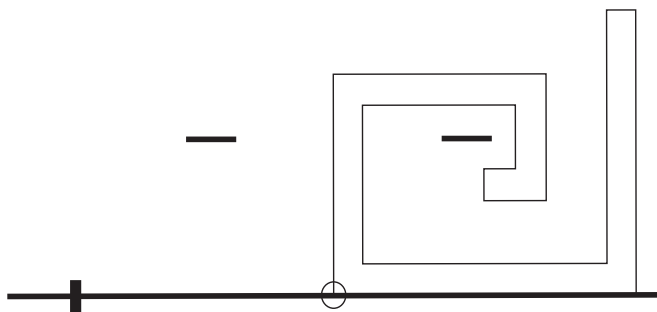
$$\begin{aligned} \bar{\mathbb{S}} &= \left\{ (x,y) \in \mathbb{R}^2, f(x,y) > 0 \text{ or } g(x,y) > 0 \right\} \\ &= \left\{ (x,y) \in \mathbb{R}^2, \max(f(x,y), g(x,y)) > 0 \right\} \end{aligned}$$

(Illustration avec Proj2D)

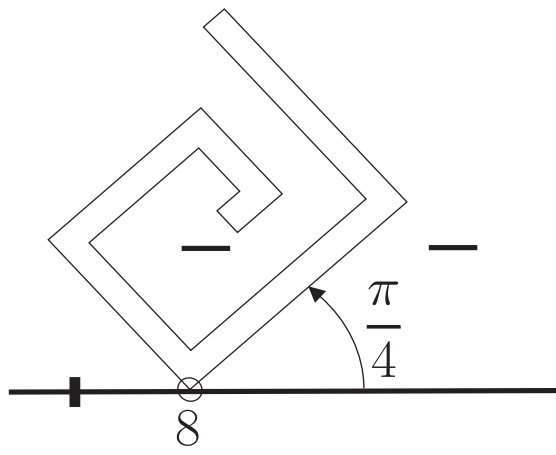
4 Planification de chemin



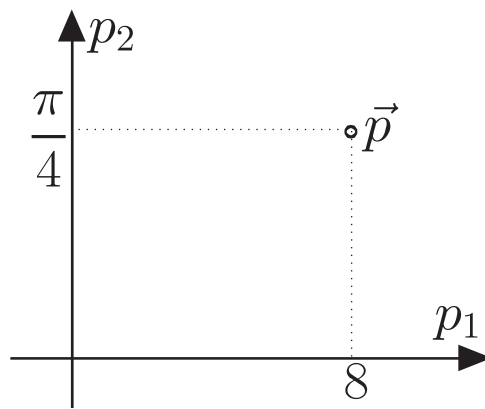
Initial configuration: $\vec{p} = (0 \ 0)^T$



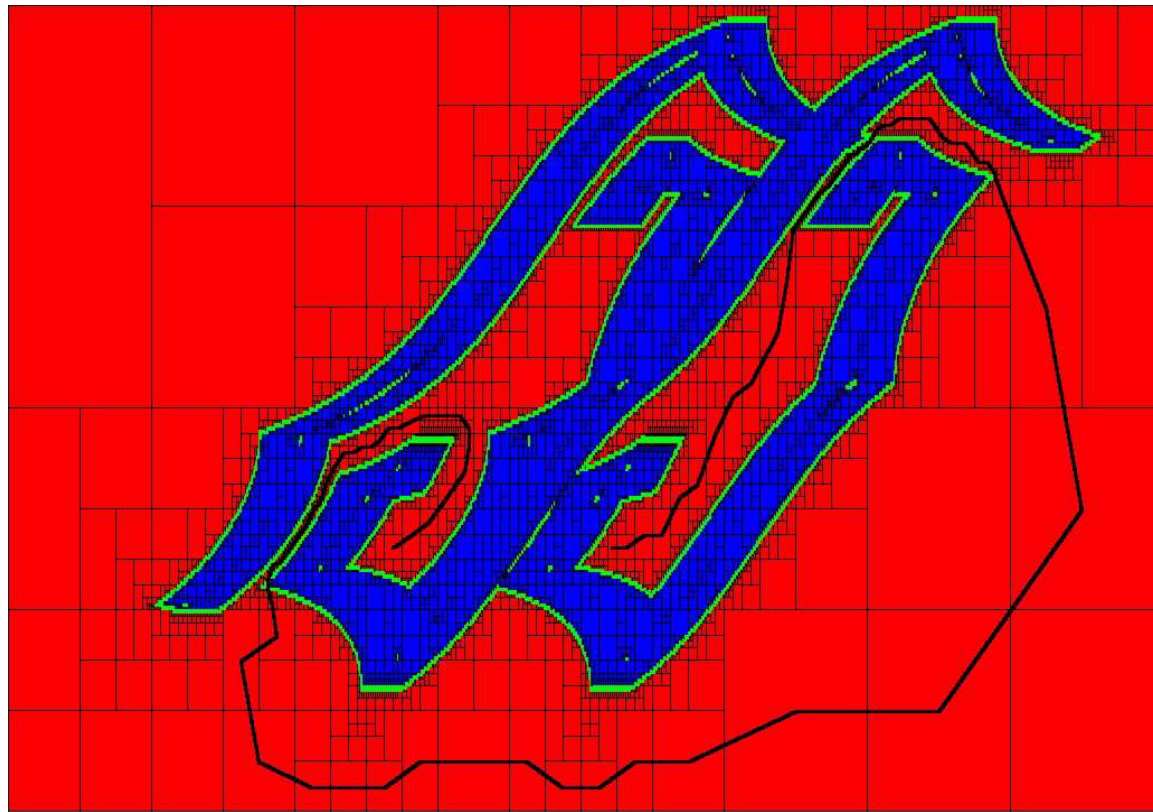
Goal configuration: $\vec{p} = (17 \ 0)^T$

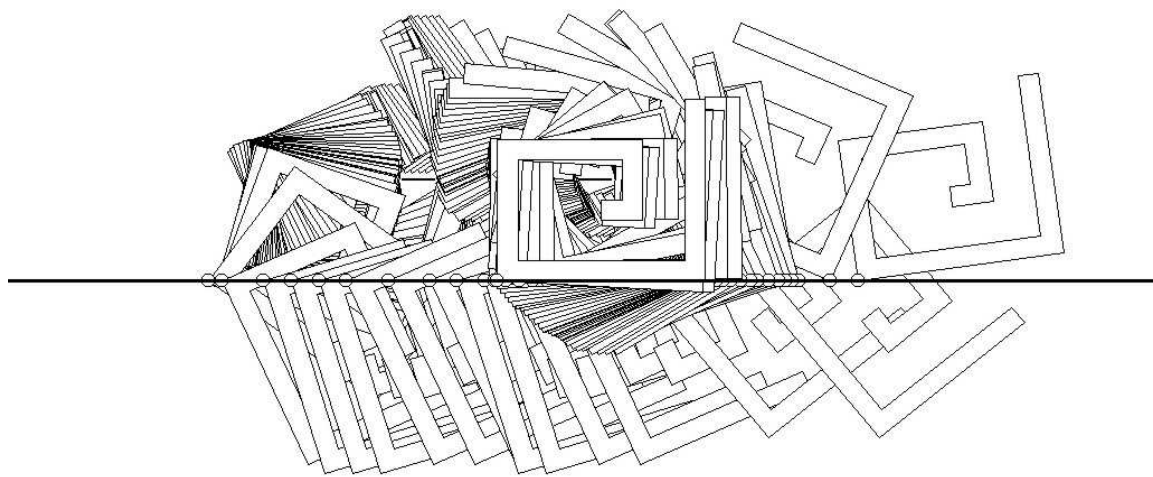


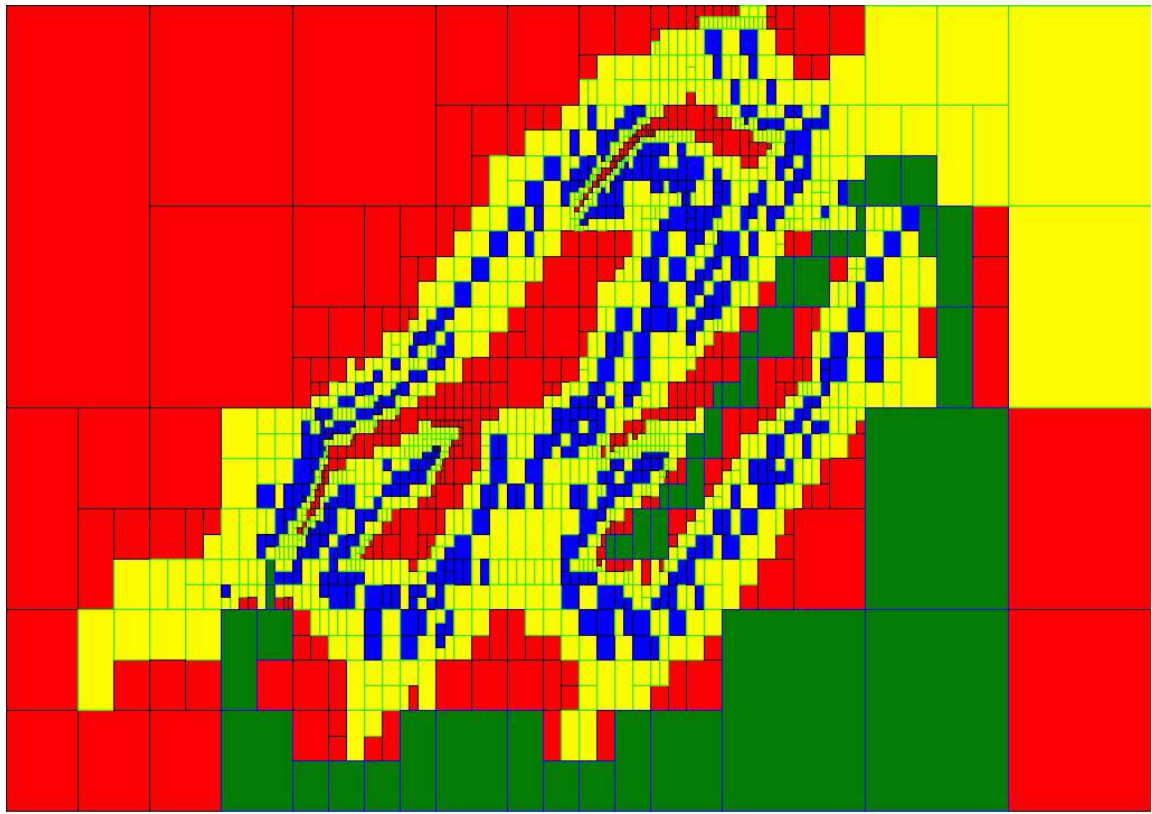
Room

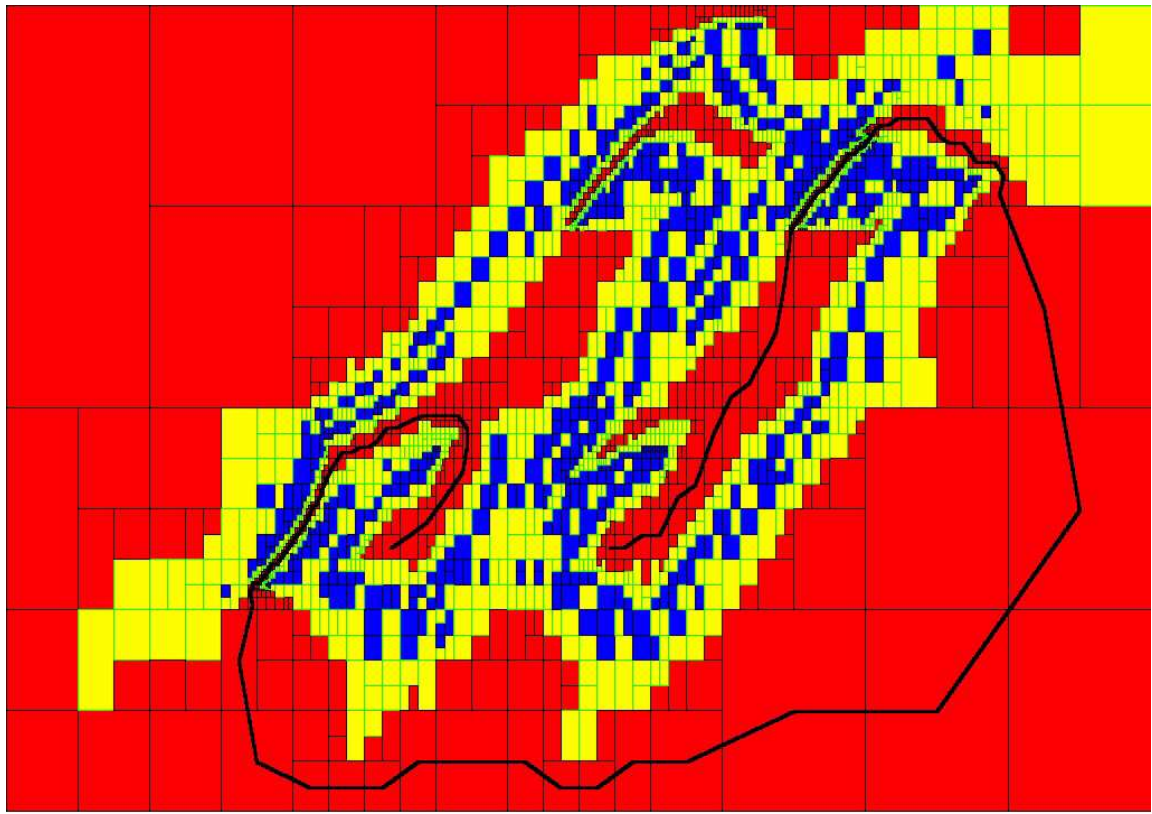


Configuration space









5 Commande d'un voilier

5.1 Equations d'état

$$\left\{ \begin{array}{lcl} \dot{x} & = & v \cos \theta \\ \dot{y} & = & v \sin \theta - \beta V \\ \dot{\theta} & = & \omega \\ \dot{\delta}_s & = & u_1 \\ \dot{\delta}_r & = & u_2 \\ \dot{v} & = & \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ \dot{\omega} & = & \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s & = & \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r & = & \alpha_r v \sin \delta_r. \end{array} \right.$$

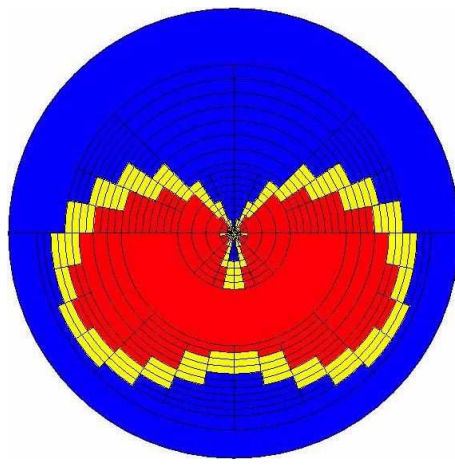
5.2 Est-il possible de l'immobiliser ?

$$\left\{ \begin{array}{lcl} 0 & = & v \cos \theta \\ 0 & = & v \sin \theta - \beta V \\ 0 & = & \omega \\ 0 & = & u_1 \\ 0 & = & u_2 \\ 0 & = & \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ 0 & = & \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s & = & \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r & = & \alpha_r v \sin \delta_r. \end{array} \right.$$

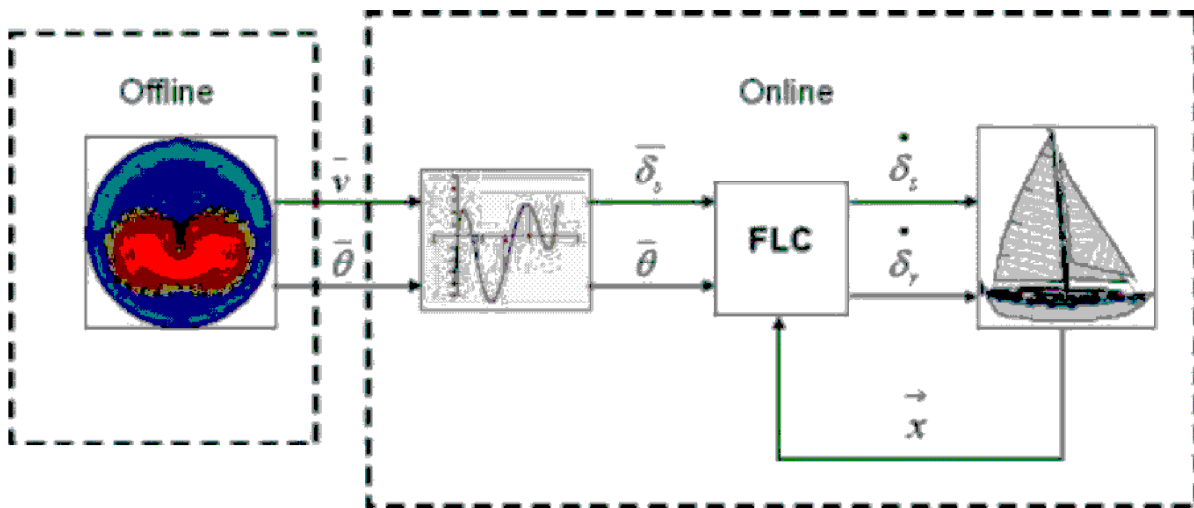
Les techniques intervalles montrent qu'aucune solution existe si $V \neq 0$.

5.3 Polaire des vitesses

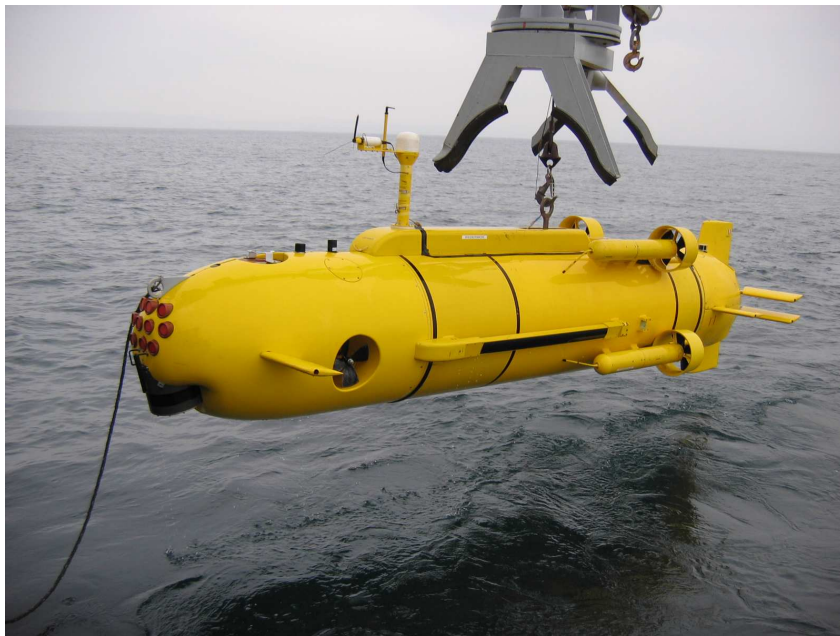
$$\mathbb{W} = \{ (\theta, v) \mid \begin{array}{l} \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\ \omega = 0, u_1 = 0, u_2 = 0 \\ \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v} = 0 \\ \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r}{J} = 0 \\ f_s = \alpha_s (V \cos(\theta + \delta_s) - v \sin \delta_s) \\ f_r = \alpha_r v \sin \delta_r \end{array} \}.$$



5.4 Controle



6 Localisation de mines



Le *Redermor*, fabriqué par le GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Montrer la simulation

Pourquoi une approche par intervalles ?

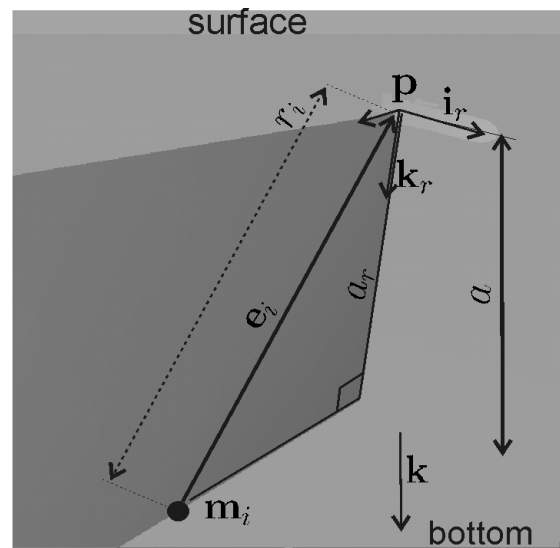
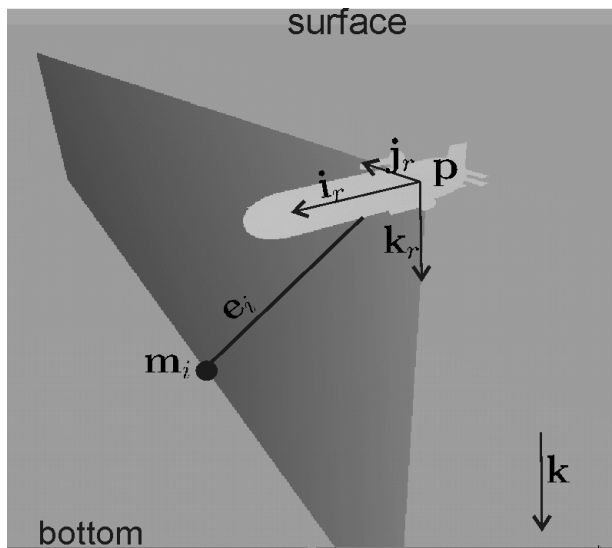
- 1) Besoin d'une approche fiable.
- 2) Les équations du robot sont non linéaires.
- 3) Les bruits de mesure sont non gaussiens.
- 4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
- 5) Beaucoup de mesures redondantes sont disponibles.

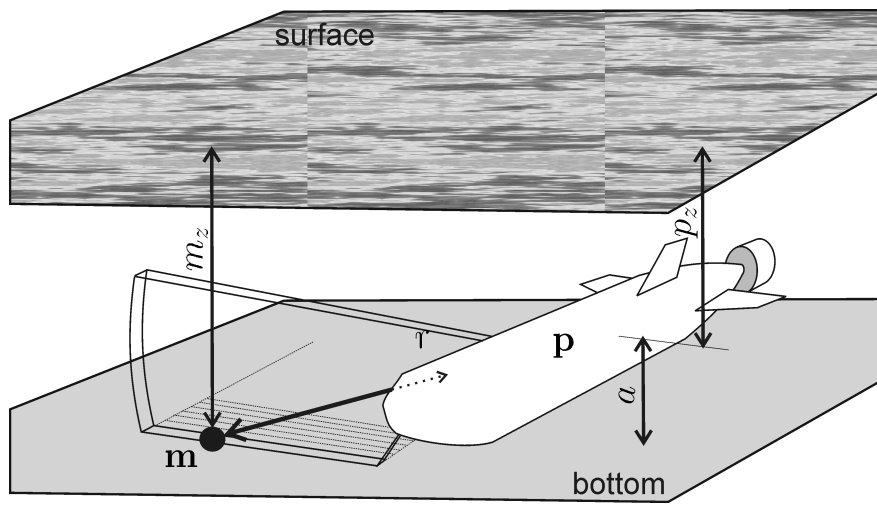
6.1 Capteurs

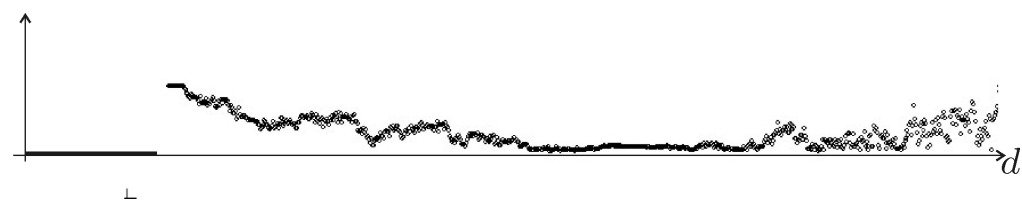
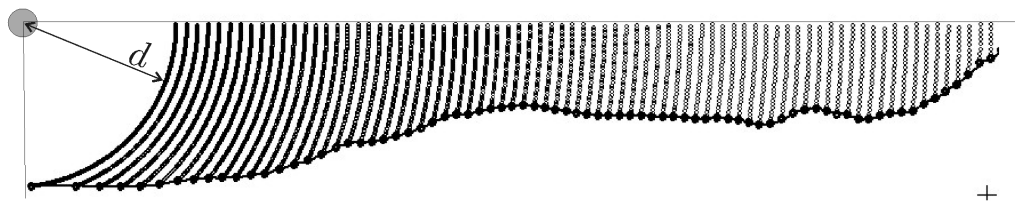
Un GPS (Global positioning system), disponible à la surface.

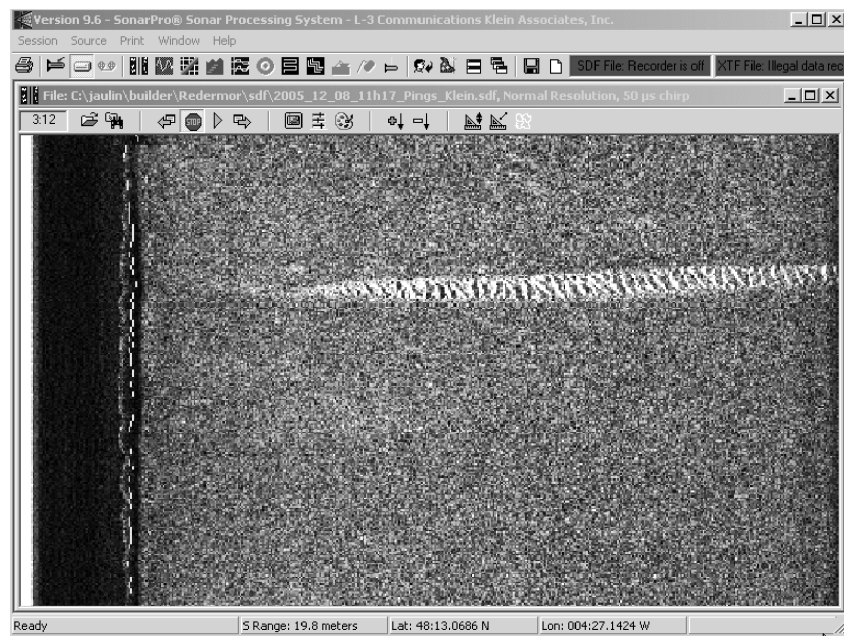
$$\begin{aligned} t_0 &= 6000 \text{ s}, & \ell^0 &= (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\ t_f &= 12000 \text{ s}, & \ell^f &= (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m \end{aligned}$$

Un sonar (KLEIN 5400 side scan sonar). Donne la distance r entre le robot et la mine

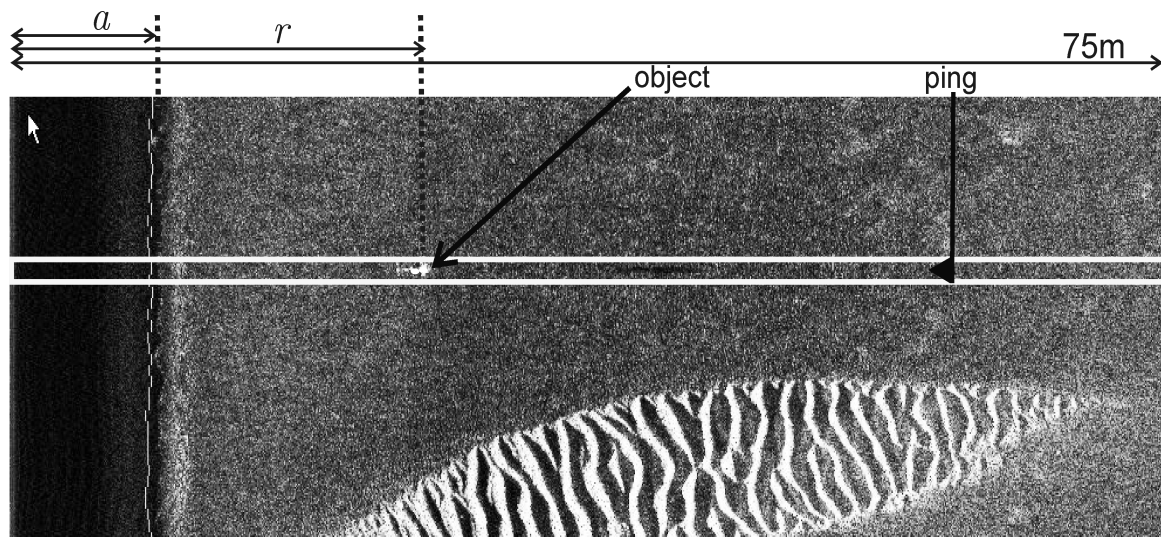








Screenshot du logiciel SonarPro

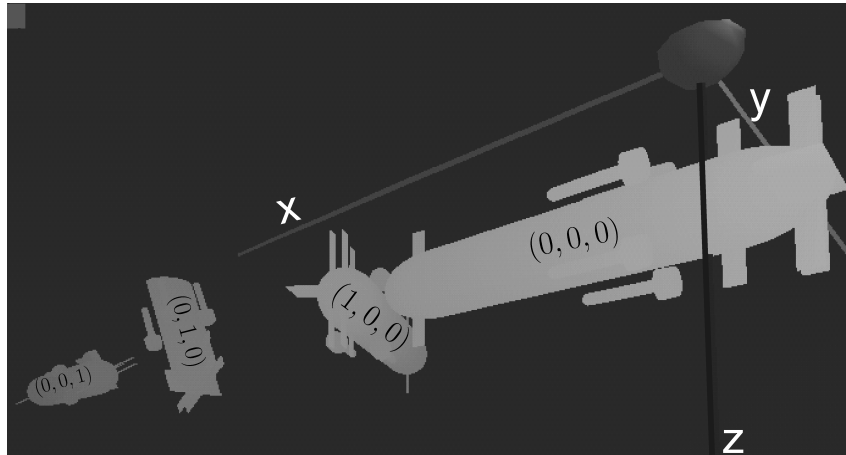


Détection d'une mine à l'aide de SonarPro

Le Loch-Doppler renvoie la vitesse du robot v_r et son altitude a ..

Une centrale inertielle (Octans III from IXSEA) renvoie le roulis ϕ , le tangage θ et le cap ψ du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$



6.2 Données

Pour chaque $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$, nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

6.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t)=\left(\begin{array}{ccc}1&0&0\\0&\cos\varphi(t)&-\sin\varphi(t)\\0&\sin\varphi(t)&\cos\varphi(t)\end{array}\right),$$

$$\mathbf{R}(t)=\mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

$$\dot{\mathbf{p}}(t)=\mathbf{R}(t).\mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))||\;=\;r(i),$$

$$\mathbf{R}^\top(\tau(i))\left(\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))\right)\in [0]\times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i))-p_z(\tau(i))-a(\tau(i))\in [-0.5,0.5]$$

6.4 Contraintes globales

Exemple 1

Contraintes

$$\left\{ \begin{array}{l} \mathbf{y} = \mathbf{A}\mathbf{x} \\ \mathbf{x} \in [\mathbf{x}] \subset \mathbb{R}^2, \mathbf{y} \in [\mathbf{y}] \subset \mathbb{R}^2 \\ \mathbf{A} \in [\mathbf{A}] \end{array} \right.$$

Décomposition :

$$\left\{ \begin{array}{l} y_1 = a_{11}x_1 + a_{12}x_2 \\ y_2 = a_{21}x_1 + a_{22}x_2 \end{array} \right.$$

i.e.

$$\left\{ \begin{array}{l} z_1 = a_{11}x_1, \quad z_2 = a_{12}x_2, \quad y_1 = z_1 + z_2 \\ z_3 = a_{21}x_1, \quad z_4 = a_{22}x_2, \quad y_2 = z_3 + z_4 \\ z_1 \in [-\infty, \infty], \dots, z_4 \in [-\infty, \infty] \end{array} \right. .$$

Exemple 2

Contraintes :

$$\begin{array}{l} y = \mathbf{R}x \\ \text{Rot}(\mathbf{R}) \end{array} , \quad \begin{array}{l} x \in [x], y \in [y] \\ \mathbf{R} \in [\mathbf{R}] \end{array}$$

Contractions :

$$\begin{array}{l} [y] : = [y] \cap [\mathbf{R}] * [x], \\ [x] : = [x] \cap [\mathbf{R}]^T * [y], \end{array}$$

Exemple 3.

Contraintes :

$$\begin{aligned} \dot{\mathbf{p}}(t) &= \mathbf{R}(t) \cdot \mathbf{v}_r(t) \\ \forall t \in [t_0, t_1], \mathbf{R}(t) &\in [\mathbf{R}], \mathbf{v}_r(t) \in [\mathbf{v}_r] \end{aligned}$$

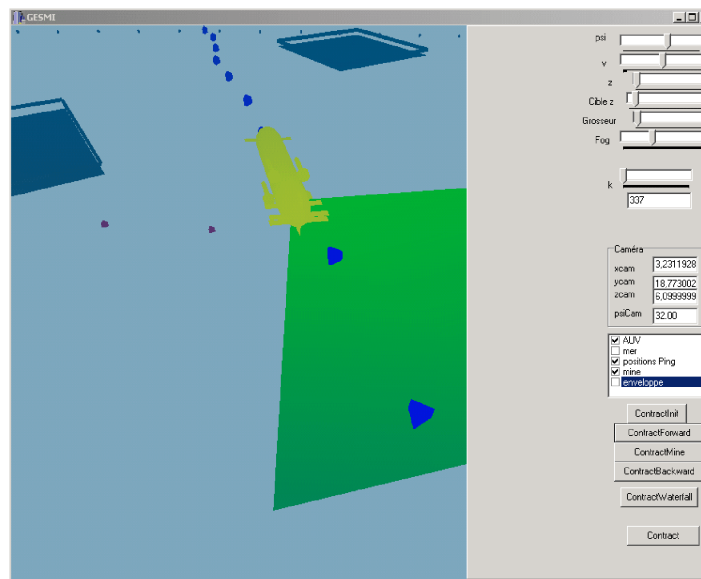
Puisque

$$\mathbf{p}(t_1) = \mathbf{p}(t_0) + \int_{t_0}^{t_1} \mathbf{R}(t) \cdot \mathbf{v}_r(t) \in \mathbf{p}(t_0) + (t_1 - t_0) \cdot [\mathbf{R}] \cdot [\mathbf{v}_r],$$

les domaines pour $\mathbf{p}(t_0)$ et $\mathbf{p}(t_1)$ sont contractés comme suit

$$\begin{aligned} [\mathbf{p}](t_1) &= [\mathbf{p}](t_1) \cap ([\mathbf{p}](t_0) + (t_1 - t_0) \cdot [\mathbf{R}] \cdot [\mathbf{v}_r]), \\ [\mathbf{p}](t_0) &= [\mathbf{p}](t_0) \cap ([\mathbf{p}](t_1) + (t_0 - t_1) \cdot [\mathbf{R}] \cdot [\mathbf{v}_r]). \end{aligned}$$

6.5 GESMI



GESMI (Guaranteed Estimation of Sea Mines with Intervals)

```

//-----
void Cmult(imatrix& C, imatrix& A, imatrix& B)
{
    for (int i=1; i<=C.dim1(); i++)
        for (int j=1; j<=C.dim2(); j++)
            {
                box a=Row(A,i);
                box b=Column(B,j);
                interval c=C.GetVal(i,j);
                CProdScalaire(c, a, b); C.SetVal(i,j,c);
                for (int k=1; k<=A.dim2(); k++)
                    {A.SetVal(i,k,a[k]);B.SetVal(k,j,b[k]);};
            }
}

//-----
void Cmult(box& c, imatrix& A, box& b)
{
    for (int i=1; i<=c.dim; i++)
        {
            box a=Row(A,i);
            CProdScalaire(c[i],a,b);
            for (int k=1; k<=b.dim; k++) A.SetVal(i,k,a[k]);
        }
}

//-----
void Crot(imatrix& R)
{
    imatrix Rt=Transpose(R);
    imatrix I=iEye(R.dim1());
    Cmult(I,R,Rt);
}

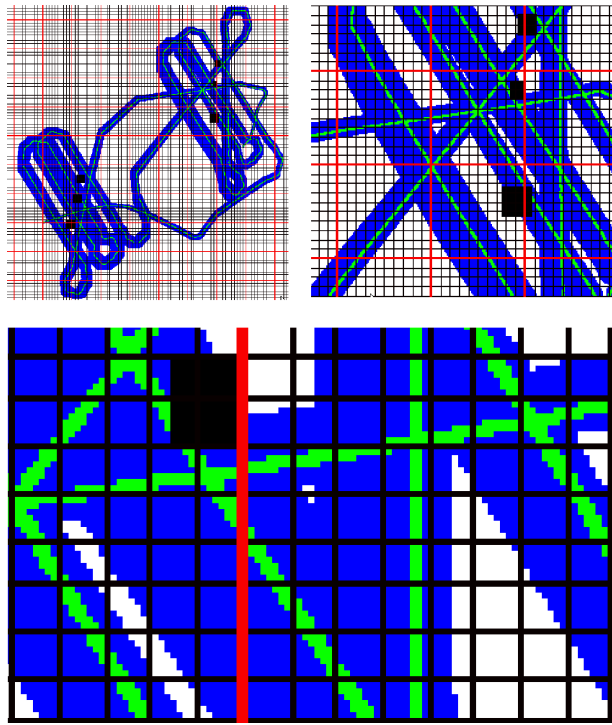
//-----
void Cantisym(imatrix& A)
{
    for (int i=1; i<=A.dim1(); i++)
        { A.SetVal(i,i,interval(-0,0));
          for (int j=i+1; j<=A.dim1(); j++)
              {
                  A.SetVal(j,i,Inter(-A(i,j),A(j,i)));
                  A.SetVal(i,j,Inter(A(i,j),-A(j,i)));
              }
        }
}
//-----

```

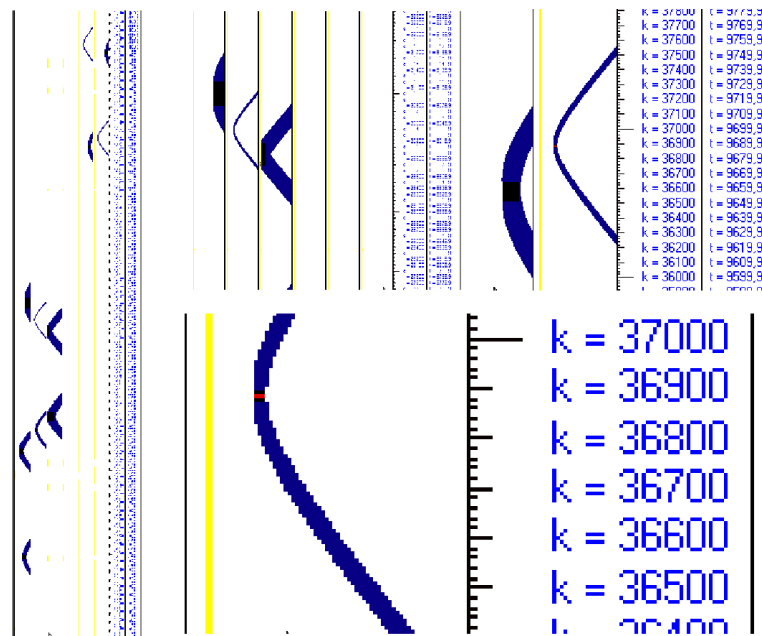
```

//-----
int TForm1::Contract_Forward(void)
{
    for (int k=0;k<kmax;k++)
        P[k+1].Intersect(P[k]+dT*Rot[k]*vr[k]);
}
//-----
int TForm1::Contract_Backward(void)
{
    for (int k=kmax-2;k>=0;k--)
        P[k].Intersect(P[k+1]-dT*Rot[k]*vr[k]);
}
//-----
int TForm1::Contract_Mine(void)
{
    for (int k=0;k<kmax;k++)
        for (int km=0;km<kmmmax;km++)
            if (W[k].vu[km])
            {
                Cplus(mines[km].P[3],P[k][3],a[k],1);
                Cdistance(W[k].r[km],P[k],mines[km].P);
                W[k].e[km].Intersect(P[k]-mines[km].P);
                W[k].e[km].Intersect(Rot[k]*W[k].er[km]);
                Cmoins(W[k].e[km],P[k],mines[km].P,-1);
            }
}
//-----

```



Trajectoire reconstruite par GESMI



Cascade reconstruite par GESMI