Image Shape Extraction using Interval Methods

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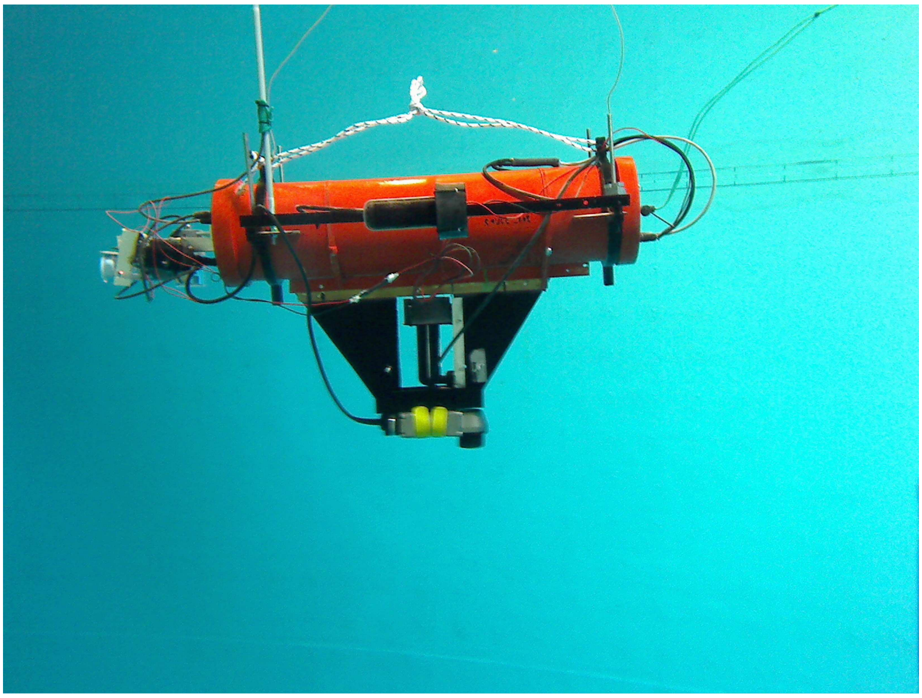
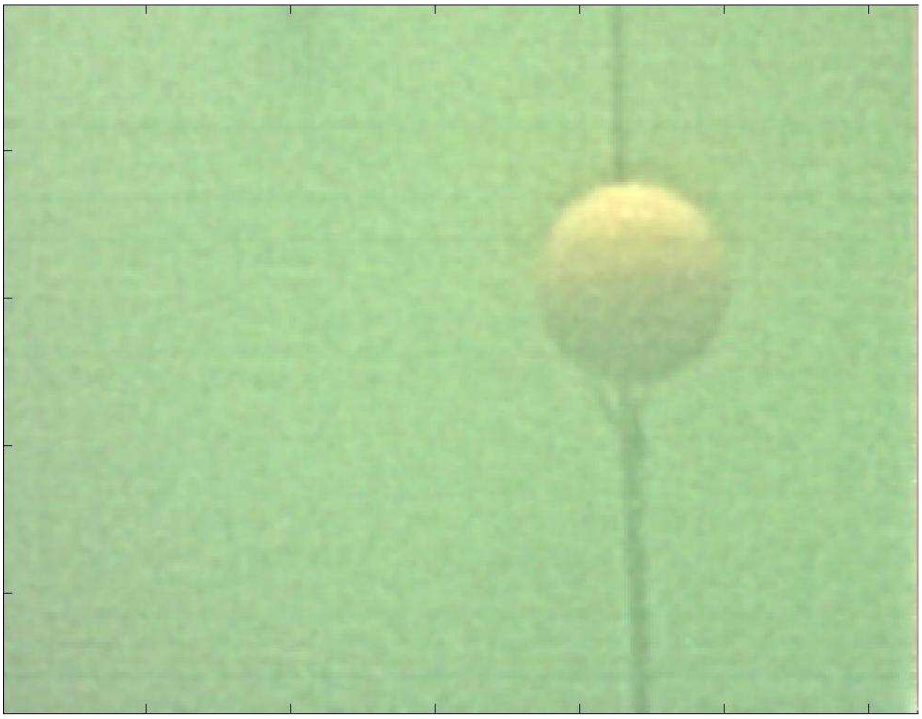
Sysid 2009
Saint Malo, Juillet 2009

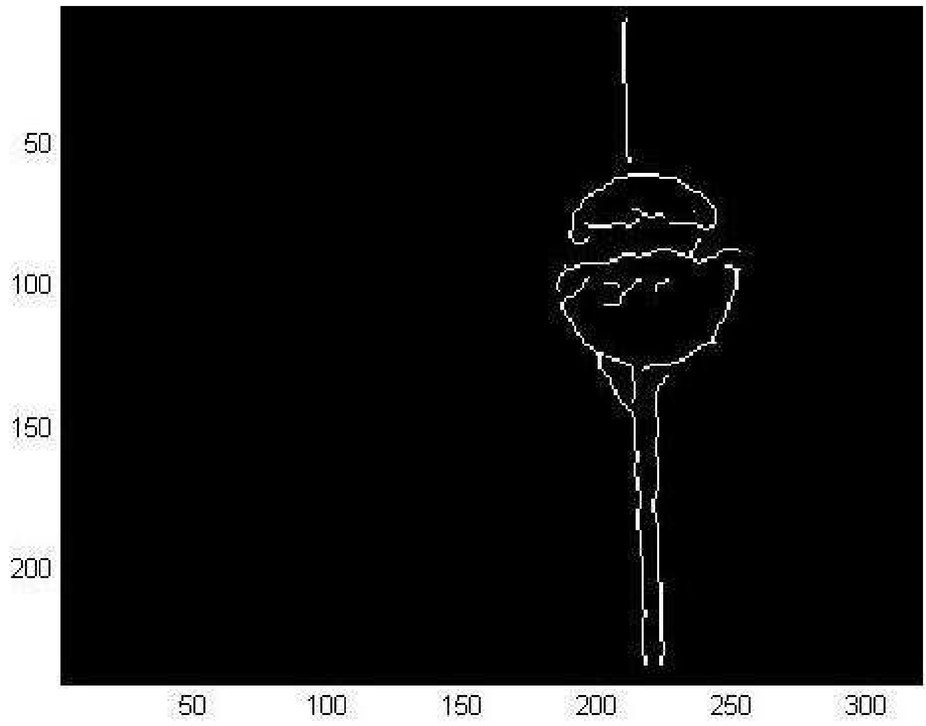
1 Shape detection problem



Sauc'isse robot swimming inside a pool



A spheric buoy seen by Sauc'isse



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2 Set estimation

An *implicit parameter set estimation problem* amounts to characterizing

$$\mathbb{P} = \bigcap_{i \in \{1,\dots,m\}} \underbrace{\left\{\mathbf{p} \in \mathbb{R}^n, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}(\mathbf{p},\mathbf{y}) = \mathbf{0}\right\}}_{\mathbb{P}_i}$$

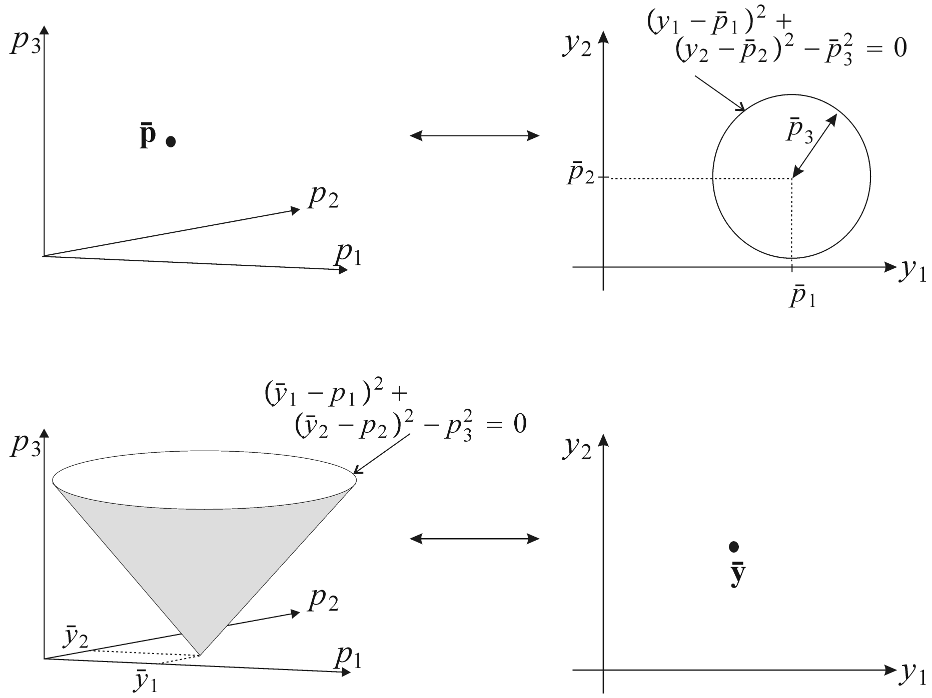
where $\mathbf{p}$ is the parameter vector, $[\mathbf{y}](i)$ is the ith measurement box and $\mathbf{f}$ is the model function.

3 Shape extraction as a set estimation problem

Consider the *shape function* $\mathbf{f}(\mathbf{p}, \mathbf{y})$, where $\mathbf{y} \in \mathbb{R}^2$ corresponds to a pixel and $\mathbf{p}$ is the shape vector.

Example (circle):

$$f(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2.$$



p_3
$\bar{\mathbf{p}}$
p_2
p_1
$(y_1 - \bar{p}_1)^2 + (y_2 - \bar{p}_2)^2 - \bar{p}_3^2 = 0$
y_2
$\bar{p}_3$
$\bar{p}_2$
y_1
$\bar{p}_1$
$(\bar{y}_1 - p_1)^2 + (\bar{y}_2 - p_2)^2 - p_3^2 = 0$
p_3
p_2
$\bar{y}_2$
p_1
$\bar{y}_1$
y_2
$\bar{\mathbf{y}}$
y_1

The *shape* associated with $\mathbf{p}$ is

$$\mathcal{S}(\mathbf{p}) \overset{\text{def}}{=} \left\{ \mathbf{y} \in \mathbb{R}^2, \mathbf{f}(\mathbf{p}, \mathbf{y}) = \mathbf{0} \right\}.$$

Consider a set of (small) boxes in the image

$$\mathcal{Y} = \{[\mathbf{y}](1), \ldots, [\mathbf{y}](m)\}.$$

Each box is assumed to intersect the shape we want to extract.

In our buoy example,

- $\mathcal{Y}$ corresponds to edge pixel boxes.

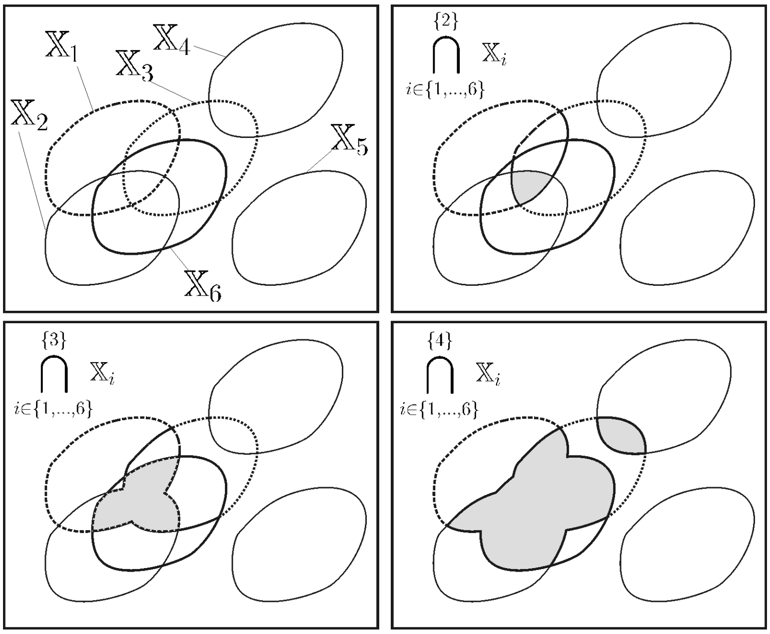
- $f(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2$.

- $\mathbf{p} = (p_1, p_2, p_3)^\mathsf{T}$ where p_1, p_2 are the coordinates of the center of the circle and p_3 its radius.

Now, in our shape extraction problem, a lot of $[\mathbf{y}](i)$ are outlier.

4 Robust set estimation

The q-relaxed intersection denoted by $\bigcap^{\{q\}} \mathbb{X}_i$ is the set of all $\mathbf{x}$ which belong to all $\mathbb{X}_i$'s, except q at most.



$\mathbb{X}_1$
$\mathbb{X}_2$
$\mathbb{X}_3$
$\mathbb{X}_4$
$\mathbb{X}_5$
$\mathbb{X}_6$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{2\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{3\}} \mathbb{X}_i$
$\bigcap_{i\in\{1,\ldots,6\}}^{\{4\}} \mathbb{X}_i$

The q relaxed feasible set is

$$\mathbb{P}^{\{q\}} \stackrel{\text{def}}{=} \bigcap_{i\in\{\mathbf{1},\ldots,m\}}^{\{q\}} \left\{\mathbf{p} \in \mathbb{R}^n, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}\left(\mathbf{p},\mathbf{y}\right) = \mathbf{0}\right\}.$$

5 Interval propagation

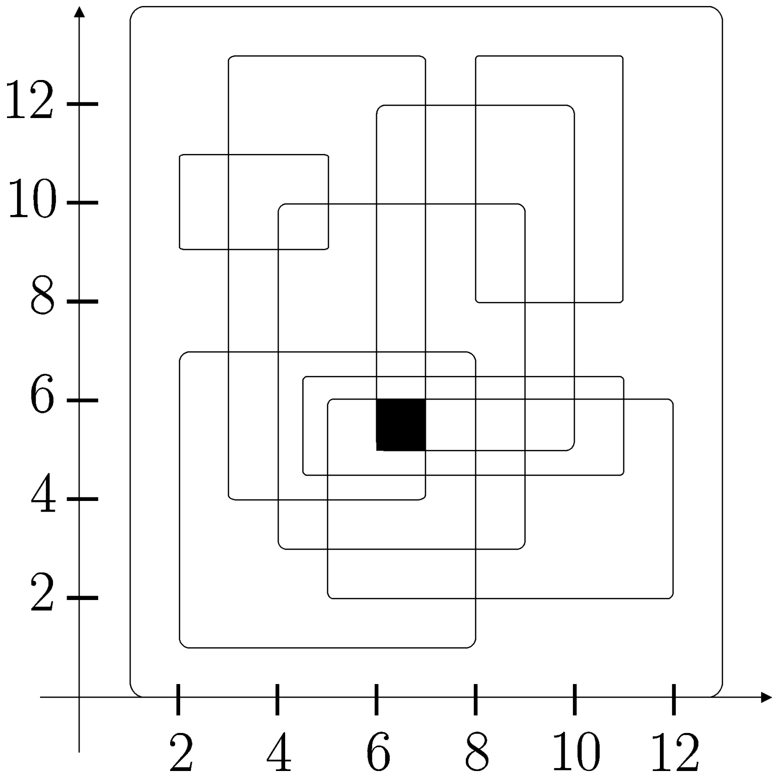
An optimal contractor for the set

$$\left\{\mathbf{p}\in[\mathbf{p}],\exists\mathbf{y}\in[\mathbf{y}],(y_1-p_1)^2+(y_2-p_2)^2-p_3^2=0\right\}.$$

FB(in: $[\mathbf{y}],[\mathbf{p}]$, out: $[\mathbf{p}]$)
$[d_1]:=[y_1]-[p_1]\,;$
$[d_2]:=[y_2]-[p_2]\,;$
$[c_1]:=[d_1]^2\,;$
$[c_2]:=[d_2]^2\,;$
$[c_3]:=[p_3]^2\,;$
$[e]:=[0,0]\cap([c_1]+[c_2]-[c_3])\,;$
$[c_1]:=[c_1]\cap([e]-[c_2]+[c_3])\,;$
$[c_2]:=[c_2]\cap([e]-[c_1]+[c_3])\,;$
$[c_3]:=[c_3]\cap([c_1]+[c_2]-[e])\,;$
$[\bar{p}_3]:=[p_3]\cap\sqrt{[c_3]};$
$[d_2]:=[d_2]\cap\sqrt{[c_2]};$
$[d_1]:=[d_1]\cap\sqrt{[c_1]};$
$[p_2]:=[p_2]\cap([y_2]-[d_2])\,;$
$[p_1]:=[p_1]\cap([y_1]-[d_1])\,;$

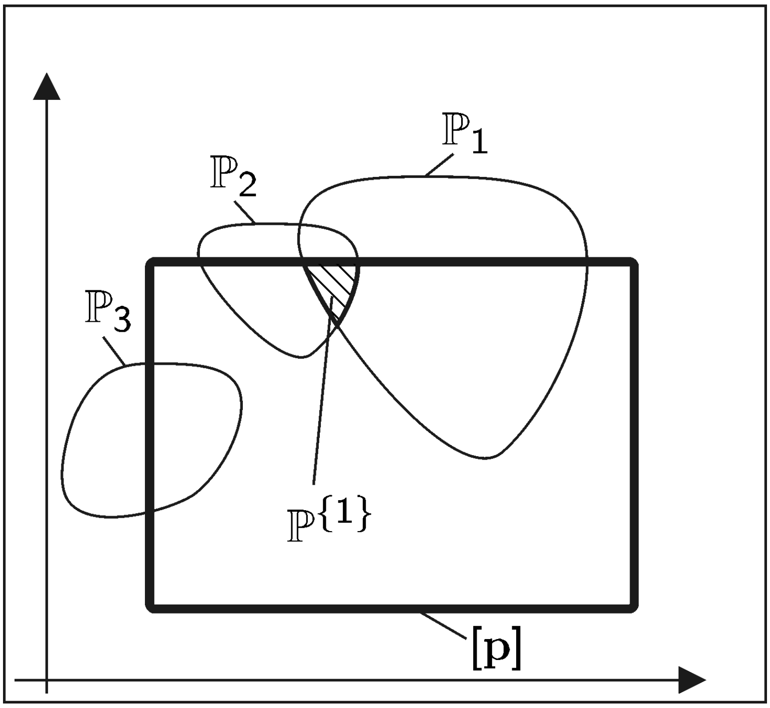
5.1 Relaxed intersection of boxes

Computing the q relaxed intersection of m boxes is tractable.

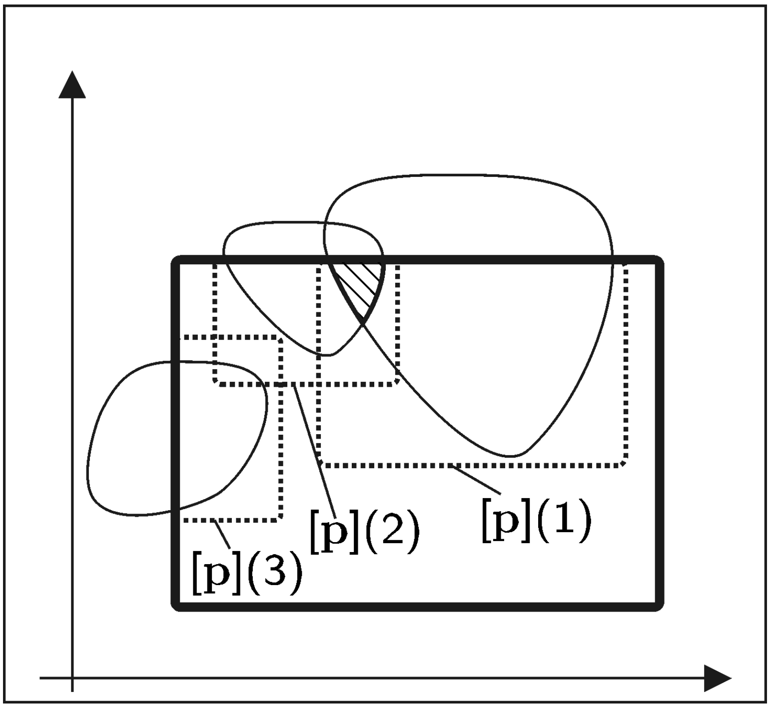


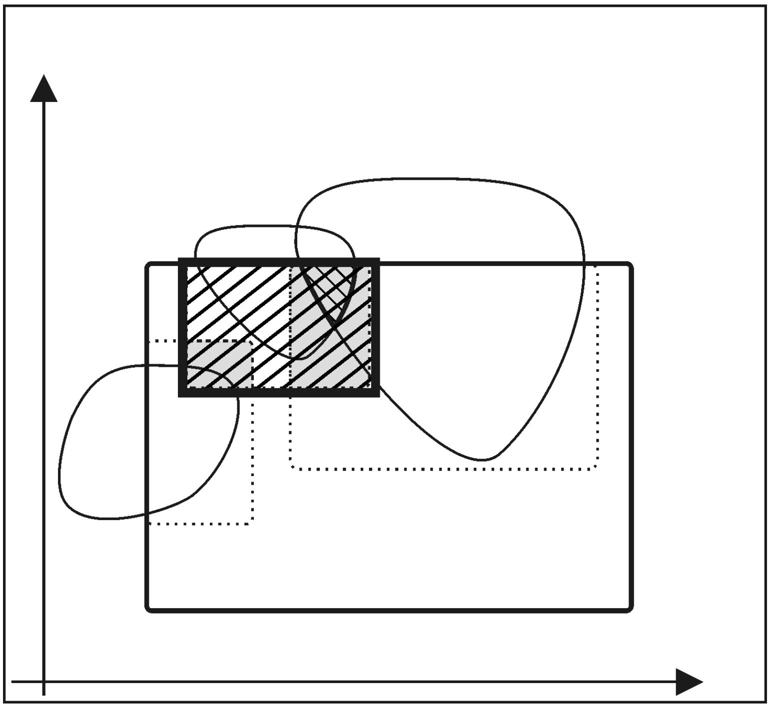
The black box is the 2-intersection of 9 boxes

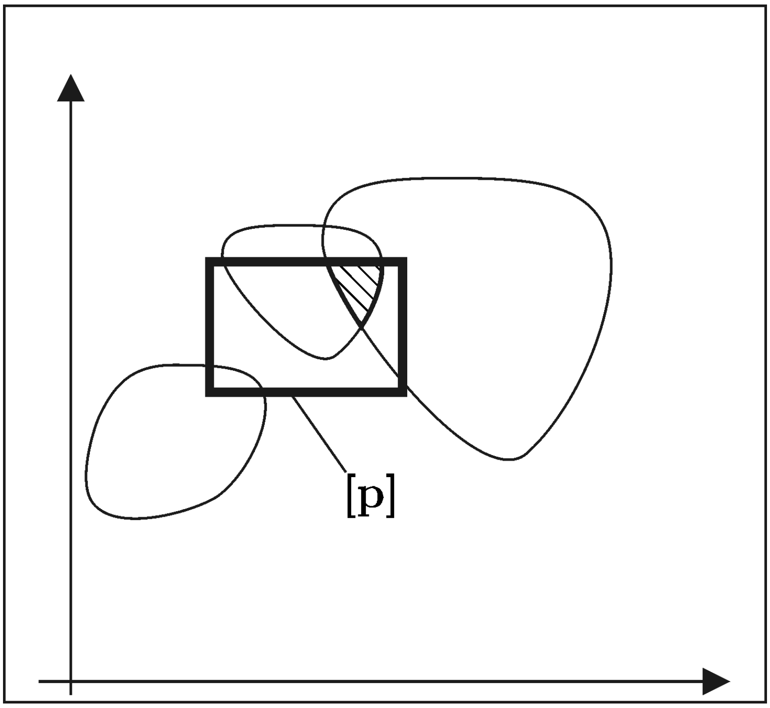
5.2 Algorithm

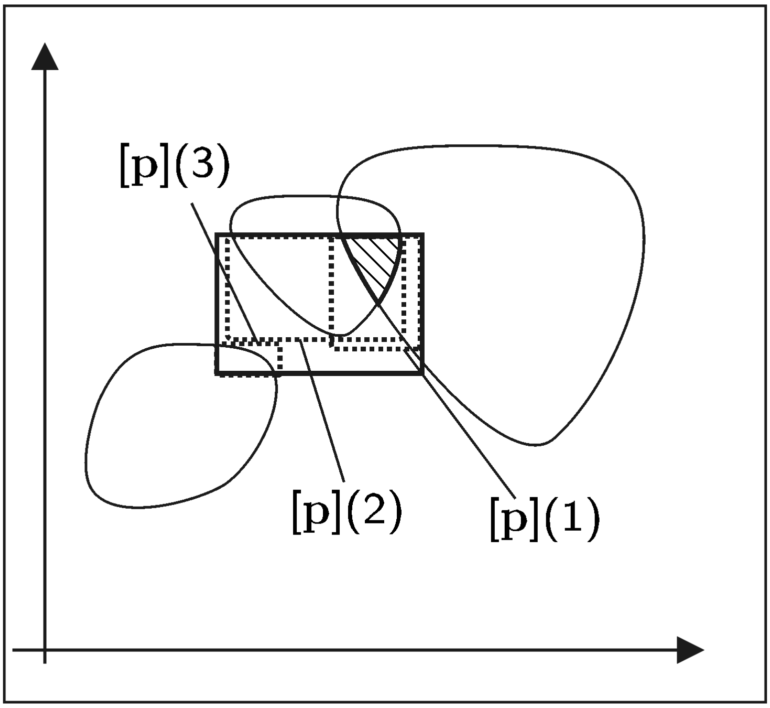


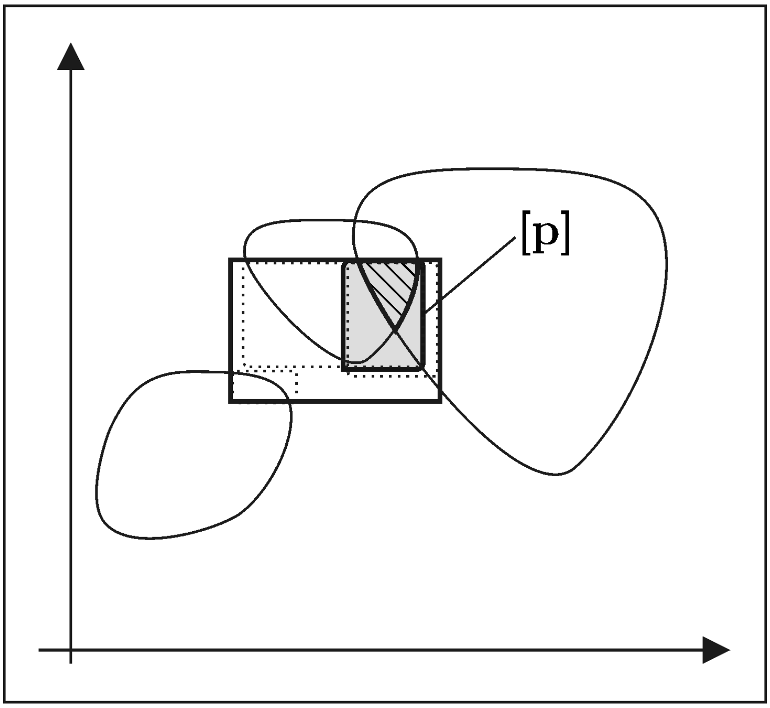
$\mathbb{P}_1$
$\mathbb{P}_2$
$\mathbb{P}_3$
$\mathbb{P}^{\{1\}}$
[p]


[p](1)
[p](2)
[p](3)




[p]


[p](3)
[p](2)
[p](1)

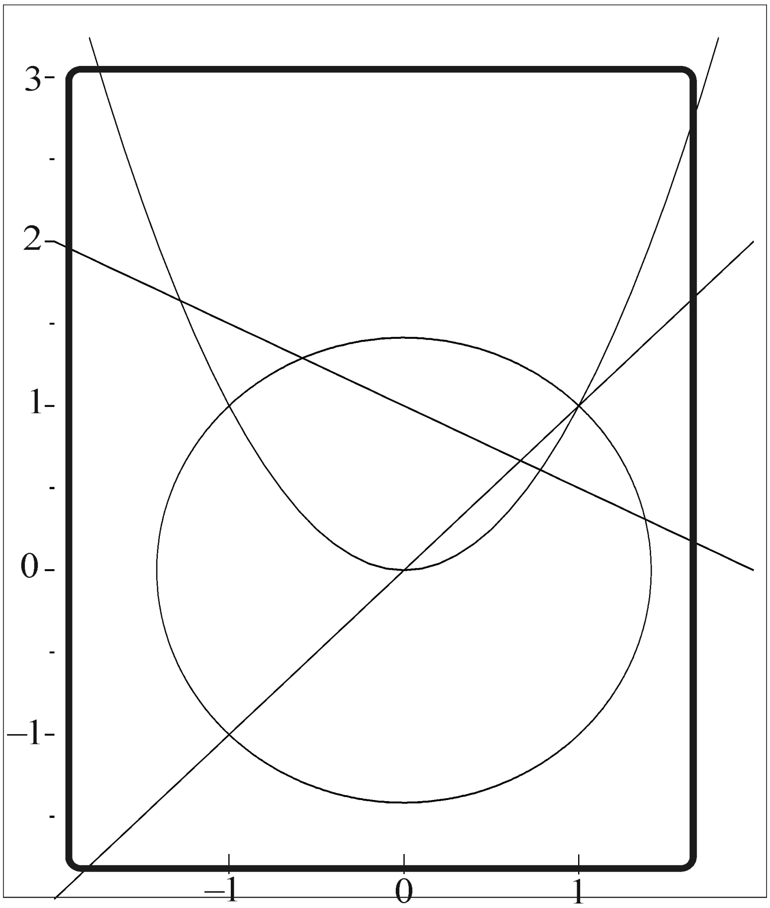

[p]

5.3 Example

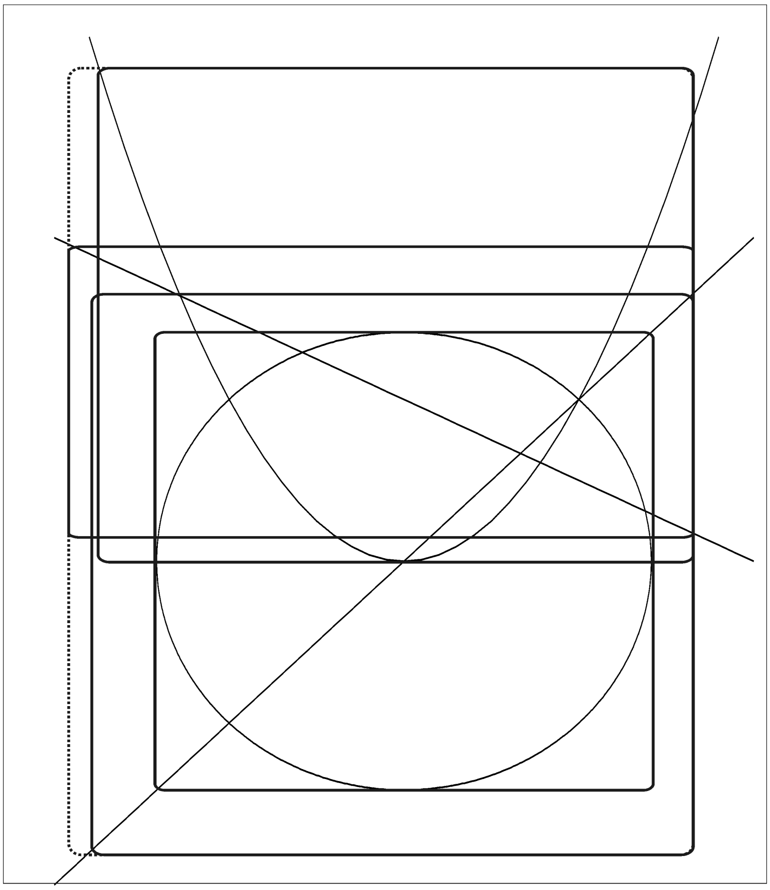
Solve

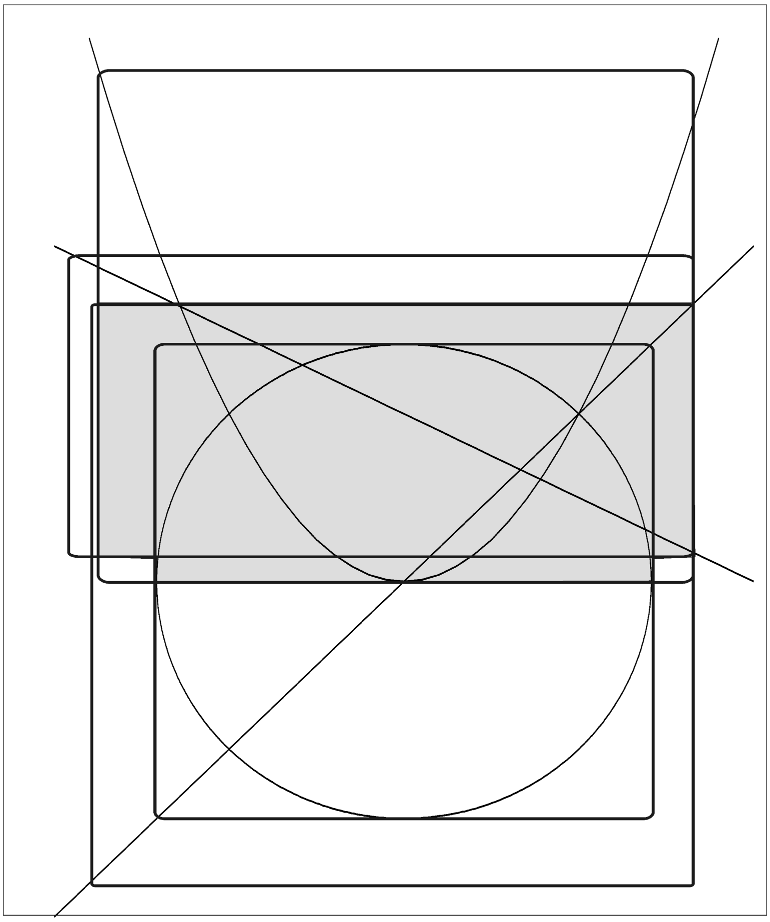
$$\begin{cases} p_2 - p_1^2 & = & 0 \\ p_2^2 + p_1^2 - 1 & = & 0 \\ p_2 - p_1 & = & 0 \\ 2p_2 + p_1 - 2 & = & 0 \end{cases}$$

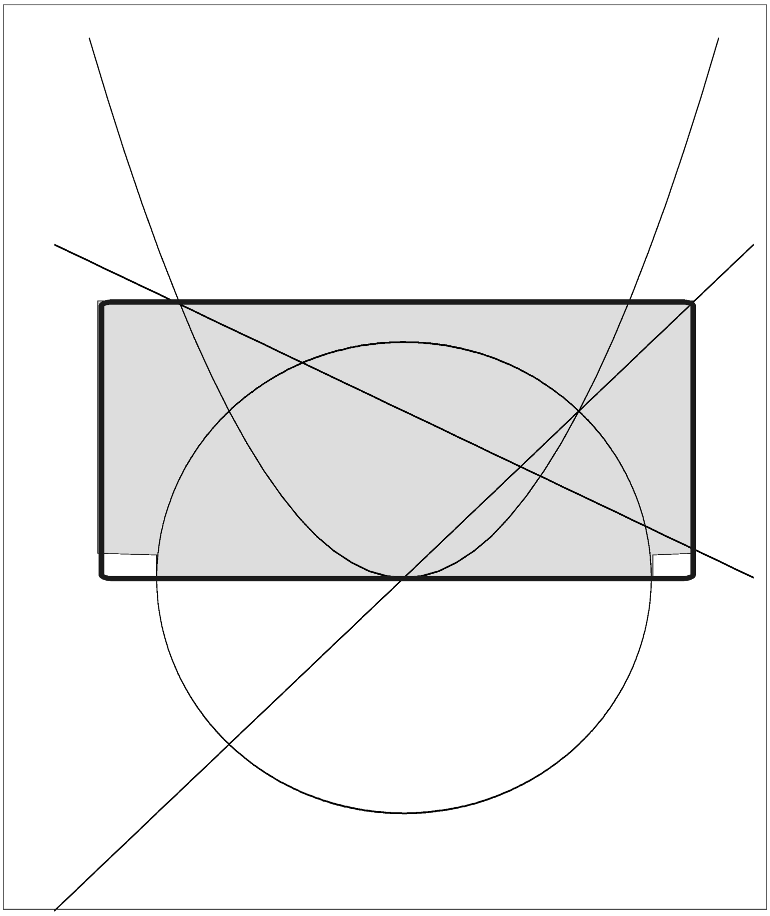
with $q = 1$.

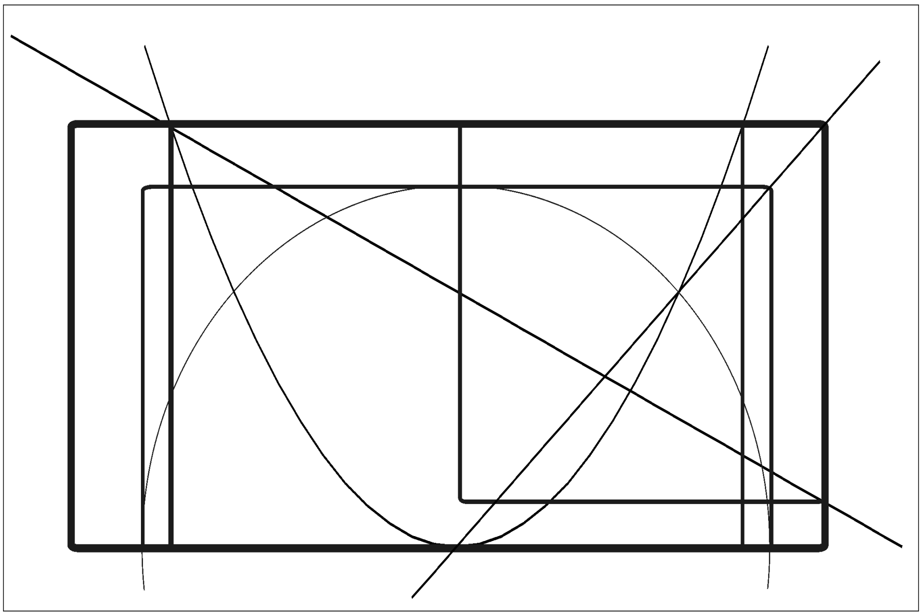


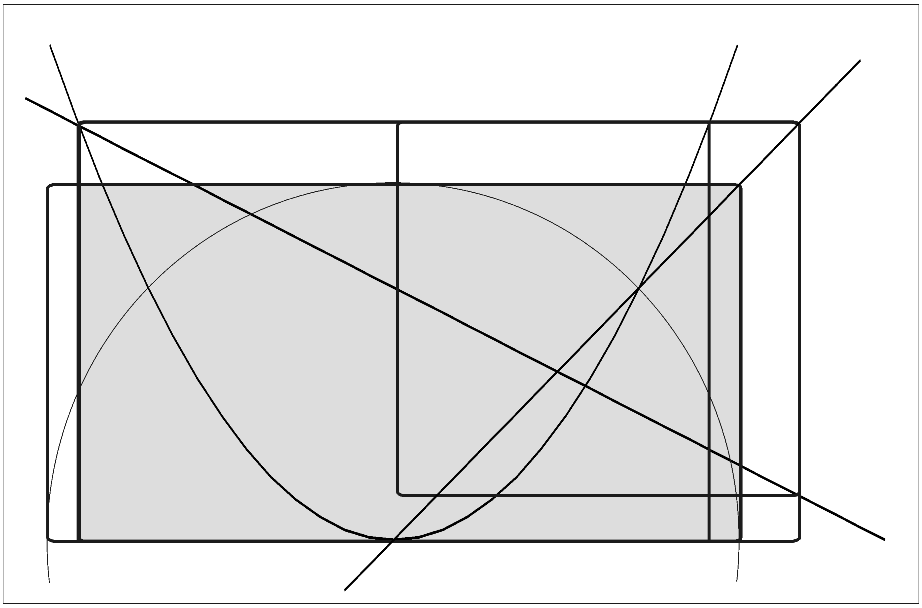
3
2
1
0
−1
−1
0
1

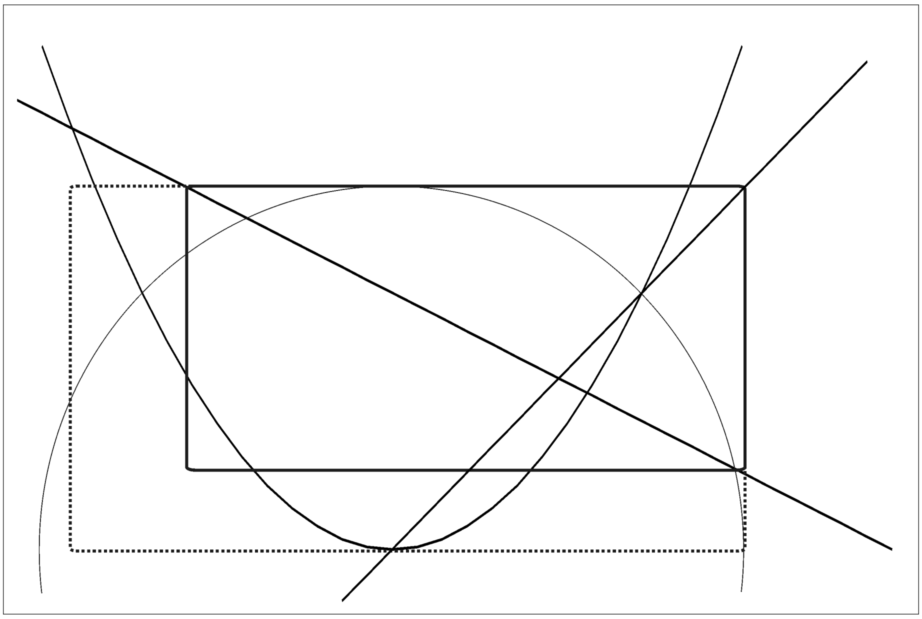


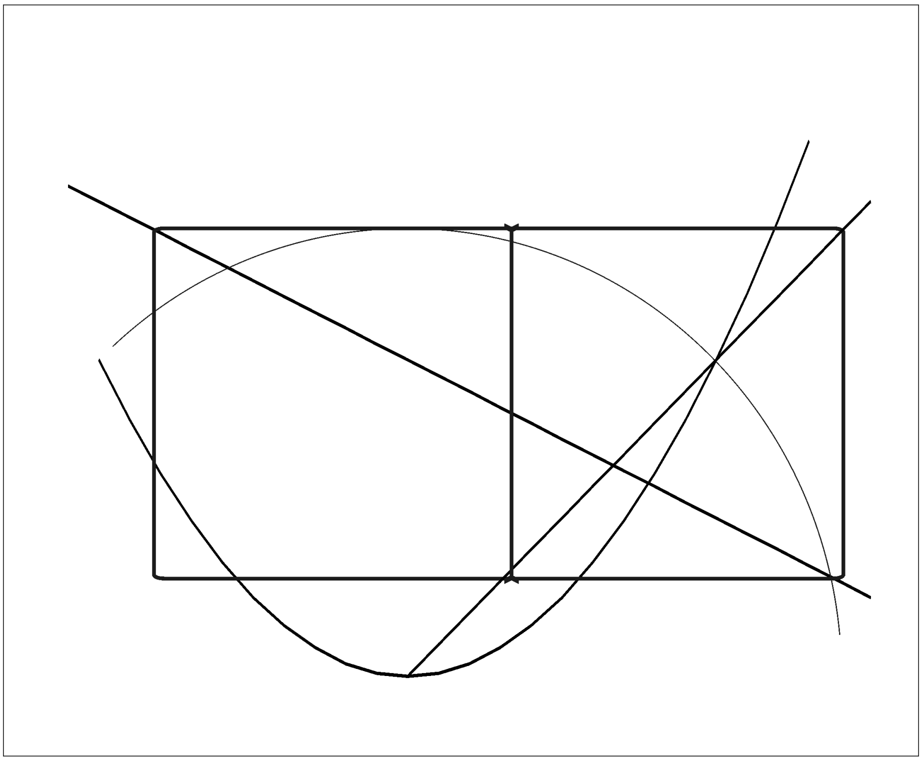


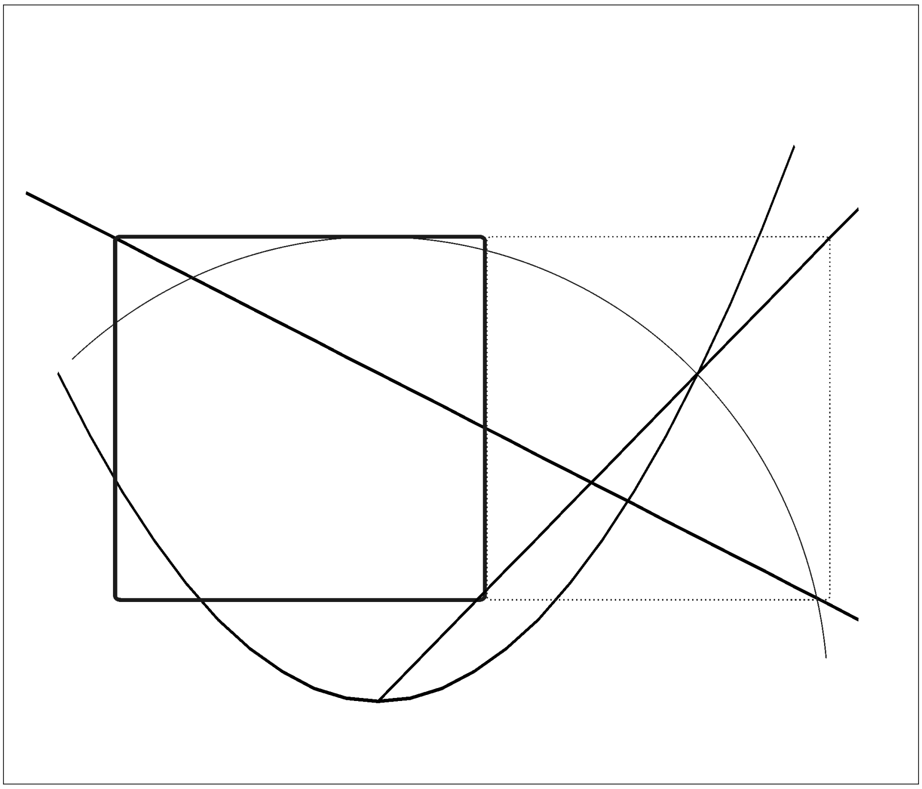


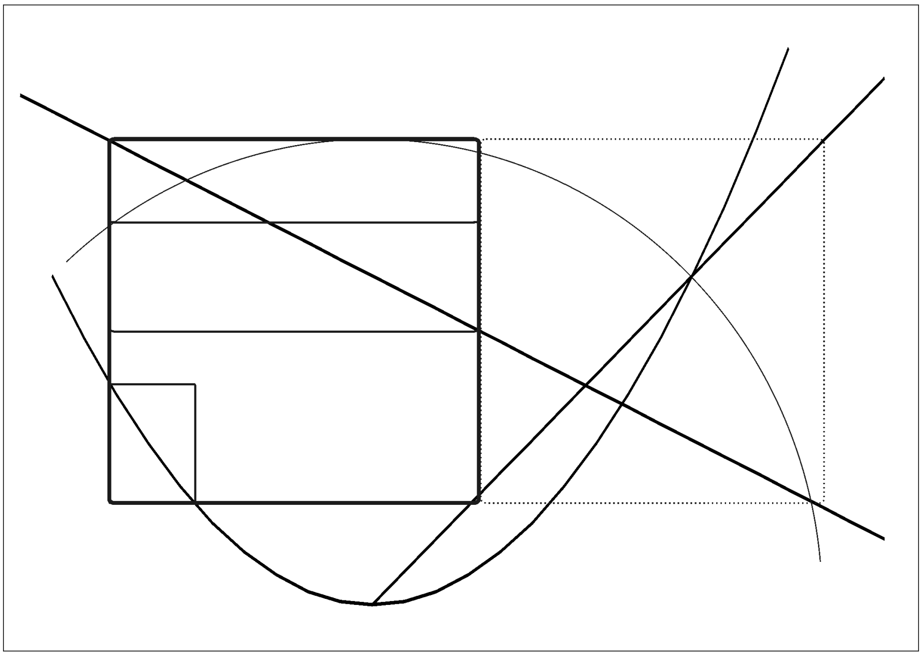


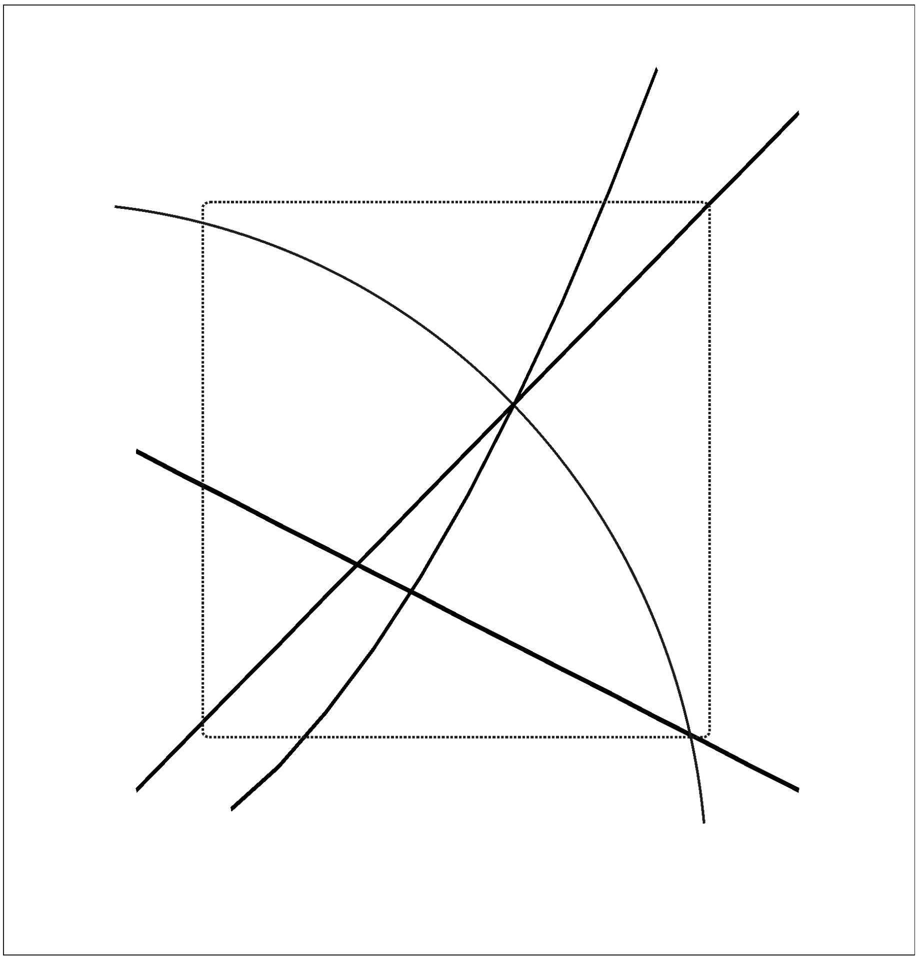


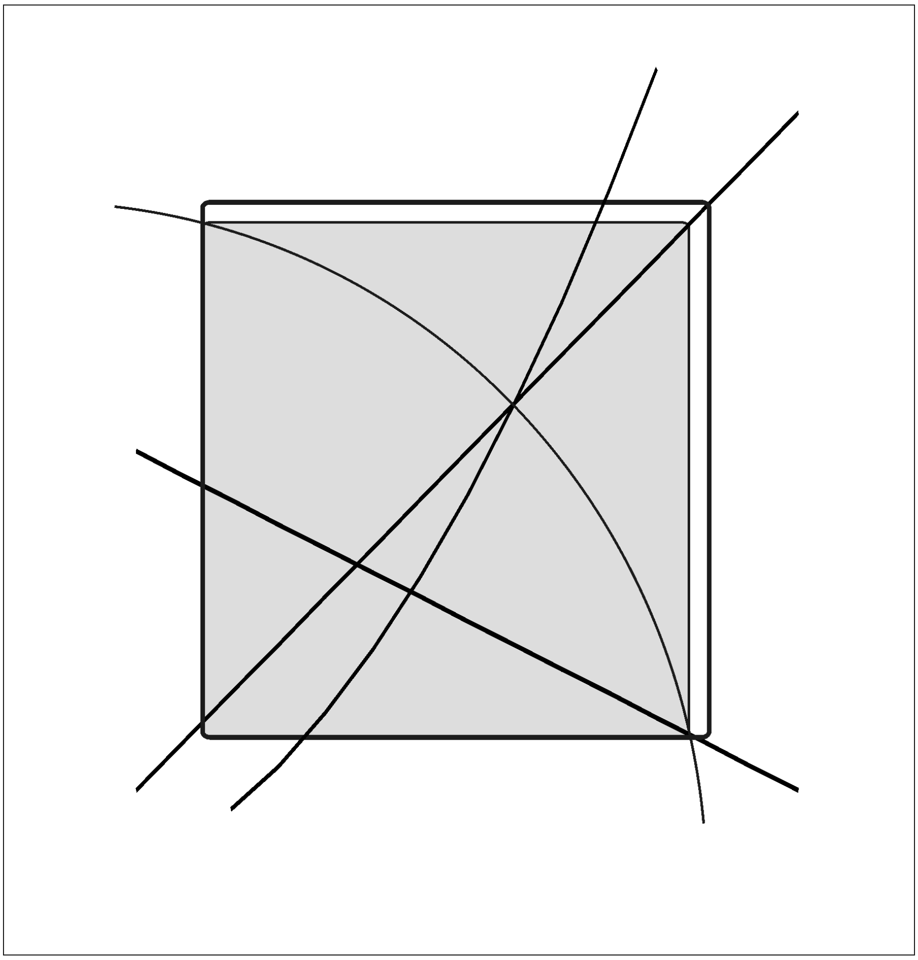


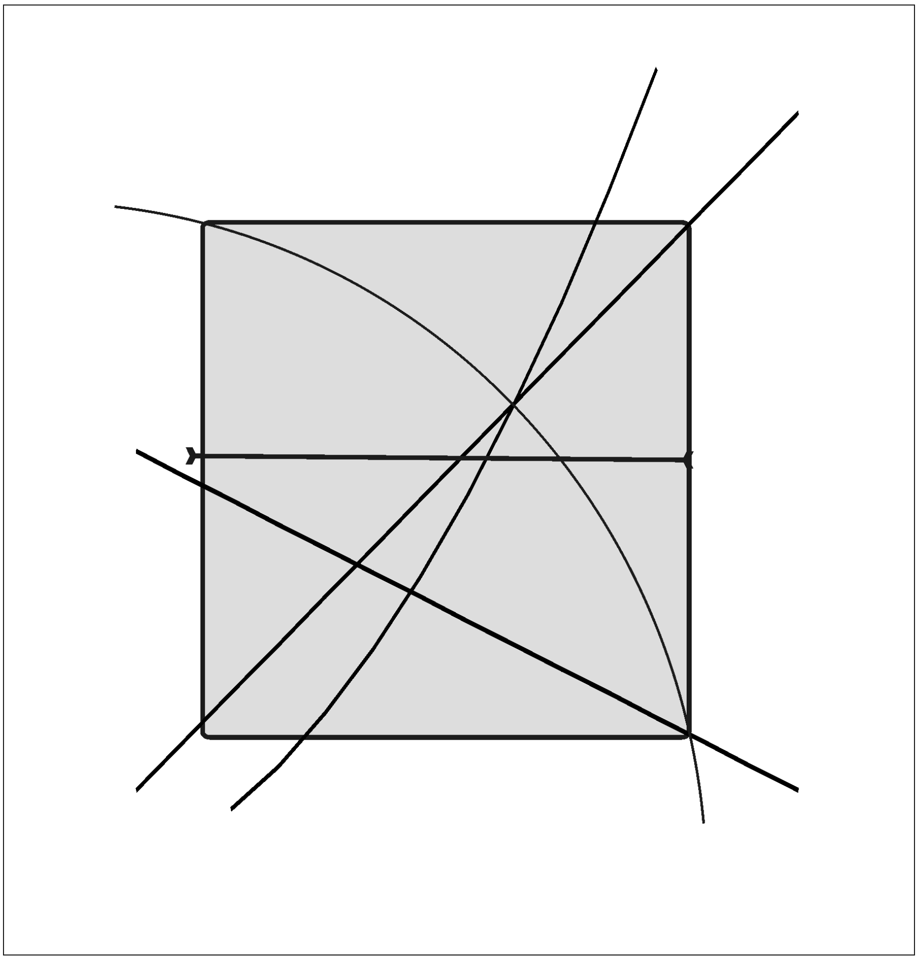


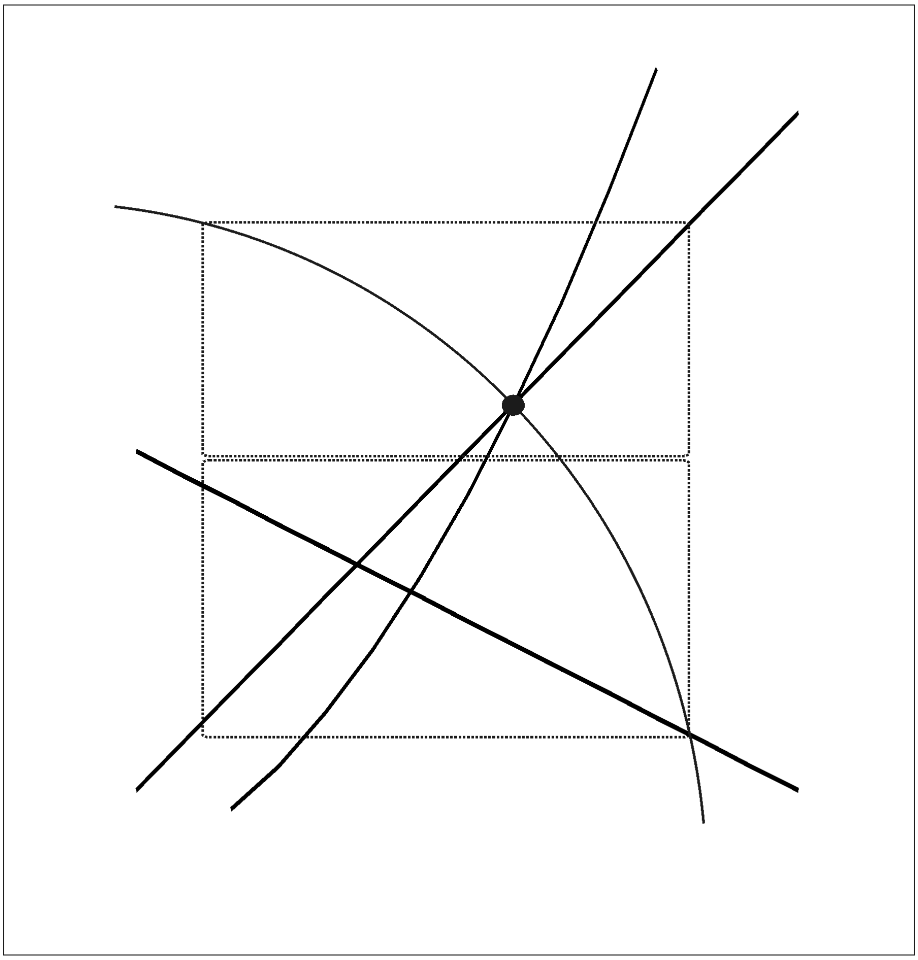




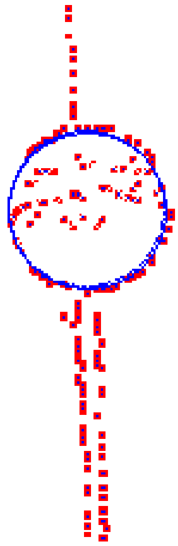




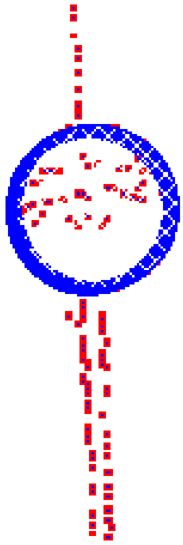




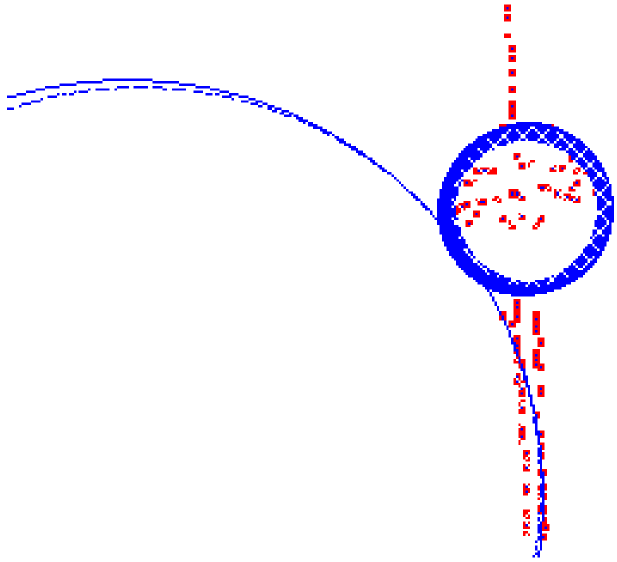
6 Results



$q = 0.70\ m$ (i.e. 70% of the data can be outlier)



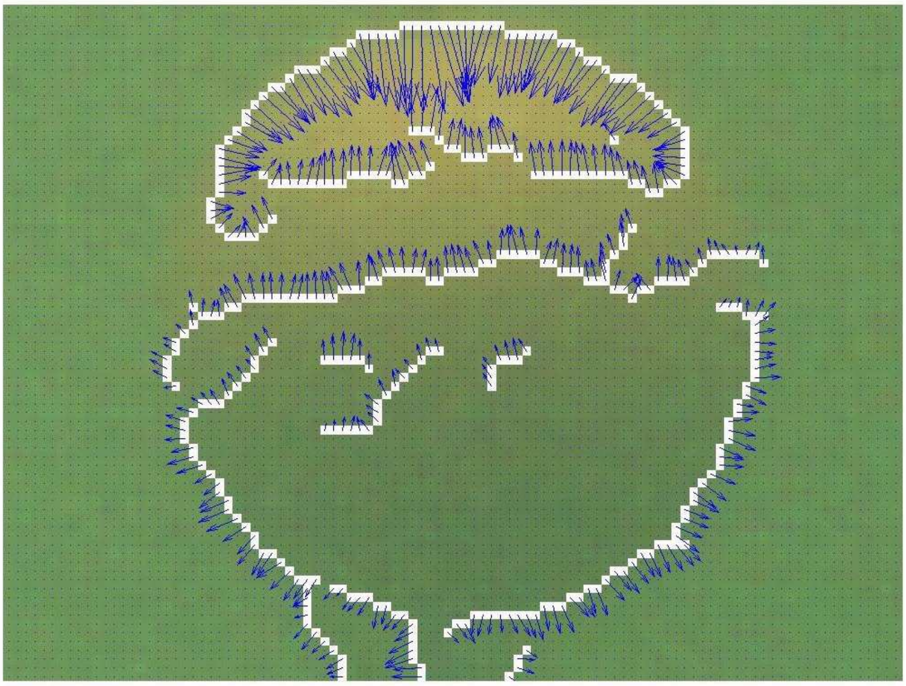
$q = 0.80\ m$ (i.e. 80% of the data can be outlier)



$q =$ 0.81 m (i.e. 81% of the data can be outlier)

O'Gorman and Clowes (1976), in the context of the Hough transform (1972):

the local gradient of the image intensity is orthogonal to the edge.



Now, $\mathbf{y} = (y_1, y_2, y_3)^\mathsf{T}$ where y_3 is the direction of the gradient.

The gradient condition is

$$\det \begin{pmatrix} \frac{\partial f(\mathbf{p},\mathbf{y})}{\partial y_1} & \cos(y_3) \\ \frac{\partial f(\mathbf{p},\mathbf{y})}{\partial y_2} & \sin(y_3) \end{pmatrix} = 0.$$

For $f(\mathbf{p}, \mathbf{y}) = (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2$, we get

$$\mathbf{f}(\mathbf{p}, \mathbf{y}) = \begin{pmatrix} (y_1 - p_1)^2 + (y_2 - p_2)^2 - p_3^2 \\ (y_1 - p_1)\sin(y_3) - (y_2 - p_2)\cos(y_3) \end{pmatrix}.$$

New outliers: the edge points that are on the shape, but that do not satisfy the gradient condition.

The computing time is now 2 seconds instead of 15 seconds.

7 Hough transform

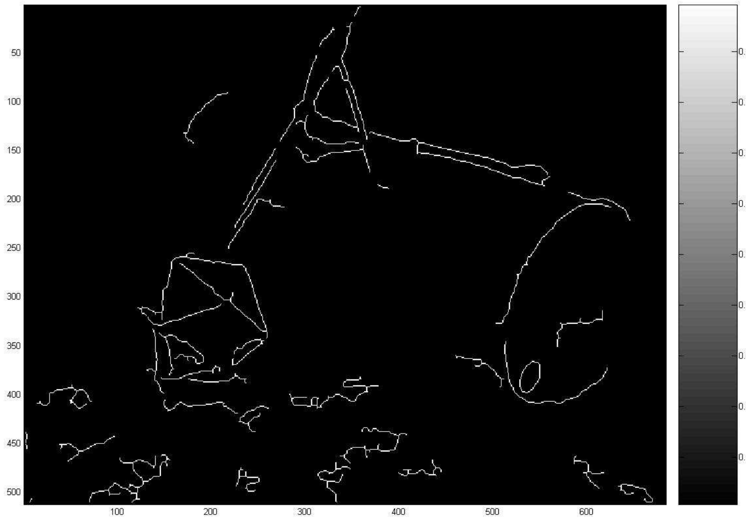
The Hough transform is defined by

$$\eta(\mathbf{p}) = \text{card}\{i \in \{\mathbf{1}, \ldots, m\}, \exists \mathbf{y} \in [\mathbf{y}](i), \mathbf{f}(\mathbf{p}, \mathbf{y}) = \mathbf{0}\}.$$

Hough method keeps all $\mathbf{p}$ such that $\eta(\mathbf{p}) \geq m - q$.

Instead, our approach solves $\eta(\mathbf{p}) \geq m - q$.

8 Perspective



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0.2
0.1