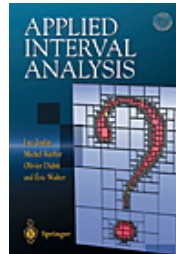


# Calcul par intervalles pour la résolution garantie de problèmes non-linéaires



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Ecole Nationale Supérieure de Physique de Strasbourg

# **1 Intervalles et propagation**

## 1.1 Calcul par intervalles

Si  $\diamond \in \{+, -, \cdot, /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3] \cdot [2, 5] &= [-5, 15], \\ [-1, 3] / [2, 5] &= [-\frac{1}{2}, \frac{3}{2}], \\ [-1, 3] \vee [2, 5] &= [2, 5]. \end{aligned}$$



$$\begin{aligned}
[x^-, x^+] + [y^-, y^+] &= [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] &= [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
&\quad x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+], \\
[x^-, x^+] \vee [y^-, y^+] &= [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}$$

Si  $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\begin{aligned}\sin([0, \pi]) &= [0, 1], \\ \text{sqr}([-1, 3]) &= [-1, 3]^2 = [0, 9], \\ \text{abs}([-7, 1]) &= [0, 7], \\ \text{sqrt}([-10, 4]) &= \sqrt{[-10, 4]} = [0, 2], \\ \log([-2, -1]) &= \emptyset.\end{aligned}$$

## 1.2 Projection de contraintes

Soient  $x, y, z$  trois variables telles que

$$\begin{aligned}x &\in [-\infty, 5], \\y &\in [-\infty, 4], \\z &\in [6, \infty], \\z &= x + y.\end{aligned}$$

Les valeurs  $< 2$  pour  $x$ ,  $< 1$  pour  $y$  et  $> 9$  pour  $z$  sont inconsistantes.

## **1.3 Méthode numérique de projection**

En effet, puisque  $x \in [-\infty, 5]$ ,  $y \in [-\infty, 4]$ ,  $z \in [6, \infty]$  et  $z = x + y$ , nous avons

$$z = x + y \Rightarrow z \in [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ = [6, \infty] \cap [-\infty, 9] = [6, 9].$$

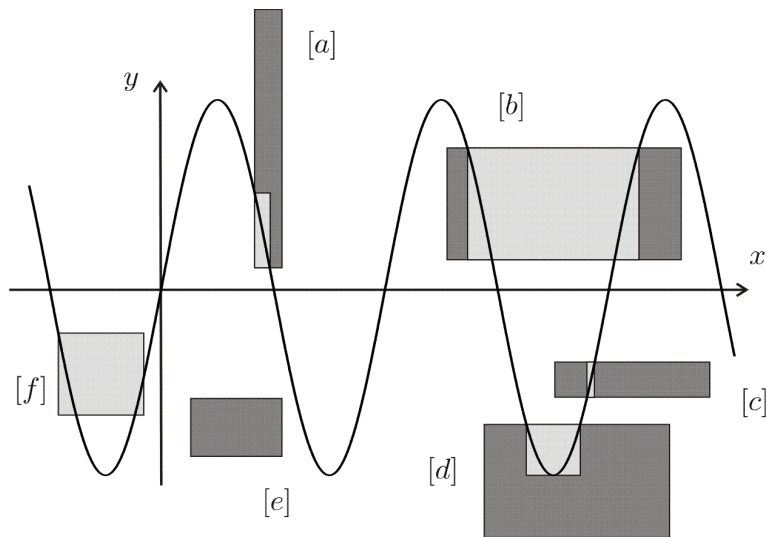
$$x = z - y \Rightarrow x \in [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ = [-\infty, 5] \cap [2, \infty] = [2, 5].$$

$$y = z - x \Rightarrow y \in [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ = [-\infty, 4] \cap [1, \infty] = [1, 4].$$

Pour la contrainte

$$y = \sin x, \quad x \in [x], y \in [y]$$

le problème est un peu plus difficile.



## 1.4 Algorithme de propagation-bissection

**Exemple.** Cherchons à résoudre.

$$y = x^2$$

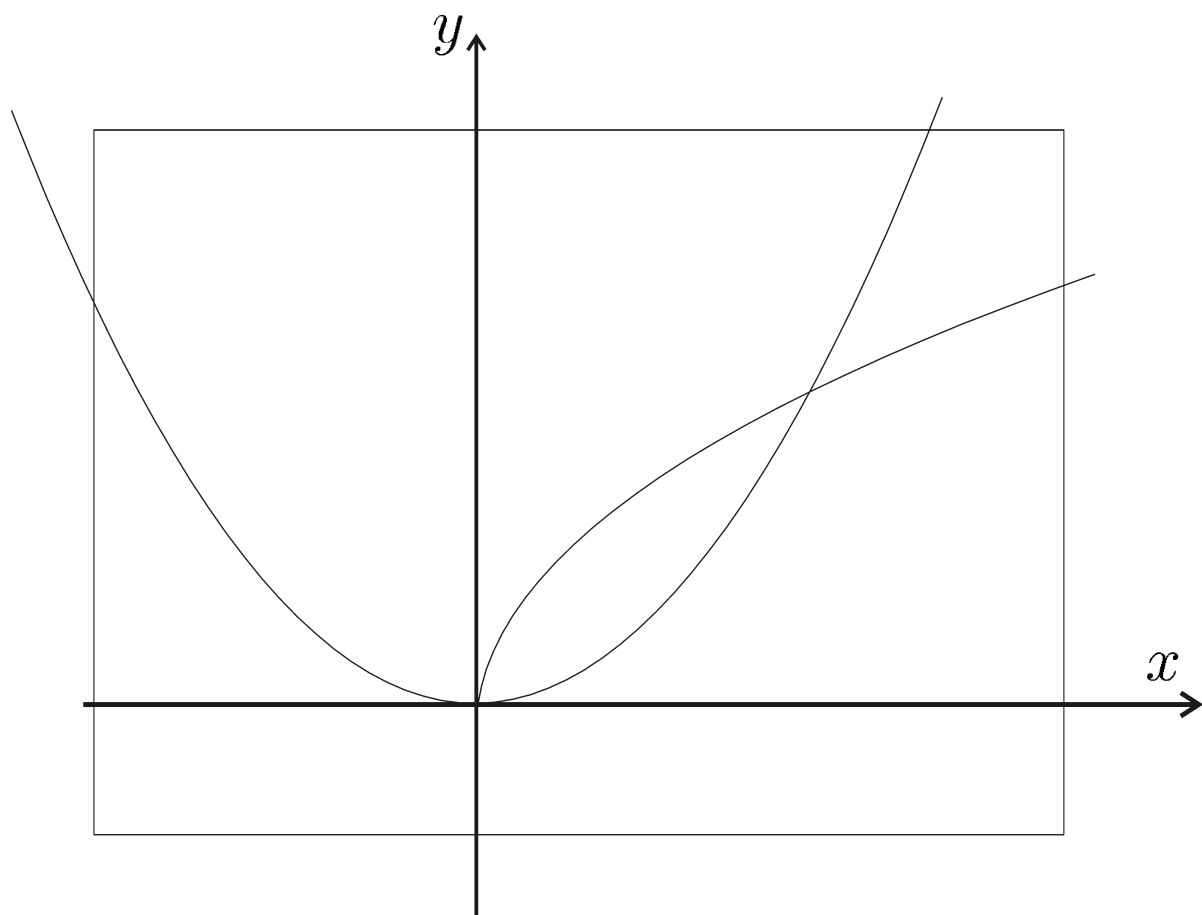
$$y = \sqrt{x}.$$

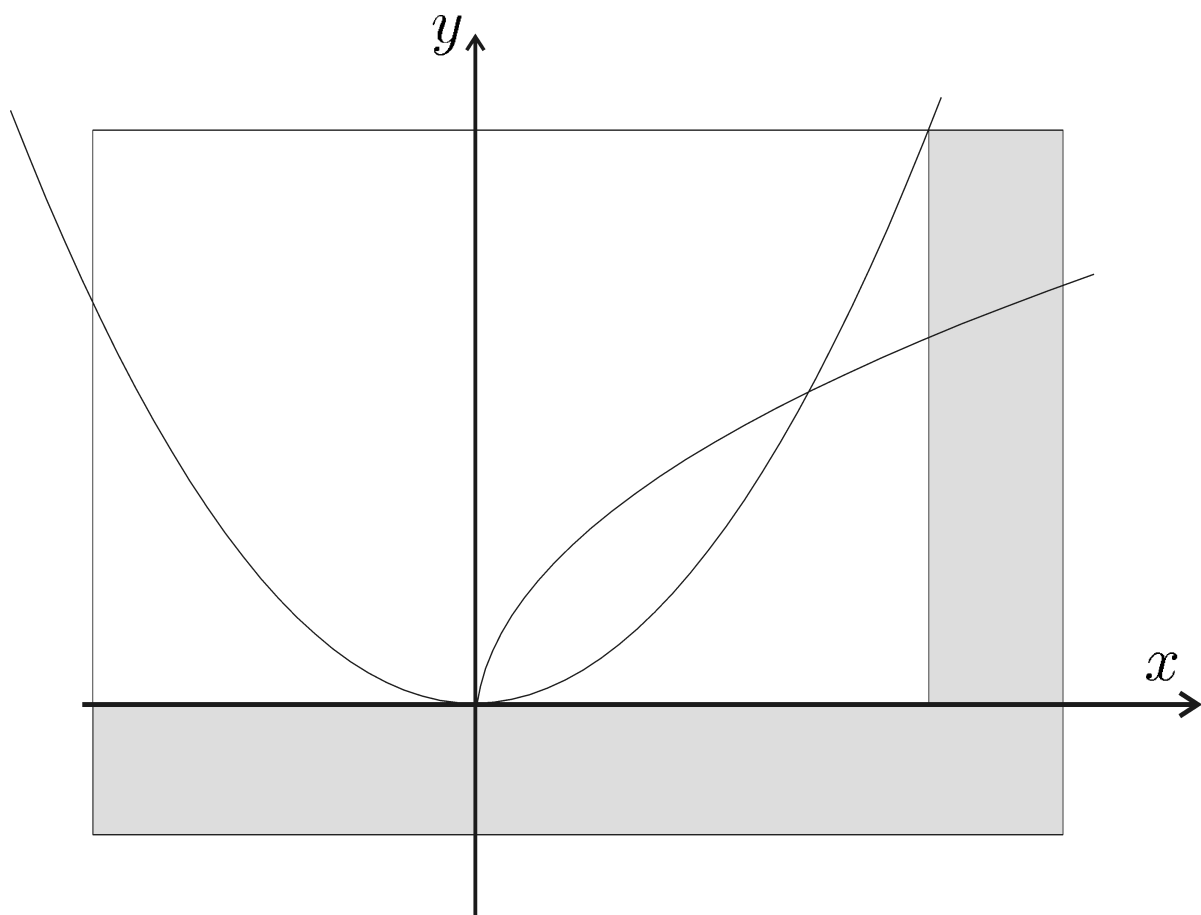
On a deux contracteurs

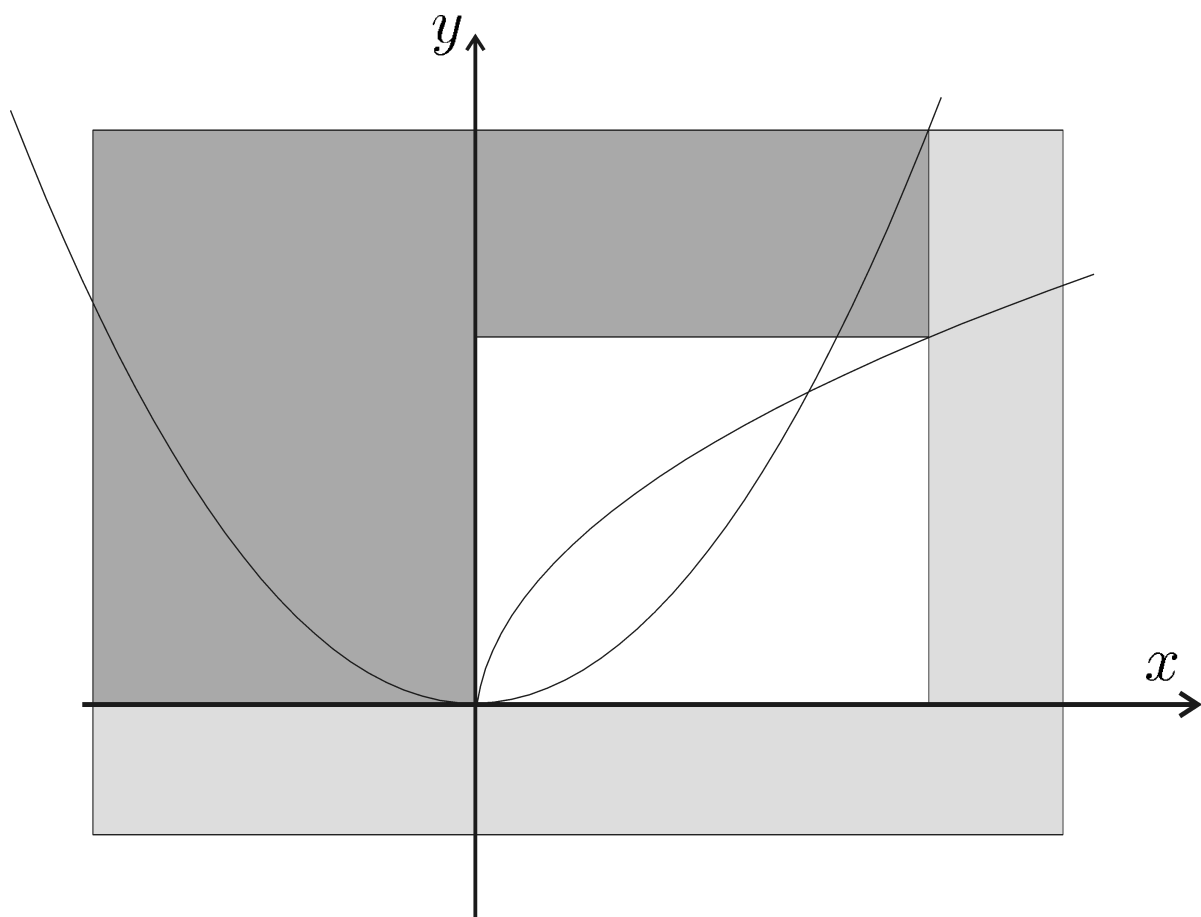
$$\mathcal{C}_1 : \begin{cases} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{cases} \quad \text{associé à } y = x^2$$

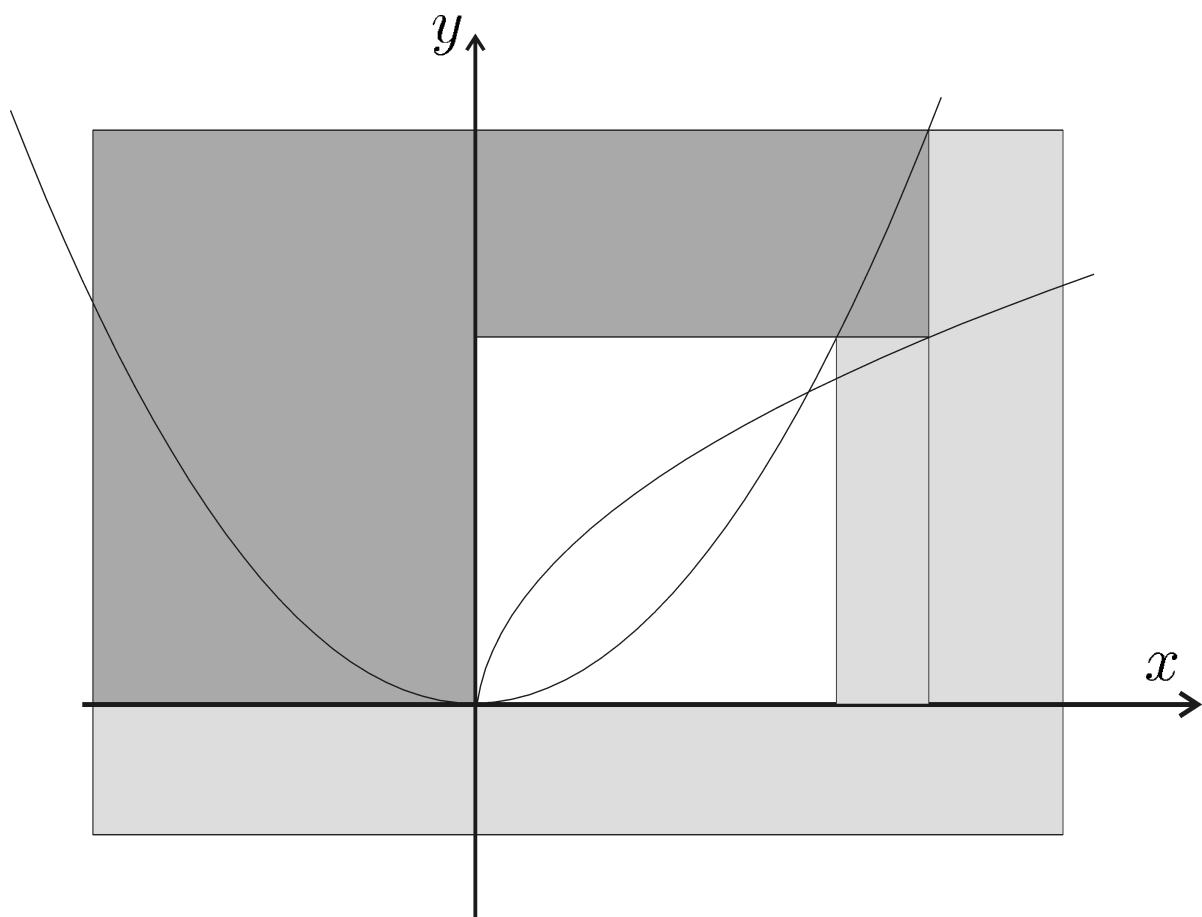
$$\mathcal{C}_2 : \begin{cases} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{cases} \quad \text{associé à } y = \sqrt{x}$$

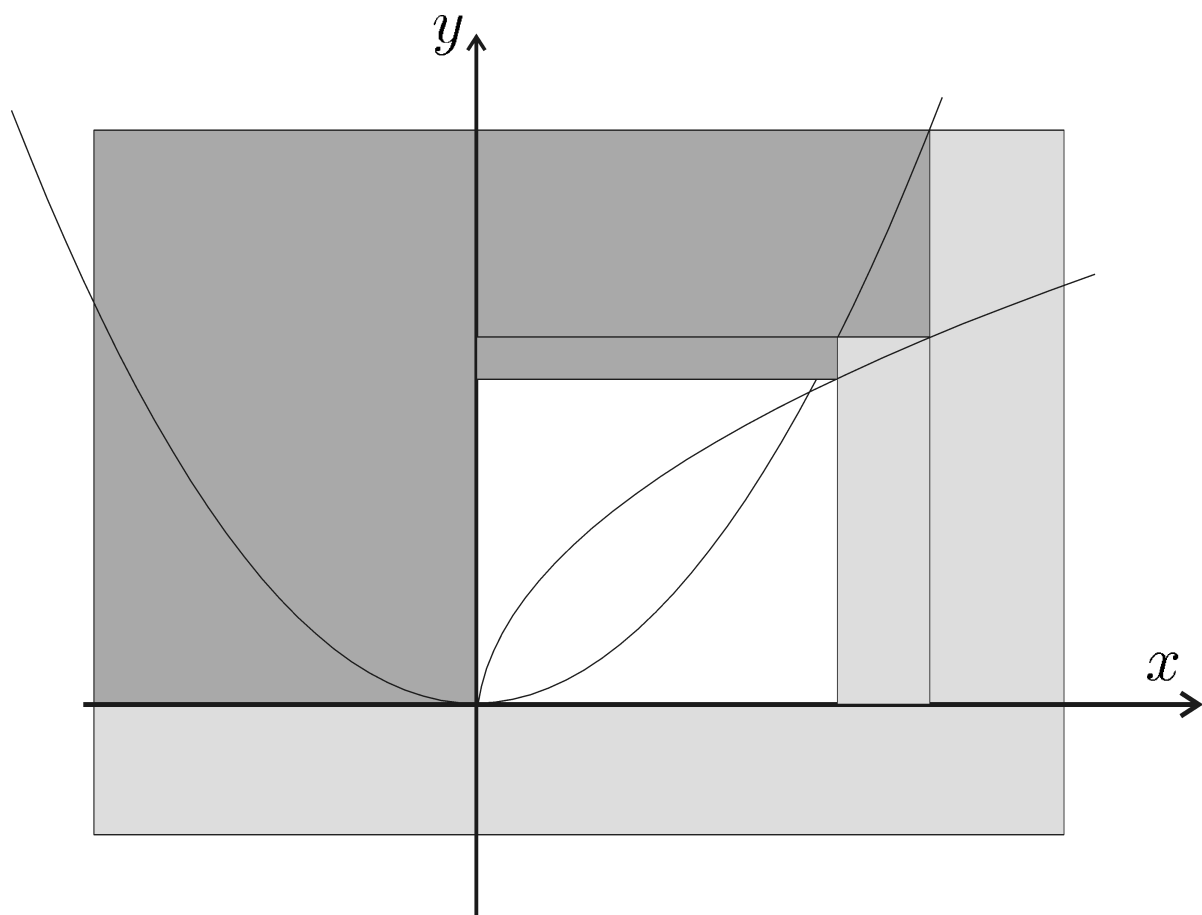


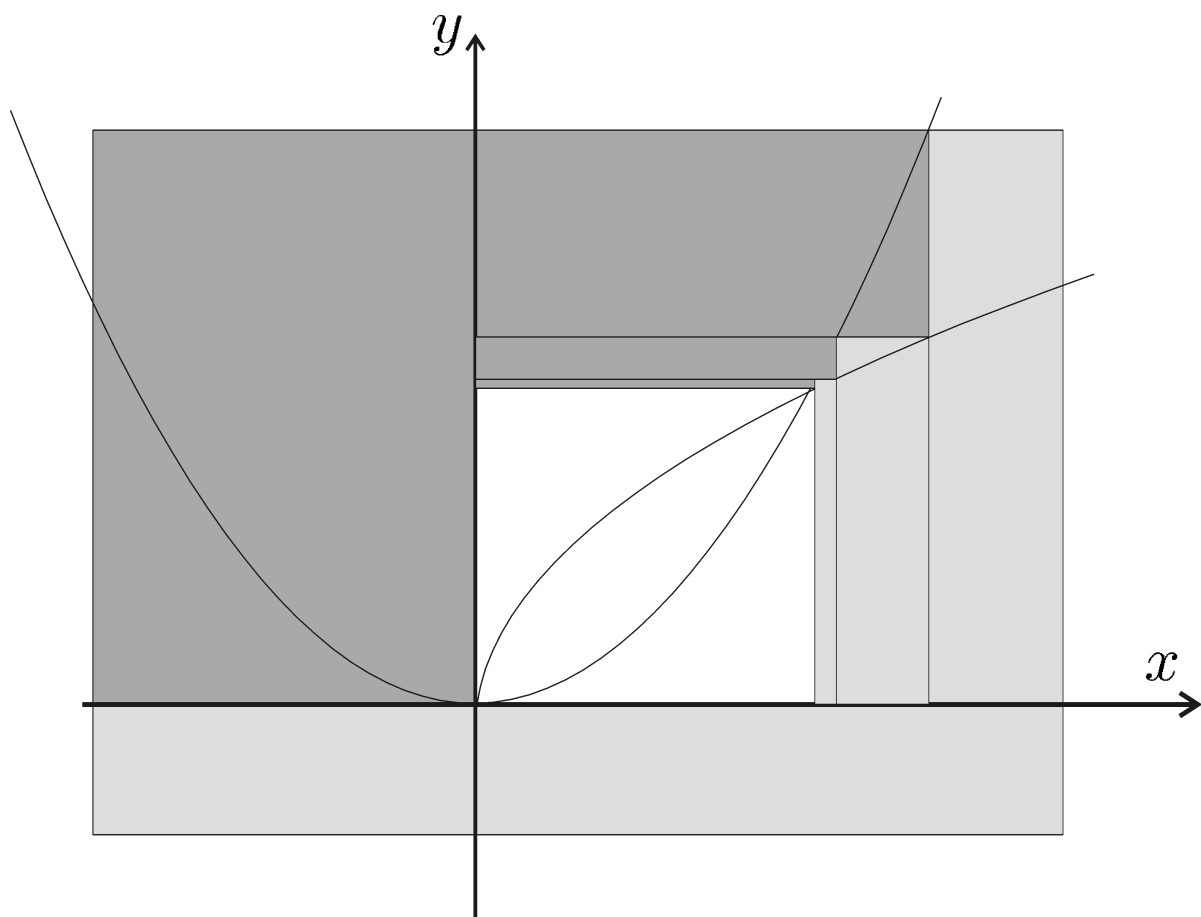


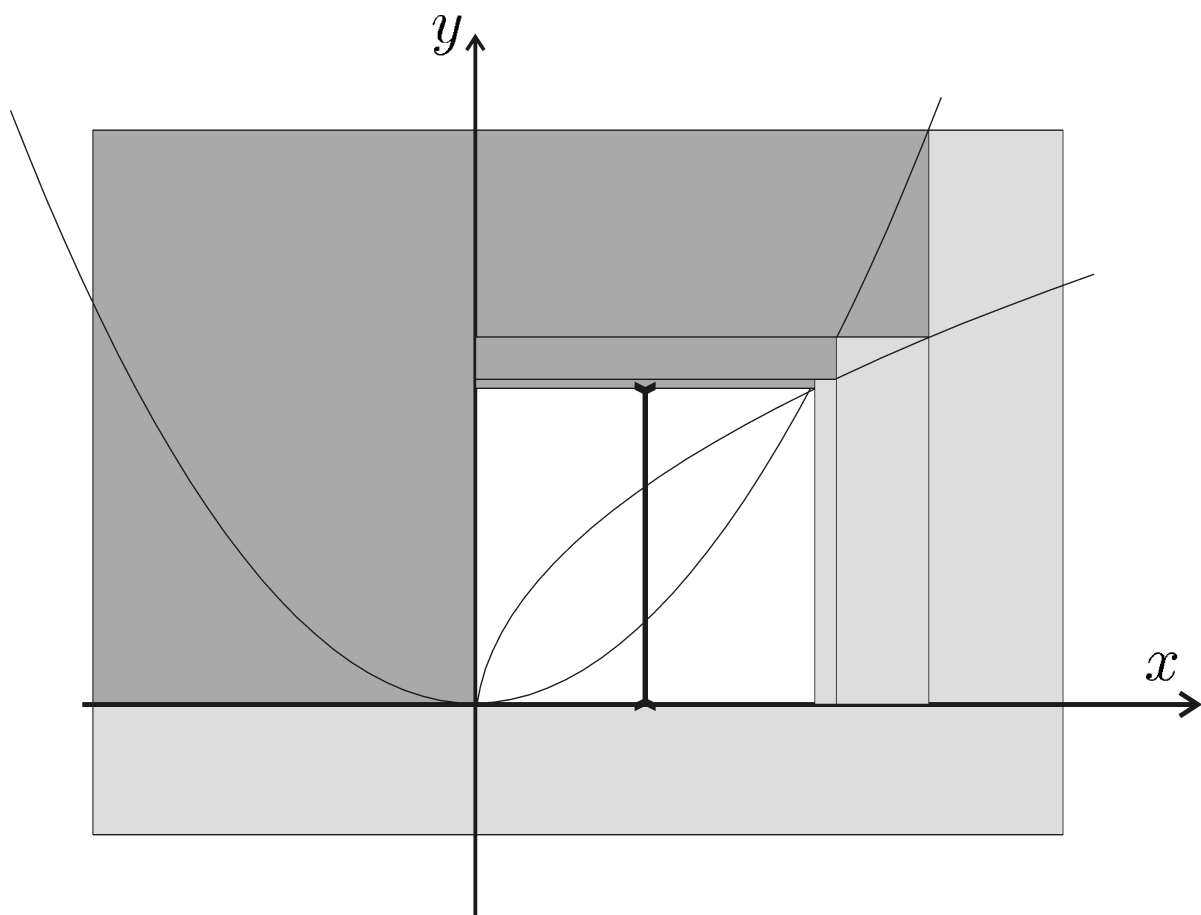


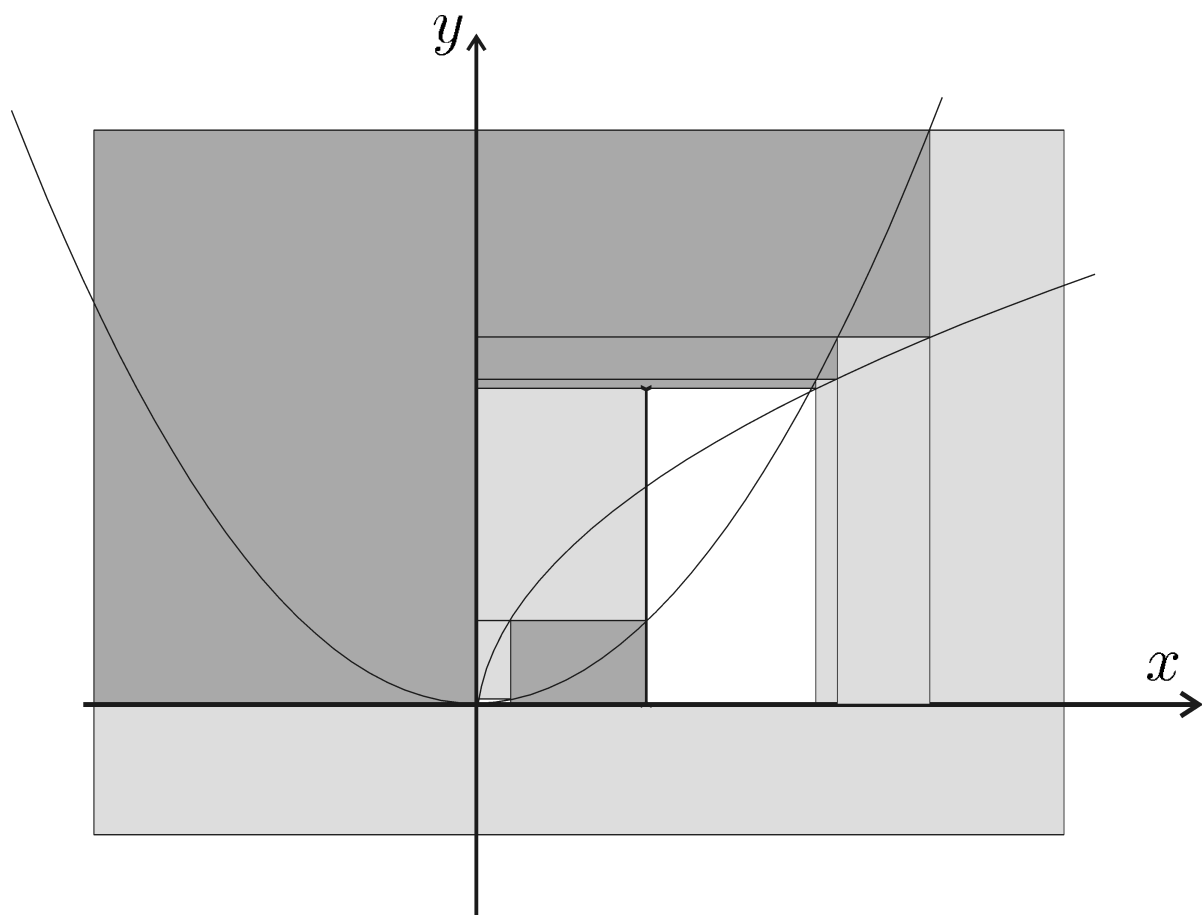




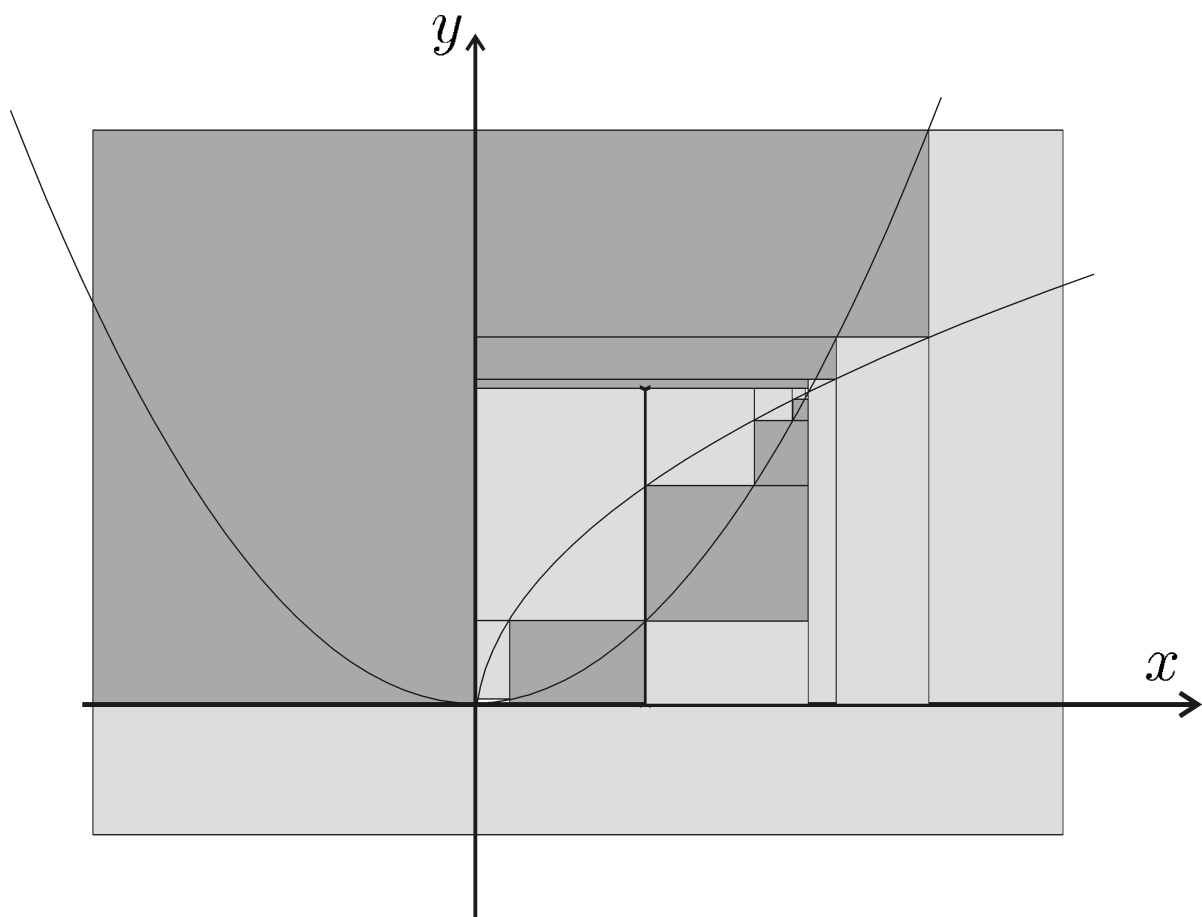












## 1.5 Décomposition

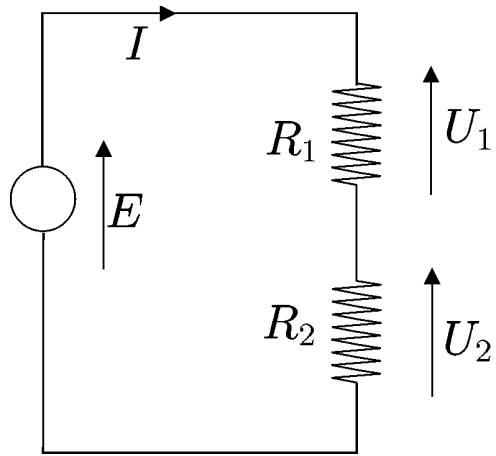
Pour les contraintes plus complexes, il nous faut effectuer une décomposition

$$\begin{aligned} x + \sin(y) - xz &\leq 0, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{aligned}$$

se décompose en

$$\left\{ \begin{array}{l} a = \sin(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array} \right., \quad \begin{array}{l} x \in [-1, 1] \\ y \in [-1, 1] \\ z \in [-1, 1] \end{array} \quad \begin{array}{l} a \in ]-\infty, \infty[ \\ b \in ]-\infty, \infty[ \\ c \in ]-\infty, \infty[ \\ d \in ]-\infty, 0] \end{array}$$

## 1.6 Un exemple académique



## Contraintes

$$\begin{aligned}P &= EI; E = (R_1 + R_2) I; \\U_1 &= R_1 I; U_2 = R_2 I; E = U_1 + U_2.\end{aligned}$$

## Domaines initiaux

$$\begin{aligned}R_1 &\in [0, \infty]\Omega, & R_2 &\in [0, \infty]\Omega, \\E &\in [23, 26]\text{V}, & I &\in [4, 8]\text{A}, \\U_1 &\in [10, 11]\text{V}, & U_2 &\in [14, 17]\text{V}, \\P &\in [124, 130]\text{W},\end{aligned}$$

La propagation retourne le pavé

$$\begin{aligned} & [24; 26] \times [1.846; 2.307] \times [2.584; 3.355] \\ & \times [4.769; 5.417] \times [10; 11] \times [14; 16] \times [124; 130] \end{aligned}$$

c'est-à-dire

$$\begin{aligned} E & \in [24; 26] , & R_1 & \in [1.846; 2.307] , \\ R_2 & \in [2.584; 3.355] , & I & \in [4.769; 5.417] , \\ U_1 & \in [10; 11] , & U_2 & \in [14; 16] , \\ P & \in [124; 130] . \end{aligned}$$

## 1.7 Contracteurs

L'opérateur  $\mathcal{C}_{\mathbb{X}} : \mathbb{R}^n \rightarrow \mathbb{R}^n$  est un *contracteur* pour l'ensemble  $\mathbb{X}$  de  $\mathbb{R}^n$  si

$$\forall [\mathbf{x}] \in \mathbb{R}^n, \begin{cases} \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance),} \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(complétude).} \end{cases}$$

Nous dirons que

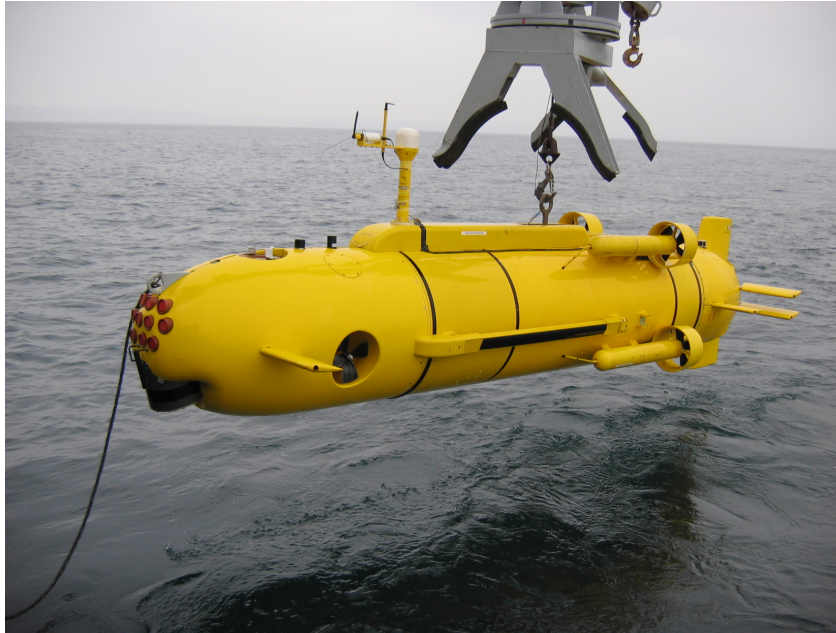
|                                                     |                                                                                                                                                       |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\mathcal{C}_{\mathbb{X}}$ est <i>monotone</i> si   | $[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset \mathcal{C}_{\mathbb{X}}([\mathbf{y}])$                 |
| $\mathcal{C}_{\mathbb{X}}$ est <i>minimal</i> si    | $\forall [\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathbb{X}]$                                     |
| $\mathcal{C}_{\mathbb{X}}$ est <i>thin</i> si       | $\forall \mathbf{x} \in \mathbb{R}^n, \mathcal{C}_{\mathbb{X}}(\{\mathbf{x}\}) = \{\mathbf{x}\} \cap \mathbb{X}$                                      |
| $\mathcal{C}_{\mathbb{X}}$ est <i>idempotent</i> si | $\forall [\mathbf{x}] \in \mathbb{IR}^n, \mathcal{C}_{\mathbb{X}}(\mathcal{C}_{\mathbb{X}}([\mathbf{x}])) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}])$ . |

$\mathcal{C}_{\mathbb{X}}$  est *convergent* si pour presque tout  $\mathbf{x}$ , et pour toute suite de pavé  $[\mathbf{x}](k)$ ,

$$[\mathbf{x}](k) \rightarrow \mathbf{x} \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}](k)) \rightarrow \{\mathbf{x}\} \cap \mathbb{X}.$$

## 2 SLAM





Le *Redermor*, fabriqué par le GESMA  
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Montrer la simulation

## **Pourquoi une approche par intervalles ?**

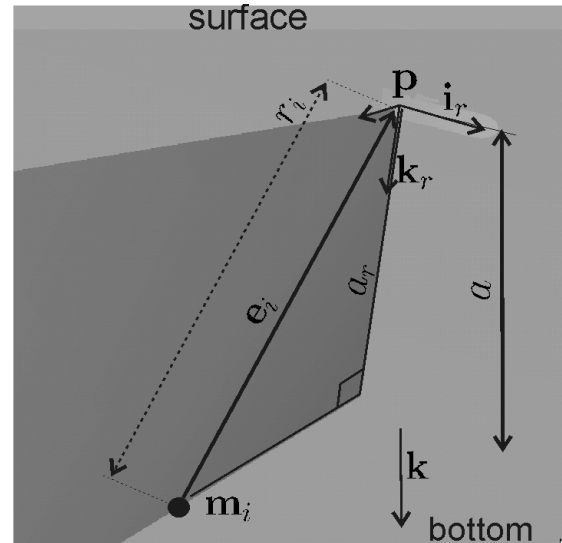
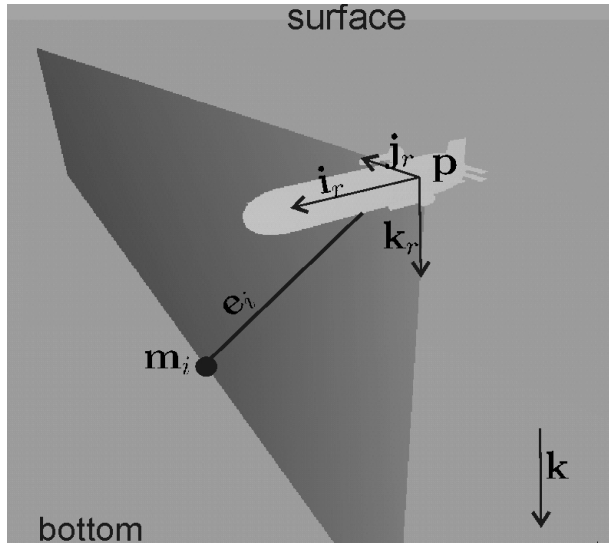
- 1) Besoin d'une approche fiable.
- 2) Les équations du robot sont non linéaires.
- 3) Les bruits de mesure sont non gaussiens.
- 4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
- 5) Beaucoup de mesures redondantes sont disponibles.

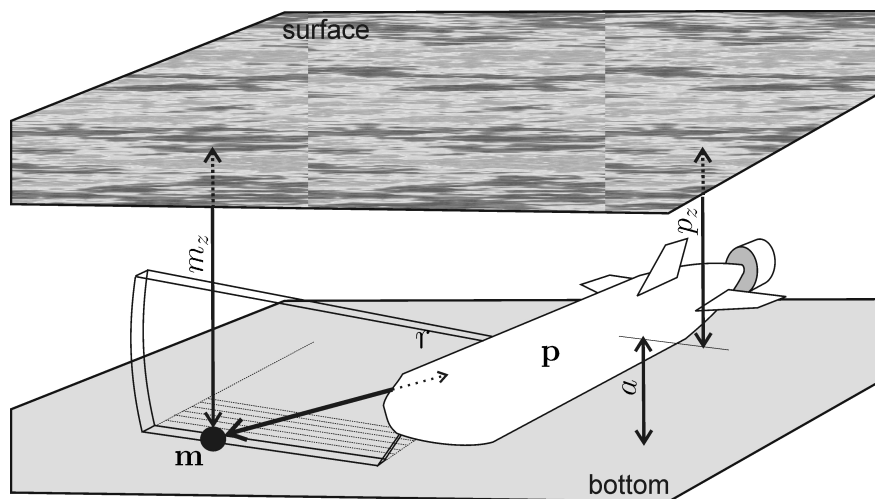
## 2.1 Capteurs

**Un GPS** (Global positioning system), disponible à la surface.

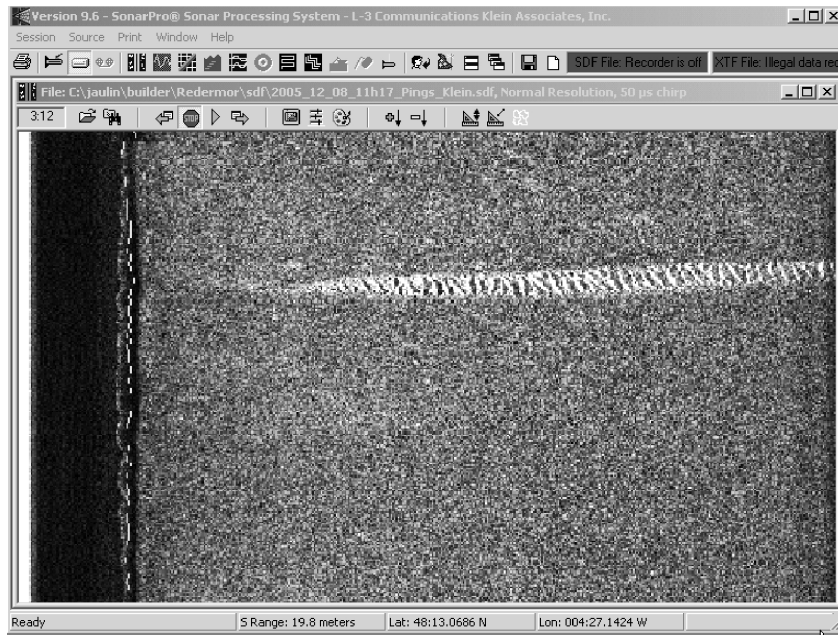
$$\begin{aligned} t_0 &= 6000 \text{ s}, & \ell^0 &= (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\ t_f &= 12000 \text{ s}, & \ell^f &= (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m \end{aligned}$$

**Un sonar** (KLEIN 5400 side scan sonar). Donne la distance  $r$  entre le robot et la mine

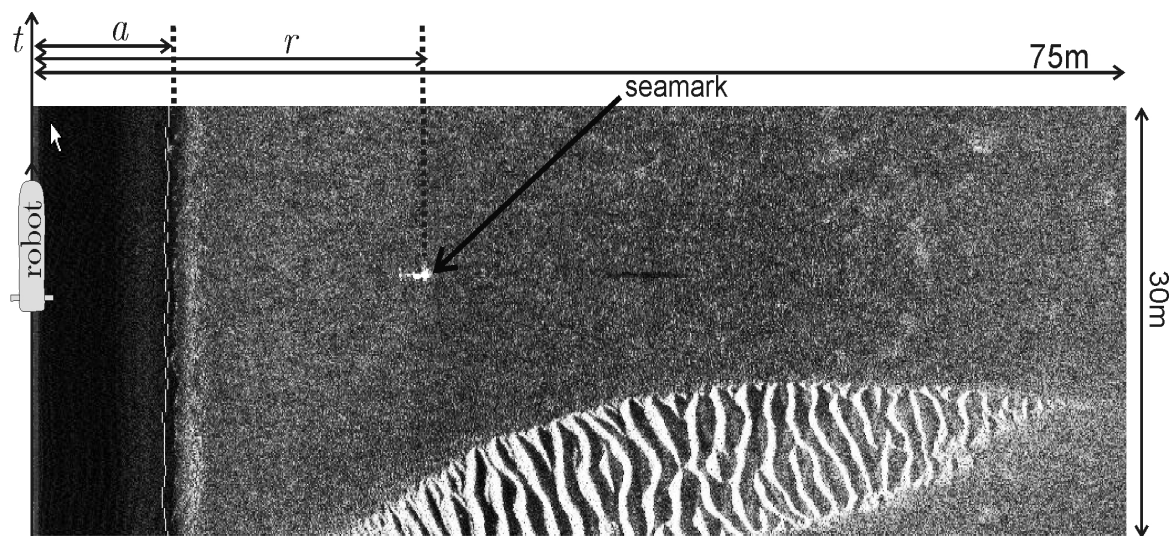








Screenshot du logiciel SonarPro



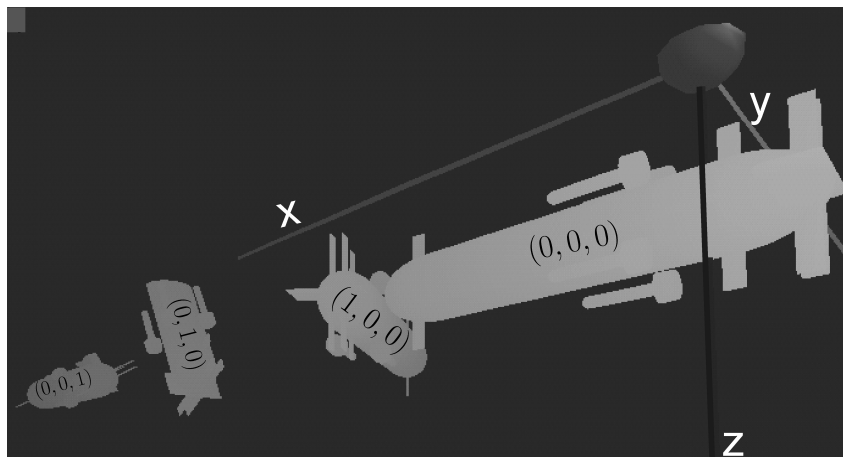
Détection d'une mine à l'aide de SonarPro

**Le Loch-Doppler** renvoie la vitesse du robot  $\mathbf{v}_r$  et son altitude  $a$ .

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] . \tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

**Une centrale inertielle** (Octans III from IXSEA) renvoie le roulis  $\phi$ , le tangage  $\theta$  et le cap  $\psi$  du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$



## 2.2 Données

Pour chaque  $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$ , nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

| $i$            | 0     | 1     | 2     | 3     | 4     | 5     |
|----------------|-------|-------|-------|-------|-------|-------|
| $\tau(i)$      | 7054  | 7092  | 7374  | 7748  | 9038  | 9688  |
| $\sigma(i)$    | 1     | 2     | 1     | 0     | 1     | 5     |
| $\tilde{r}(i)$ | 52.42 | 12.47 | 54.40 | 52.68 | 27.73 | 26.98 |

| 6     | 7     | 8     | 9     | 10    | 11    |
|-------|-------|-------|-------|-------|-------|
| 10024 | 10817 | 11172 | 11232 | 11279 | 11688 |
| 4     | 3     | 3     | 4     | 5     | 1     |
| 37.90 | 36.71 | 37.37 | 31.03 | 33.51 | 15.05 |

## 2.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t)=\left(\begin{array}{ccc}1&0&0\\0&\cos\varphi(t)&-\sin\varphi(t)\\0&\sin\varphi(t)&\cos\varphi(t)\end{array}\right),$$

$$\mathbf{R}(t)=\mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

$$\dot{\mathbf{p}}(t)=\mathbf{R}(t).\mathbf{v}_r(t),$$

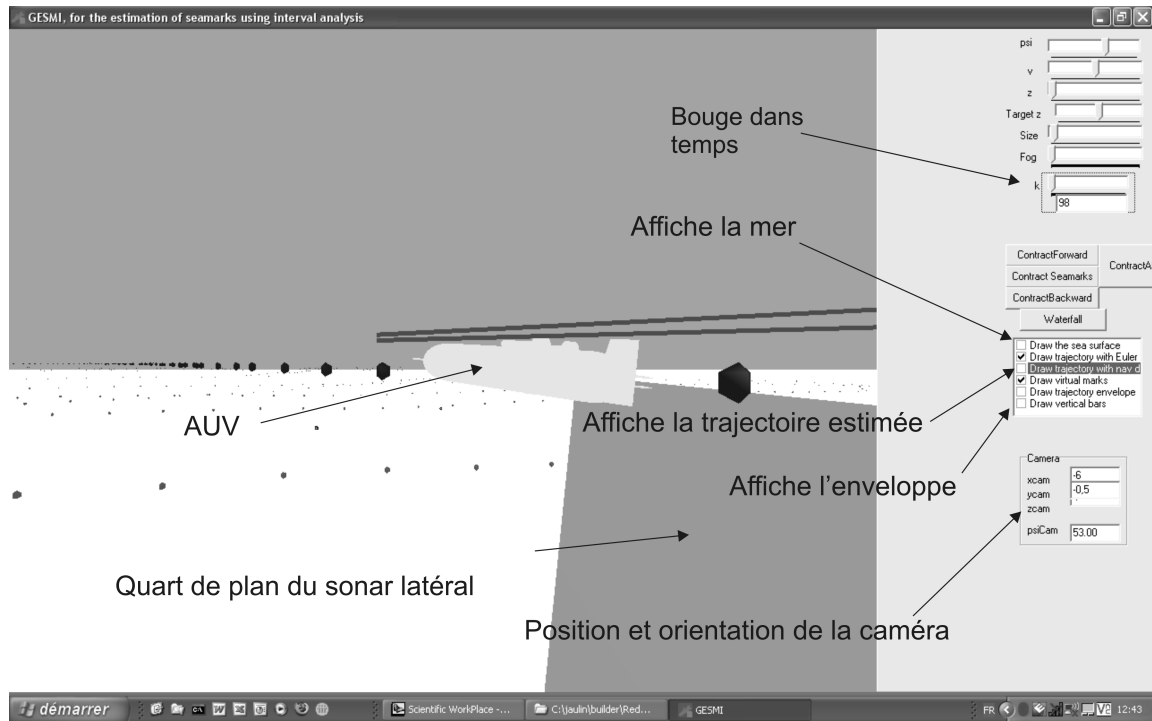
$$||\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))||\;=\;r(i),$$

$$\mathbf{R}^\top(\tau(i))\left(\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))\right)\in [0]\times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i))-p_z(\tau(i))-a(\tau(i))\in [-0.5,0.5]$$



## 2.4 Logiciel GESMI

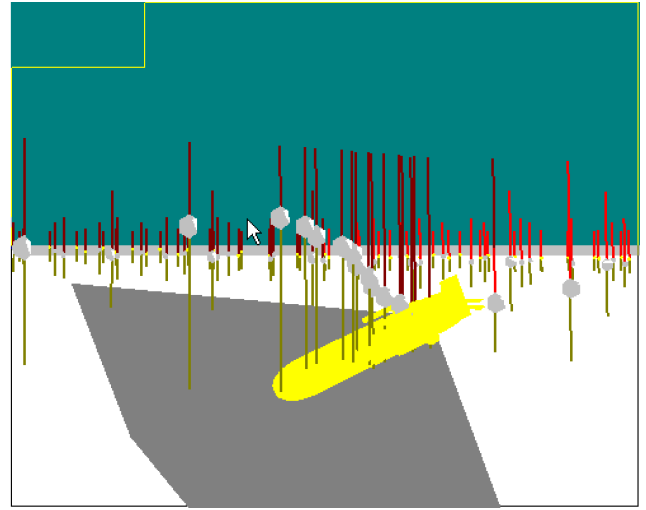
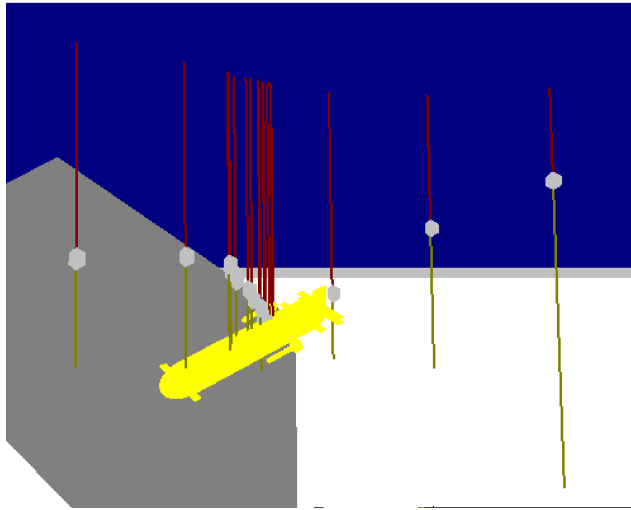


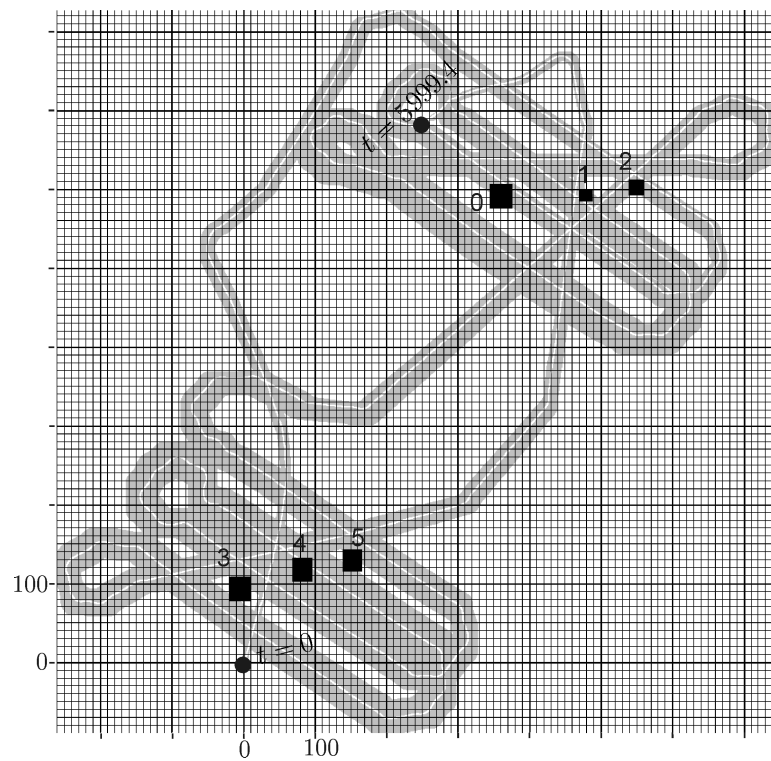
```

% This file has been generated by a generator
% and will be used by GESMI to solve a SLAM problem
% Note : every line starting by a '%' is considered as a comment by GESMI.
% -----
%
% This file contains
% 1) Some basic information about the scope of the sonar and the sampling time
% 2) The prior domains for the seamarks
% 3) The coordinates for some virtual marks
% 4) sensor data (angles, speeds, depth, altitude, position) with bounds
% 5) ping table : at the end of this file
%
% -----
% maximal distance of the lateral sonar and error interval [min,max]
75.0    1.0
% Sampling time
0.1
% -----
% Domain for seamarks detected using the software SonarPro
% -----
% Initial domains for the mines
% xmin    xmax    ymin    ymax    zmin    zmax
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
-10000    10000    -10000    10000    0        100
$
% -----
% Virtual marks (only for graphism, not used for computation)
% -----
% x        y        z        colorRed    ColorGreen    ColorBlue    LocalFrame
594.0533723    374.7293035    19    0.1    0    0    % seamark 0
599.6093723    484.5528643    19    0.1    0.1    0    % seamark 1
601.4613723    557.3573086    19    0.1    0    0.1    % seamark 2
94.01337232    -2.868189501    19    0    0.1    0    % seamark 3
119.9413723    84.75155672    20    0    0    0.1    % seamark 4
127.3493723    156.3280366    20    0    0    0    % seamark 5
0        0    0    1    0    0    1    % origin of the local frame
689.96    264.10    0    0    0    1    1    % final GPS
$
% -----
%
%t        phi        dphi        theta        dtheta        psi        dpsi        vx        dvx
0    -0.011505    0.0001745329252    0.033556    0.0001745329252    0.415613    0.0052679
0.1    -0.012272    0.0001745329252    0.034994    0.0001745329252    0.416284    0.00526
0.2    -0.012847    0.0001745329252    0.037583    0.0001745329252    0.417051    0.00526
0.3    -0.013422    0.0001745329252    0.040938    0.0001745329252    0.418202    0.00526
0.4    -0.014093    0.0001745329252    0.044773    0.0001745329252    0.419831    0.00526
0.5    -0.014285    0.0001745329252    0.0488    0.0001745329252    0.422228    0.0052679
0.6    -0.013998    0.0001745329252    0.052539    0.0001745329252    0.425488    0.00526
0.7    -0.012943    0.0001745329252    0.055703    0.0001745329252    0.42961    0.005267
0.8    -0.011121    0.0001745329252    0.0581    0.0001745329252    0.434308    0.0052679
0.9    -0.008725    0.0001745329252    0.059825    0.0001745329252    0.439773    0.00526
1    -0.005944    0.0001745329252    0.060496    0.0001745329252    0.445334    0.0052679
1.1    -0.003547    0.0001745329252    0.0604    0.0001745329252    0.450511    0.0052679
1.2    -0.001917    0.0001745329252    0.059729    0.0001745329252    0.455017    0.00526
1.3    -0.001438    0.0001745329252    0.058867    0.0001745329252    0.458564    0.00526
1.4    -0.002013    0.0001745329252    0.058387    0.0001745329252    0.461057    0.00526
1.5    -0.003356    0.0001745329252    0.058483    0.0001745329252    0.462303    0.00526
1.6    -0.005561    0.0001745329252    0.059346    0.0001745329252    0.462591    0.00526
1.7    -0.007958    0.0001745329252    0.060976    0.0001745329252    0.462112    0.00526

```







### **3 Inversion ensembliste**

## **3.1 Fonction d'inclusion**

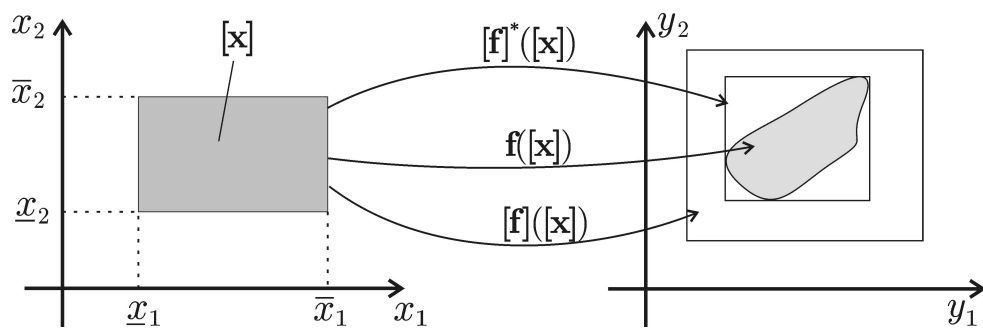
Soit  $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ . La fonction  $[\mathbf{f}] : \mathbb{IR}^n \rightarrow \mathbb{IR}^m$ , est une *fonction d'inclusion* de  $\mathbf{f}$  si

$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \quad \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}]).$$

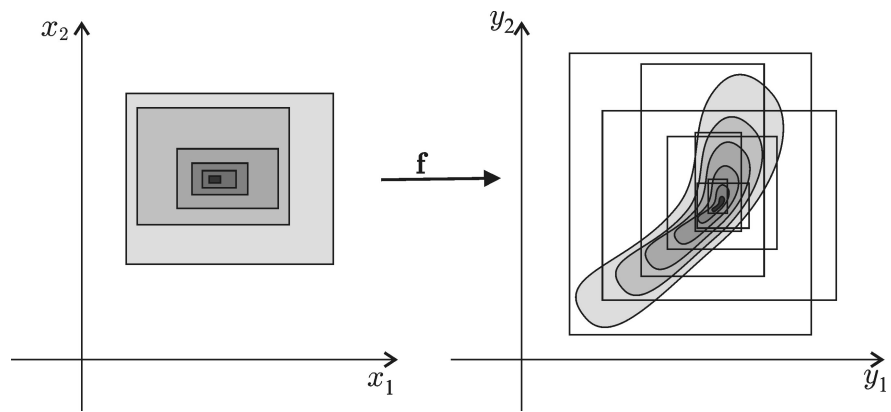
On dira que  $[\mathbf{f}]$  est

|                    |    |                                                                                                                   |
|--------------------|----|-------------------------------------------------------------------------------------------------------------------|
| <i>monotone</i>    | si | $([\mathbf{x}] \subset [\mathbf{y}]) \Rightarrow ([\mathbf{f}]([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{y}]))$ |
| <i>minimale</i>    | si | $\forall [\mathbf{x}] \in \mathbb{IR}^n, [\mathbf{f}]([\mathbf{x}]) = [\mathbf{f}([\mathbf{x}])]$                 |
| <i>convergente</i> | si | $w([\mathbf{x}](k)) \rightarrow 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}](k)) \rightarrow 0$                      |

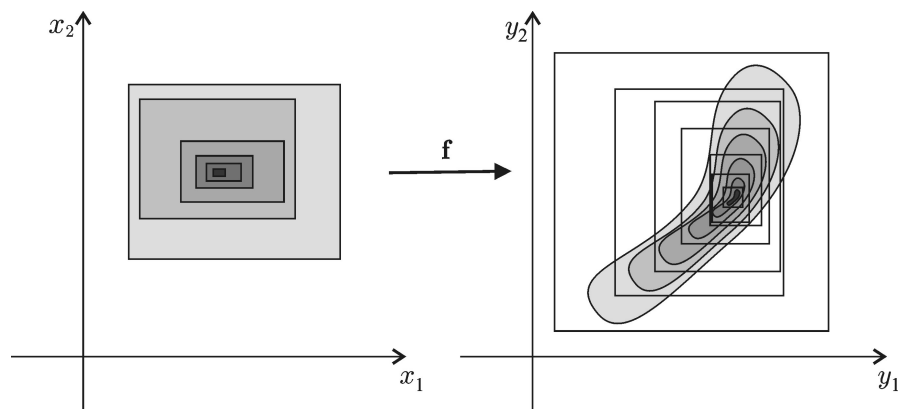




Fonctions d'inclusion  $[f]$  et  $[f]^*$ . Ici,  $[f]^*$  est minimale.



Fonction d'inclusion convergente mais pas monotone



Fonction d'inclusion convergente et monotone

## 3.2 Fonction d'inclusion naturelle

Il suffit de remplacer dans l'expression de  $f$ , tous les  $x_i$ 's par  $[x_i]$ .

**Exemple 1.** Si  $f(x) = x^2 + 2x + 4$  alors, sa fonction d'inclusion naturelle est

$$[f]([x]) = [x]^2 + 2[x] + 4.$$

Pour  $[x] = [-3, 4]$ , nous avons

$$\begin{aligned}[f]([-3, 4]) &= [-3, 4]^2 + 2[-3, 4] + 4 \\ &= [0, 16] + [-6, 8] + 4 = [-2, 28].\end{aligned}$$

Notons que

$$f([-3, 4]) = [3, 28] \subset [f]([-3, 4]) = [-2, 28].$$

**Exemple 2.** Soit

$$\begin{aligned} \mathbf{f} : \mathbb{R}^2 &\rightarrow \mathbb{R}^3 \\ (x_1, x_2) &\mapsto (x_1x_2, x_1^2, x_1 - x_2). \end{aligned}$$

Sa fonction d'inclusion naturelle  $[f]$  est

$$\begin{array}{ccc} \mathbb{R}^2 & \longrightarrow & \mathbb{R}^3 \\ ([x_1], [x_2]) & \longrightarrow & ([x_1] * [x_2], [x_1]^2, [x_1] - [x_2]) . \end{array}$$

Puisque dans les  $f_i$ , les  $x_i$  n'apparaissent qu'une seule fois, la fonction d'inclusion est minimale.



**Exemple 3** : Si  $f$  est donnée par

|                                                                                                 |
|-------------------------------------------------------------------------------------------------|
| <b>Algo</b> $f(\text{in: } \mathbf{x} = (x_1, x_2, x_3), \text{out: } \mathbf{y} = (y_1, y_2))$ |
|-------------------------------------------------------------------------------------------------|

|                          |
|--------------------------|
| 1 $z := x_1;$            |
| 2    for $k := 0$ to 100 |
| 3 $z := x_2(z + kx_3);$  |
| 4    next;               |
| 5 $y_1 := z;$            |
| 6 $y_2 := \sin(zx_1);$   |

Quelle est sa fonction d'inclusion naturelle ?

Sa fonction d'inclusion naturelle est

| <b>Algo</b> $[f](\text{in: } [x] = ([x_1], [x_2], [x_3]), \text{out: } [y] = ([y_1], [y_2]))$ |                                     |
|-----------------------------------------------------------------------------------------------|-------------------------------------|
| 1                                                                                             | $[z] := [x_1];$                     |
| 2                                                                                             | for $k := 0$ to 100                 |
| 3                                                                                             | $[z] := [x_2] * ([z] + k * [x_3]);$ |
| 4                                                                                             | next;                               |
| 5                                                                                             | $[y_1] := [z];$                     |
| 6                                                                                             | $[y_2] := \sin([z] * [x_1]);$       |

Ici,  $[f]$  est convergente et monotone.

### 3.3 Sous-pavages

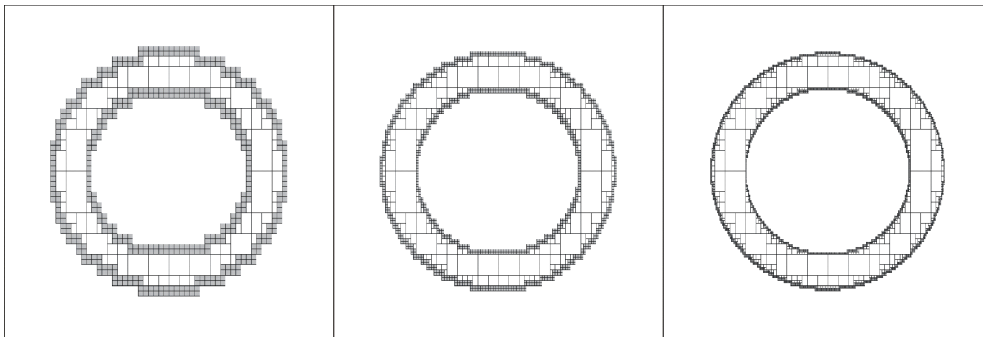
Les ensembles  $\mathbb{X}$  peuvent être encadrés par deux sous-pavages  $\mathbb{X}^-$  et  $\mathbb{X}^+$  :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Par exemple, l'ensemble

$$\mathbb{X} = \{(x_1, x_2) \mid x_1^2 + x_2^2 \in [1, 2]\}$$

peut être encadré comme ci-dessous



## 3.4 Algorithme

Il nous faut encadrer l'ensemble

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}),$$

où  $\mathbb{Y} \subset \mathbb{R}^m$ . et  $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ , par deux sous-pavages

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Tests pour montrer si  $[\mathbf{x}]$  à l'intérieur ou à l'extérieur de  $\mathbb{X}$  :

- (i)  $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y} \Rightarrow [\mathbf{x}] \subset \mathbb{X}$
- (ii)  $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset \Rightarrow [\mathbf{x}] \cap \mathbb{X} = \emptyset.$

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Algo</b> Sivia(in: $[x]$ ; out: $\mathcal{L}^-, \mathcal{L}^+$ )                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 1 $\mathcal{L} := \{[x]\}; \mathcal{L}^- = \emptyset; \mathcal{L}^+ := \emptyset;$<br>2    if $\mathcal{L} \neq \emptyset$ , $[x] := \text{pop}(\mathcal{L})$ , else end;<br>3    if $[f]([x]) \subset \mathbb{Y}$ , $\text{stack}(\mathcal{L}^-, \mathcal{L}^+, [x]);$ goto 2;<br>4    if $[f]([x]) \cap \mathbb{Y} = \emptyset$ , goto 2;<br>5    if $w([x]) < \varepsilon$ , $\text{stack}(\mathcal{L}^+, [x]);$ goto 2;<br>6 $\text{stack}(\mathcal{L}, \text{left}([x]), \text{right}([x]));$ goto 2. |

## 3.5 Estimation de paramètres

Modèle :  $\phi(\mathbf{p}, t) = p_1 e^{-p_2 t}$ ,

Paramètres :  $p_1, p_2$ ,

Pavé initial pour les paramètres :  $[\mathbf{p}] \subset \mathbb{R}^2$ ,

Temps de mesure :  $t_1, t_2, \dots, t_m$ ,

Barres des données :  $[y_1^-, y_1^+], [y_2^-, y_2^+], \dots, [y_m^-, y_m^+]$ ,

Ensemble de vraisemblance :

$$\mathbb{S} = \{\mathbf{p} \in [\mathbf{p}], \phi(\mathbf{p}, t_1) \in [y_1^-, y_1^+], \dots, \phi(\mathbf{p}, t_m) \in [y_m^-, y_m^+]\}.$$



Posons

$$\phi(\mathbf{p}) = \begin{pmatrix} \phi(\mathbf{p}, t_1) \\ \phi(\mathbf{p}, t_m) \end{pmatrix}$$

et

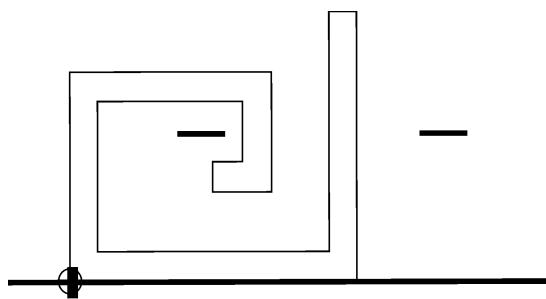
$$[\mathbf{y}] = [y_1^-, y_1^+] \times \cdots \times [y_m^-, y_m^+]$$

alors

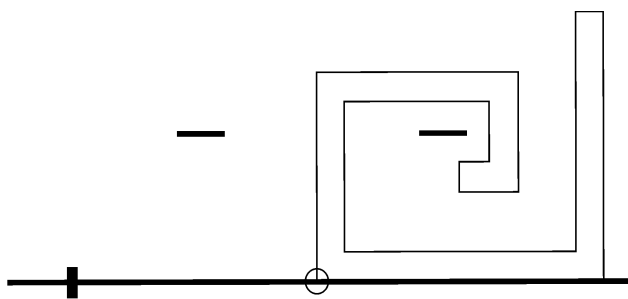
$$\mathbb{S} = [\mathbf{p}] \cap \phi^{-1}([\mathbf{y}]).$$

*Lancer Setdemo (fait par Guillaume Baffet)*

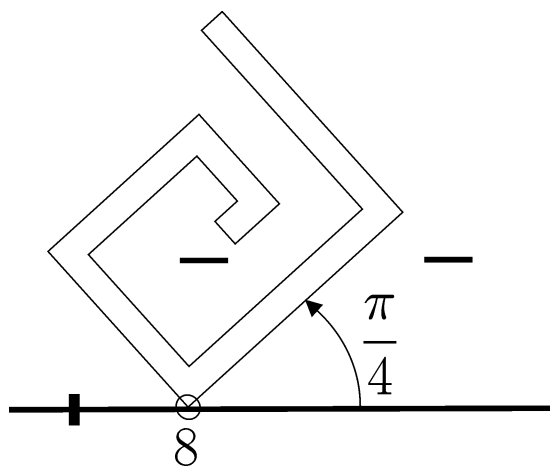
## **3.6 Planification de chemin**



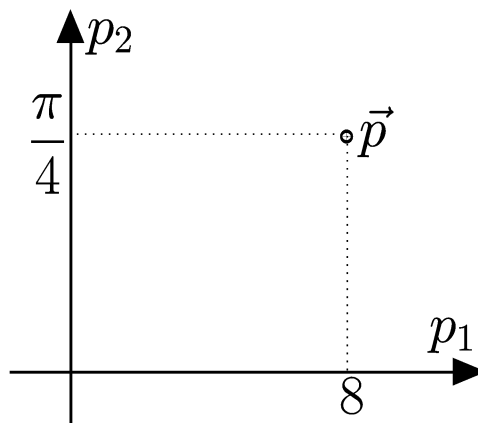
Initial configuration:  $\vec{p} = (0 \ 0)^T$



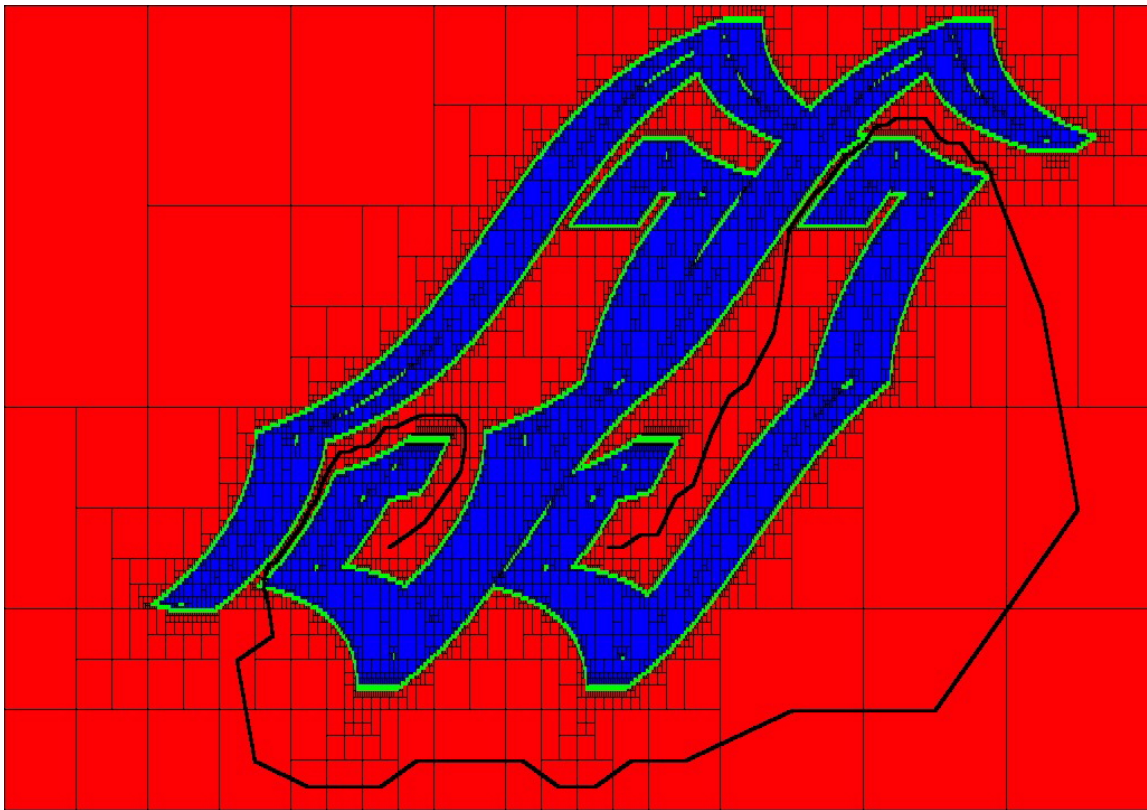
Goal configuration:  $\vec{p} = (17 \ 0)^T$

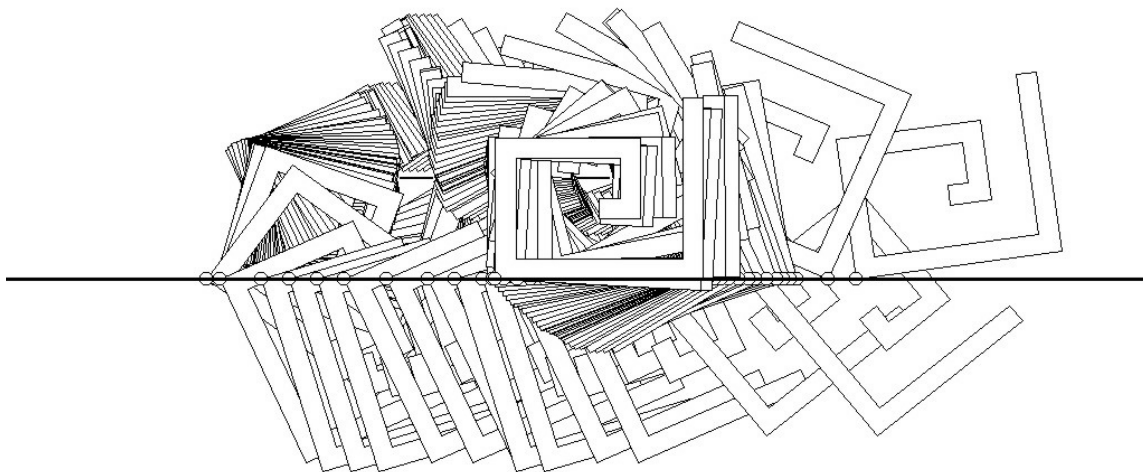


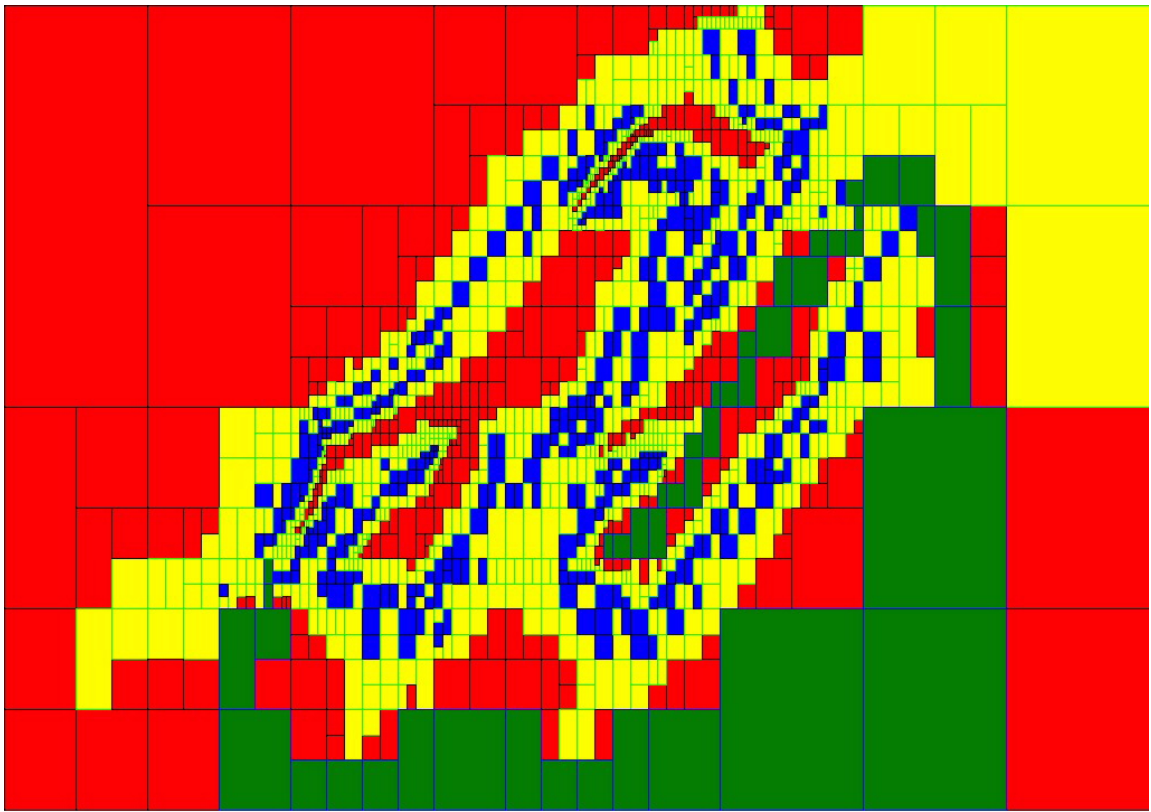
Room



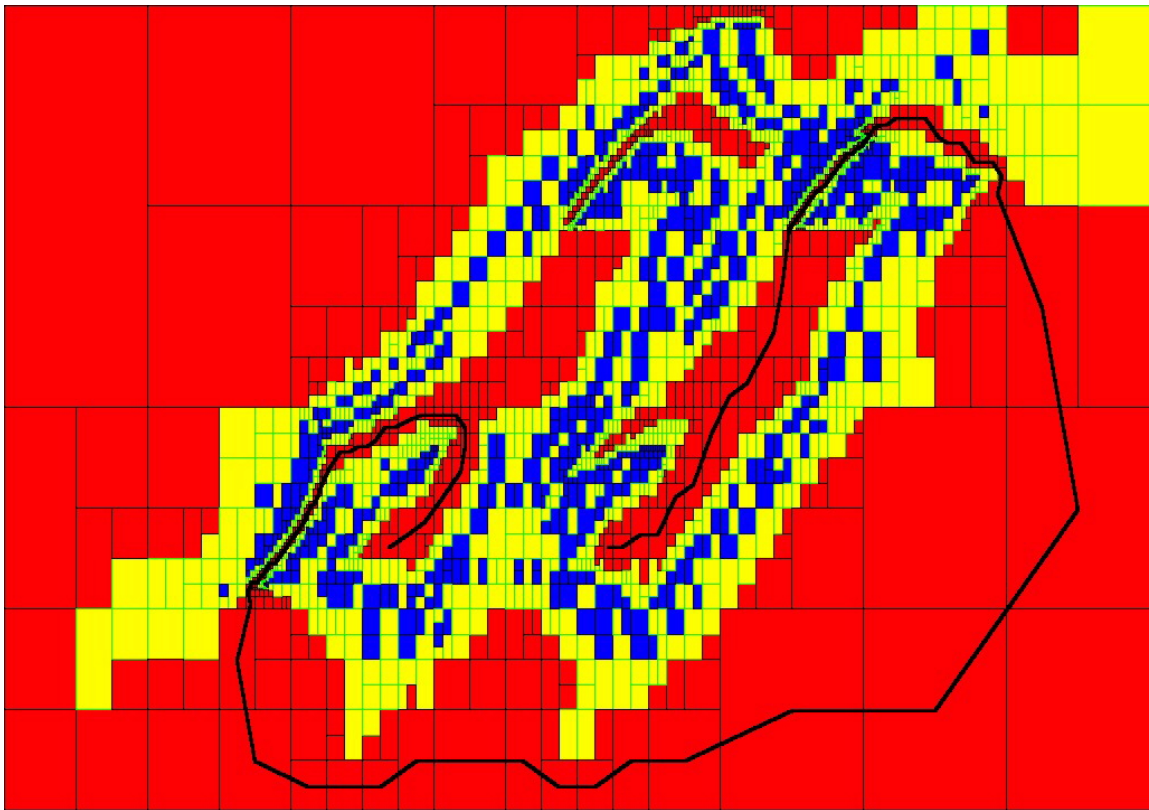
Configuration space











## **3.7 Commande d'un voilier**

$$\left\{ \begin{array}{lcl} \dot{x} & = & v \cos \theta \\ \dot{y} & = & v \sin \theta - \beta V \\ \dot{\theta} & = & \omega \\ \dot{\delta}_s & = & u_1 \\ \dot{\delta}_r & = & u_2 \\ \dot{v} & = & \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ \dot{\omega} & = & \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s & = & \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r & = & \alpha_r v \sin \delta_r. \end{array} \right.$$

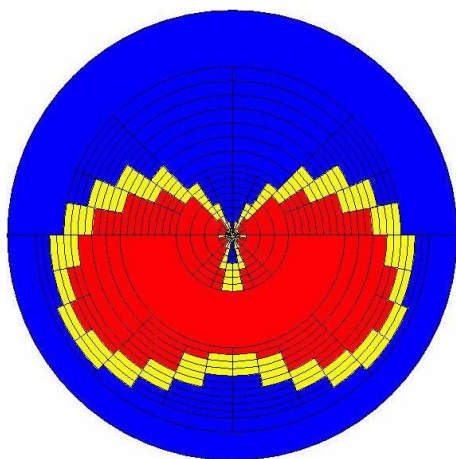
Est-il possible de l'immobiliser ?

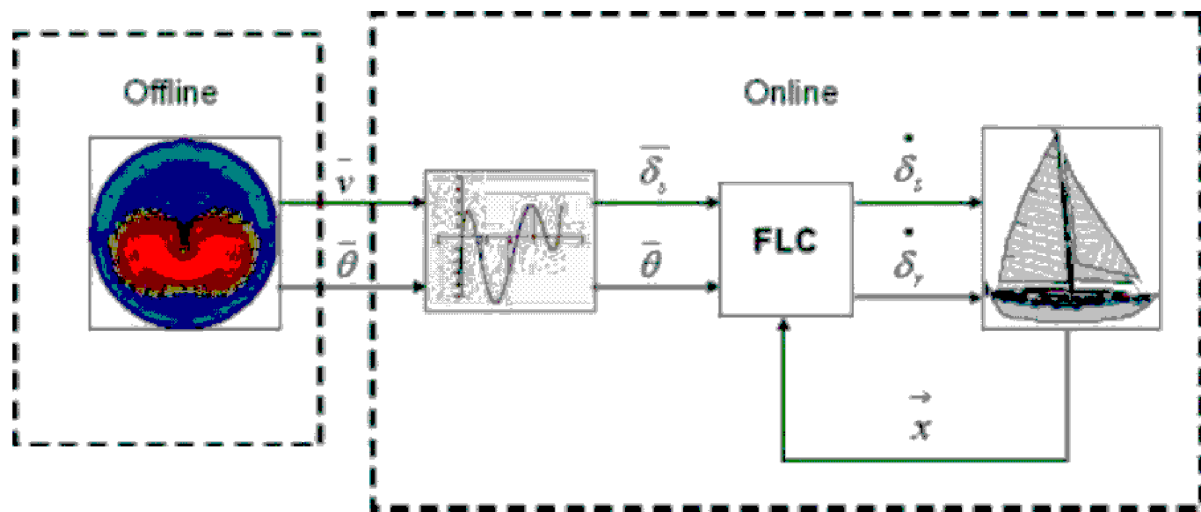
$$\left\{ \begin{array}{lcl} 0 & = & v \cos \theta \\ 0 & = & v \sin \theta - \beta V \\ 0 & = & \omega \\ 0 & = & u_1 \\ 0 & = & u_2 \\ 0 & = & \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ 0 & = & \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s & = & \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r & = & \alpha_r v \sin \delta_r. \end{array} \right.$$

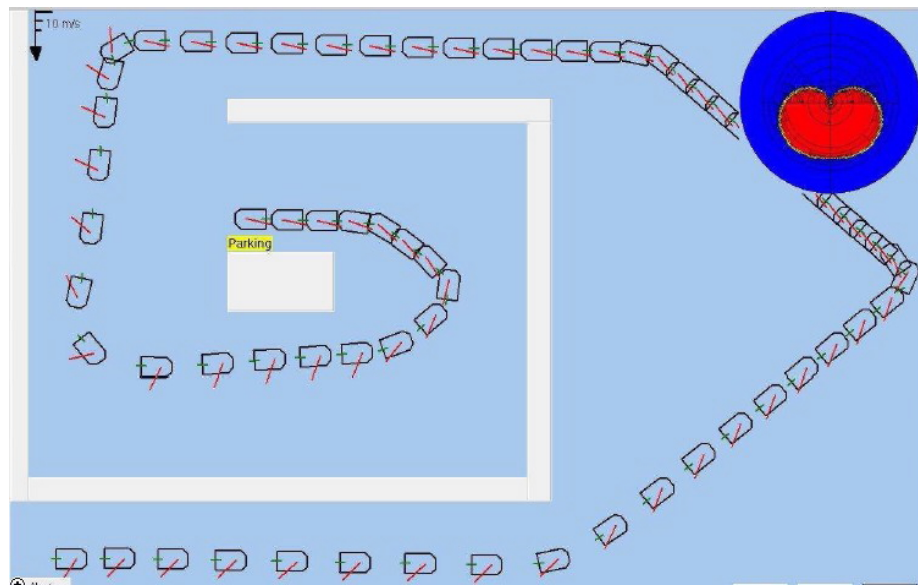
Les techniques intervalles montrent qu'aucune solution existe si  $V \neq 0$ .

## Polaire des vitesses

$$\mathbb{W} = \{ (\theta, v) \mid \begin{array}{l} \exists (\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\ \omega = 0, u_1 = 0, u_2 = 0 \\ \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{(\ell - r_s \cos \delta_s)^m} = 0 \\ \frac{f_s - r_r \cos \delta_r f_r}{J} = 0 \\ f_s = \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r = \alpha_r v \sin \delta_r \end{array} \}.$$









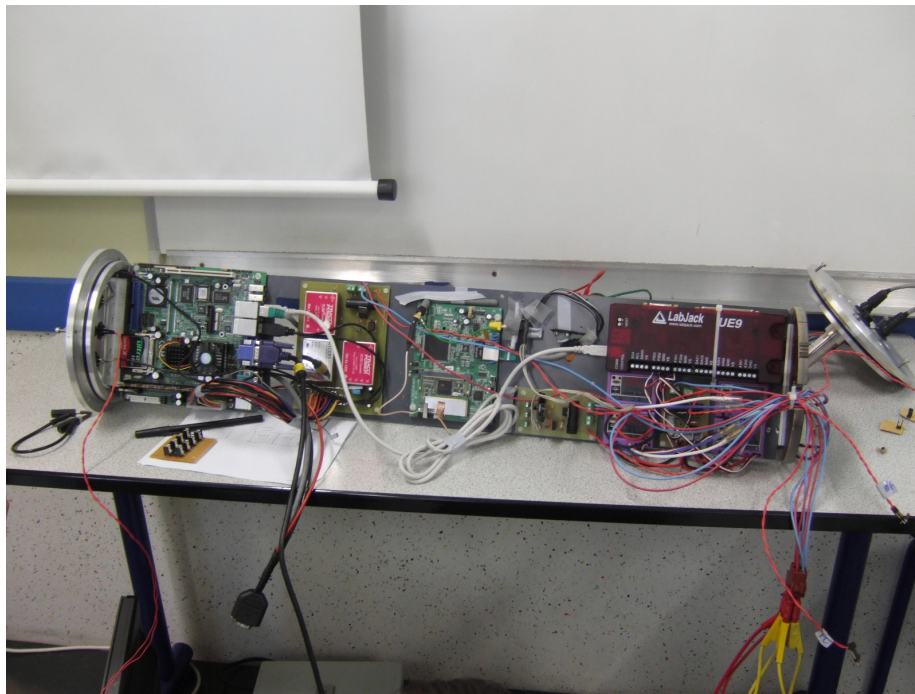
## 4 Robustesse

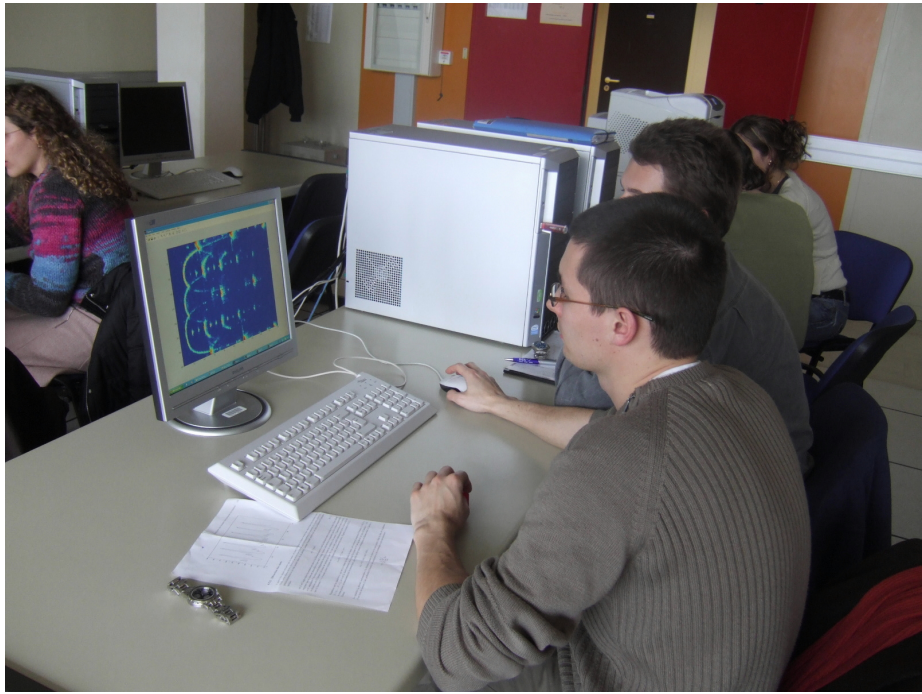
*Montrer le logiciel de démo de Jan Sliwka*



Portsmouth, July 12-15, 2007.





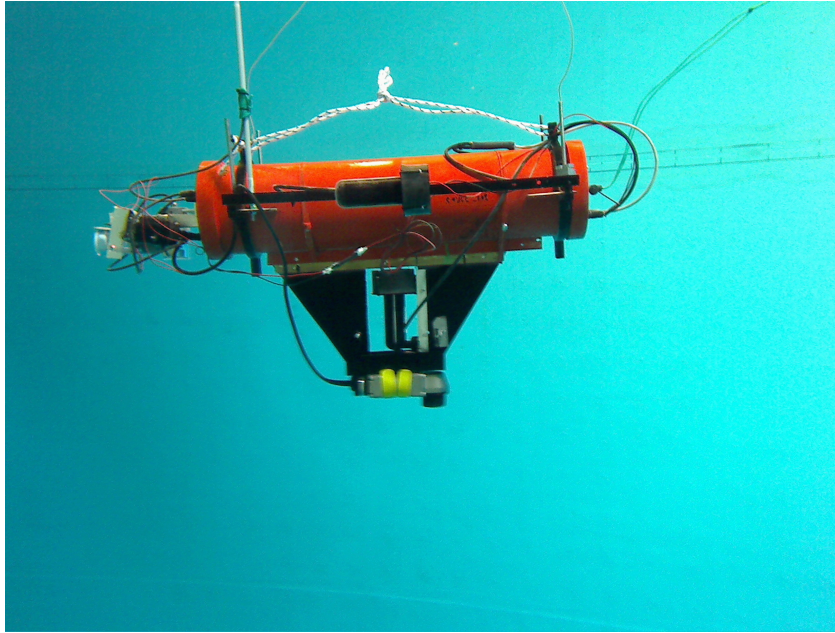












Robot Sauc'isse dans une piscine

