Intervalles et applications

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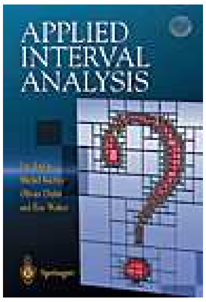
ENSIETA, Brest



Séminaire pour le GT calcul ensembliste du GDR Macs

Lyon, le 1er octobre 2007.

1 Propagation de contraintes sur les intervalles

1.1 Arithmétique sur les intervalles

Si $\diamond \in \{+, -, ., /, \max, \min\}$, on a

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}] .$$

Par exemple,

$$\begin{aligned} [-1,3]+[2,5] &= [1,8], \\ [-1,3].[2,5] &= [-5,15], \\ [-1,3]/[2,5] &= [-\tfrac{1}{2},\tfrac{3}{2}], \\ [-1,3]\vee[2,5] &= [2,5]. \end{aligned}$$

Si $f \in \{\text{cos, sin,sqr, sqrt, log, exp, } \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}] \,.$$

Par exemple,

$$\begin{array}{rcl}
\sin([0,\pi]) & = & [0,1], \\
\text{sqr}([-1,3]) & = & [-1,3]^2 = [0,9], \\
\text{abs}([-7,1]) & = & [0,7], \\
\text{sqrt}([-10,4]) & = & \sqrt{[-10,4]} = [0,2], \\
\log([-2,-1]) & = & \emptyset.
\end{array}$$

1.2 Projection d'une contrainte

Soient x, y, z trois variables telles que

$$\begin{array}{rcl} x & \in & [-\infty, 5], \\ y & \in & [-\infty, 4], \\ z & \in & [6, \infty], \\ z & = & x + y. \end{array}$$

Les valeurs < 2 pour x, < 1 pour y et > 9 pour z sont inconsistantes.

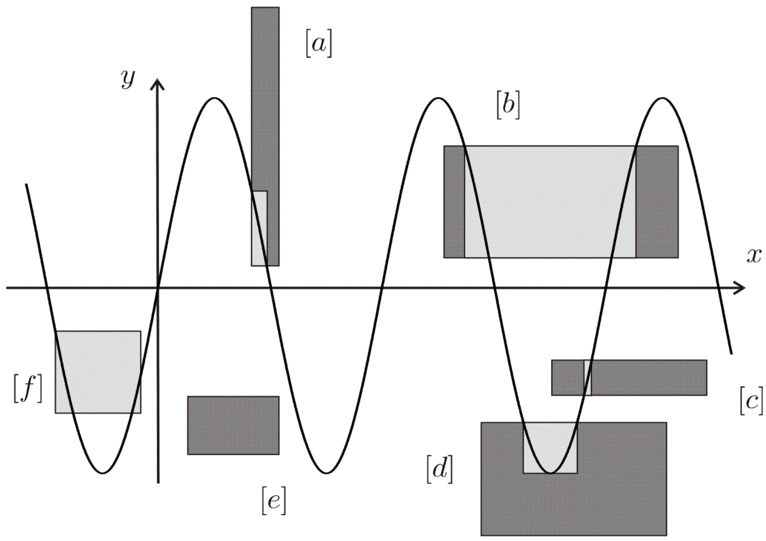
En effet, puisque $x \in [-\infty, 5], y \in [-\infty, 4], z \in [6, \infty]$ et $z = x + y$, nous avons

$$
\begin{aligned}
z = x + y \Rightarrow \quad z \in \quad & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\
& = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\
x = z - y \Rightarrow \quad x \in \quad & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\
& = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\
y = z - x \Rightarrow \quad y \in \quad & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\
& = [-\infty, 4] \cap [1, \infty] = [1, 4].
\end{aligned}
$$

Pour la contrainte

$$y = \mathsf{sin}\, x,\ x \in [x], y \in [y]$$

le problème est un peu plus difficile.



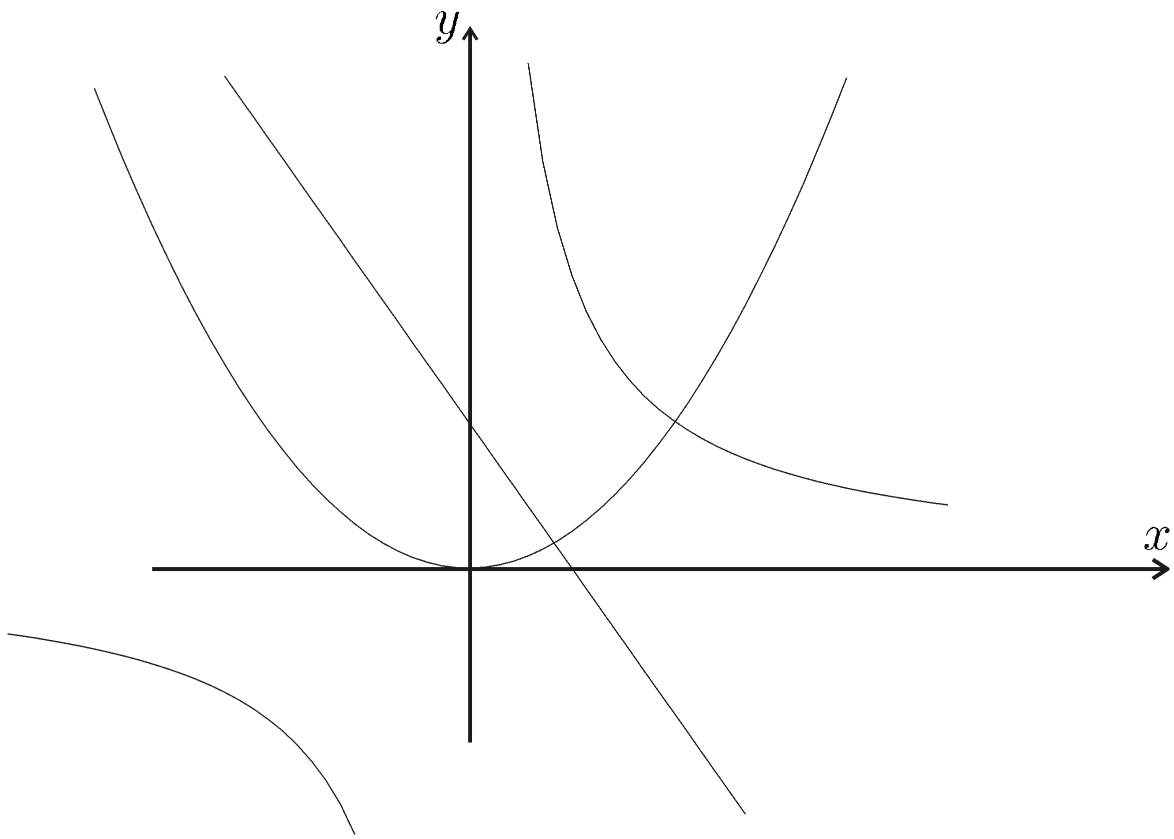
1.3 Propagation

Considérons le problème à trois contraintes

$$\left\{\begin{array}{lr} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{array}\right.$$

Pour chacune de ces variables, nous affectons le domaine $[-\infty, \infty]$.

La *propagation* consiste à projeter ces contraintes jusqu'à l'équilibre.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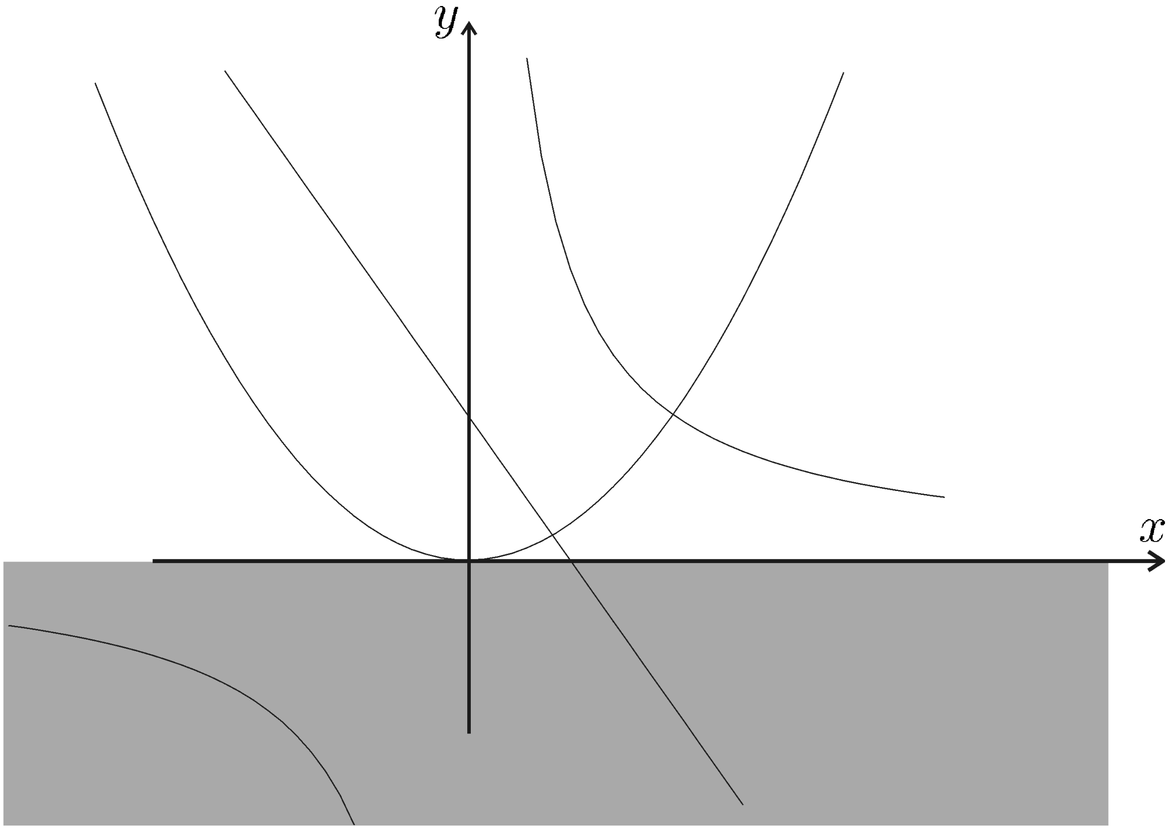

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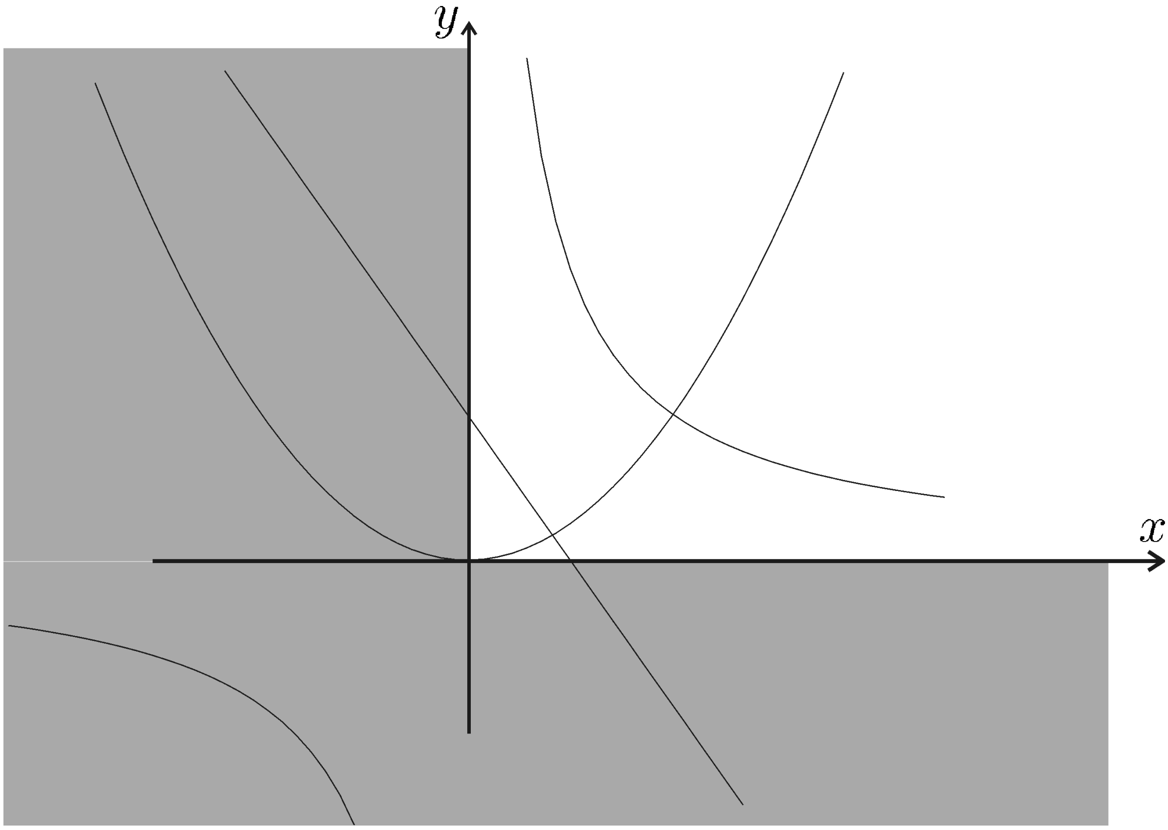

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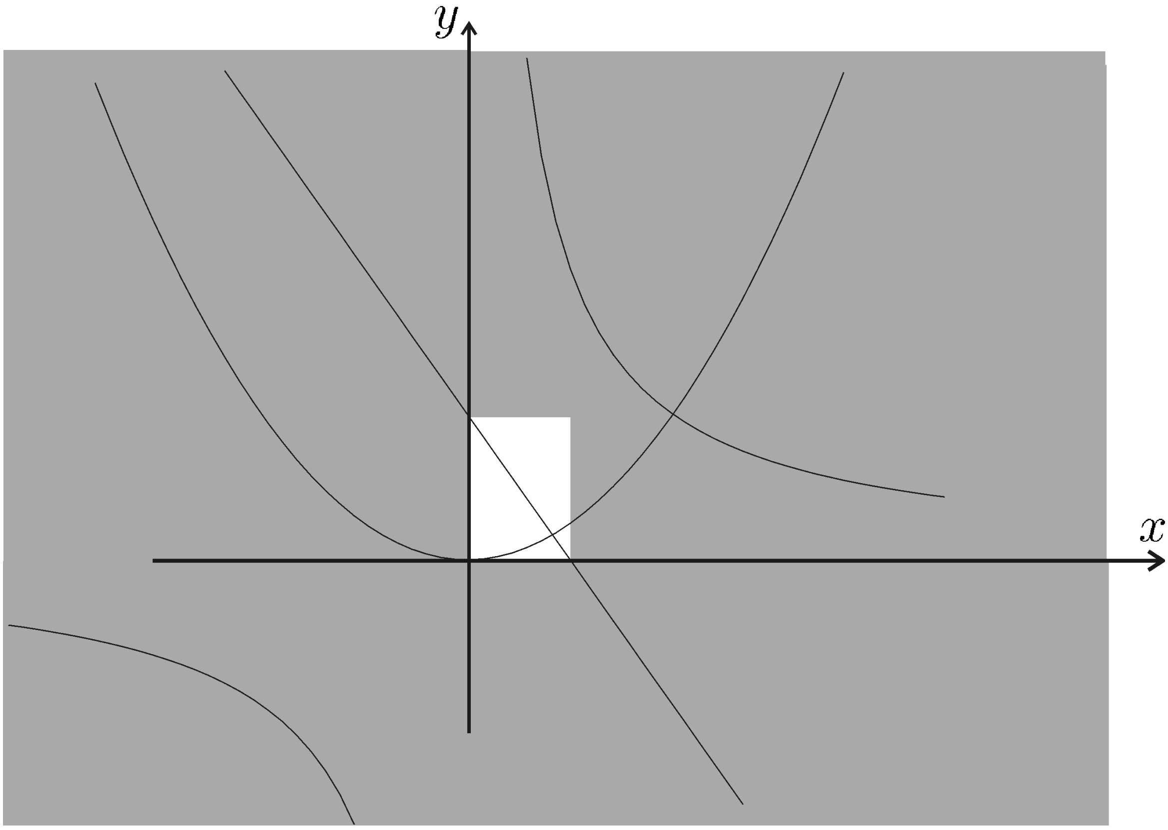

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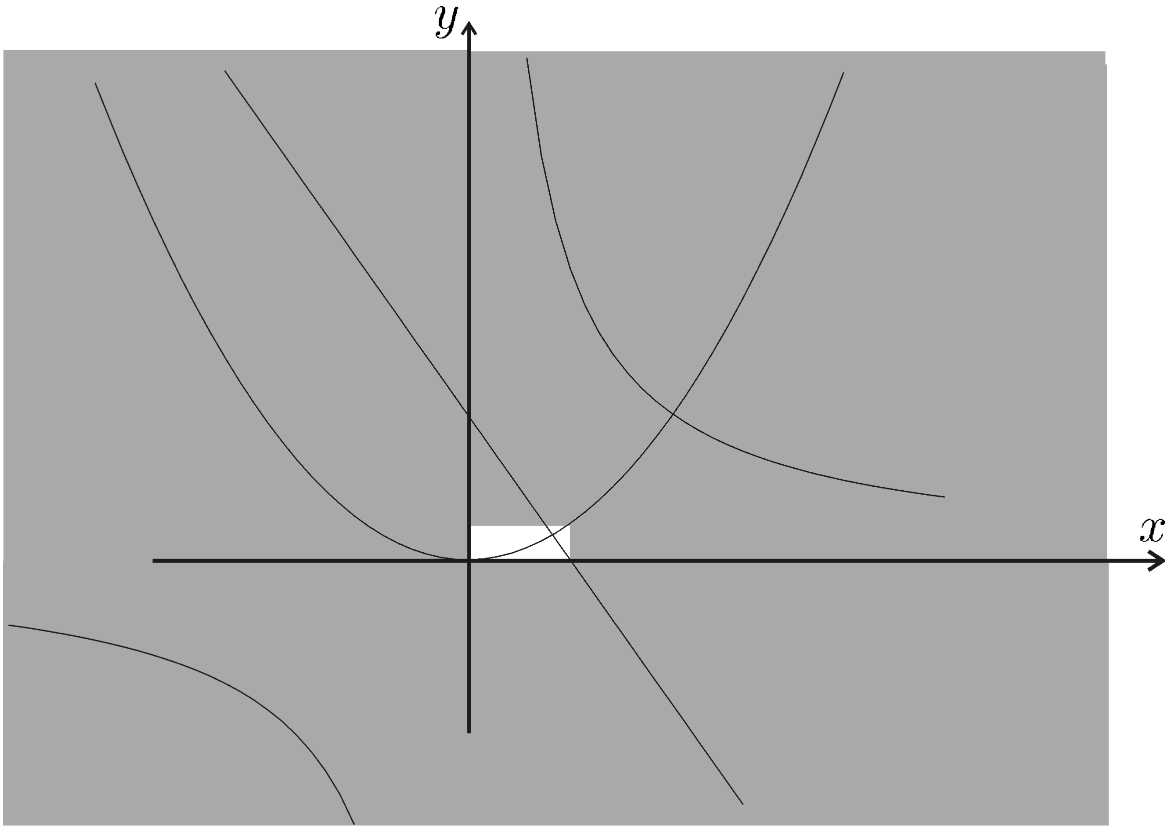

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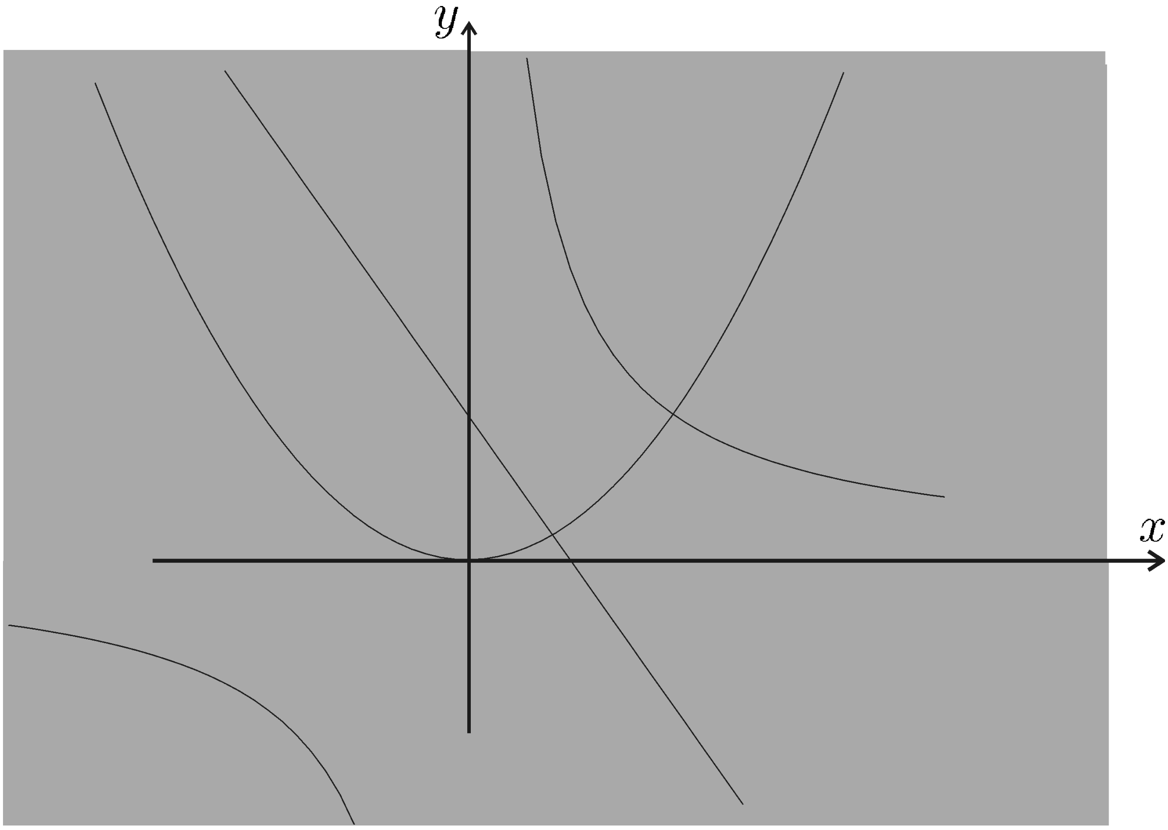
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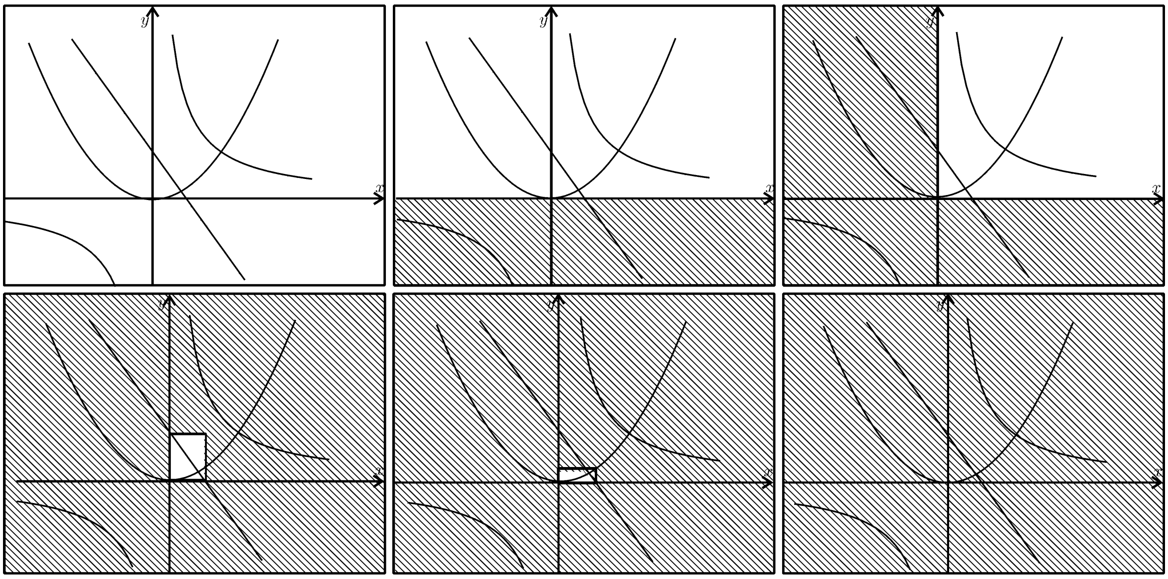
y
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$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty,\infty]^2 \ = \ [0,\infty] \\
(C_2) \Rightarrow & x \in & 1/[0,\infty] \ = [0,\infty] \\
(C_3) \Rightarrow & y \in & [0,\infty] \ \cap \ ((-2)\,.[0,\infty]+1) \\
 & & = [0,\infty] \ \cap \ ([-\infty,1]) = [0,1] \\
 & x \in & [0,\infty] \ \cap \ (-[0,1]/2+1/2) = [0,\frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0,1] \cap [0,1/2]^2 = [0,1/4] \\
(C_2) \Rightarrow & x \in & [0,1/2] \ \cap \ 1/[0,1/4] = \emptyset \\
 & y \in & [0,1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

1.4 Décomposition

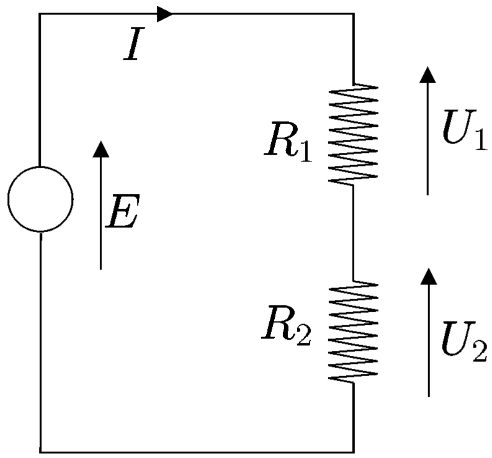
Pour des contraintes plus complexes une décomposition est nécessaire. Par exemple

$$\begin{array}{l} x + \mathsf{sin}(y) - xz \leq 0, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{array}$$

se décompose comme suit.

$$\left\{ \begin{array}{l} a = \mathsf{sin}(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array} \right. , \qquad \begin{array}{ll} x \in [-1, 1] & a \in]-\infty, \infty[\\ y \in [-1, 1] & b \in]-\infty, \infty[\\ z \in [-1, 1] & c \in]-\infty, \infty[\\ & d \in]-\infty, 0] \end{array}$$

1.5 Problème d'estimation



Contraintes

$$\begin{aligned} P &= EI;\ E = (R_1 + R_2)\, I; \\ U_1 &= R_1 I;\ U_2 = R_2 I;\ E = U_1 + U_2. \end{aligned}$$

Domaines initiaux

$$\begin{array}{cc} R_1 \in [0, \infty]\Omega, & R_2 \in [0, \infty]\Omega, \\ E \in [23, 26]\mathrm{V}, & I \in [4, 8]\mathrm{A}, \\ U_1 \in [10, 11]\mathrm{V}, & U_2 \in [14, 17]\mathrm{V}, \\ P \in [124, 130]\mathrm{W}. & \end{array}$$

Les domaines contractés sont

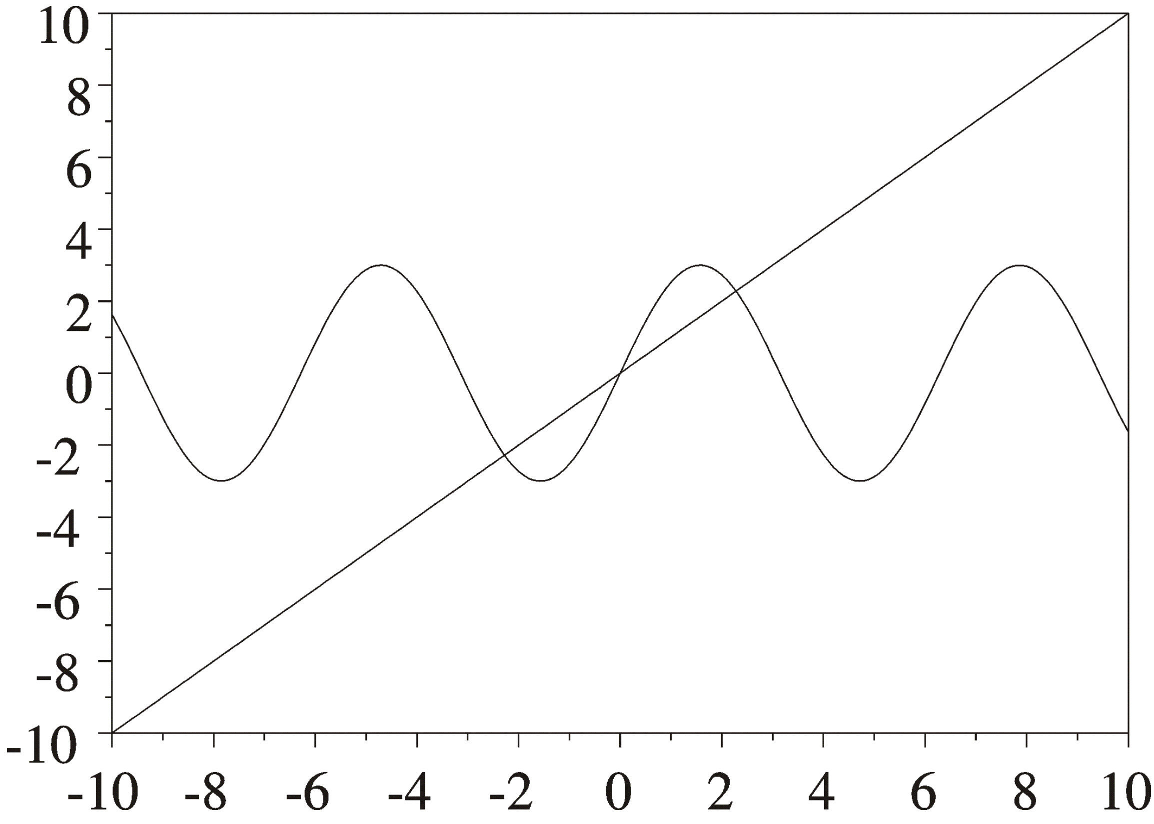
$$\begin{array}{cc} R_1 \in [\mathbf{1.84}, \mathbf{2.31}]\,\Omega, & R_2 \in [\mathbf{2.58}, \mathbf{3.35}]\Omega, \\ E \in [\mathbf{24}, 26]\mathsf{V}, & I \in [\mathbf{4.769}, \mathbf{5.417}]\,\mathsf{A}, \\ U_1 \in [10, 11]\mathsf{V}, & U_2 \in [14, \mathbf{16}]\mathsf{V}, \\ P \in [124, 130]\mathsf{W}. & \end{array}$$

2 Résolution d'équations

2.1 Principe

Considérons le système

$$\left\{\begin{array}{lcl} y & = & 3\sin(x) \\ y & = & x \end{array}\right. \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$



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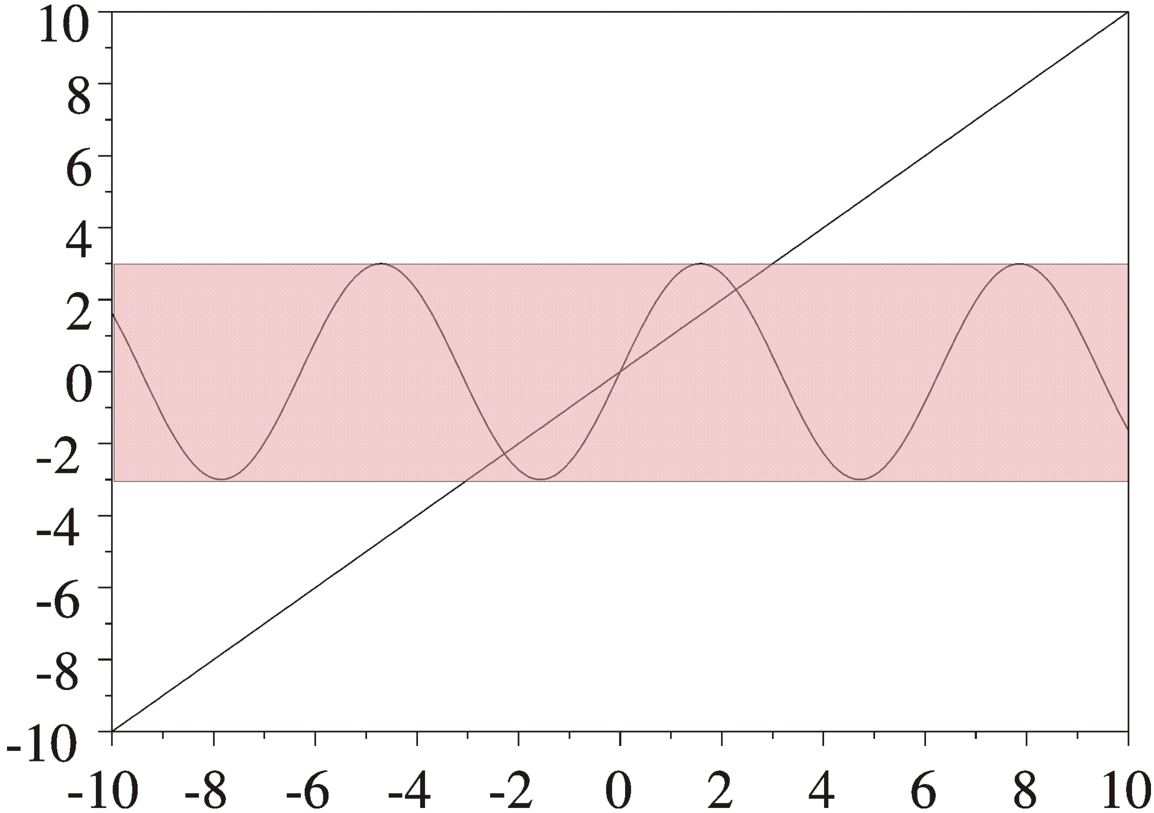

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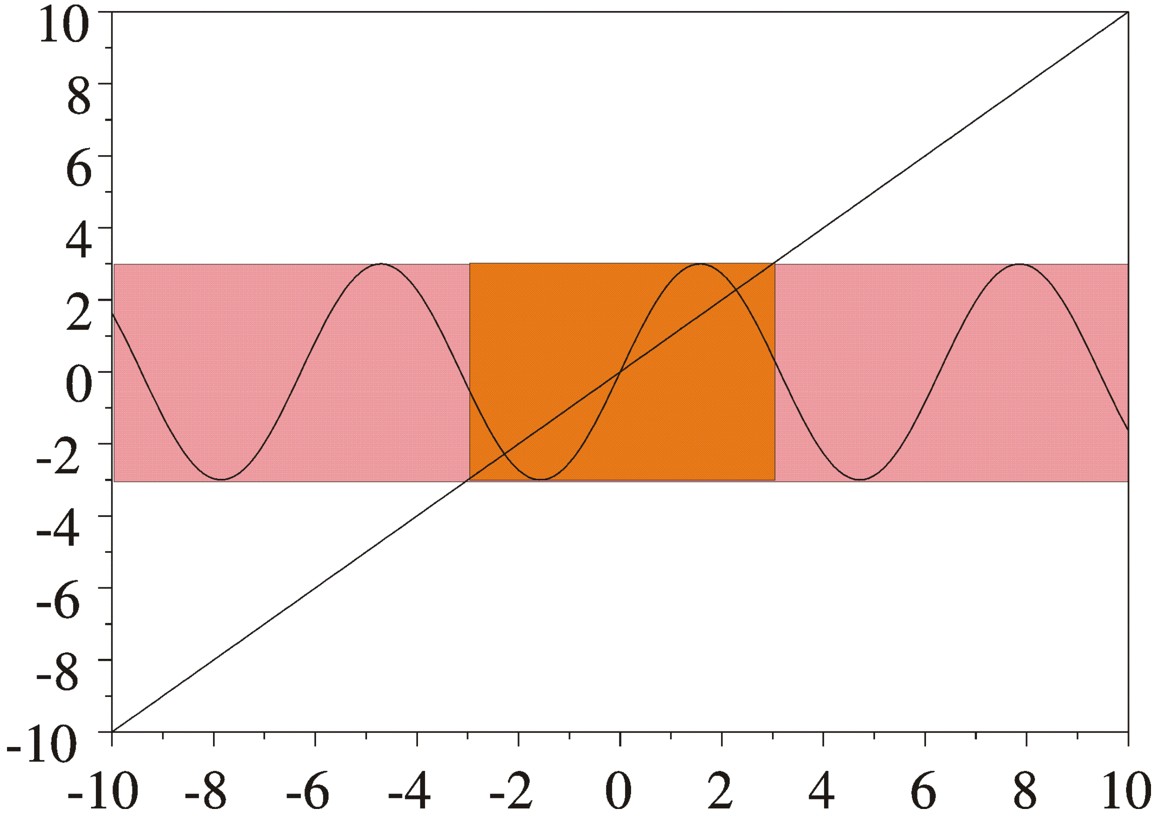

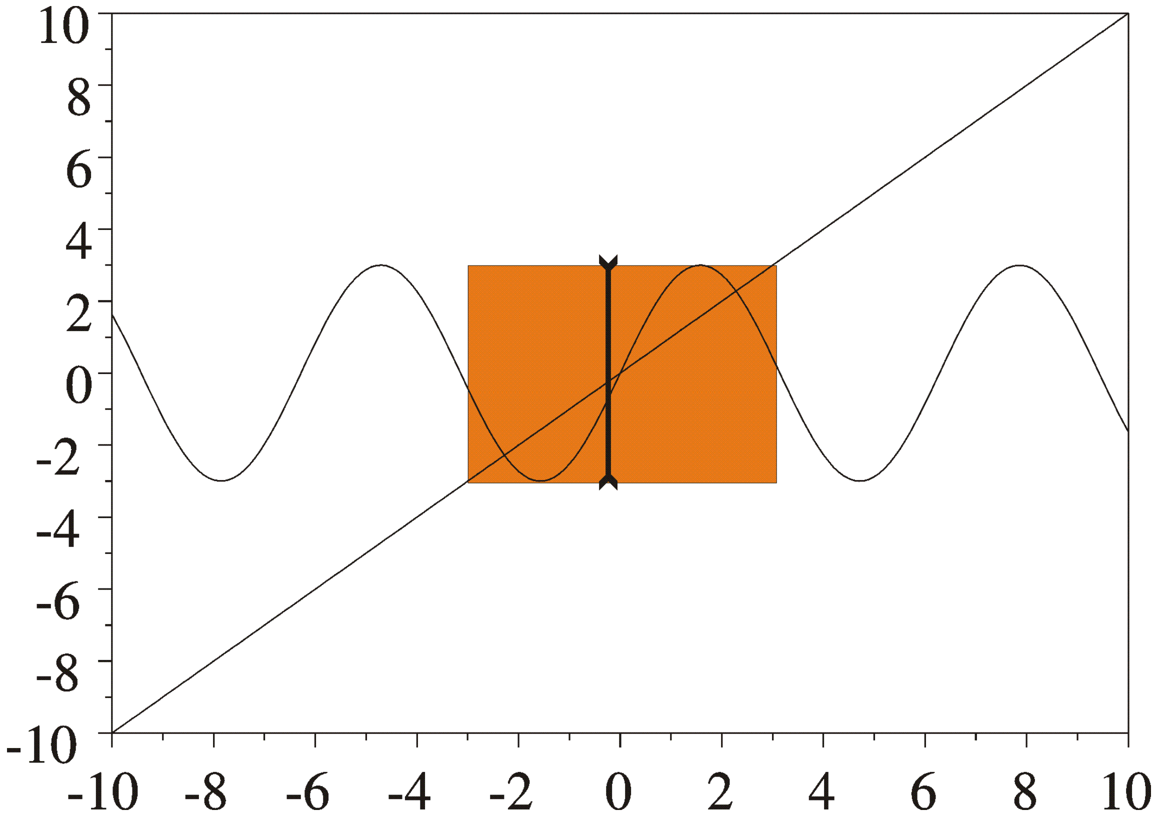
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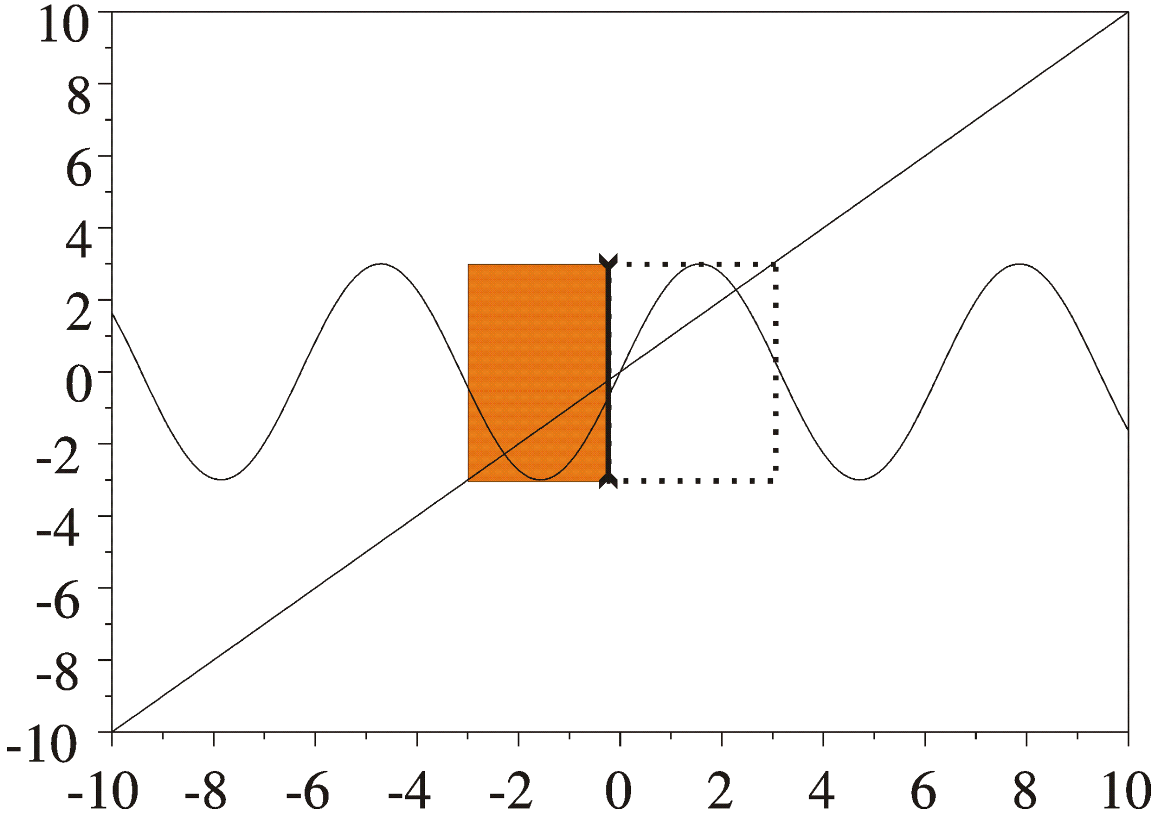
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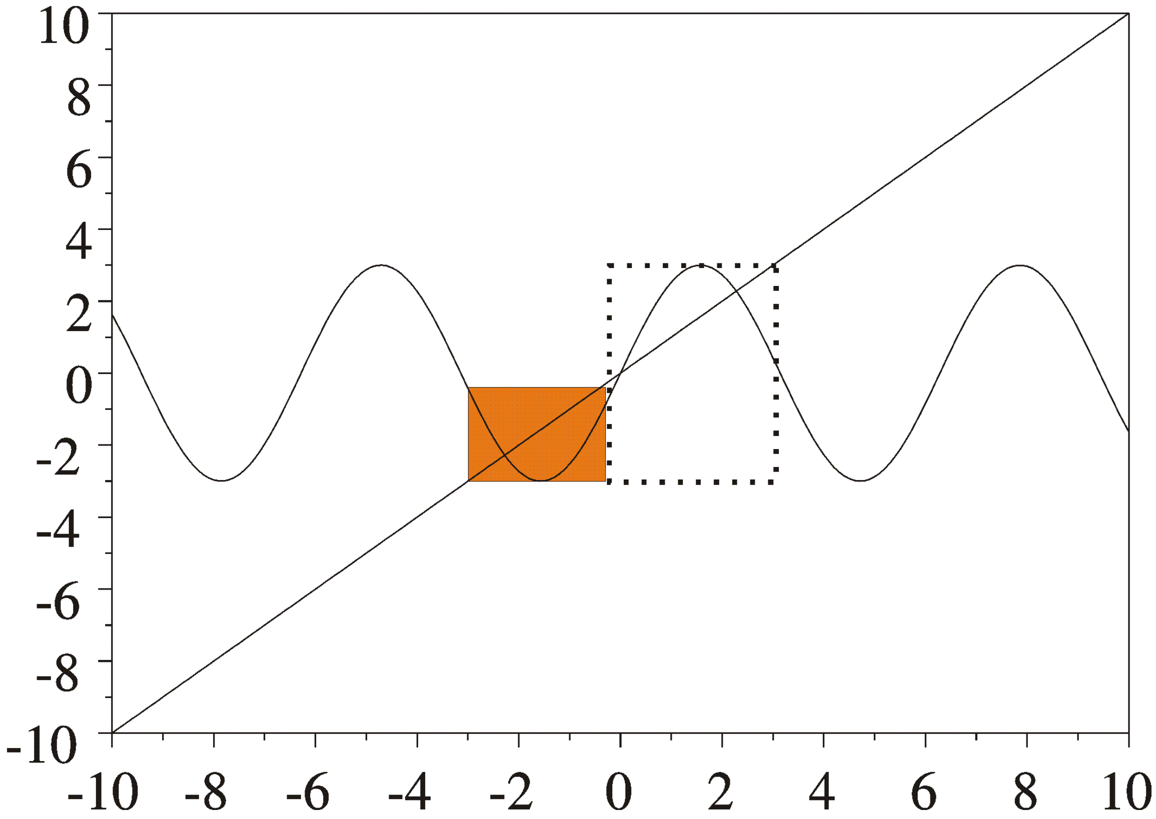
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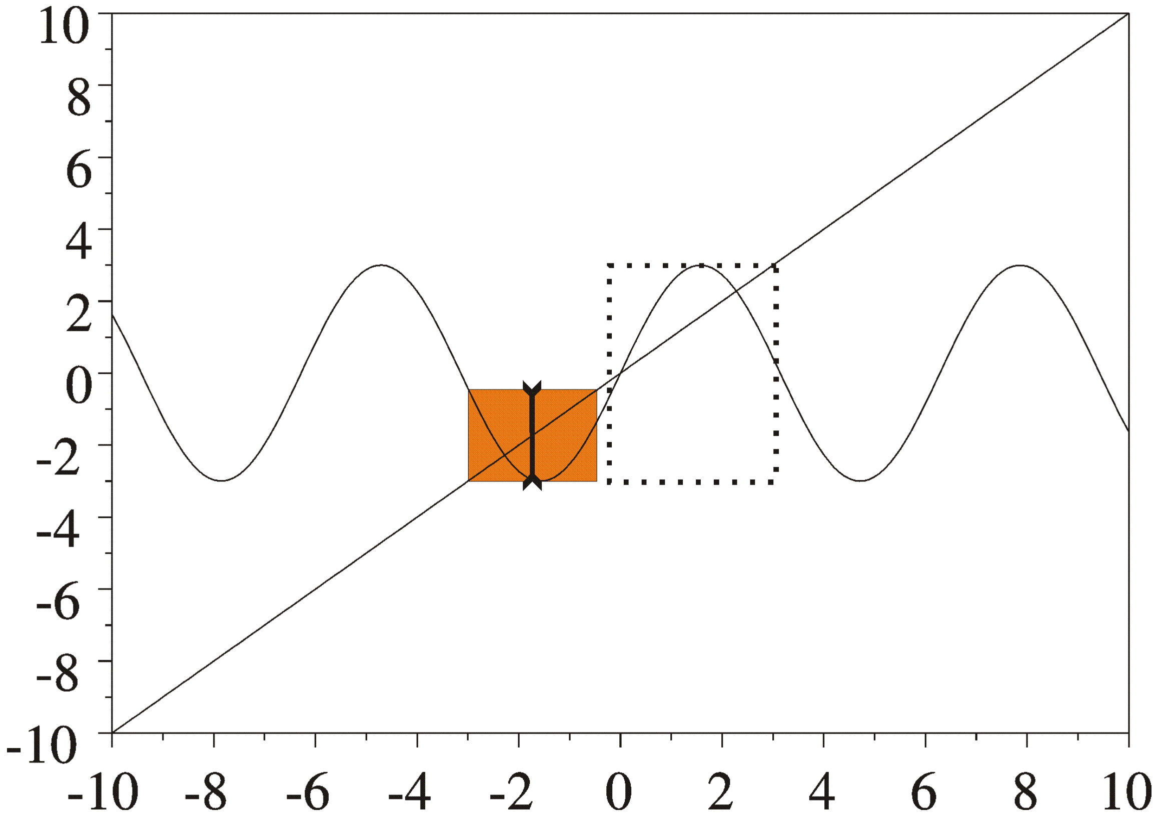

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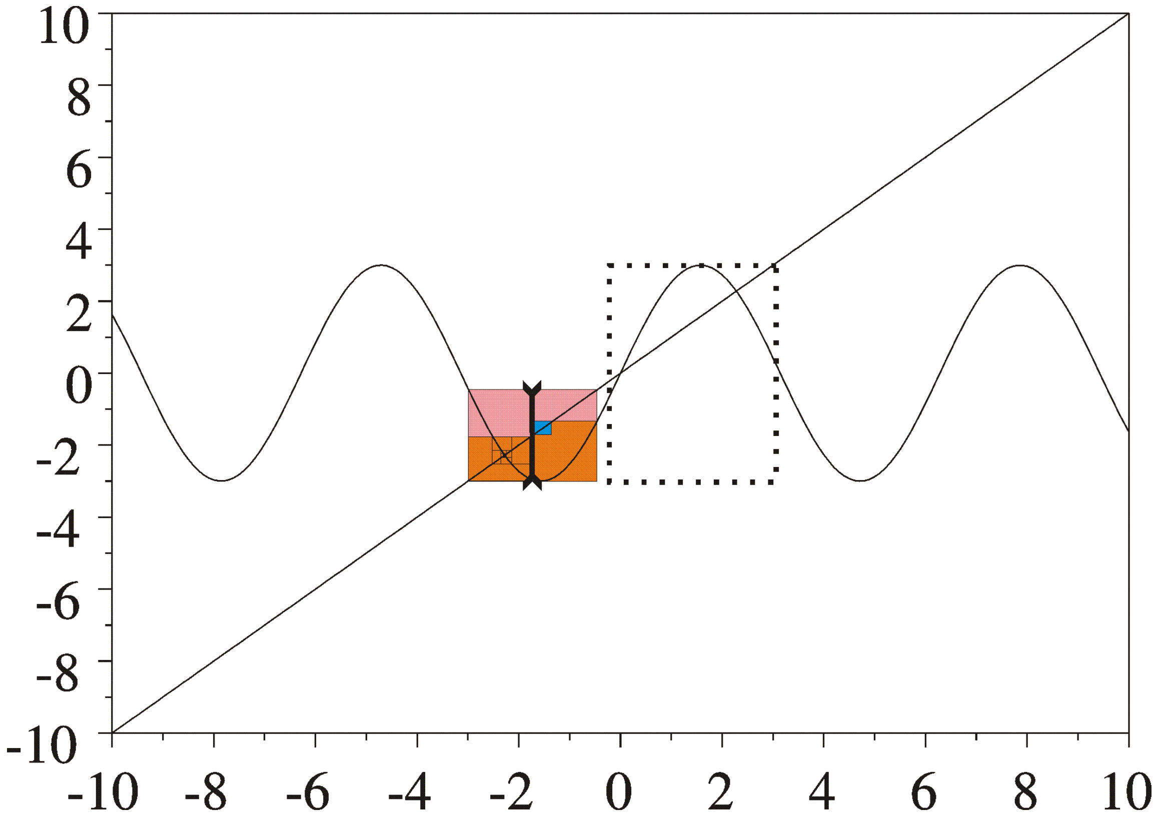

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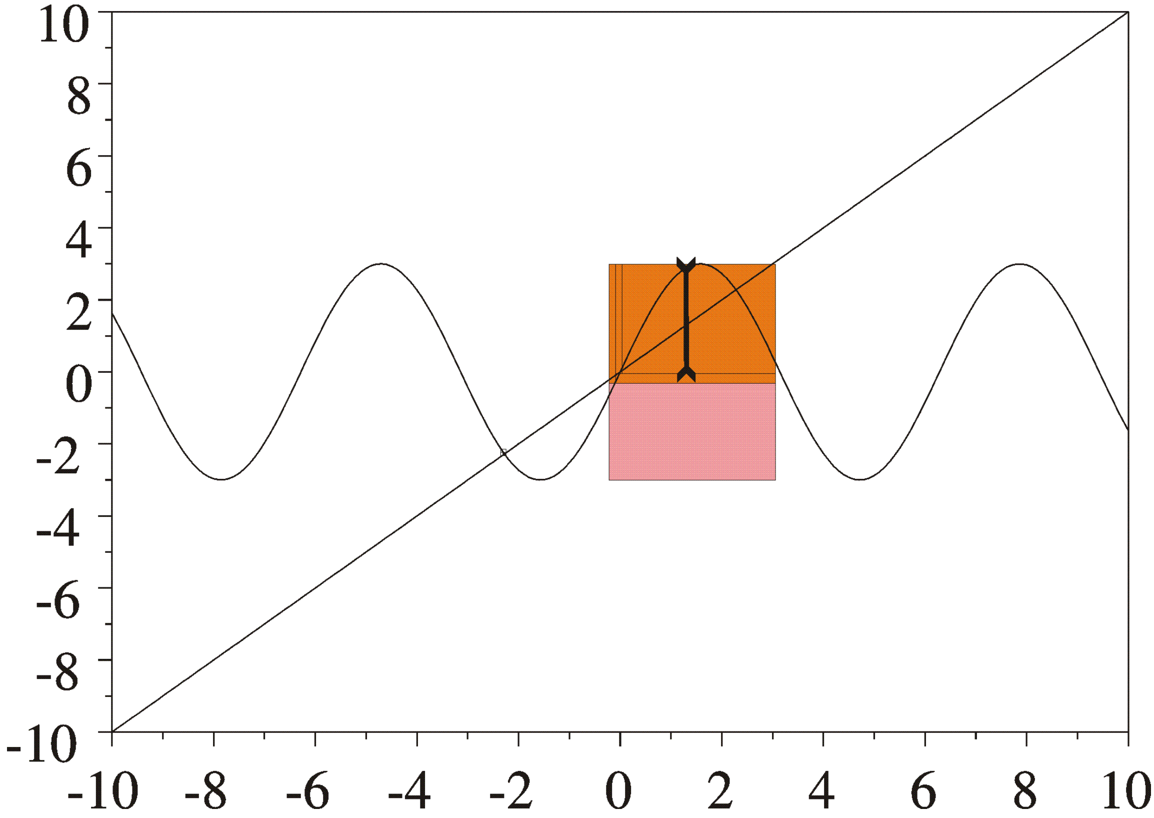
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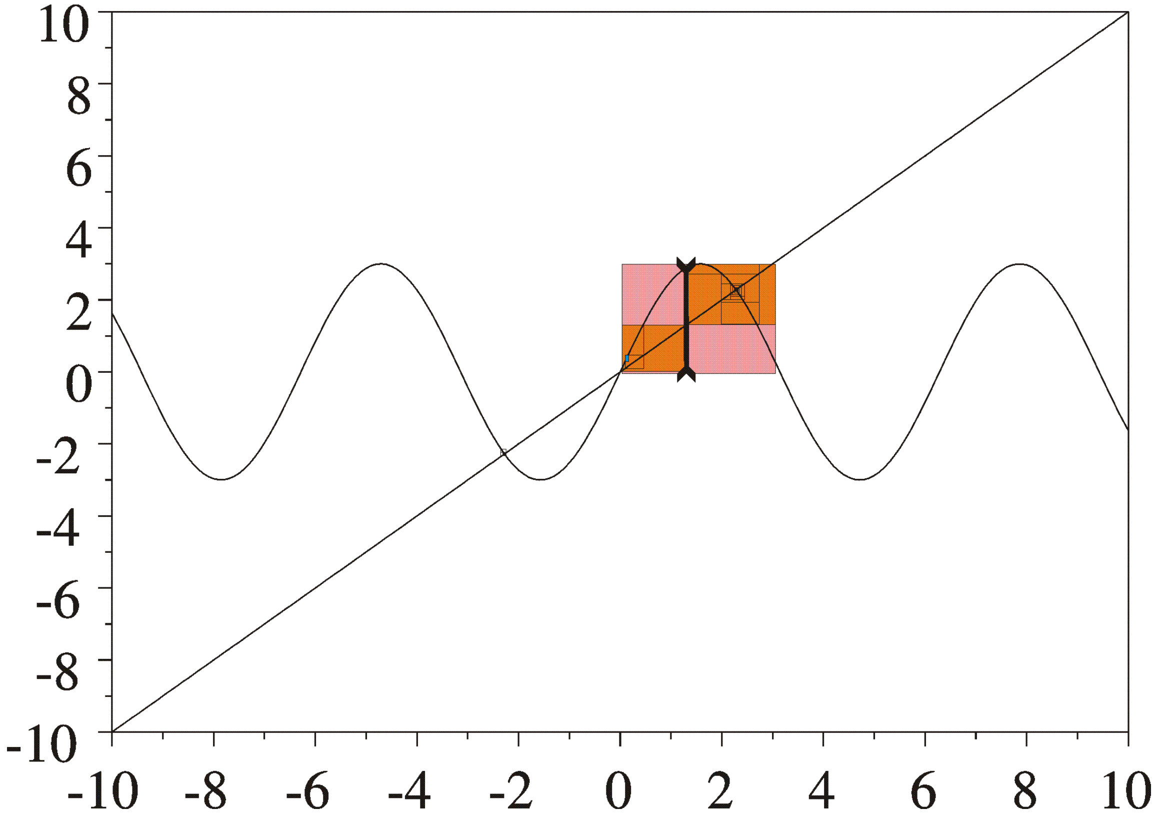

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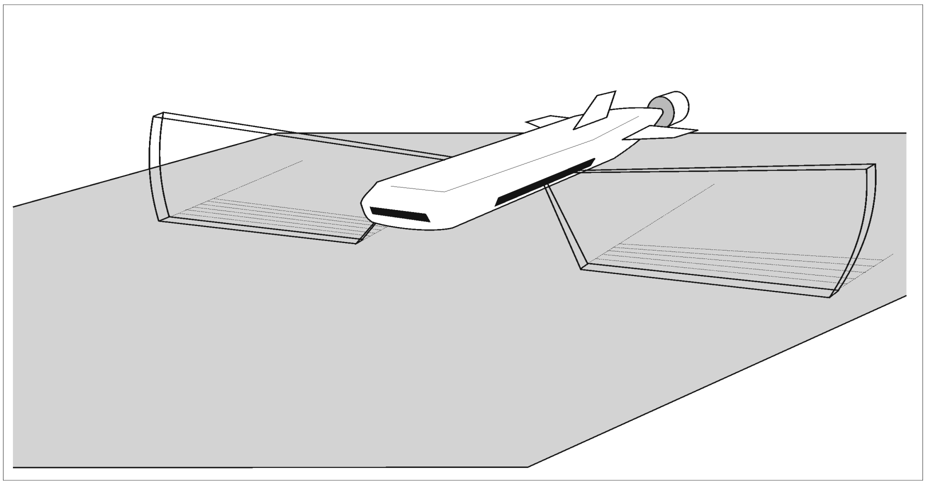
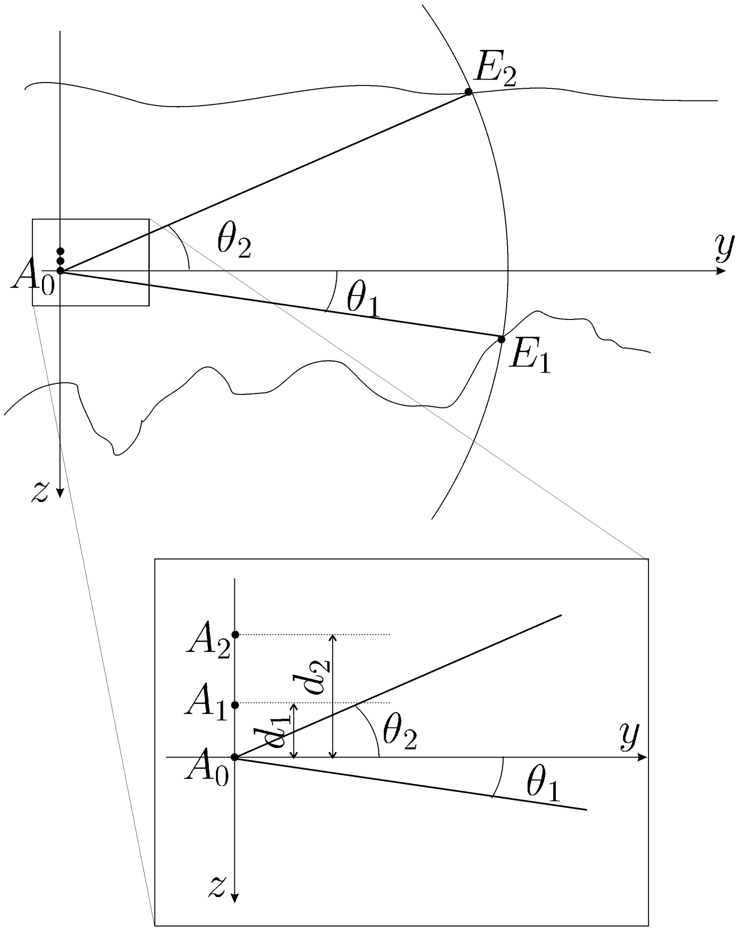
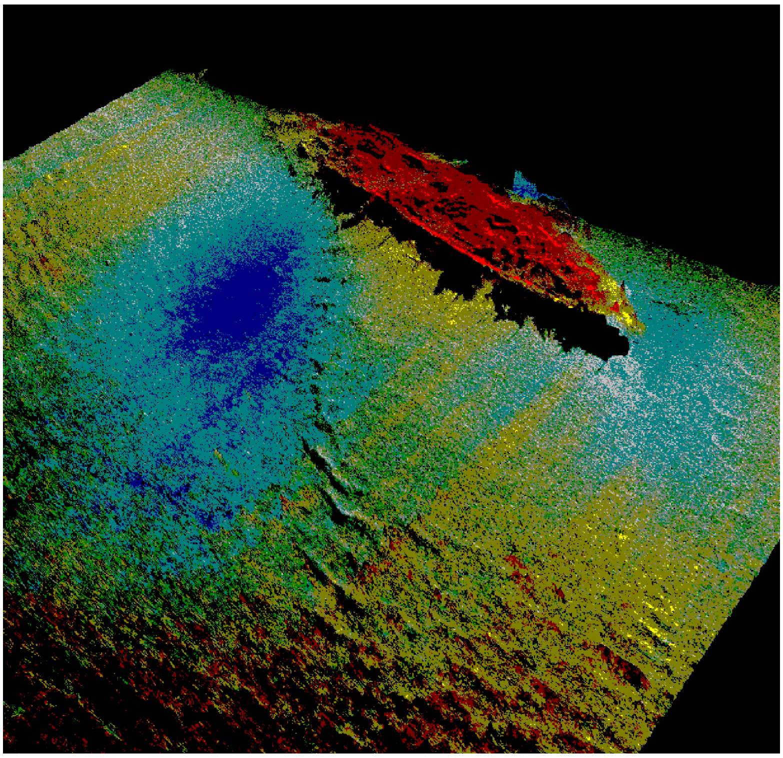
2.2 Imagerie 3d du fond de l'océan



Chaque sonar possède trois antennes A_0, A_1, A_2. L'onde émise par A_0 est $s(t) = e^{2\pi f_0 t}$.
On a $c = 1500$ ms^{-1}, $\lambda = 3$ mm, $f_0 = 455$ kHz.



E_2
θ_2
y
A_0
θ_1
E_1
z
A_2
A_1
d_2
d_1
θ_2
y
A_0
θ_1
z



3 Approximation intérieure

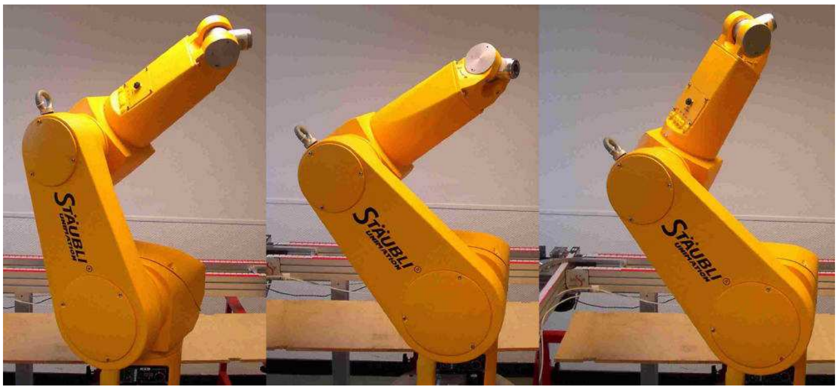
On passe au complémentaire. Par exemple, si

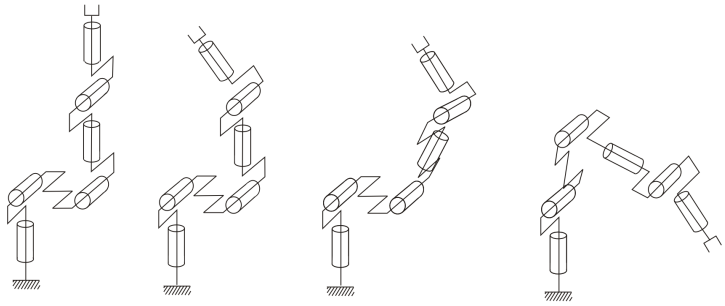
$$\mathbb{S} = \left\{ (x,y) \in \mathbb{R}^2,\ f(x,y) \leq 0 \text{ et } g(x,y) \leq 0 \right\}.$$

Son complémentaire est

$$\begin{aligned} \overline{\mathbb{S}} &= \left\{ (x,y) \in \mathbb{R}^2,\ f(x,y) > 0 \text{ or } g(x,y) > 0 \right\} \\ &= \left\{ (x,y) \in \mathbb{R}^2,\ \max\left(f(x,y), g(x,y)\right) > 0 \right\} \end{aligned}$$

4 Calibration de robots


STÄUBLI



Algorithm f

inputs : $\mathbf{q} = (q_1, ..., q_6)^t$,

$$\mathbf{p} = \left(\alpha_j, d_j, r_j, \theta_0, \theta_j^o, b_x^i, b_y^i, b_z^i, ...\right)^t.$$

outputs : $\mathbf{x} = \left(a_x^1, a_y^1, a_z^1, a_x^2, a_y^2, a_z^2, a_x^3, a_y^3, a_z^3\right)^t$.

$$\mathbf{M} := \begin{pmatrix} \cos q_6 & -\sin q_6 \cos\alpha_6 & \sin q_6 \sin\alpha_6 & d_6 \cos q_6 \\ \sin q_6 & \cos q_6 \cos\alpha_6 & -\cos q_6 \sin\alpha_6 & d_6 \sin q_6 \\ 0 & \sin\alpha_6 & \cos\alpha_6 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

for $j := 5$ to 1,

$\theta := \theta_j^o + q_j$;

$$\mathbf{M} := \begin{pmatrix} \cos\theta & -\sin\theta & 0 & d_j \\ \sin\theta\cos\alpha_j & \cos\theta\cos\alpha_j & -\sin\alpha_j & -r_j\sin\alpha_j \\ \sin\theta\sin\alpha_j & \cos\theta\sin\alpha_j & \cos\alpha_j & r_j\cos\alpha_j \\ 0 & 0 & 0 & 1 \end{pmatrix} .\mathbf{M}$$

endfor

$$\mathbf{M} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 0 \\ 0 & 0 & 1 & r_0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .\mathbf{M}$$

for $i := 1$ to 3, $\mathbf{b}^i = \begin{pmatrix} b_x^i & b_y^i & b_z^i & 1 \end{pmatrix}^\mathsf{T}$;

$$\mathbf{x} := \begin{pmatrix} \mathbf{M} & 0 & 0 \\ 0 & \mathbf{M} & 0 \\ 0 & 0 & \mathbf{M} \end{pmatrix} \begin{pmatrix} \mathbf{b}^1 \\ \mathbf{b}^2 \\ \mathbf{b}^3 \end{pmatrix}.$$

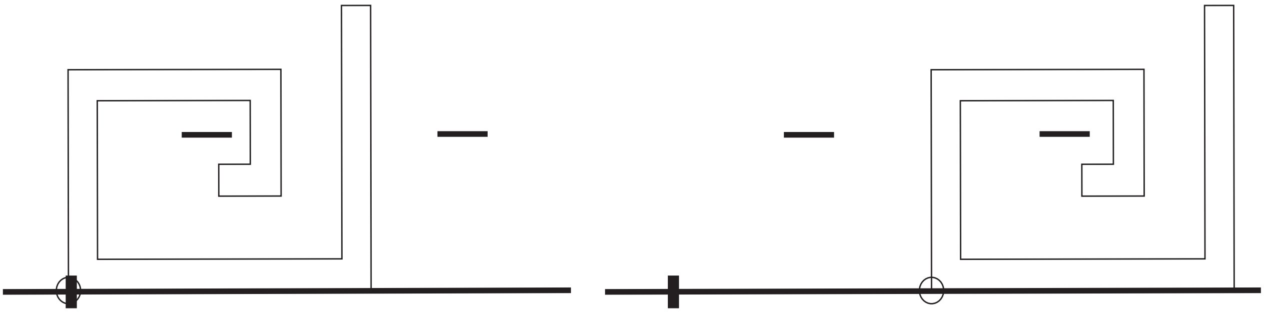
j	α_j	d_j	θ_j^o	r_j
0	-	-	$\frac{\pi}{2}$	0.5
1	0.1	0	0	0
2	-$\frac{\pi}{2}$	0	0	0
3	0	0.5	0	0
4	$\frac{\pi}{2}$	0	0	0.5
5	-$\frac{\pi}{2}$	0	0	0
6	$\frac{\pi}{2}$	0	-	-

i	b_x^i	b_y^i	b_z^i
1	0.1	0.2	0.1
2	0.1	0.1	0.2
3	0.2	0.1	0.1

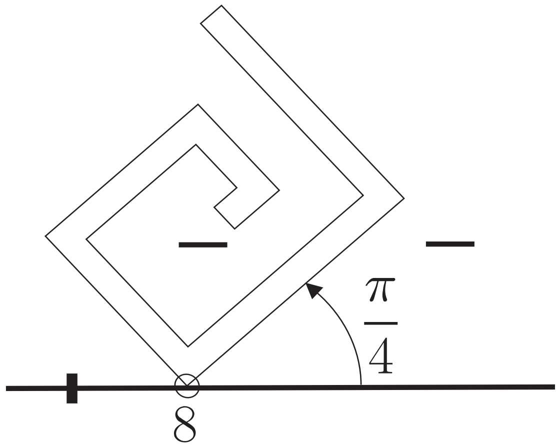
	domaines initiaux	domaines contractés
r_0	$[0.4, 0.6]$	$[0.494046, 0.50101]$
d_1	$[0, 0.1]$	$[0, 0.000558009]$
r_1	$[0, 0.1]$	$[0, 0.00693694]$
d_3	$[0.49, 0.51]$	$[0.498385, 0.501133]$
r_4	$[0.49, 0.51]$	$[0.499216, 0.50114]$
b_x^1	$[0, 0.2]$	$[0.0996052, 0.100629]$
b_y^1	$[0.1, 0.3]$	$[0.199502, 0.200455]$
b_z^1	$[0, 0.2]$	$[0.0997107, 0.100714]$
b_x^2	$[0, 0.2]$	$[0.0996747, 0.100712]$
b_y^2	$[0, 0.2]$	$[0.0994585, 0.10031]$
b_z^2	$[0.1, 0.3]$	$[0.199535, 0.200642]$
b_x^3	$[0.1, 0.3]$	$[0.199689, 0.200578]$
b_y^3	$[0, 0.2]$	$[0.0997562, 0.100319]$
b_z^3	$[0, 0.2]$	$[0.0995661, 0.100557]$

5 Planification de chemin

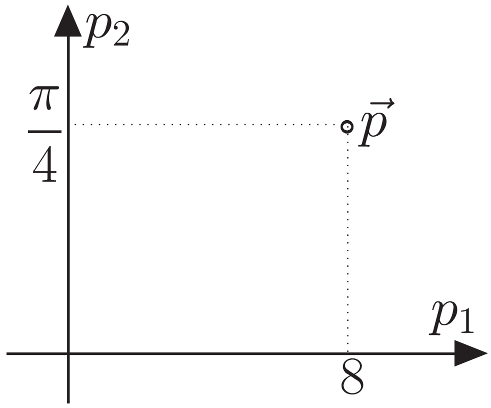
5.1 Principe



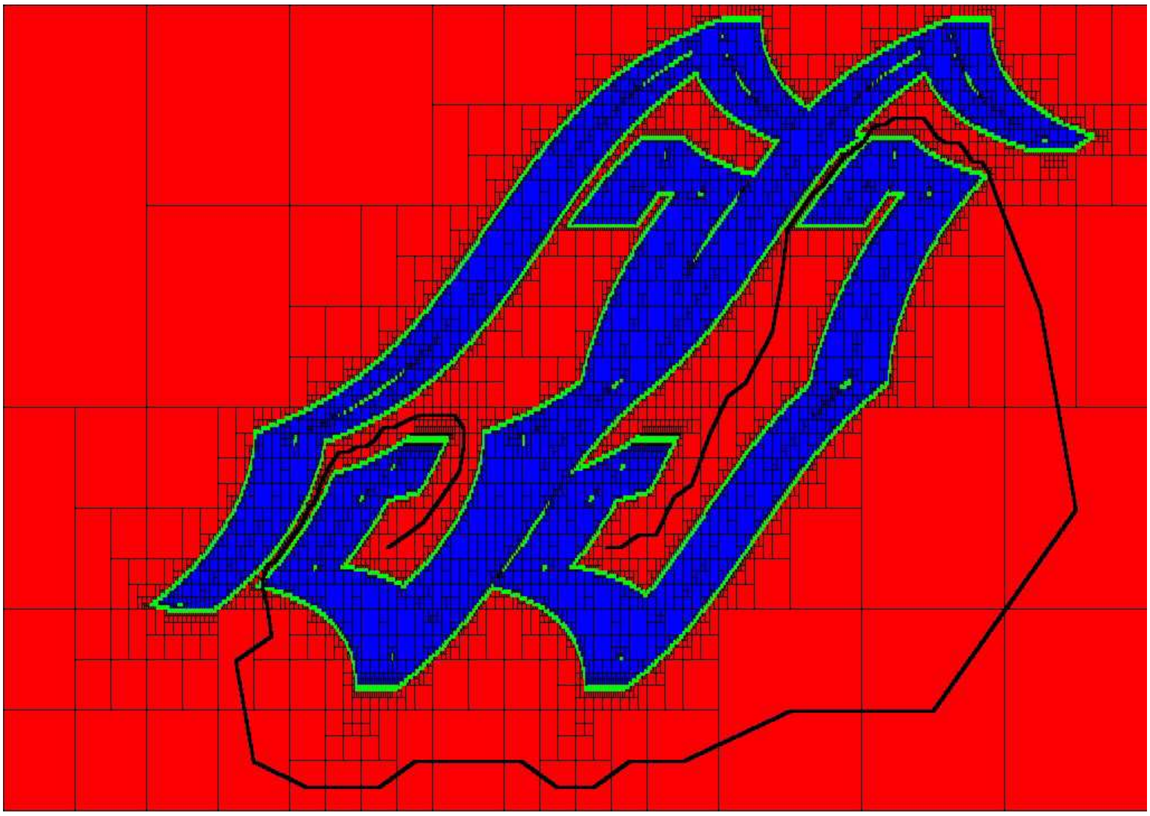
Initial configuration: $\vec{p} = (0\ 0)^{\mathrm{T}}$ Goal configuration: $\vec{p} = (17\ 0)^{\mathrm{T}}$

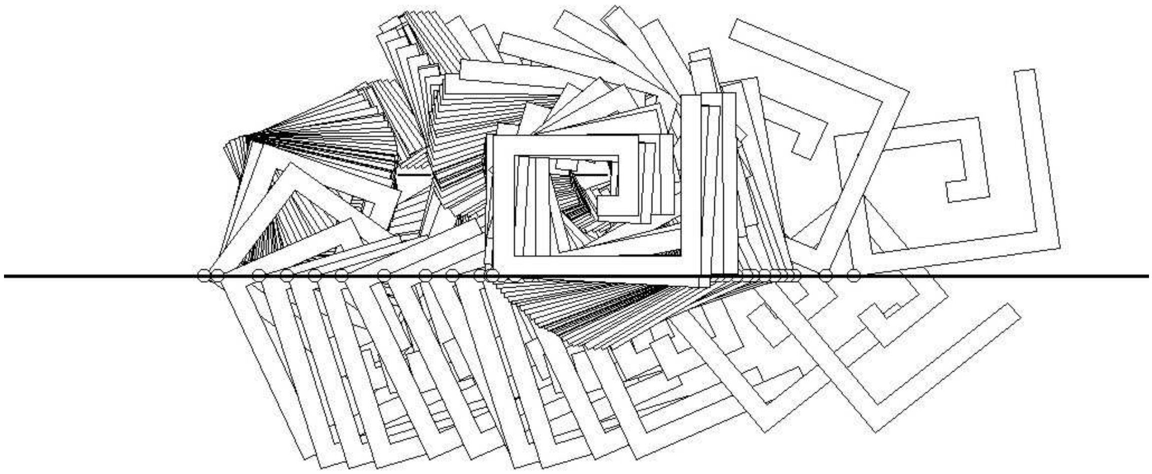


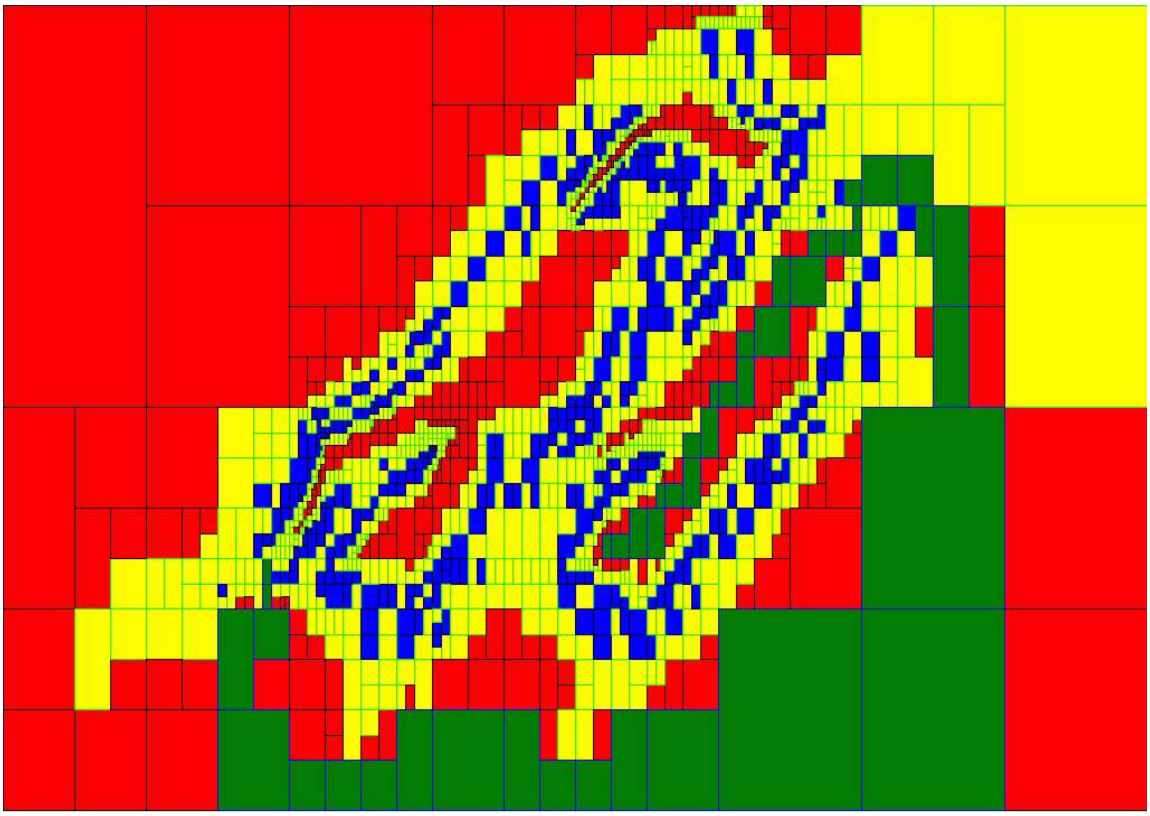
Room

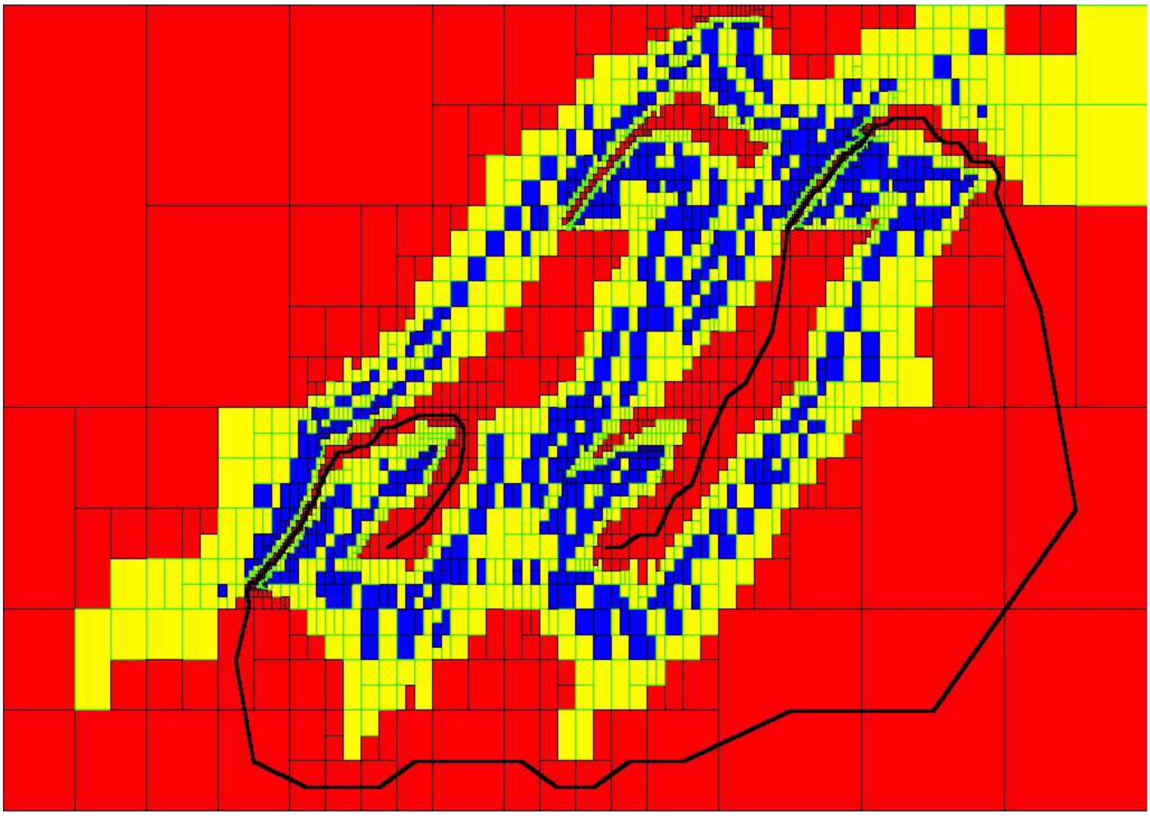


Configuration space





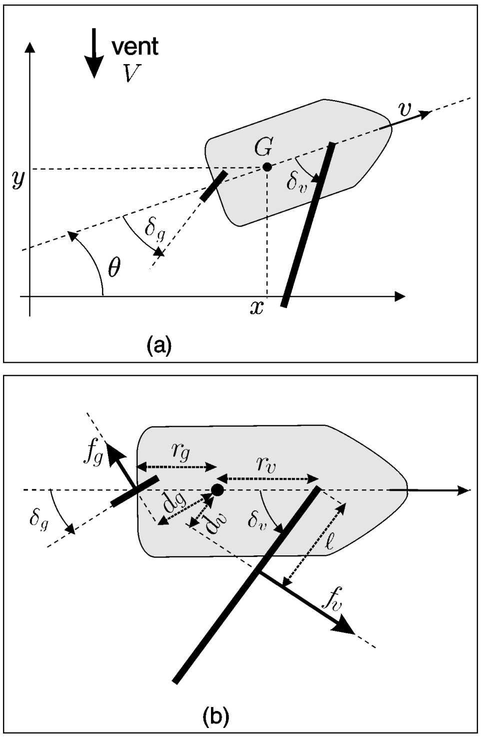




5.2 Utilisation pour la conception de mécanismes

(montrer Robot2d et CIA de N. Delanoue)

6 Analyse et conception d'un voilier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6.1 Equations d'état

$$\left\{\begin{array}{lll}
\dot{x} & = & v\cos\theta \\
\dot{y} & = & v\sin\theta-\beta V \\
\dot{\theta} & = & \omega \\
\dot{\delta}_s & = & u_1 \\
\dot{\delta}_r & = & u_2 \\
\dot{v} & = & \frac{f_s\sin\delta_s-f_r\sin\delta_r-\alpha_f v}{m} \\
\dot{\omega} & = & \frac{(\ell-r_s\cos\delta_s)f_s-r_r\cos\delta_r f_r-\alpha_\theta\omega}{J} \\
f_s & = & \alpha_s\left(V\cos\left(\theta+\delta_s\right)-v\sin\delta_s\right) \\
f_r & = & \alpha_r v\sin\delta_r.
\end{array}\right.$$

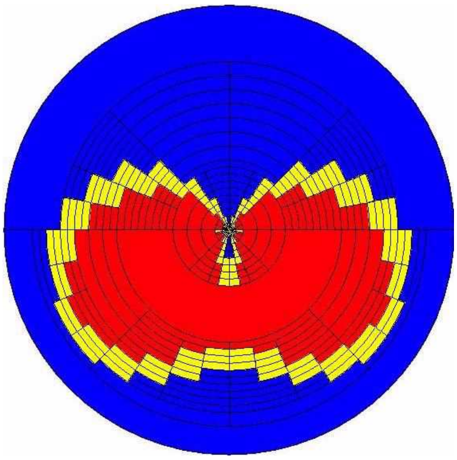
6.2 Est-il possible de l'immobiliser ?

$$\left\{ \begin{array}{lll} 0 & = & v \cos \theta \\ 0 & = & v \sin \theta - \beta V \\ 0 & = & \omega \\ 0 & = & u_1 \\ 0 & = & u_2 \\ 0 & = & \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ 0 & = & \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s & = & \alpha_s \left(V \cos \left(\theta + \delta_s \right) - v \sin \delta_s \right) \\ f_r & = & \alpha_r v \sin \delta_r . \end{array} \right.$$

Les techniques intervalles montrent qu'aucune solution existe si $V \neq 0$.

6.3 Polaire des vitesse

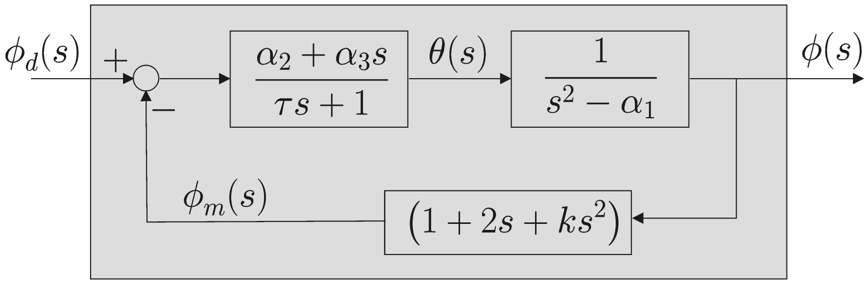
$$\mathbb{W} = \left\{ \quad (\theta, v) \mid \quad \begin{array}{l} \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\ \omega = 0, u_1 = 0, u_2 = 0 \\ \frac{f_s \sin\delta_s - f_r \sin\delta_r - \alpha_f v}{m} = 0 \\ \frac{(\ell - r_s \cos\delta_s) f_s - r_r \cos\delta_r f_r}{J} = 0 \\ f_s = \alpha_s \left(V \cos\left(\theta + \delta_s\right) - v \sin\delta_s\right) \\ f_r = \alpha_r v \sin\delta_r \end{array} \right\}.$$



7 Conception de régulateurs robustes

7.1 Stabilité robuste

On considère un système de régulation pour une moto à 1km/h.



avec

$$\begin{aligned} \alpha_1 &\in [\alpha_1] = [8.8; 9.2]\,, \alpha_2 \in [\alpha_2] = [2.8; 3.2]\,, \\ \alpha_3 &\in [\alpha_3] = [0.8; 1.2]\,, \\ \tau &\in [\tau] = [1.8; 2.2]\,, k \in [k] = [-3.2; -2.8]. \end{aligned}$$

Le système est-il stable à coup-sûr ?

Sa fonction de transfert est

$$\frac{\phi(s)}{\phi_d(s)} = \frac{\alpha_2 + \alpha_3 s}{\left(s^2 - \alpha_1\right)(\tau s + 1) + (\alpha_2 + \alpha_3 s)\left(1 + 2s + ks^2\right)}.$$

Le critère de Routh nous dit que le système est stable si

$$\begin{pmatrix} \tau + \alpha_3 k \\ \alpha_2 k + 2\alpha_3 + 1 \\ \alpha_3 - \alpha_1 \tau + 2\alpha_2 - \frac{(\tau+\alpha_3 k)(-\alpha_1+\alpha_2)}{\alpha_2 k+2\alpha_3+1} \\ -\alpha_1 + \alpha_2 \end{pmatrix}$$

sont de même signe.

Or, nous avons l'équivalence suivante

$$
\begin{array}{rc}
 & b_1,\ b_2,\ b_3,\ b_4 \text{ n'ont pas tous le même signe} \\
\Leftrightarrow & \left\{ \begin{array}{c} \min\left(b_1, b_2, b_3, b_4\right) \leq 0 \\ -\max\left(b_1, b_2, b_3, b_4\right) \leq 0 \end{array} \right. .
\end{array}
$$

Donc, nous avons la stabilité robuste si

$$\exists \alpha_1 \in [\alpha_1], \exists \alpha_2 \in [\alpha_2], \exists \alpha_3 \in [\alpha_3], \exists \tau \in [\tau], \exists k \in [k],$$
$$\begin{cases} \min \begin{pmatrix} \tau + \alpha_3 k \\ \alpha_2 k + 2\alpha_3 + 1 \\ \alpha_3 - \alpha_1 \tau + 2\alpha_2 - \frac{(\tau+\alpha_3 k)(-\alpha_1+\alpha_2)}{\alpha_2 k + 2\alpha_3 + 1} \\ -\alpha_1 + \alpha_2 \end{pmatrix} \leq 0 \\ -\max \begin{pmatrix} \tau + \alpha_3 k \\ \alpha_2 k + 2\alpha_3 + 1 \\ \alpha_3 - \alpha_1 \tau + 2\alpha_2 - \frac{(\tau+\alpha_3 k)(-\alpha_1+\alpha_2)}{\alpha_2 k + 2\alpha_3 + 1} \\ -\alpha_1 + \alpha_2 \end{pmatrix} \leq 0 \end{cases}$$

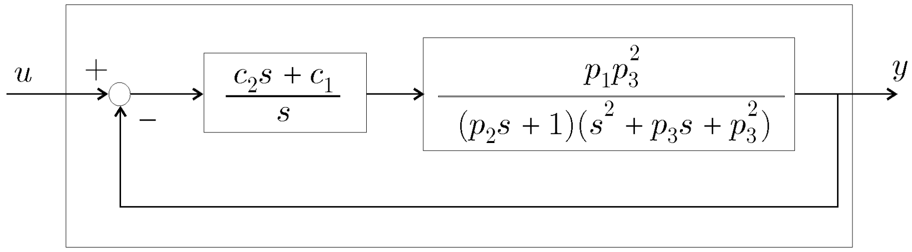
est une proposition fausse.

C'est-à-dire que le CSP

$$
\begin{array}{rcl}
\mathcal{V} & = & \{a_0, a_1, a_2, a_3, \alpha_1, \alpha_2, \alpha_3, \tau, k\}, \\
\mathcal{D} & = & \{[\alpha_0], [\alpha_1], [\alpha_2], [\alpha_3], [\alpha_2], [\alpha_3], [\tau], [k]\}, \\
\mathcal{C} & = & \left\{ \begin{array}{l}
a_3 = \tau + \alpha_3 k \text{ ; } a_2 = \alpha_2 k + 2\alpha_3 + 1 \text{ ;} \\
a_1 = \alpha_3 - \alpha_1 \tau + 2\alpha_2, \\
a_0 = -\alpha_1 + \alpha_2 \text{ ; } b = \frac{a_2 a_1 - a_3 a_0}{a_2}; \\
\quad \min(a_3, a_2, b, a_0), \leq 0. \\
-\max(a_3, a_2, b, a_0) \leq 0
\end{array} \right\}
\end{array}
$$

n'a pas de solution.

7.2 Conception d'un régulateur robuste



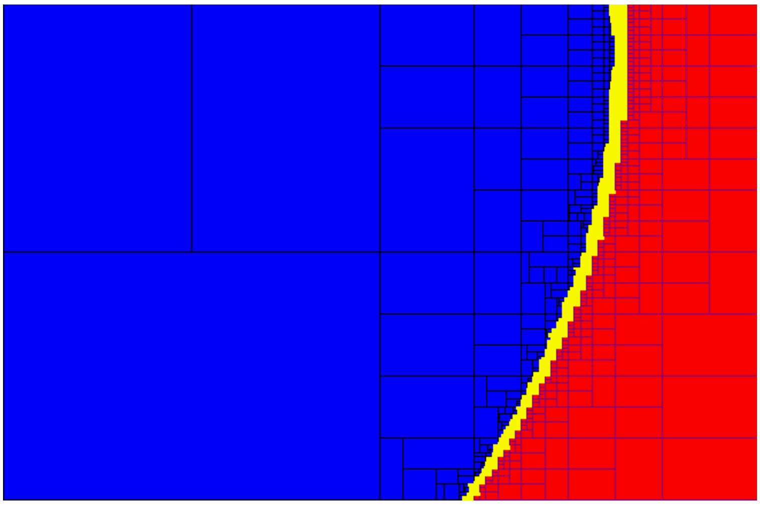
u
$+$
$-$
$\frac{c_2 s + c_1}{s}$
$\frac{p_1 p_3^2}{(p_2 s + 1)(s^2 + p_3 s + p_3^2)}$
y

La fonction de transfert du système bouclé est

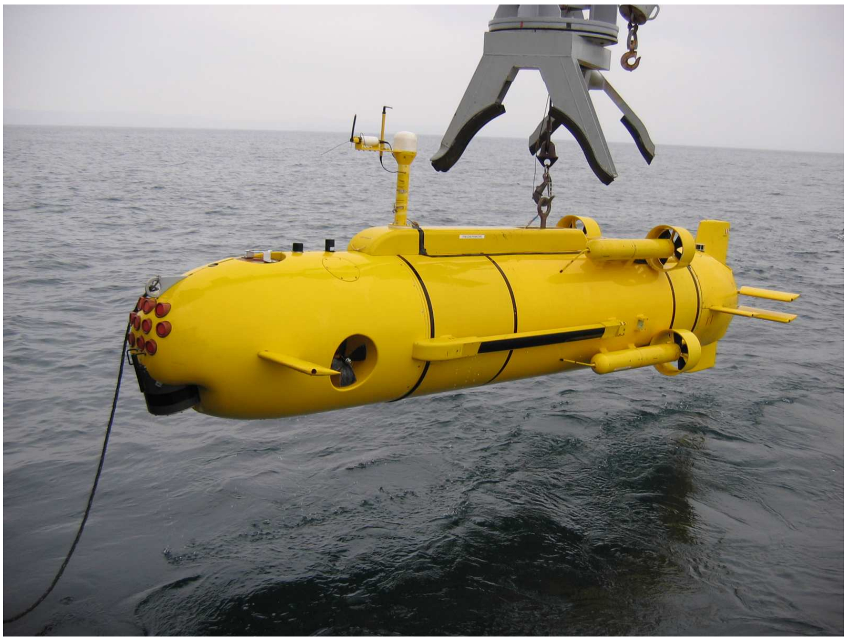
$$\frac{\left(c_2 s + c_1\right)\frac{p_1 p_3^2}{p_2}}{s^4 + \left(p_3 + \frac{1}{p_2}\right)s^3 + \left(p_3^2 + \frac{p_3}{p_2}\right)s^2 + \frac{p_3^2 + c_2 p_1 p_3^2}{p_2}s + \frac{c_1 p_1 p_3^2}{p_2}}.$$

Le critère de Routh nous dit qu'on a la stabilité si tous les éléments suivants ont le même signe.

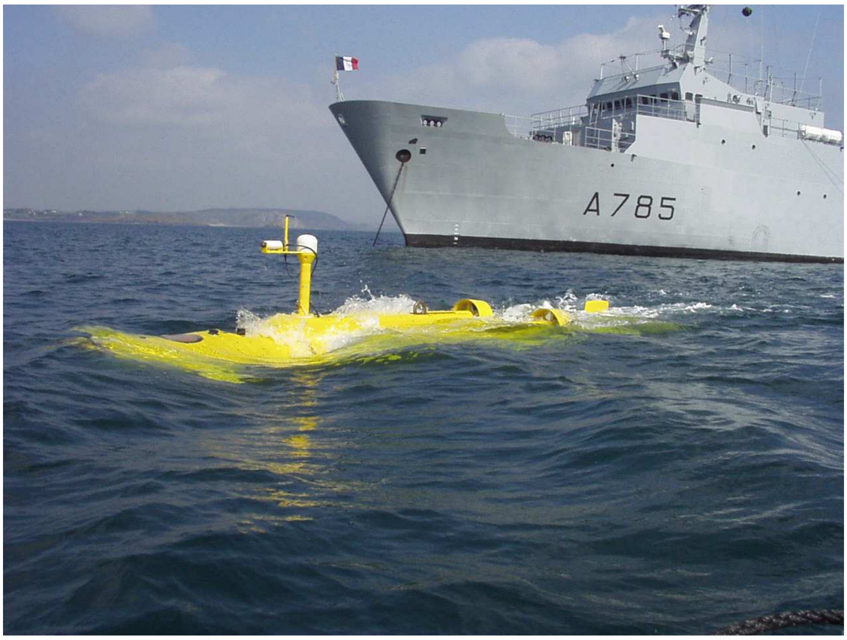
$$\begin{pmatrix} p_2 \\ p_2 p_3 + 1 \\ p_2 p_3^2 + p_3 - \frac{p_2\left(p_3^2 + c_2 p_1 p_3^2\right)}{p_2 p_3 + 1} \\ p_3^2 + c_2 p_1 p_3^2 - \frac{\left(p_2 p_3 + 1\right)^2\left(c_1 p_1 p_3^2\right)}{\left(p_2 p_3^2 + p_3\right)\left(p_2 p_3 + 1\right) - p_2\left(p_3^2 + c_2 p_1 p_3^2\right)} \\ c_1 p_1 p_3^2 \end{pmatrix}$$



8 Localisation de mines



Le *Redermor*, fabriqué par le GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Pourquoi une approche par intervalles ?

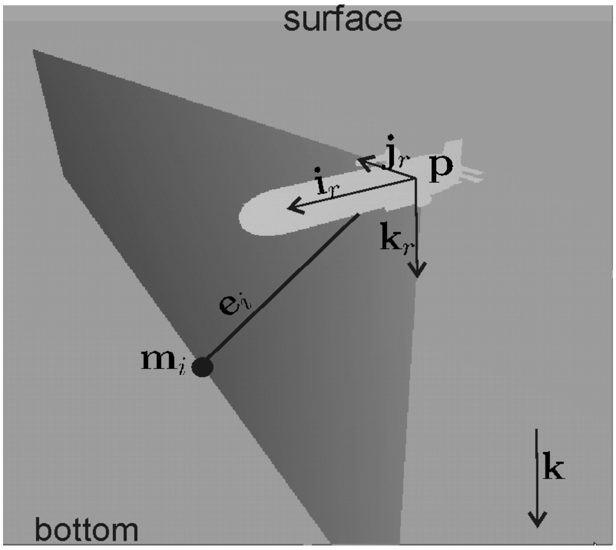
1) Besoin d'une approche fiable.
2) Les équations du robot sont non linéaires.
3) Les bruits de mesure sont non gaussiens.
4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
5) Beaucoup de mesures redondantes sont disponibles.

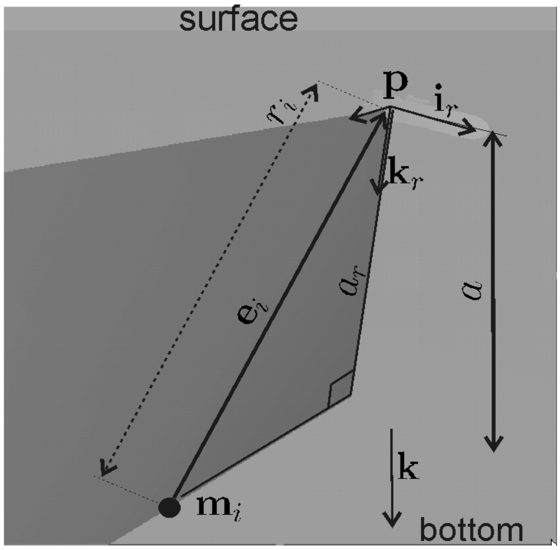
8.1 Capteurs

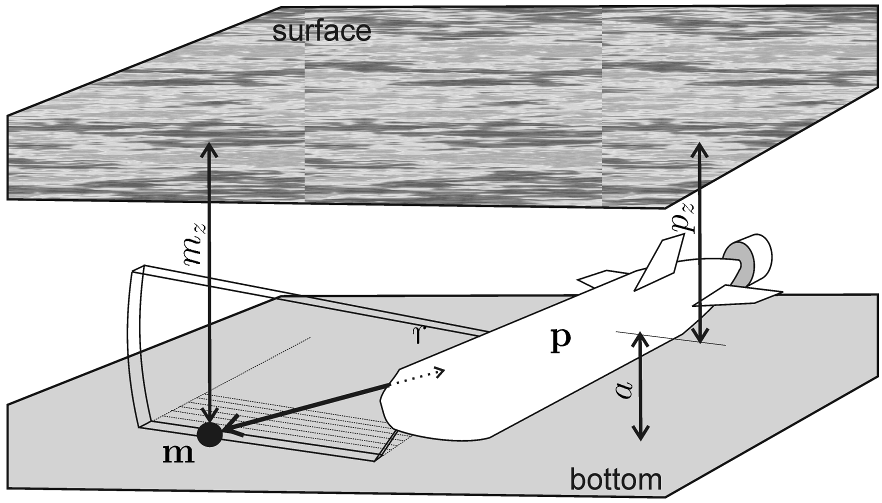
Un GPS (Global positioning system), disponible à la surface.

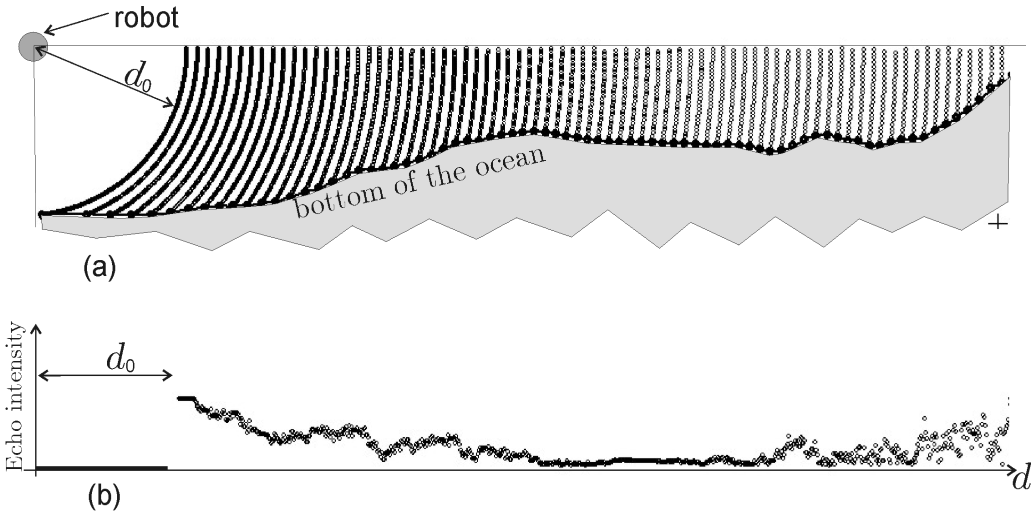
$$
\begin{array}{ll}
t_0 = 6000 \text{ s}, & \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\
t_f = 12000 \text{ s}, & \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m
\end{array}
$$

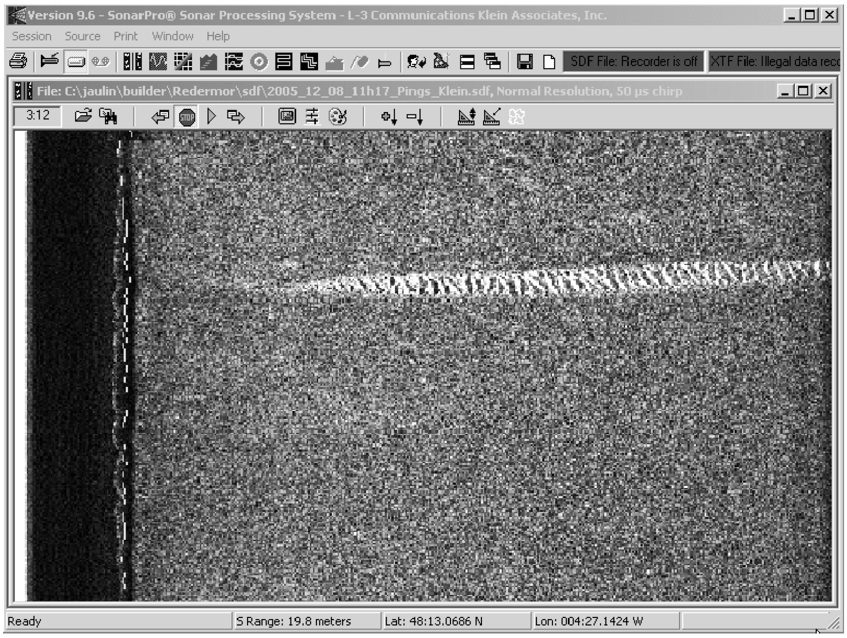
Un sonar (KLEIN 5400 side scan sonar). Donne la distance r entre le robot et la mine



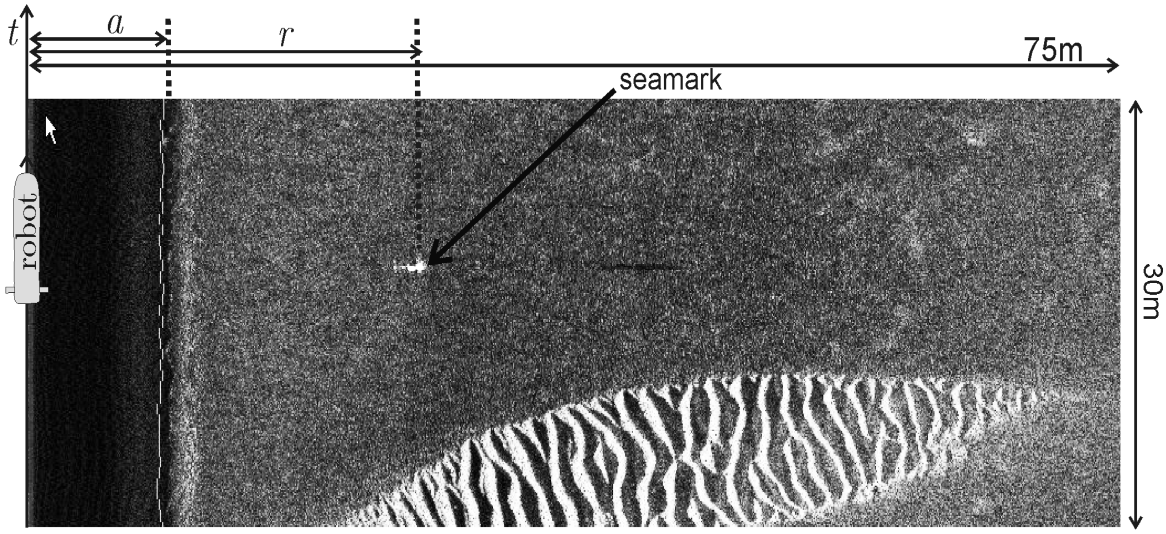



surface
m_z
p_z
r
$\mathbf{p}$
a
$\mathbf{m}$
bottom


robot
d_0
bottom of the ocean
(a)
Echo intensity
d_0
d
(b)



Screenshot du logiciel SonarPro

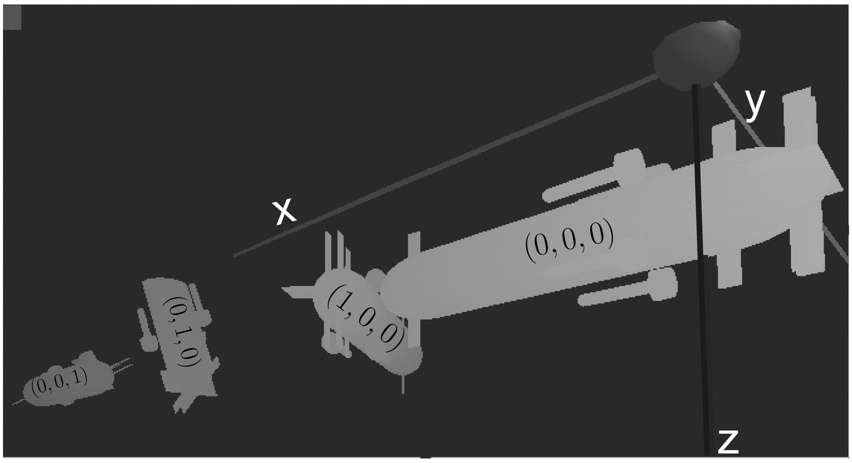


Détection d'une mine à l'aide de SonarPro

Le Loch-Doppler renvoie la vitesse du robot $\mathbf{v}_r$ et son altitude a.

Une centrale inertielle (Octans III from IXSEA) renvoie le roulis ϕ, le tangage θ et le cap ψ du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



8.2 Données

Pour chaque $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

8.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

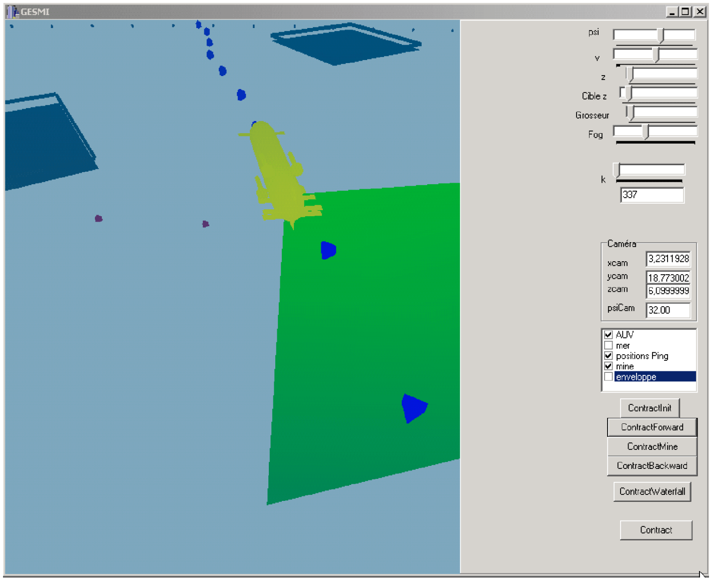
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

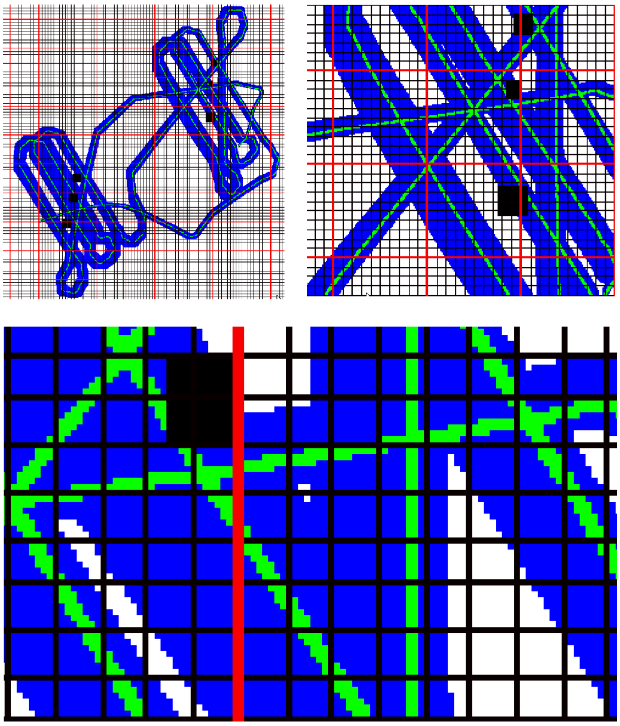
$$\mathbf{R}^\mathsf{T}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5]$$

8.4 GESMI



GESMI (Guaranteed Estimation of Sea Mines with Intervals)



Trajectoire reconstruite par GESMI