Towing with sailboat robots

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1 Introduction

How to tow a large object (the *load*) with sailboat robots
?

Problems. currents, winds, . . .

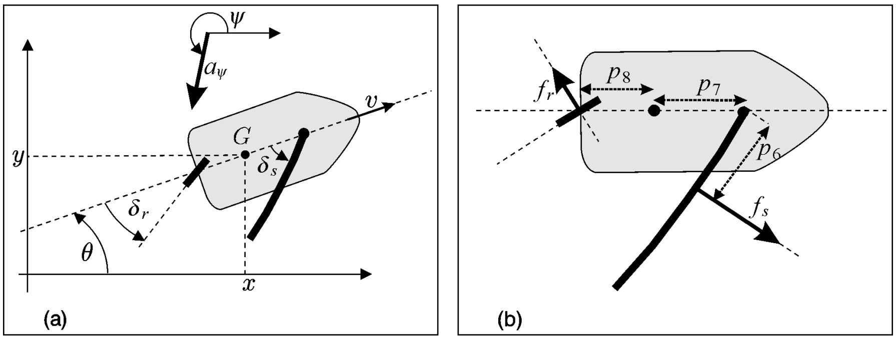
1) The main propulsion of the load will be governed by the currents (as for space exporers).

2) The sailboat influence the direction of the load perpendicular to the current.

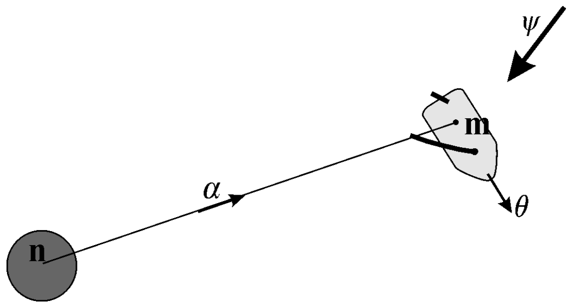
3) Find a zero effort path using techniques from optimal control.

4) The sailboat plays the role of a regulator controlling the load around the reference trajectory.

2 Model



ψ
a_ψ
v
G
y
δ_s
δ_r
θ
x
(a)
f_r
p_8
p_7
p_6
f_s
(b)


ψ
m
α
θ
n

Model for the sailboat and the load

$$\begin{cases} \dot{\mathbf{m}} &= v.\mathbf{u}_\theta + p_1 a_\psi \mathbf{u}_\psi - f_c \mathbf{u}_\alpha \\ \dot{\theta} &= \omega \\ \dot{v} &= \frac{f_s \sin \delta_s - f_r \sin u_1 - p_2 v.|v| - f_c \cos(\alpha - \theta)}{p_9} \\ \dot{\omega} &= \frac{f_s (p_6 - p_7 \cos \delta_s) - p_8 f_r \cos u_1 - p_3 \omega v}{p_{10}} \\ \dot{\mathbf{s}} &= \frac{f_c.\mathbf{u}_\alpha - p_{12}.\|\mathbf{s}\|.\mathbf{s}}{p_{11}} \\ \dot{\mathbf{n}} &= \mathbf{s} \end{cases}$$

State variables. $(m_x, m_y, \theta, v, \omega)$: boat ; (n_x, n_y, s_x, s_y): load.

The link variables

$$\begin{cases} (a) & \mathbf{w}^{\mathsf{ap}} & = & \begin{pmatrix} a_\psi \cos(\psi - \theta) - v \\ a_\psi \sin(\psi - \theta) \end{pmatrix} \\ (b) & \psi^{\mathsf{ap}} & = & \mathsf{atan2}(\mathbf{w}^{\mathsf{ap}}) \\ (c) & \gamma_{\mathsf{s}} & = & \cos\psi^{\mathsf{ap}} + \cos u_2 \\ (d) & \delta_s & = & \begin{cases} \pi + \psi^{\mathsf{ap}} & \text{if } \gamma_{\mathsf{s}} \leq 0 \\ -u_2 \,\mathsf{sign}(\sin\psi^{\mathsf{ap}}) & \text{otherwise} \end{cases} \\ (e) & f_s & = & p_4 \|\mathbf{w}^{\mathsf{ap}}\| \sin(\delta_s - \psi^{\mathsf{ap}}) \\ (f) & f_r & = & p_5 v \sin u_1 \\ (g) & f_c & = & \exp(\|\mathbf{m} - \mathbf{n}\| - L_0) \\ (h) & \alpha & = & \mathsf{atan2}(\mathbf{m} - \mathbf{n}) \end{cases}$$

3 Controller

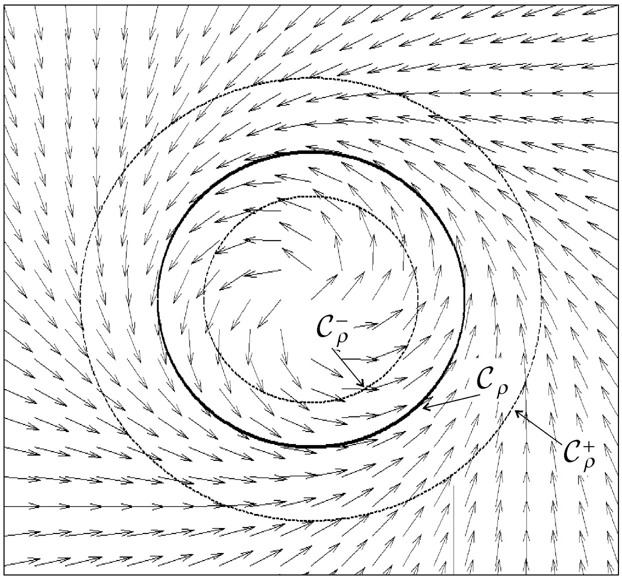
Moving the load with a proportional control

We have to pool toward

$$\alpha^* = \alpha_0 + \sin(\alpha_0 - \mathrm{atan2}(\dot{\mathbf{n}})) = \alpha_0 + \frac{\cos\theta \dot{n}_y - \sin\theta \dot{n}_x}{\|\dot{\mathbf{n}}\|}$$

where α_0 is the desired direction.

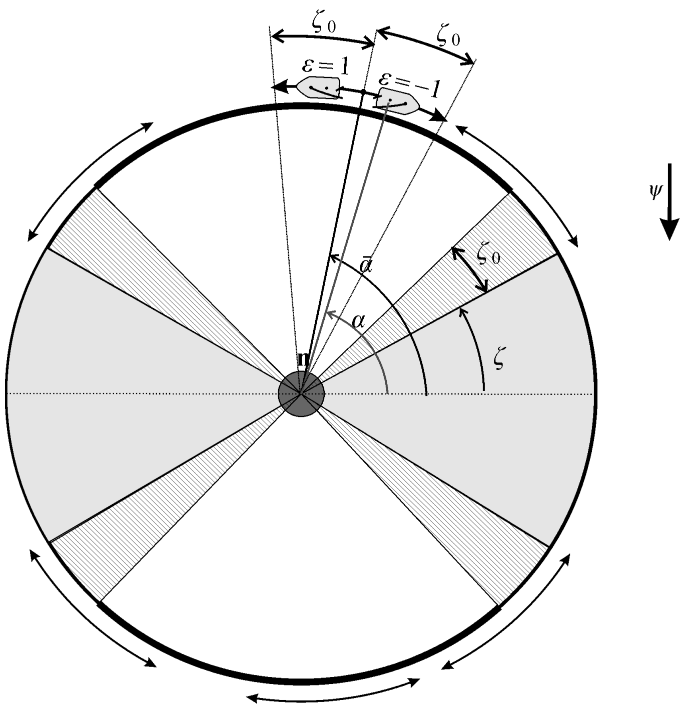
Vector field. It is a function from $\mathbb{R}^2$ to $[-\pi, \pi]$ which associates to $\mathbf{n}$, the direction α_0.



Segment. A *segment* is an arc of the circle $(\mathbf{n}, L_0)$ with half angle ζ_0. A segment with middle angle $\bar{\alpha}$ is feasible if the two directions (direct $\varepsilon = 1$ and indirect $\varepsilon = -1$) are feasible, i.e,

$$|\sin(\bar{\alpha} - \psi)| \leq \cos(\zeta + \zeta_0),$$

The value for ε changes once the boat has reached the vertex of the segment, *i.e.*, when $\cos(\bar{\alpha} - \alpha) < \cos \zeta_0$.

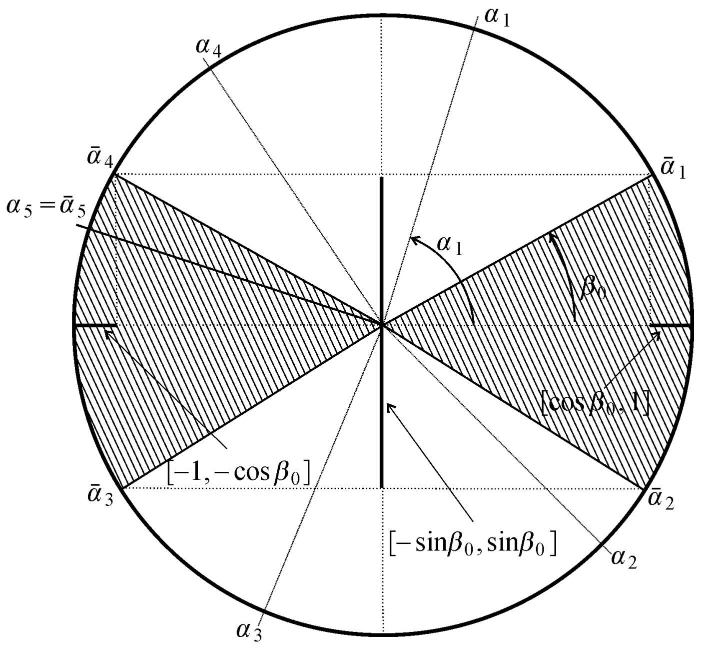


Projection. The projection of α onto

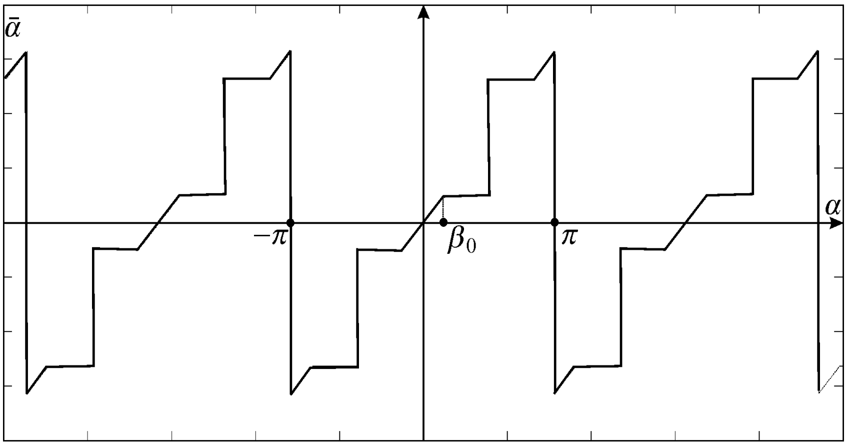
$$\mathbb{B} = \{\beta \text{ such that } \cos|\beta| > \cos\beta_0\}$$

is

$$\bar{\alpha} = \operatorname{atan2}\begin{pmatrix} \max(|\cos\alpha|, \cos\beta_0).\operatorname{sign}(\cos\alpha) \\ \min(|\sin\alpha|, \sin\beta_0).\operatorname{sign}(\sin\alpha) \end{pmatrix},$$

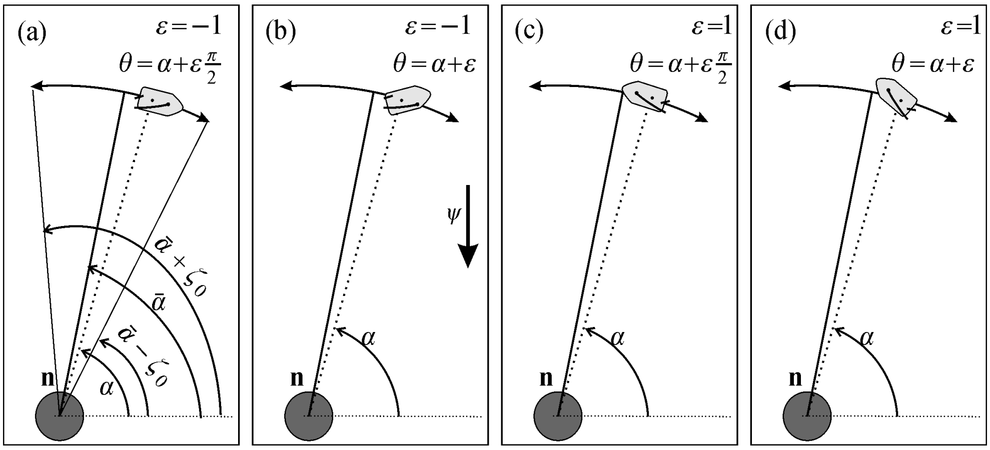


For $\beta = 0.5$, the graph of the projection is



The robot has to pull

To follow the segment, take $\bar{\theta} = \alpha + \varepsilon\frac{\pi}{2}$.
To pull the load, an attack angle should be taken: $\bar{\theta} = \alpha + \varepsilon$
.



Heading control.

To avoid any loop, always tack with the cable on the back,

$$u_1 = \begin{cases} \frac{\pi}{4}.\text{sign}\left(\textbf{sin}\left(\alpha - \bar{\theta}\right)\right) & \text{if } (\ \textbf{cos}\left(\theta - \bar{\theta}\right) \leq 0) \quad \text{(tacking)} \\ \frac{\pi}{4}.\,\text{sin}\left(\theta - \bar{\theta}\right) & \text{otherwise.} \end{cases}$$

Controller. in: $\mathbf{m}, \mathbf{n}, \dot{\mathbf{n}}, \theta, \psi$; out: u_1, u_2; inout: ε

$\alpha_0 = \text{field}(\mathbf{n})$; $\alpha^* = \alpha_0 + \sin(\alpha_0 - \text{atan2}(\dot{\mathbf{n}}))$;

$\tilde{\alpha} = \alpha^* - \psi$; $\beta_0 = \frac{\pi}{2} - \zeta - \zeta_0$;

$\bar{\alpha} = \psi + \text{atan2}\begin{pmatrix} \max(|\cos\tilde{\alpha}|, \cos\beta_0).\text{sign}(\cos\tilde{\alpha}) \\ \min(|\sin\tilde{\alpha}|, \sin\beta_0).\text{sign}(\sin\tilde{\alpha}) \end{pmatrix}$

$\alpha = \text{atan2}(\mathbf{m} - \mathbf{n})$;

if $\cos(\bar{\alpha} - \alpha) < \cos\zeta_0$ then $\varepsilon = \text{sign}(\sin(\bar{\alpha} - \alpha))$;

$\bar{\theta} = \alpha + \varepsilon$;

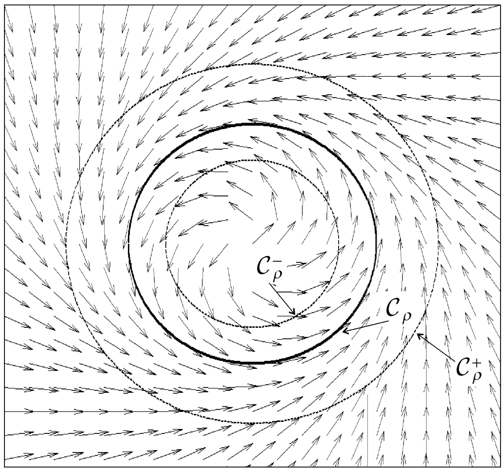
$u_1 = \begin{cases} \frac{\pi}{4}.\text{sign}\left(\sin\left(\alpha - \bar{\theta}\right)\right) & \text{if } (\cos(\theta - \bar{\theta}) \leq 0) \\ \frac{\pi}{4}.\sin\left(\theta - \bar{\theta}\right) & \text{otherwise.} \end{cases}$

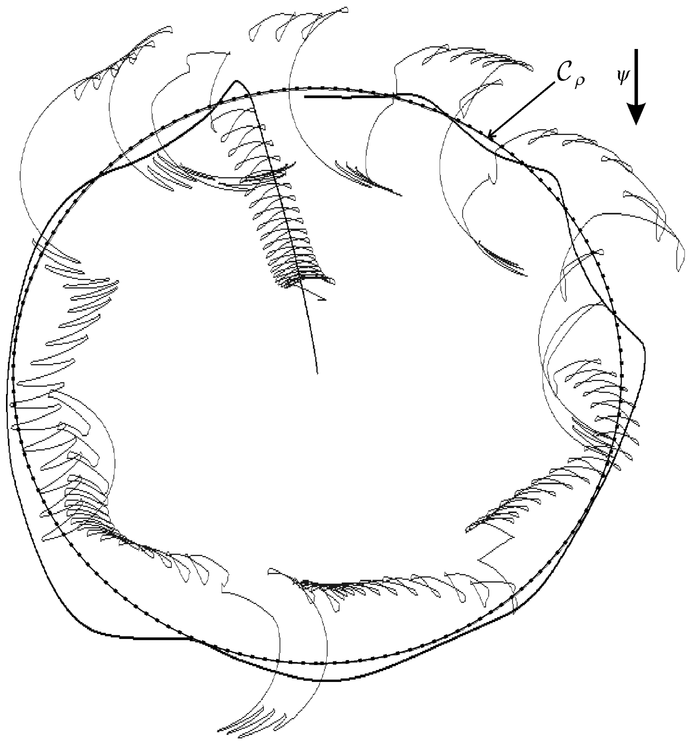
$u_2 = \pi.\frac{\cos(\psi - \bar{\theta}) + 1}{4}$.

4 Test-case

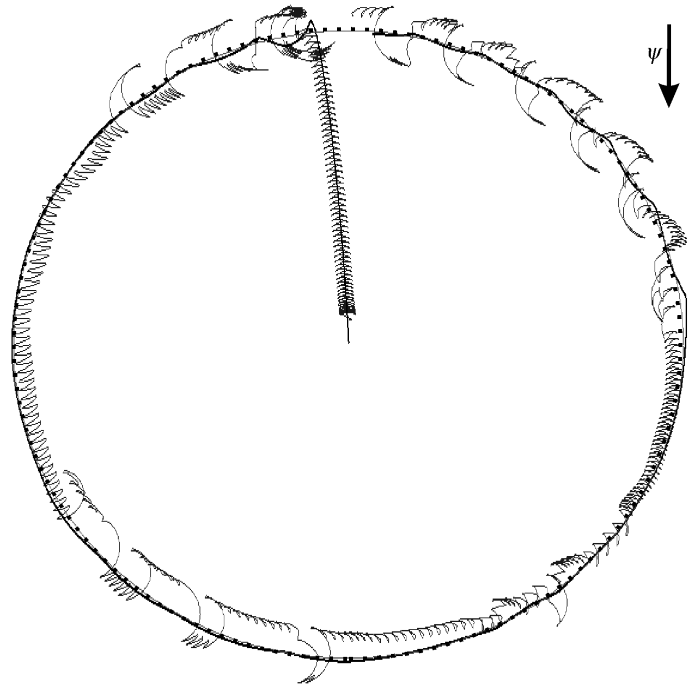
The load has to follow a circle $\mathcal{C}_\rho$. Choose the vector field

$$\varphi(\mathbf{n}) = \mathrm{atan2}(\mathbf{n}) + \frac{\pi}{2} + \mathrm{atan}\left(\frac{\|\mathbf{n}\| - \rho}{\delta}\right),$$





The load follows a circle with radius $\rho = 150$m,



The load follows a circle with radius $\rho = 500$m.