

# Explore and return problem in a minimalist environment

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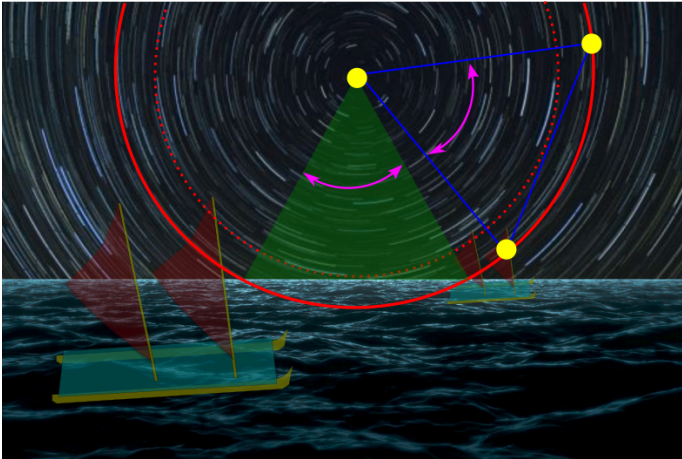
Toulouse, réunion MRIS, June 18, 2019



# Polynesian navigation

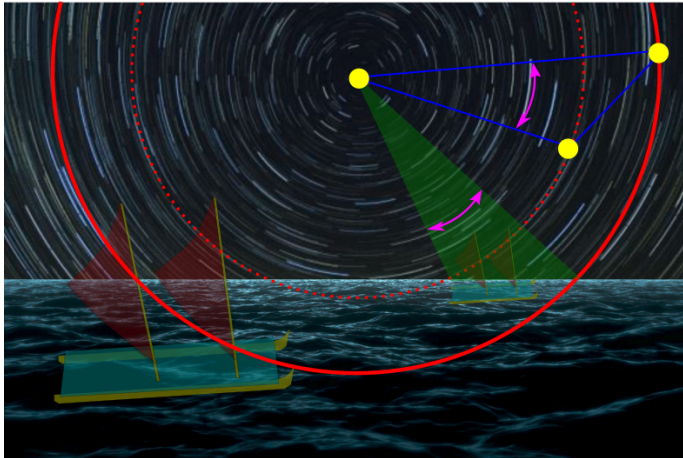


Find the route without GPS, compass and clocks



## Pair of stars technique





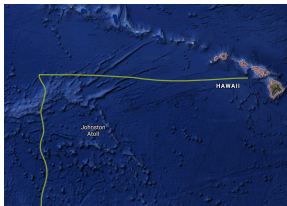
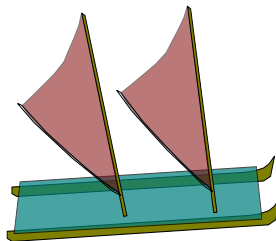
Pair of stars technique

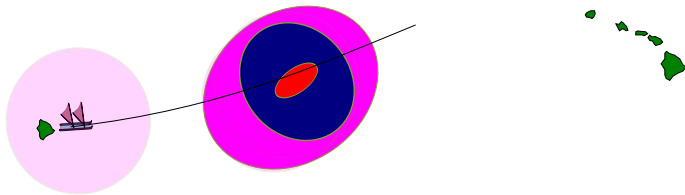
# Polynesian navigation

## Exploration

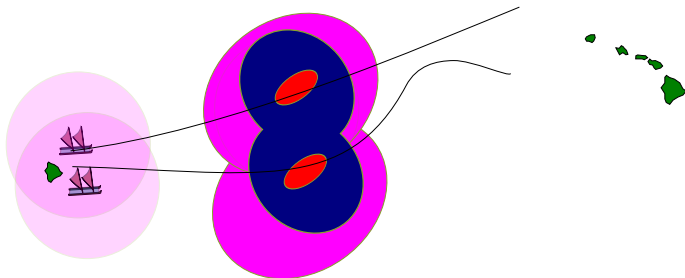
### Reach an island

### No-lost zone





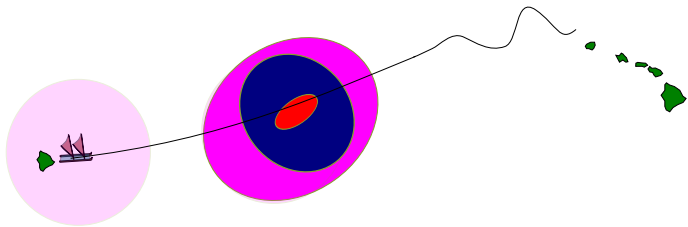
Prove that islands will be reached by one boat



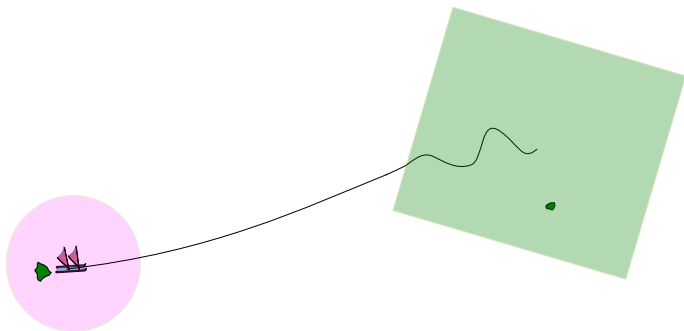
Prove that islands will be reached by the  $n$  boats



Alignment to keep the heading in case of clouds



Find a control to reach the geo-localized islands



Explore a given area entirely to find new islands

# Our problem

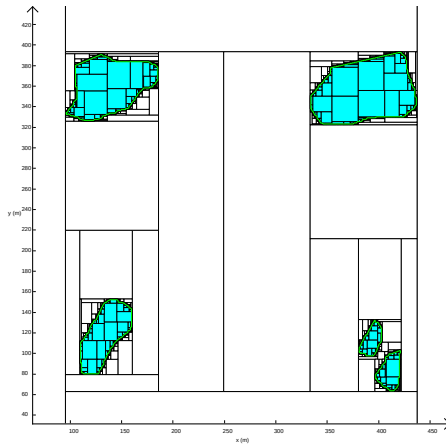


- Given a set of geo-localized islands  $\mathbf{m}_i, i \geq 0$ .
- The  $i$ th coastal area is:

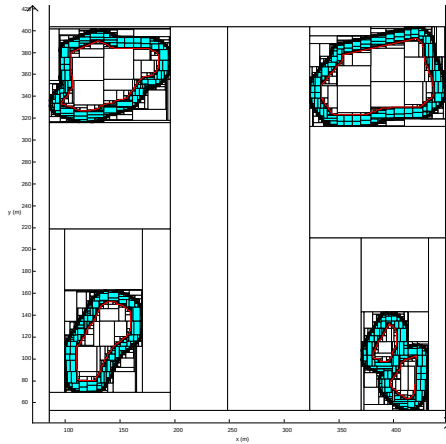
$$\mathbb{C}_i = \{\mathbf{x} \mid c_i(\mathbf{x}) \leq 0\}.$$

- A robot has to move in this environment without being lost.

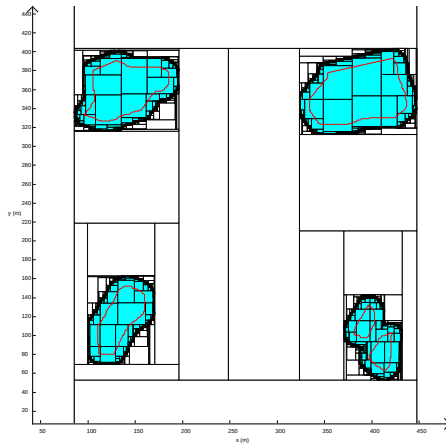
Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



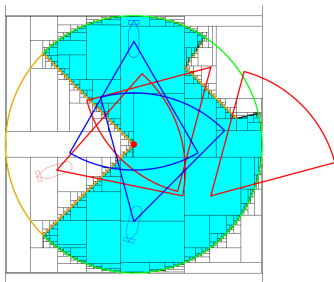
Polynesian navigation  
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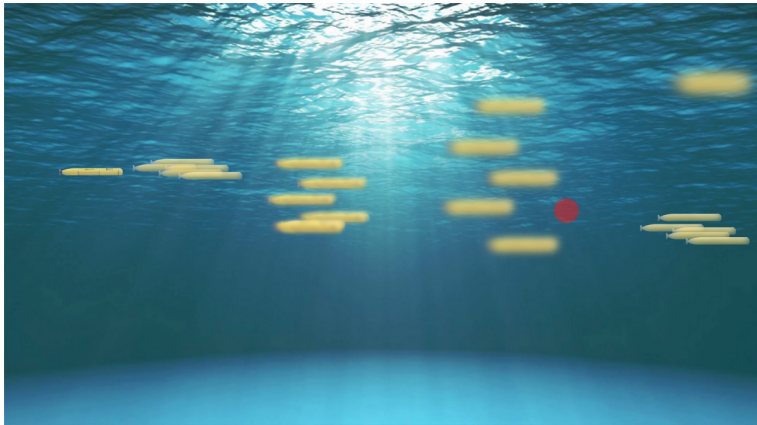
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$C_1, C_2, C_3, C_4$



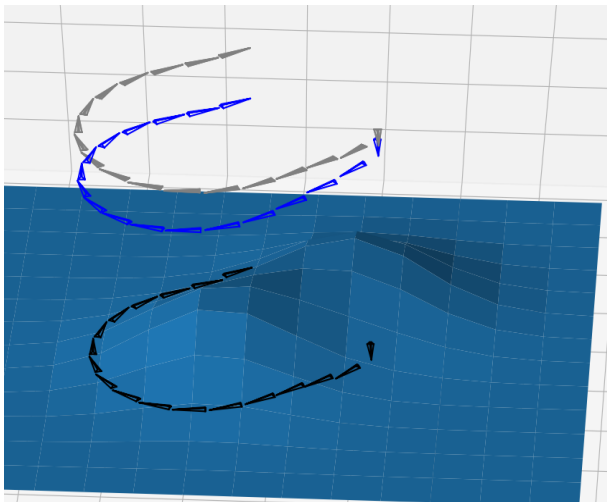
$$\mathbb{C}_1 = \{\mathbf{x} = (x, y, \theta) | (x, y) \in \text{blue and } \theta \text{ toward the buoy}\}$$



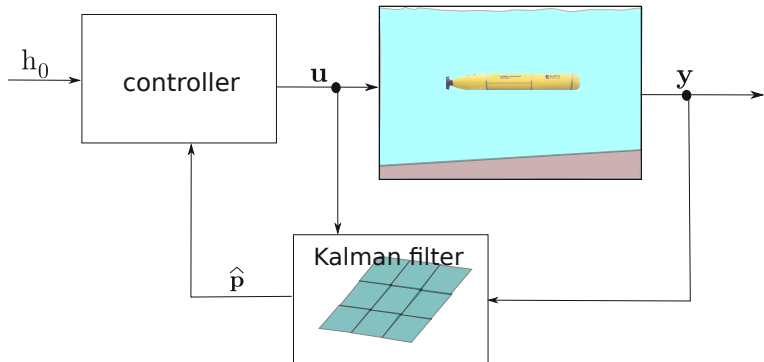
[3]

# Follow isobaths

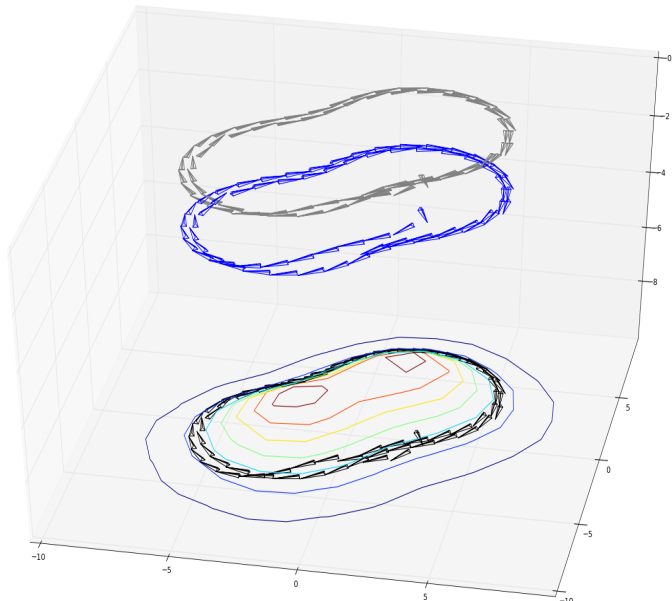
Polynesian navigation  
Exploration  
Reach an island  
No-lost zone







Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



# Exploration

# Blind observer

Given

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), & \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}), & \mathbf{y}(t) \in [\mathbf{y}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$

An observer is *blind* if  $\dim \mathbf{y} = 0$ .

# Predict is a blind estimation problem

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t)) + \mathbf{e}_y(t) \\ \mathbf{u}(t) &= \mathbf{r}(\mathbf{y}(t)) + \mathbf{e}_u(t) \end{cases} \quad \begin{array}{l} \mathbf{e}_y(t) \in [\mathbf{e}_y] \\ \mathbf{e}_u(t) \in [\mathbf{e}_u] \end{array}$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{r}(\mathbf{g}(\mathbf{x}(t)) + \mathbf{e}_y(t)) + \mathbf{e}_u(t)) \\ \mathbf{e}_y(t) \in [\mathbf{e}_y], \mathbf{e}_u(t) \in [\mathbf{e}_u] \end{cases}$$



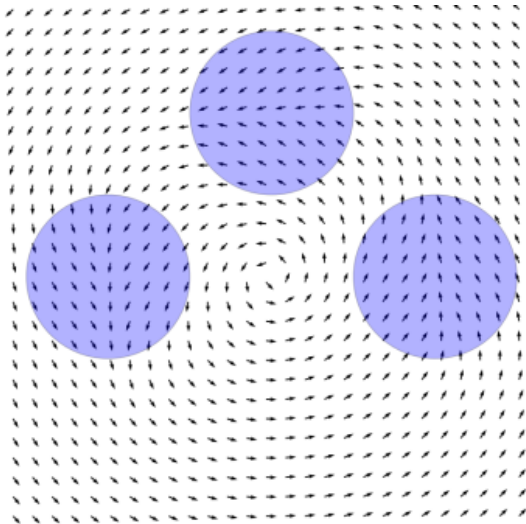
$$\dot{\mathbf{x}}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{v}(t))$$
$$\mathbf{v}(t) \in [\mathbf{v}]$$

with

$$\mathbf{v} = (\mathbf{e}_y, \mathbf{e}_u)$$

$$\mathbf{h}(\mathbf{x}, \mathbf{v}) = \mathbf{f}(\mathbf{x}, \mathbf{r}(\mathbf{g}(\mathbf{x}) + \mathbf{e}_y) + \mathbf{e}_u)$$

$$[\mathbf{v}] = [\mathbf{e}_y] \times [\mathbf{e}_u]$$



# Visible area

The robot has a state  $\mathbf{x}$ . The visible area is  $\mathbb{V}(\mathbf{x})$

**Example.** The robot is able to see all up to 3 meters

$$\mathbb{V}(\mathbf{x}) = \left\{ (z_1, z_2) \mid (z_1 - x_1)^2 + (z_2 - x_2)^2 \leq 9 \right\}.$$

# Blind exploration

The explored zone  $\mathbb{Z}$  is defined by [1]

$$\left\{ \begin{array}{l} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbb{Z} = \bigcup_{t \geq 0} \mathbb{V}(\mathbf{x}(t)) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{array} \right.$$

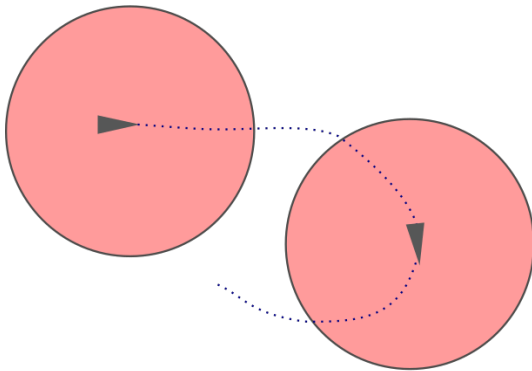
We have

$$\underbrace{\bigcap_{\mathbf{x}(\cdot) \in \mathcal{X}(\cdot)} \bigcup_{t \geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^-} \subset \mathbb{Z} \subset \underbrace{\bigcup_{\mathbf{x}(\cdot) \in \mathcal{X}(\cdot)} \bigcup_{t \geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^+}.$$

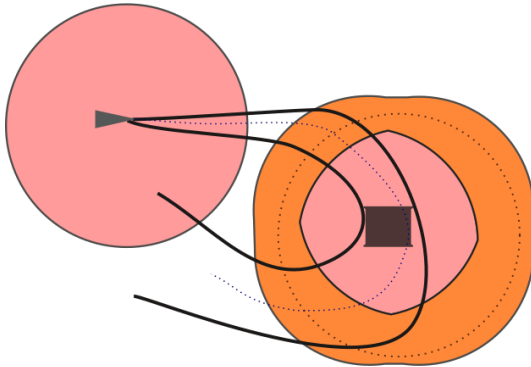
$\mathbb{Z}^-$  is the *certainly explored zone*.

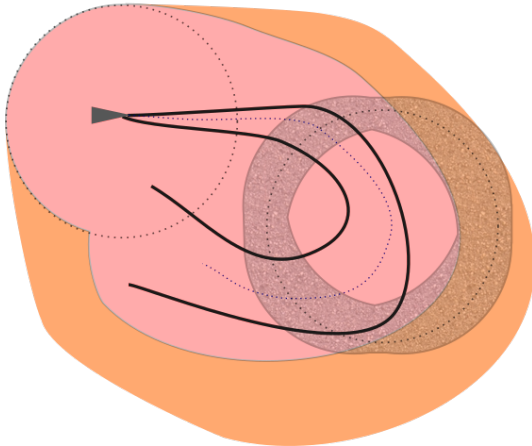
$\mathbb{Z}^+$  is the *maybe explored zone*.

$\mathbb{Z}^+ \setminus \mathbb{Z}^-$  is the *penumbra*.









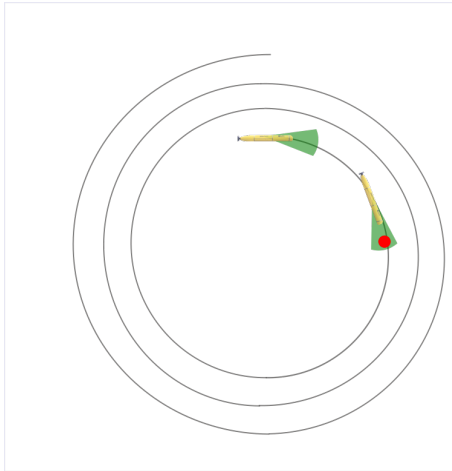


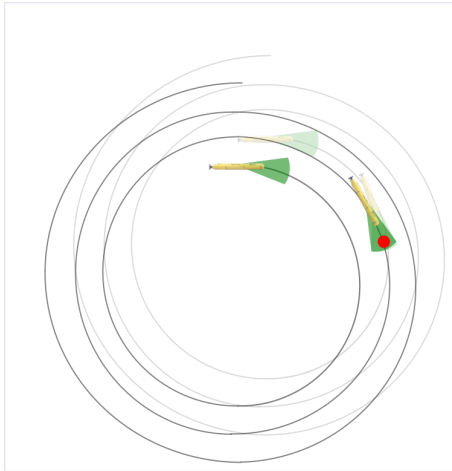
# Spiral scan



We have [2]

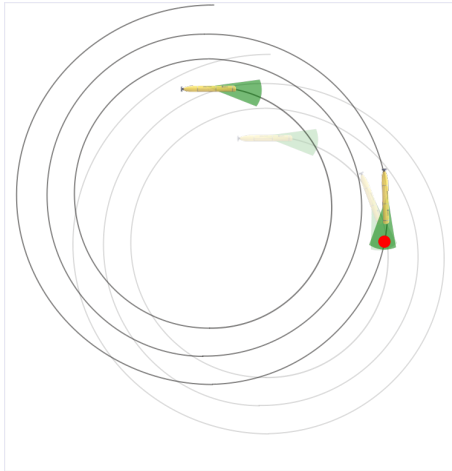
$$\underbrace{\bigcup_{t \geq 0} \bigcap_{\mathbf{x} \in \mathcal{X}(t)} \mathbb{V}(\mathbf{x})}_{\{\mathbf{z} \mid \exists t \ \forall \mathbf{x} \in \mathcal{X}(t), \mathbf{z} \in \mathbb{V}(\mathbf{x})\}} \subset \mathbb{Z}^- = \underbrace{\bigcap_{\mathbf{x}(\cdot) \in \mathcal{X}(\cdot)} \bigcup_{t \geq 0} \mathbb{V}(\mathbf{x}(t))}_{\{\mathbf{z} \mid \forall \mathbf{x}(\cdot) \in \mathcal{X}(\cdot), \exists t, \mathbf{z} \in \mathbb{V}(\mathbf{x})\}}$$

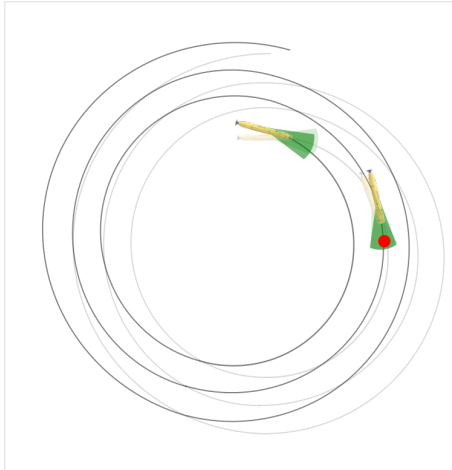


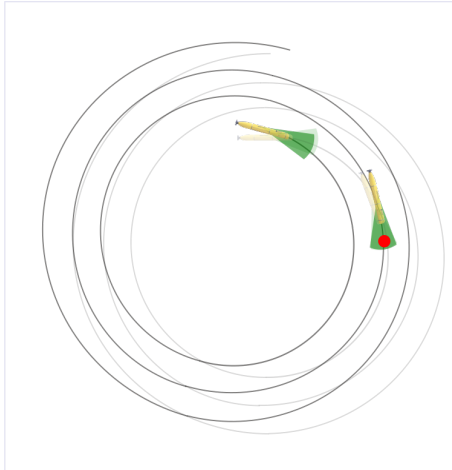


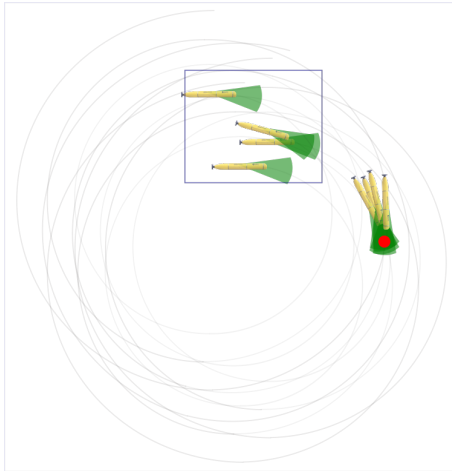


Polynesian navigation  
**Exploration**  
Reach an island  
No-lost zone



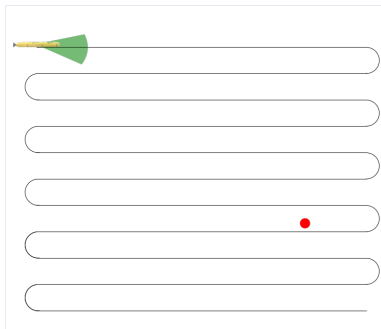


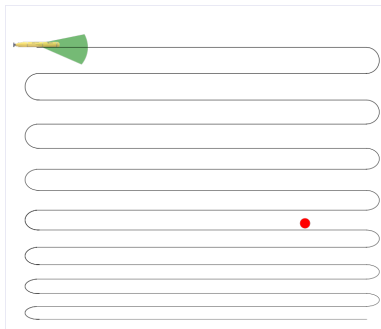




# Boustrophedon

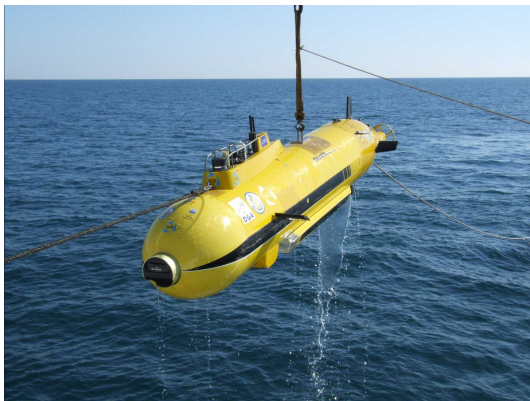
Which pattern is the best for exploration?



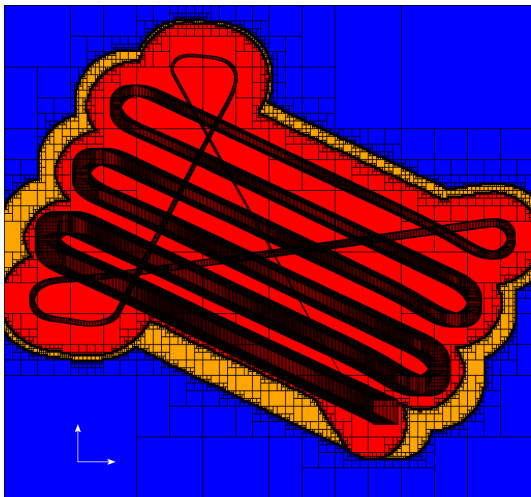


# Applications





Daurade DGA-TN



During its boustrophedon Daurade explored  $\mathbb{Z} \in \llbracket \mathbb{Z} \rrbracket$

# Reach an island

# Assumptions

- The coastal areas are small compare to the offshore area.
- In the coastal area, the robot knows its state
- Offshore, the robot is blind

# Robot

The robot is described by

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), & \mathbf{u}(\cdot) \in [\mathbf{u}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$

Offshore, the robot is blind and an open loop strategy.

The set flow  $\Phi : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathcal{P}(\mathbb{R})$  is defined as:

$$\Phi(t_1, \mathbf{x}_0) = \{\mathbf{a} | \exists \mathbf{u}(\cdot) \in [\mathbf{u}](t), \mathbf{a} = \mathbf{x}(t_1), \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), \mathbf{x}(0) = \mathbf{x}_0\}$$

*i.e.*, if at  $t = 0$ , the robot is at  $\mathbf{x}_0$ , we have  $\mathbf{x}(t_1) \in \Phi(t_1, \mathbf{x}_0)$ .



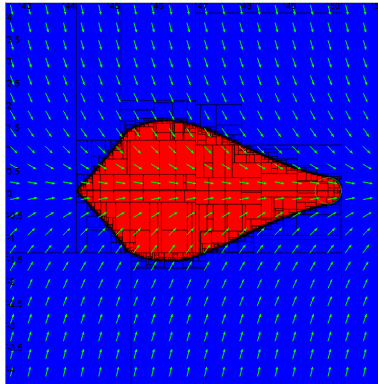
## Backward reach set

Given the set  $\mathbb{A}$ , the backward reach set is defined by

$$\text{Back}(\mathbb{A}) = \{\mathbf{x} \mid \forall \varphi \in \Phi, \exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{A}\}.$$

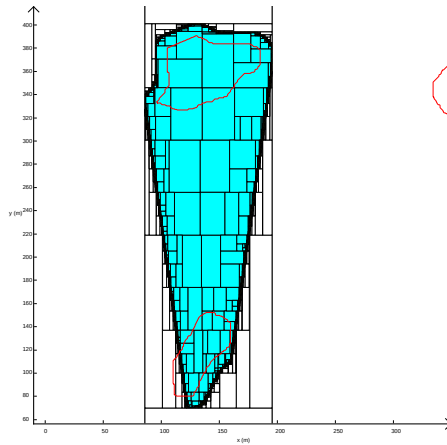
Example.

$$\Phi(t, \mathbf{x}) = \begin{pmatrix} x_1 + t \\ e^{-t} \cdot x_2 \end{pmatrix} + t(1 + |x_2|) \begin{pmatrix} [-1, 1] \\ [-1, 1] \end{pmatrix}$$



Safe start points with a East strategy to reach the circle

Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



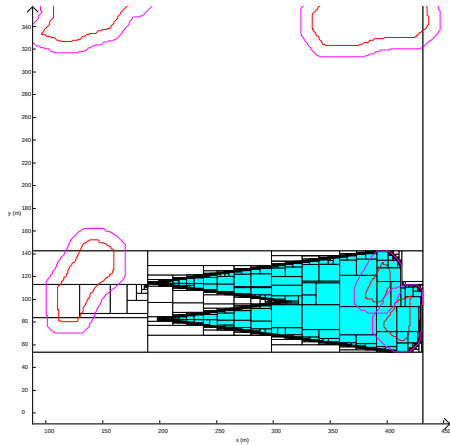
# Archipelagic effect

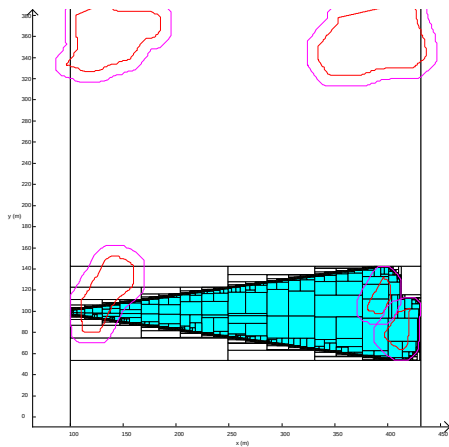
We have

$$\text{Back}(\mathbb{A} \cup \mathbb{B}) \supset \text{Back}(\mathbb{A}) \cup \text{Back}(\mathbb{B})$$

**Proof.**

$$\begin{aligned} & \mathbf{x} \in \text{Back}(\mathbb{A}) \cup \text{Back}(\mathbb{B}) \\ \Leftrightarrow & (\forall \varphi \in \Phi, \exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{A}) \vee (\forall \varphi \in \Phi, \exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{B}) \\ \Leftrightarrow & \forall \varphi \in \Phi, (\exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{A}) \vee (\exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{B}) \\ \Rightarrow & \forall \varphi \in \Phi, \exists t \geq 0, (\varphi(t, \mathbf{x}) \in \mathbb{A} \vee \varphi(t, \mathbf{x}) \in \mathbb{B}) \\ \Leftrightarrow & \forall \varphi \in \Phi, \exists t \geq 0, (\varphi(t, \mathbf{x}) \in \mathbb{A} \cup \mathbb{B}) \\ \Leftrightarrow & \mathbf{x} \in \text{Back}(\mathbb{A} \cup \mathbb{B}) \end{aligned}$$







# No-lost zone

## Moving between coastal zones

- We have  $m$  coastal sets  $\mathbb{C}_1, \mathbb{C}_2, \dots, i \in \{1, 2, \dots\}$
- We have open loop control strategies  $\mathbf{u}_j, j \in \{1, 2, \dots\}$ ,
- Equivalently, we have set flows  $\Phi_j(t, \mathbf{x}_0)$ .
- The control strategy cannot change offshore.

# Graph

From  $\mathbb{C}_1$  we can reach  $\mathbb{C}_2$  with the  $j$ th control strategy if

$$\mathbb{C}_1 \cap \text{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From  $\mathbb{C}_1$  we can reach  $\mathbb{C}_2$  with at least one control strategy if

$$\mathbb{C}_1 \cap \bigcup_j \text{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From  $\mathbb{C}_1$  we can reach  $\mathbb{C}_2 \cup \mathbb{C}_3$  with at least one control strategy if

$$\mathbb{C}_1 \cap \bigcup_j \text{Back}(j, \mathbb{C}_2 \cup \mathbb{C}_3) \neq \emptyset.$$

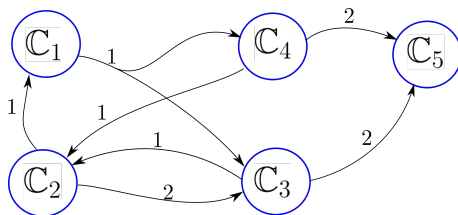
We define  $\hookrightarrow$  as:

- $\mathbb{C}_a \hookrightarrow \mathbb{C}_b$  if from  $\mathbb{C}_a$  we can reach  $\mathbb{C}_b$  with at least one control strategy  $j$ .
- $\hookrightarrow$  is the smallest transitive relation such that

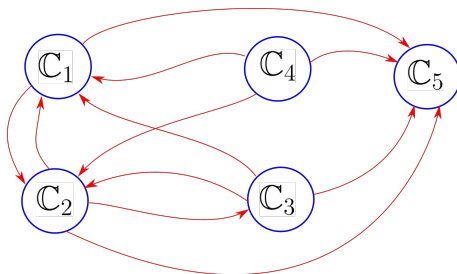
$$\left\{ \begin{array}{l} \forall k \in \mathbb{K}, \mathbb{C}_{i_k} \hookrightarrow \mathbb{C}_b \\ \exists j, \mathbb{C}_a \cap \text{Back}(j, \bigcup_{k \in \mathbb{K}} \mathbb{C}_{i_k}) \neq \emptyset \end{array} \right. \Rightarrow \mathbb{C}_a \hookrightarrow \mathbb{C}_b$$

# Graph





$$\mathbb{C}_1 \cap \text{Back}(1, \mathbb{C}_3 \cup \mathbb{C}_4) \neq \emptyset \Rightarrow \mathbb{C}_1 \rightarrow (\mathbb{C}_3, \mathbb{C}_4)$$



Graph of  $\hookrightarrow$

# Cycle

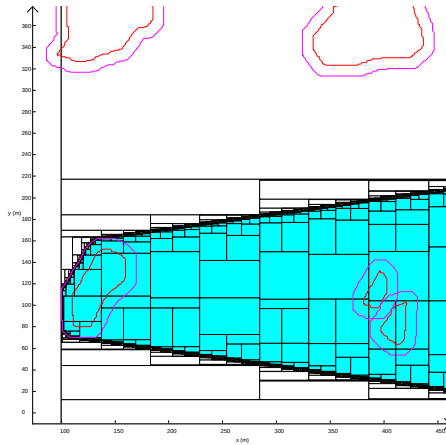
If

$$\left\{ \begin{array}{l} \mathbb{C}_{i_1} \cap \mathbb{C}_{i_2} = \emptyset \\ \mathbb{C}_{i_1} \hookrightarrow \mathbb{C}_{i_2} \\ \mathbb{C}_{i_2} \hookrightarrow \mathbb{C}_{i_1} \end{array} \right.$$

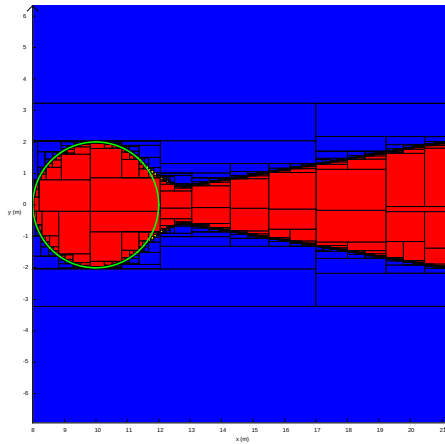
then we can revisit  $\mathbb{C}_{i_1}$  and  $\mathbb{C}_{i_2}$  for ever.

# Forward

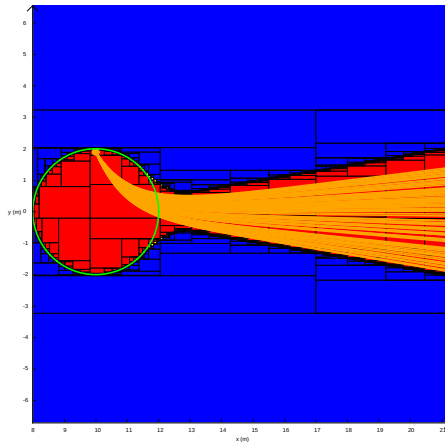
Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



Polynesian navigation  
Exploration  
Reach an island  
No-lost zone





# No lost zone

It is the set  $\mathbb{S}$  of all states we may visit from a coastal area without being lost with the available control strategy.

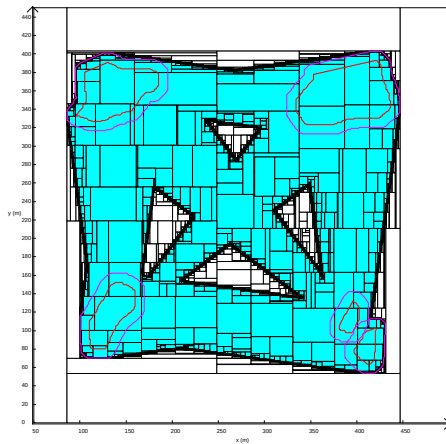
Define

$$\mathcal{J}_j = \{k | \mathbb{C}_k \cap \text{Back}(j, \bigcup_{i \neq k} \mathbb{C}_i) \neq \emptyset\}.$$

Thus

$$\begin{cases} \mathbf{x} \in \text{Back}(j, \cup_i \mathbb{C}_i) \\ \mathbf{x} \in \text{Forw}(j, \mathbb{C}_k), k \in \mathcal{J}_j \end{cases} \Rightarrow \mathbf{x} \in \mathbb{S}$$

Polynesian navigation  
Exploration  
Reach an island  
No-lost zone



## Open question

The robot at position **a** is lost for all feasible control strategies the robot cannot guarantee that it will reach an island.  
How to prove that the robot is lost?



B. Desrochers and L. Jaulin.

Computing a guaranteed approximation the zone explored by a robot.

*IEEE Transaction on Automatic Control*, 62(1):425–430, 2017.



V. Drevelle, L. Jaulin, and B. Zerr.

Guaranteed characterization of the explored space of a mobile robot by using subpavings.

In *Proc. Symp. Nonlinear Control Systems (NOLCOS'13)*,  
Toulouse, 2013.



S. Rohou.

*Reliable robot localization: a constraint programming approach over dynamical systems.*

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