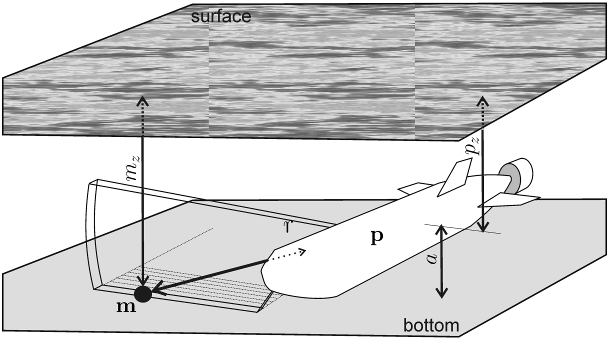
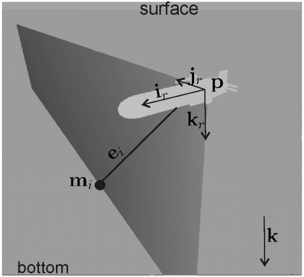
Sweep function

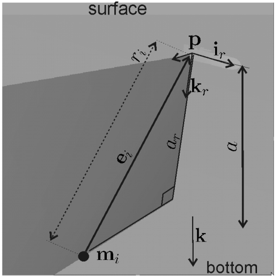
Brest (virtual) 33
2021, May 5

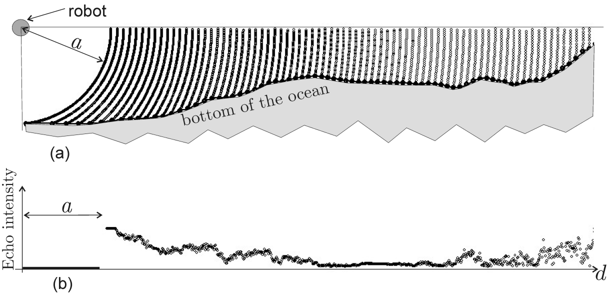
Lateral sonar

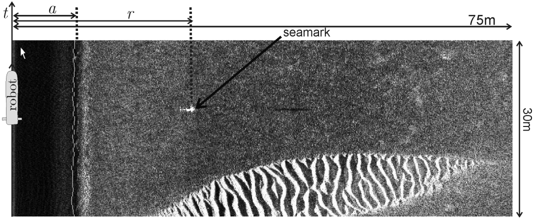
We want to formalize the problem of counting for exploration
Slides
in the context of lateral sonars.


surface
m_z
p_z
r
$\mathbf{p}$
a
$\mathbf{m}$
bottom


surface
$\mathbf{j}_r$
$\mathbf{p}$
$\mathbf{i}_r$
$\mathbf{k}_r$
$\mathbf{e}_i$
$\mathbf{m}_i$
$\mathbf{k}$
bottom

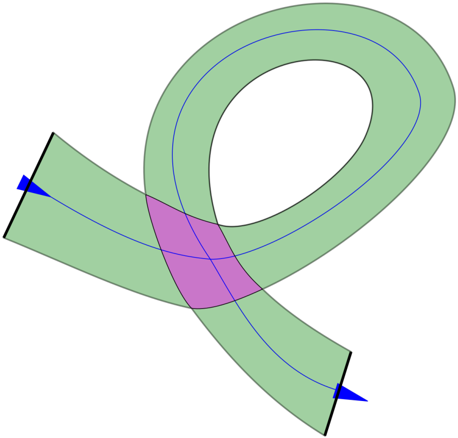

surface
$\mathbf{p}$
$\mathbf{i}_r$
r_i
$\mathbf{k}_r$
$\mathbf{e}_i$
a_r
a
$\mathbf{k}$
$\mathbf{m}_i$
bottom

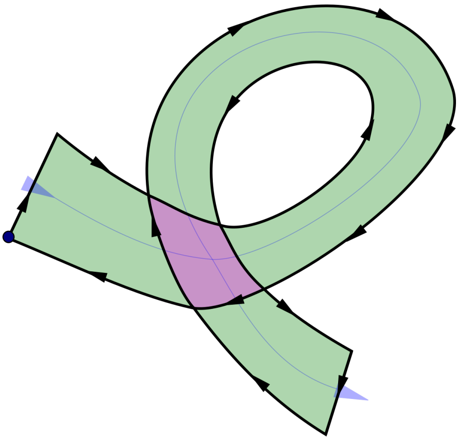

robot
a
bottom of the ocean
(a)
Echo intensity
a
d
(b)

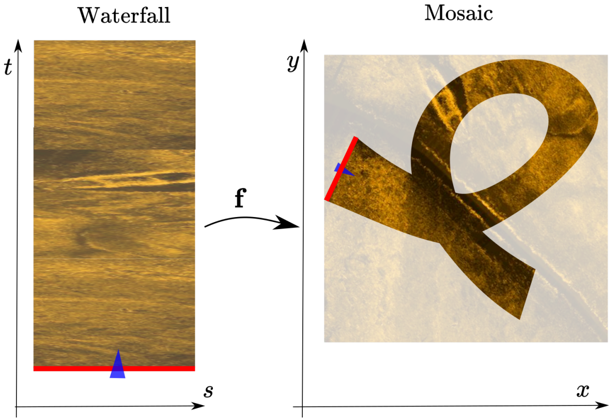


A waterfall : https://youtu.be/l3PM-fXGNPw

Sweep function






Waterfall
Mosaic
t
s
f
y
x

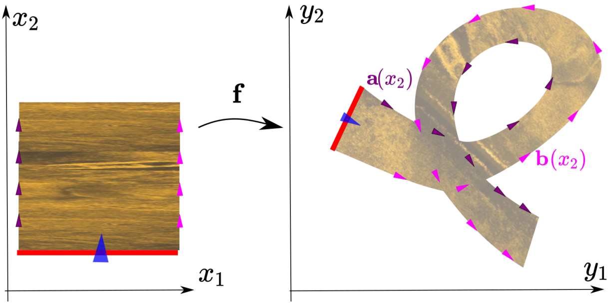
A continuous function

$$\mathbf{f}: \begin{array}{ccc} \mathbb{R}^2 & \mapsto & \mathbb{R}^2 \\ \mathbf{x} & \mapsto & \mathbf{y} = \mathbf{f}(\mathbf{x}) \end{array}$$

is a sweep function if it is affine in x_1, i.e., $\mathbf{f}(\mathbf{x}) = \mathbf{g}(x_2) \cdot x_1 + \mathbf{a}(x_2)$ or equivalently

$$\mathbf{f}(\mathbf{x}) = \mathbf{a}(x_2) \cdot (1 - x_1) + \underbrace{(\mathbf{g}(x_2) + \mathbf{h}(x_2))}_{=\mathbf{b}(x_2)} \cdot x_1$$

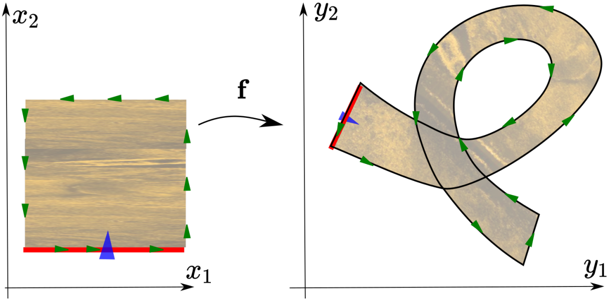
For simplicity, we assume that $x_1 \in [0,1], x_2 \in [0,1]$.


x_2
x_1
$\mathbf{f}$
y_2
y_1
$\mathbf{a}(x_2)$
$\mathbf{b}(x_2)$

We define the waterfall contour

$$\gamma(\tau) = \begin{cases} (\tau, 0) & \text{if } \tau \in [0,1] \\ (1, \tau - 1) & \text{if } \tau \in [1,2] \\ (3 - \tau, 1) & \text{if } \tau \in [2,3] \\ (0, 4 - \tau) & \text{if } \tau \in [3,4] \end{cases}$$

and the mosaic contour as $\mathbf{f}(\gamma(\tau)), \tau \in [0,4]$.


x_2
x_1
f
y_2
y_1

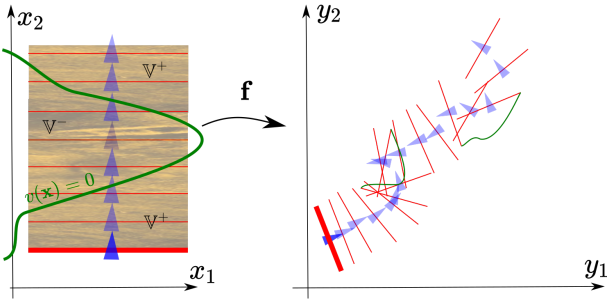
Proposition. If for all $x_2 \in [0,1]$,

$$\det\left(\mathbf{b}(x_2)-\mathbf{a}(x_2),\frac{d}{dx_2}\mathbf{a}(x_2)\right)\geq 0$$
$$\det\left(\mathbf{b}(x_2)-\mathbf{a}(x_2),\frac{d}{dx_2}\mathbf{b}(x_2)\right)\geq 0$$

then

$$\text{card}\{\ker \mathbf{f}\}=\frac{1}{2\pi}\oint_{\mathbf{f}(\gamma(\tau))}\left(\frac{y_1}{y_1^2+y_2^2}dy_2-\frac{y_2}{y_1^2+y_2^2}dy_1\right)$$

Positive and negative sweep zones


x_2
$\mathbb{V}^+$
$\mathbb{V}^-$
$v(\mathbf{x}) = 0$
$\mathbb{V}^+$
x_1
$\mathbf{f}$
y_2
y_1

The sweep speed is

$$v(\mathbf{x}) = \det\left(\frac{\mathbf{b}(x_2)-\mathbf{a}(x_2)}{\|\mathbf{b}(x_2)-\mathbf{a}(x_2)\|}, \frac{d\mathbf{a}(x_2)}{dx_2}\cdot(1-x_1)+\frac{d\mathbf{b}(x_2)}{dx_2}\cdot x_1\right).$$

We define

$$\begin{array}{rcl} \mathbb{V}^- & = & \left\{\mathbf{x}\in\mathbb{R}^2 \,|\, v(\mathbf{x})\leq 0\right\} \\ \mathbb{V}^+ & = & \left\{\mathbf{x}\in\mathbb{R}^2 \,|\, v(\mathbf{x})\geq 0\right\} \end{array}$$

By decomposing the box into the two sweep zones, we can generalize the proposition, by removing the inequality condition.