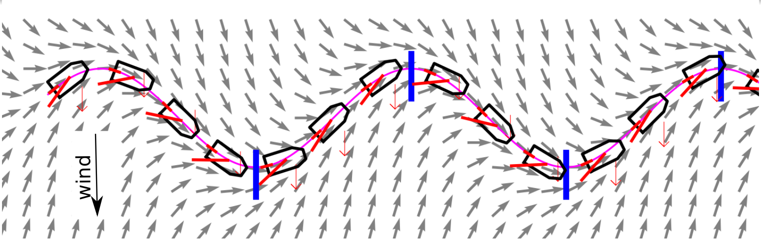
Tight Slalom Control for Sailboat Robots

M. Le Gallic, J. Tillet, F. Le Bars, L. Jaulin

IRSC 2018, Southampton
September 1, 2018

Tight slalom


wind

We consider a mobile robot [1]

$$\begin{cases} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x},\mathbf{u}) \\ \mathbf{p} & = & \mathbf{g}(\mathbf{x}) \end{cases}$$

with an input vector $\mathbf{u} = (u_1, \ldots, u_m)$ and a pose vector $\mathbf{p} = (p_1, \ldots, p_{m+1})$.

Control $m+1$ state variables and not only m of them.

Perform a path following instead of a trajectory tracking.

The controller makes $\dot{\mathbf{p}}$ collinear (instead of equal) to the required field.

Method

Dubins car:

$$\begin{cases} \dot{x}_1 &= \cos x_3 \\ \dot{x}_2 &= \sin x_3 \\ \dot{x}_3 &= u \end{cases}$$

Output

$$e = x_3 + \operatorname{atan} x_2.$$

Thus

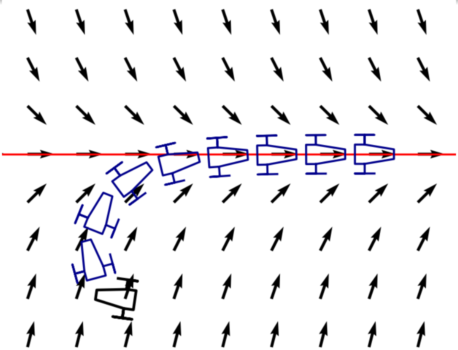
$$\dot{e} = \dot{x}_3 + \frac{\dot{x}_2}{1 + x_2^2} = u + \frac{\sin x_3}{1 + x_2^2}.$$

We want

$$\dot{e} + e = 0.$$

We get:

$$u = -x_3 - \operatorname{atan} x_2 - \frac{\sin x_3}{1+x_2^2}$$



Generalization

We want to follow the field $\psi(\mathbf{p})$.
This can be translated into $\varphi(\psi(\mathbf{p}), \dot{\mathbf{p}}) = \mathbf{0}$, where

$$\varphi(\mathbf{r}, \mathbf{s}) = \mathbf{0} \Leftrightarrow \exists \lambda > 0, \lambda \mathbf{r} = \mathbf{s}.$$

We define

$$\mathbf{e} = \varphi\left(\psi(\mathbf{p}), \underline{\mathbf{p}}\right) = \varphi\left(\psi(\mathbf{g}(\mathbf{x})), \frac{\partial \mathbf{g}}{\partial \mathbf{x}}(\mathbf{x}) \cdot \mathbf{f}(\mathbf{x}, \mathbf{u})\right).$$

Since $\dim \mathbf{e} = \dim \mathbf{u} = m$, we can make $\mathbf{e} \to \mathbf{0}$.

Van der Pol cycle

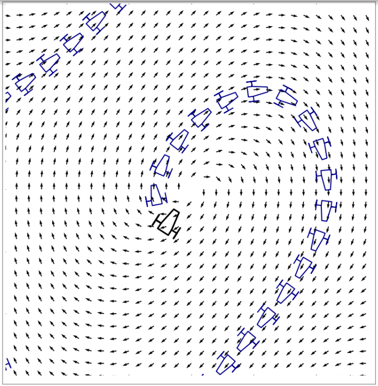
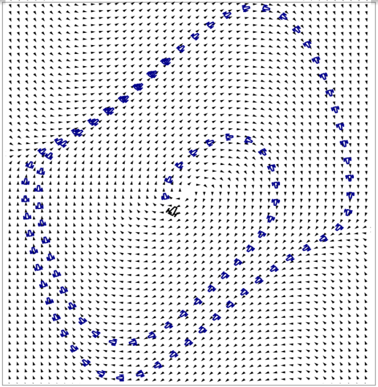
The car has to follow the limit cycle of the Van der Pol equation:

$$\psi(\mathbf{p}) = \begin{pmatrix} p_2 \\ -\left(0.01\ p_1^2 - 1\right) p_2 - p_1 \end{pmatrix}.$$

We take $\mathbf{g}(\mathbf{x}) = (x_1, x_2)^\mathsf{T}$ to build paths in the (x_1, x_2)-space.

We get that final controller is

$$u = -\text{sawtooth}\left(x_3 - \text{atan2}\left(-\left(\frac{x_1^2}{100} - 1\right)x_2 - x_1, x_2\right)\right) + \frac{\left(\left(\frac{x_1^2}{100}-1\right)x_2+x_1\right)\cdot\sin x_3 + x_2\cdot\left(\frac{x_1 x_2 \cos x_3}{50} + \left(\frac{x_1^2}{100}-1\right)\sin x_3 + \cos x_3\right)}{x_2^2 + \left(\left(\frac{x_1^2}{100}-1\right)x_2+x_1\right)^2}$$

Controller

We consider sailboat [2]:

$$\begin{cases} \dot{x}_1 & = & v\cos\theta \\ \dot{x}_2 & = & v\sin\theta \\ \dot{\theta} & = & -\rho_2 v \sin 2u_1 \\ \dot{v} & = & \rho_3 \left\|\mathbf{w}_{\text{ap}}\right\| \sin\left(\delta_s - \psi_{\text{ap}}\right) \sin\delta_s - \rho_1 v^2 \\ \sigma & = & \cos\psi_{\text{ap}} + \cos u_2 \\ \delta_s & = & \begin{cases} \pi + \psi_{\text{ap}} & \text{if } \sigma \leq 0 \\ -\text{sign}\left(\sin\psi_{\text{ap}}\right)\cdot u_2 & \text{otherwise} \end{cases} \\ \mathbf{w}_{\text{ap}} & = & \begin{pmatrix} -a\ \sin(\theta) - v \\ -a\ \cos(\theta) \end{pmatrix} \\ \psi_{\text{ap}} & = & \text{angle } \mathbf{w}_{\text{ap}} \end{cases}$$

The path to follow is

$$e(\mathbf{p}) = 10\sin\left(\frac{p_1}{10}\right) - p_2 = 0$$

where $e(\mathbf{p})$ is the error.
We want $\dot{e} = -0.1\,e$.

Thus

$$\underbrace{\cos\left(\frac{p_1}{10}\right)\dot{p}_1 - \dot{p}_2}_{\dot{e}(\mathbf{p})} = -\frac{1}{10}\underbrace{\left(10\sin\left(\frac{p_1}{10}\right) - p_2\right)}_{e(\mathbf{p})}$$

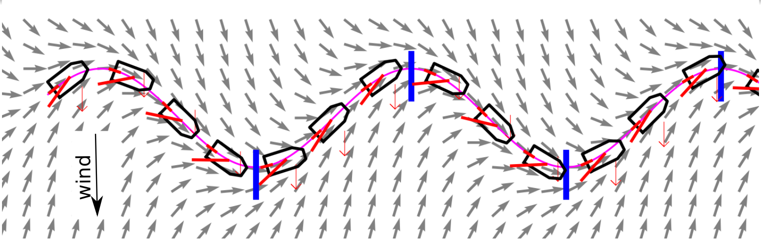
We take $\dot{p}_1 = 1$, to go to the right. Thus:

$$\psi(\mathbf{p}) = \begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \cos\left(\frac{p_1}{10}\right) + \frac{1}{10}\left(10\sin\left(\frac{p_1}{10}\right) - p_2\right) \end{pmatrix} \quad (1)$$

which is attracted by the curve $p_2 = 10\sin\left(\frac{p_1}{10}\right)$.

The controller is

$$
\begin{aligned}
u_1 &= -\frac{1}{2}\arcsin\left(\tanh\left(\frac{\hat{\omega}}{\rho_2 v}\right)\right) \\
\hat{\omega} &= -(\text{sawtooth}(\theta - \text{atan2}(b,1)) + \frac{\dot{b}}{1+b^2} \\
\dot{b} &= \frac{1}{10}\cos x_3 \cdot \left(\cos\left(\frac{x_1}{10}\right) - \sin\left(\frac{x_1}{10}\right)\right) - \frac{1}{10}\sin x_3 \\
b &= \cos\left(\frac{x_1}{10}\right) + \sin\left(\frac{x_1}{10}\right) - \frac{1}{10}x_2
\end{aligned}
$$


wind

L. Jaulin.
Automation for Robotics.
ISTE editions, 2015.

L. Jaulin and F. Le Bars.
An Interval Approach for Stability Analysis; Application to Sailboat Robotics.
IEEE Transaction on Robotics, 27(5), 2012.