Underwater exploration by an autonomous robot with the method of stable cycles

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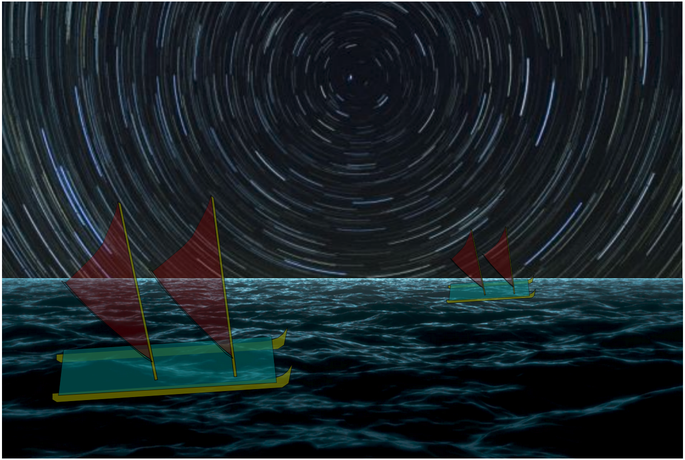
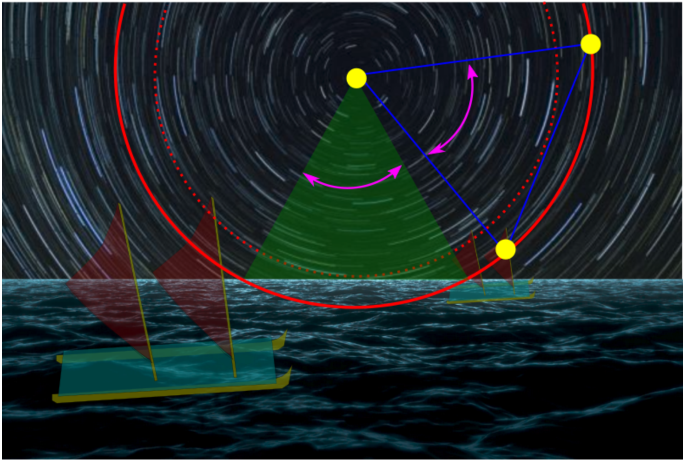
Ancestral method of navigation

Submeeting 2018

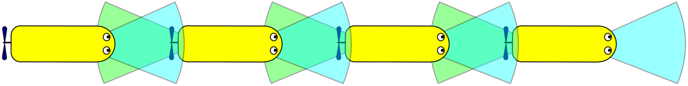
Polynesian navigation



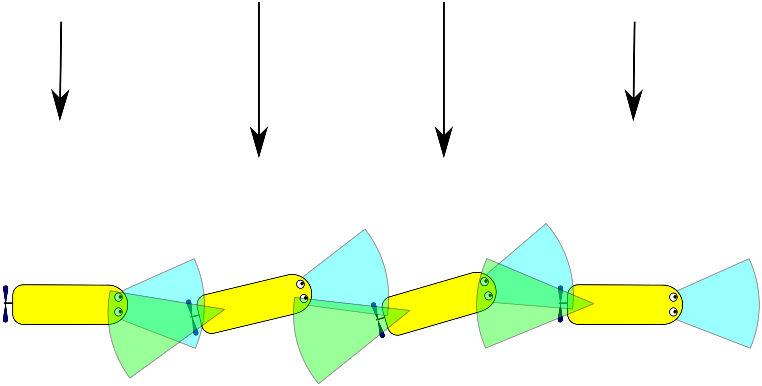
Find the route without GPS, compass and clocks with *wa'a kaulua*[3]



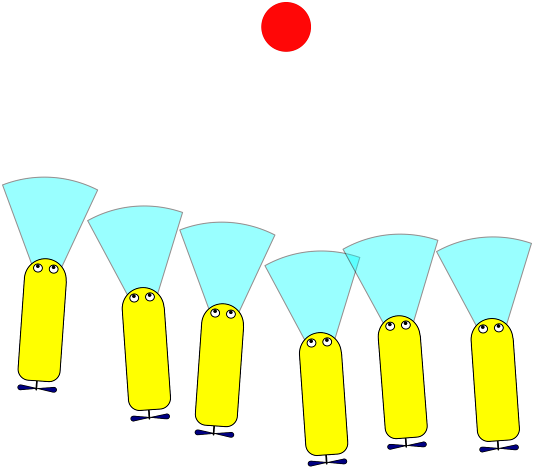
Alignment to keep the heading in case of clouds



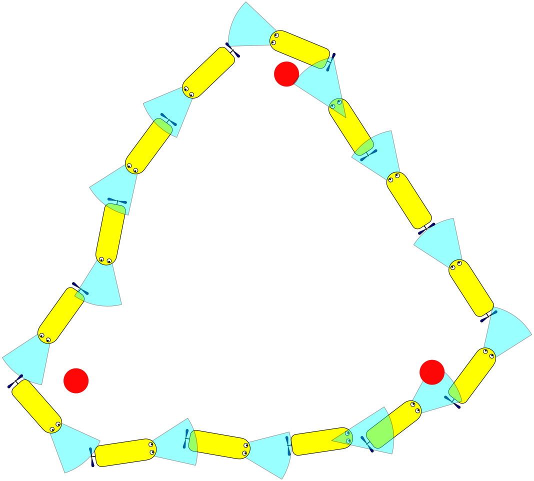
More inertia, more predictable



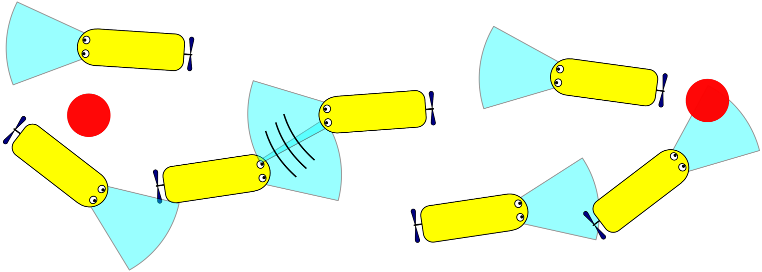
Internal deformations provide information



Explore further



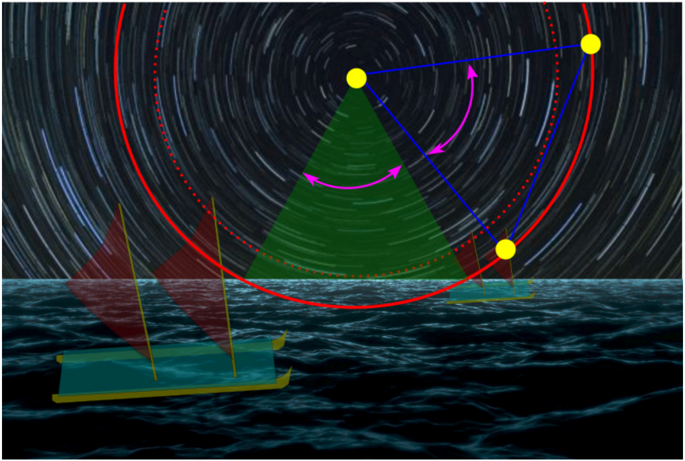
Virtual chain: localization ↔ proprioception

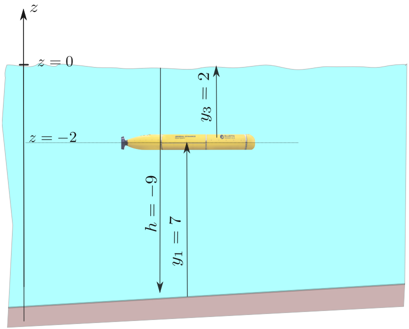


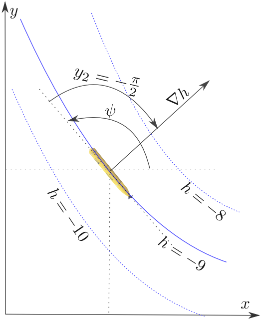
With communication we can do more

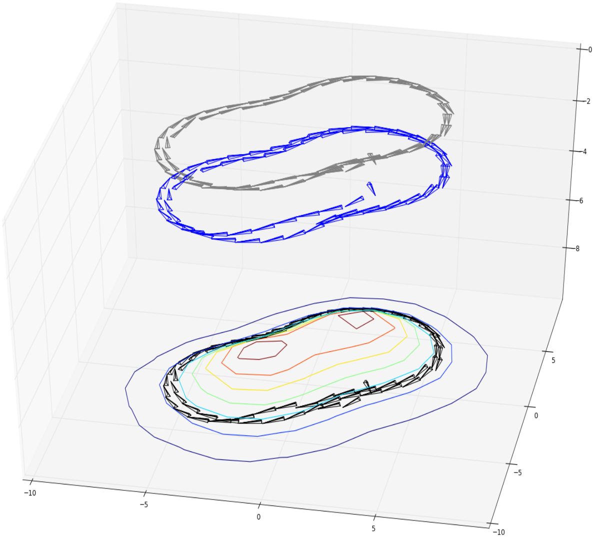
Follow a route

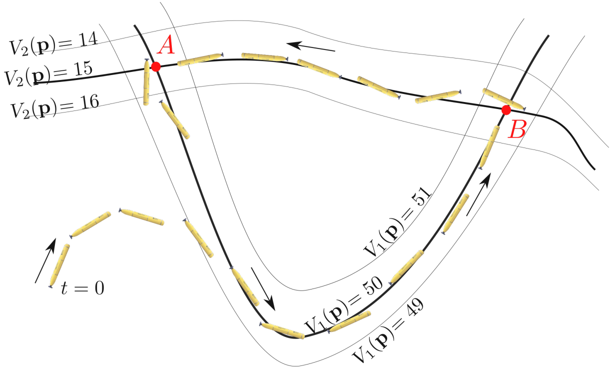
Given a function $h : \mathbb{R}^2 \mapsto \mathbb{R}$, a route in defined by $h(\mathbf{p}) = 0$.
h could be the temperature, the radiation, the pressure, the altitude, the time shift between two periodic events.

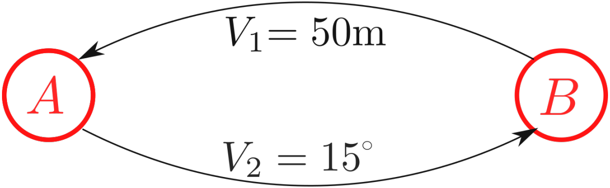



z
$z = 0$
$z = -2$
$y_3 = 2$
$h = -9$
$y_1 = 7$

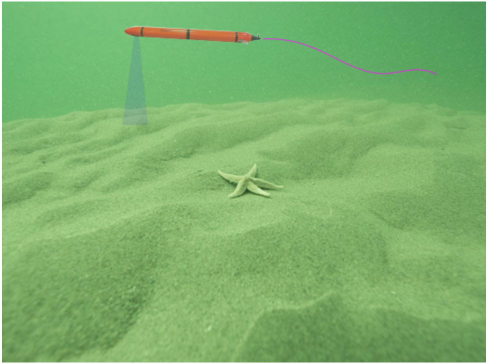

y
x
$y_2 = -\frac{\pi}{2}$
ψ
∇h
$h = -8$
$h = -9$
$h = -10$


0
2
4
6
8
5
0
−5
−10
−10
−5
0
5

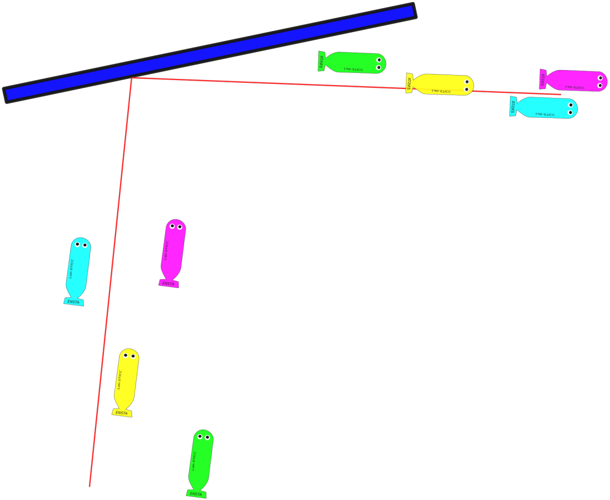

$V_2(\mathbf{p}) = 14$
$V_2(\mathbf{p}) = 15$
$V_2(\mathbf{p}) = 16$
A
B
$V_1(\mathbf{p}) = 51$
$V_1(\mathbf{p}) = 50$
$V_1(\mathbf{p}) = 49$
$t = 0$

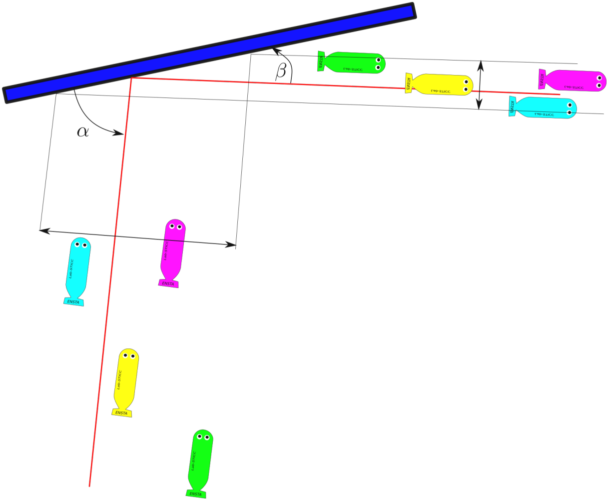

$V_1 = 50\text{m}$
A
B
$V_2 = 15^\circ$

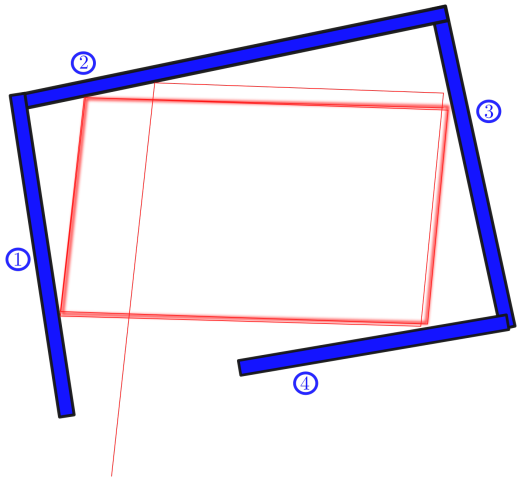
Stable cycles

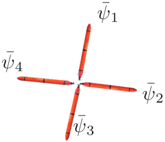


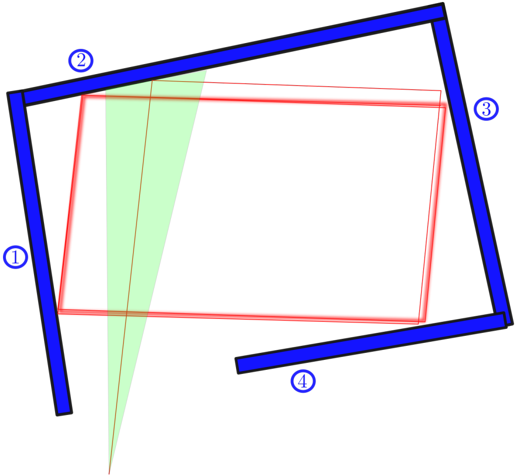
No route exist

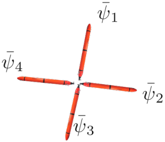


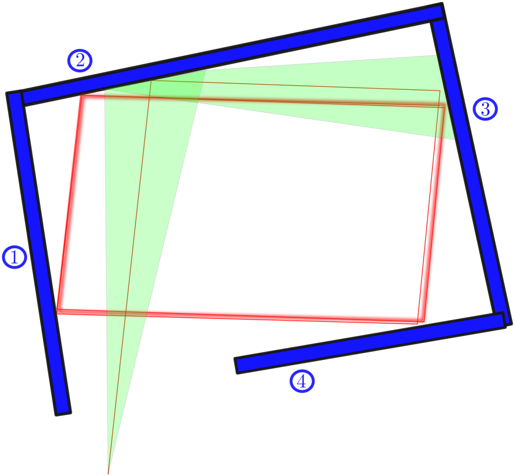

β
α
ENSTA
ENSTA
ENSTA
ENSTA

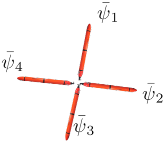

②
③
①
④

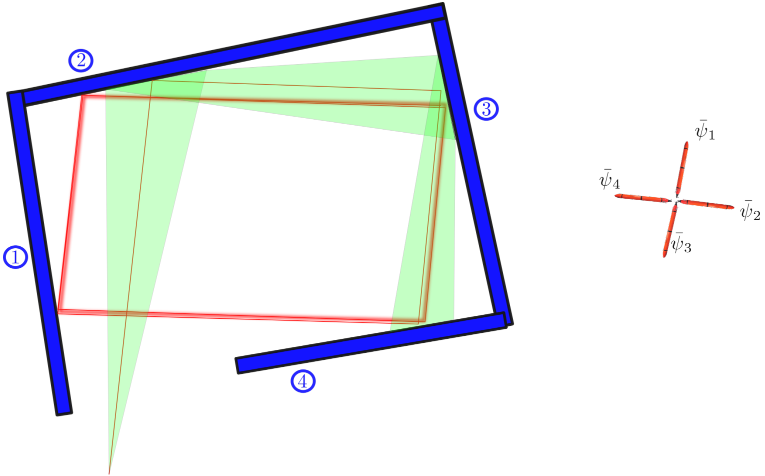

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

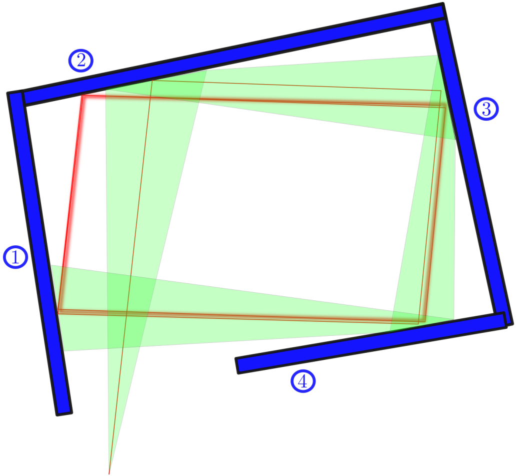

②
③
①
④

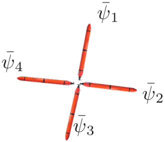

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

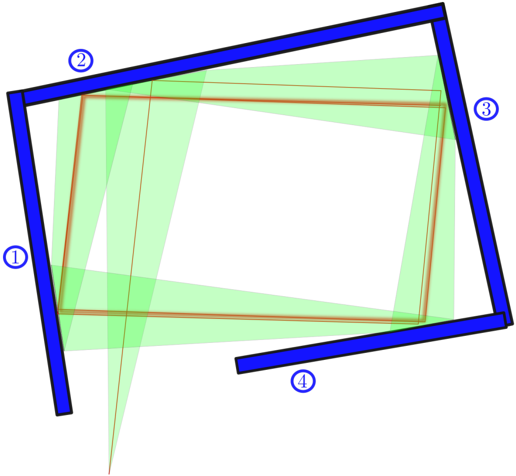

1
2
3
4

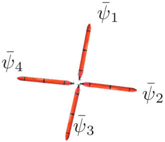

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

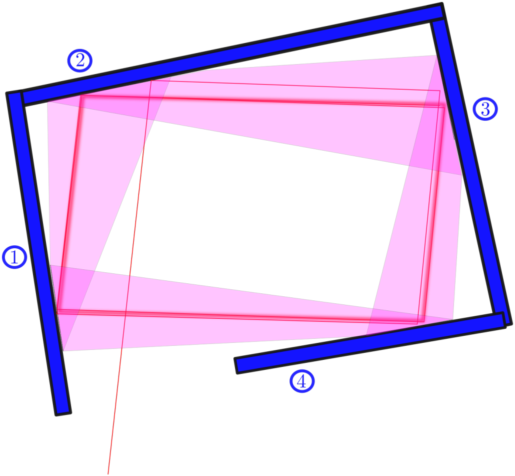

2
3
1
4
$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

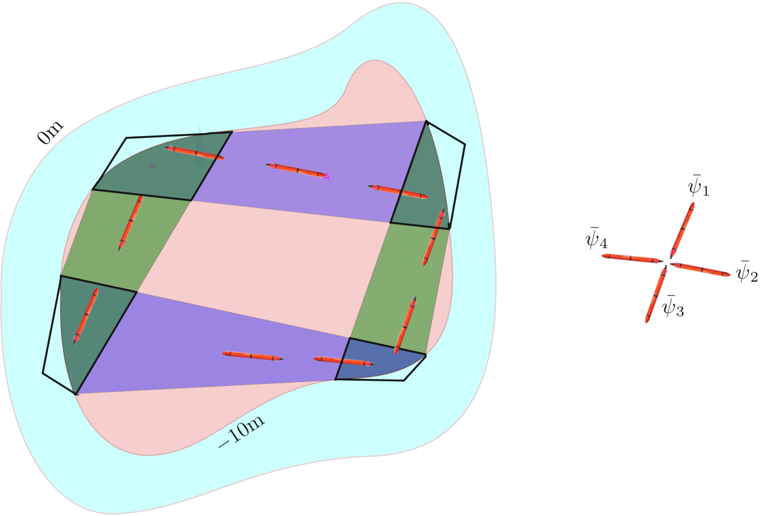

2
3
1
4

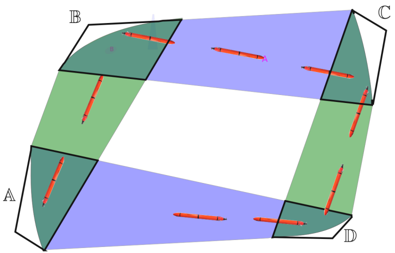

$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$

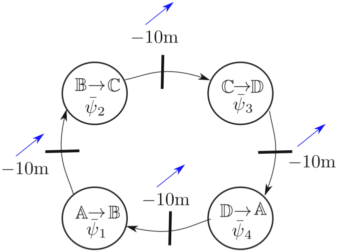

②
③
①
④


$\bar{\psi}_1$
$\bar{\psi}_4$
$\bar{\psi}_2$
$\bar{\psi}_3$


2
3
1
4


0m
−10m
A
$\bar{\psi}_1$
$\bar{\psi}_2$
$\bar{\psi}_3$
$\bar{\psi}_4$


B
C
A
D

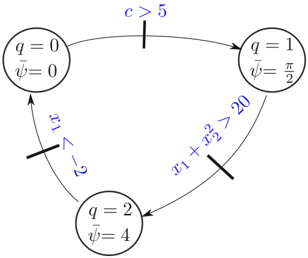

−10m
B→C
$\bar{\psi}_2$
C→D
$\bar{\psi}_3$
−10m
−10m
−10m
A→B
$\bar{\psi}_1$
D→A
$\bar{\psi}_4$

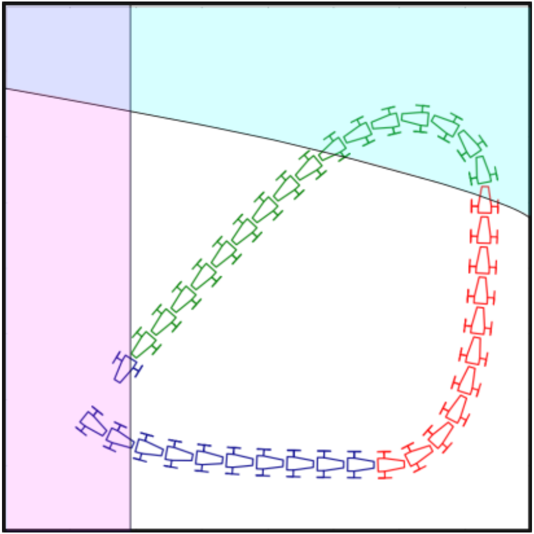
Test-case

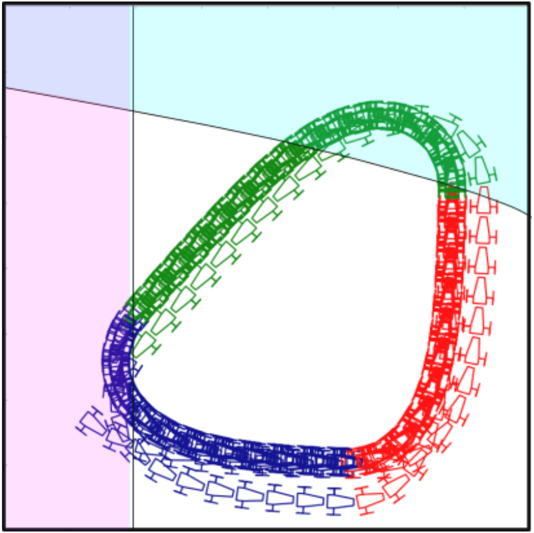
Consider the robot [2]

$$\begin{cases} \dot{x}_1 = \cos x_3 \\ \dot{x}_2 = \sin x_3 \\ \dot{x}_3 = u \end{cases}$$

with the heading control $u = \sin(\bar{\psi} - x_3)$.

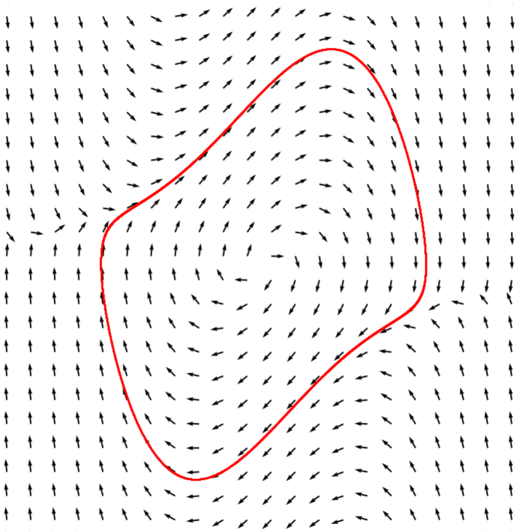






Stability with Poincaré map

System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
How to prove that the system has a cycle ?
How to prove that the system is stable ? [1][5]

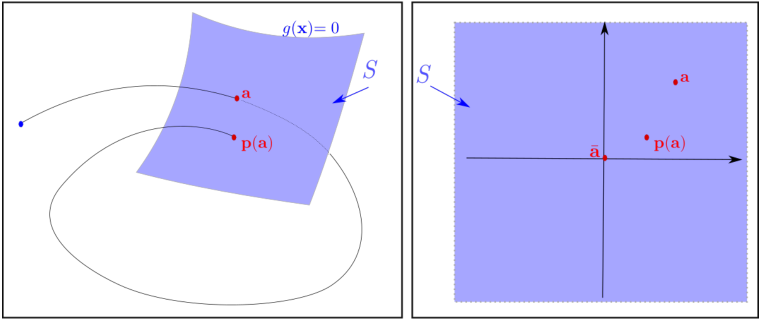


System: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$
Poincaré section $\mathscr{G}$: $g(\mathbf{x}) = 0$

We define

$$\mathbf{p} : \begin{array}{ccc} \mathscr{G} & \rightarrow & \mathscr{G} \\ \mathbf{a} & \mapsto & \mathbf{p}(\mathbf{a}) \end{array}$$

where $\mathbf{p}(\mathbf{a})$ is the point of $\mathscr{G}$ such that the trajectory initialized at $\mathbf{a}$ intersects $\mathscr{G}$ for the first time.

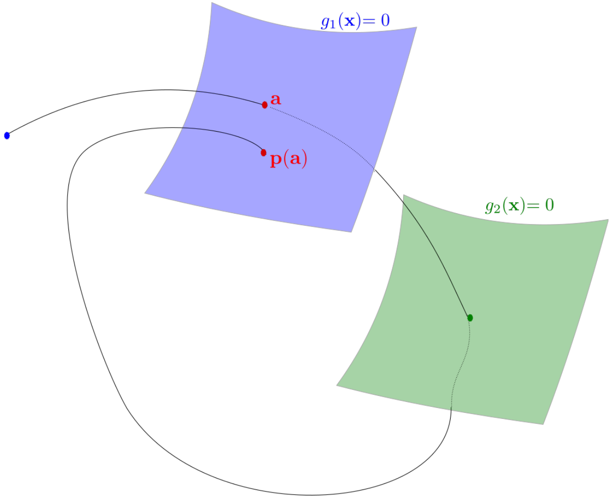

g(x)= 0
S
a
p(a)
S
a
ā
p(a)

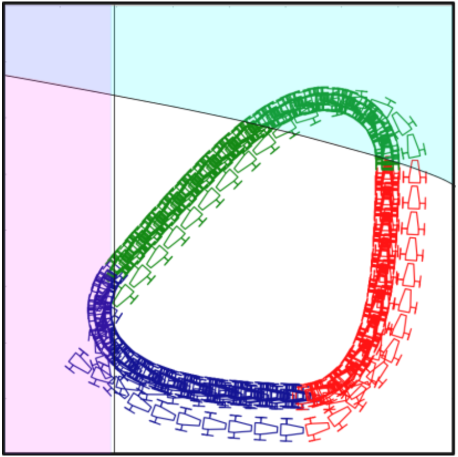
The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$

With hybrid systems

Systems: $\dot{\mathbf{x}} = \mathbf{f}_i(\mathbf{x}), i \in \{1, \ldots, m\}$
Section i: $g_i(\mathbf{x}) = 0$


$g_1(\mathbf{x})= 0$
$\mathbf{a}$
$\mathbf{p}(\mathbf{a})$
$g_2(\mathbf{x})= 0$

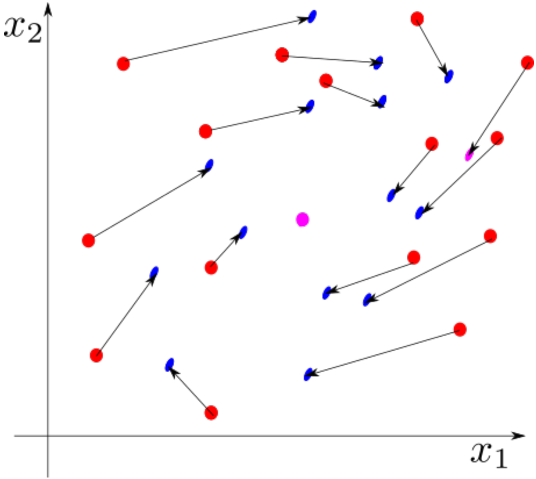


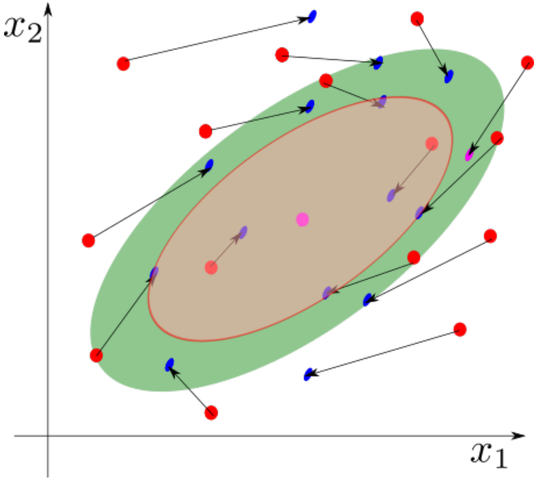
Basin of attraction

Consider the discrete time system

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k)$$

with $\mathbf{f}(\mathbf{0}) = \mathbf{0}$.


x_2
x_1


x_2
x_1

We have to find

$$\mathscr{E}_{\mathbf{x}} : \mathbf{x}^{\mathsf{T}} \cdot \mathbf{P} \cdot \mathbf{x} \leq \varepsilon$$

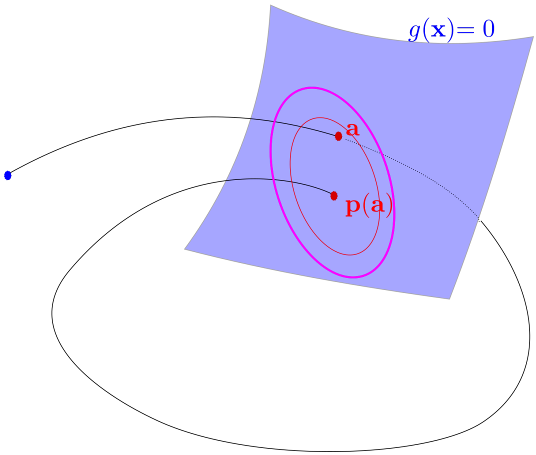
Such that

$$\mathbf{f}(\mathscr{E}_{\mathbf{x}}) \subset \mathscr{E}_{\mathbf{x}}$$

Stability of cycles

The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$



See [4]

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