

# Depth control of a float

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# Surface drifting buoys



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# Shepherd



[www.ensta-bretagne.fr/jaulin/shepherd.html](http://www.ensta-bretagne.fr/jaulin/shepherd.html)

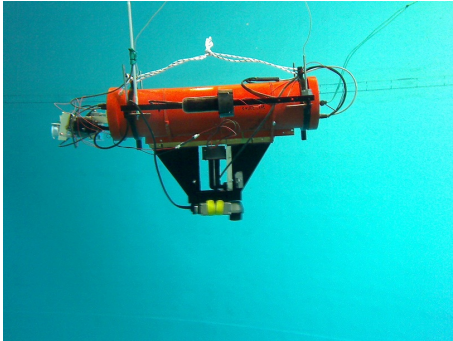
# Control

# Three examples

- The crank
- The car
- The traffic jam



# Depth control

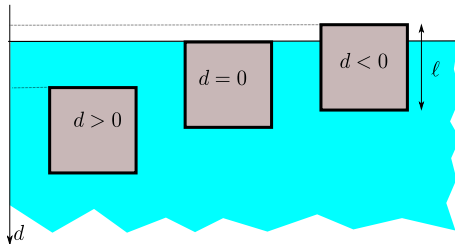


PID control is fine, since the relative degree is  $\rho = 2$ . With a ballast  $\rho = 3$

For the buoy, we need to include the model inside the controller

- 1 To be precise
- 2 To have a low consumption

We consider an underwater buoy, a cube of width  $\ell$ , immersed in the water of density  $\rho_0(z)$ .



Underwater buoy controlled by a ballast

The total force applied to the buoy is

$$f = \underbrace{mg}_{f_g \downarrow} - \underbrace{\rho_0(z) g \ell_z^2 \cdot \max(0, \ell_z + \min(d, 0))}_{f_a \uparrow} - \underbrace{\frac{1}{2} \rho_0(z) v \cdot |v| \ell_z^2 c_x}_{f_d \updownarrow}$$

A ballast allows us to control the density of the buoy which is  $(1 + \beta b) \rho_b(z)$ , where  $\beta = 0.1$ .

Since the mass of the buoy is  $m = (1 + \beta b) \rho_b(z) \ell_z^3$ . The second Newton law tells us that

$$m\dot{v} = mg - \rho_0 g \ell_z^2 \cdot \max(0, \ell_z + \min(d, 0)) - \frac{1}{2} \rho_0(z) v \cdot |v| \ell_z^2 c_x.$$

The state equations are thus

$$\begin{cases} \dot{d} &= v \\ \dot{v} &= g - \frac{\rho_0(z) g \ell_z^2 \cdot \max(0, \ell_z + \min(d, 0)) + \frac{1}{2} \rho_0(z) v \cdot |v| \ell_z^2 c_x}{(1 + \beta b) \rho_b(z) \ell_z^3} \\ \dot{b} &= u \end{cases}$$

When no saturation exists, we have

$$\begin{cases} \dot{d} &= v \\ \dot{v} &= g - \frac{\rho_0(z)}{\rho_b(z)} \cdot \frac{g^{\ell_z + \frac{1}{2}v \cdot |v| c_x}}{(1 + \beta b)^{\ell_z}} \\ \dot{b} &= u \end{cases}$$

We have

$$\begin{aligned}\dot{y} &= \dot{d} = v \\ \ddot{y} &= g - \frac{\rho_0(z)}{\rho_b(z)} \cdot \frac{g^{\ell_z + \frac{1}{2}} v \cdot |v| c_x}{(1 + \beta b)^{\ell_z}}\end{aligned}$$



In our particular case, since here,  $\ddot{y}$  is monotonic with respect to  $b$ , we do not need to compute the third derivative  $\ddot{\ddot{y}}$ .

The sliding surface is

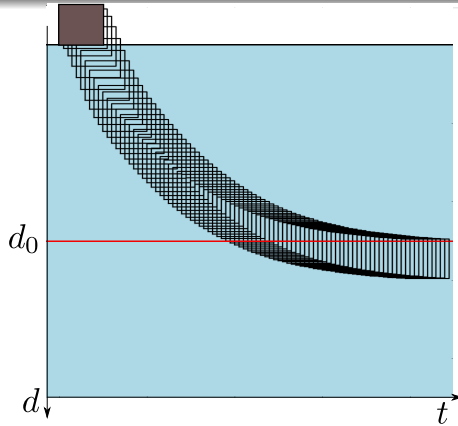
$$s(\mathbf{x}, t) = \underbrace{\ddot{y}_d - \ddot{y}}_{\ddot{e}} + \alpha_1 \underbrace{(\dot{y}_d - \dot{y})}_{\dot{e}} + \alpha_0 \underbrace{(y_d - y)}_e = 0,$$

If may take  $\alpha_1 = 2, \alpha_0 = 1$  to have the stability, we get:

$$\ddot{d}_0 - \left( g - \frac{\rho_0(z)}{\rho_b(z)} \cdot \frac{g\ell_z + \frac{1}{2}v \cdot |v| c_x}{(1 + \beta b)\ell_z} \right) + 2(\dot{d}_0 - v) + (d_0 - d) = 0.$$

We choose

$$u = \text{sign} \left( \ddot{d}_0 - \left( g - \frac{\rho_0(z)}{\rho_b(z)} \cdot \frac{g\ell_z + \frac{1}{2}v \cdot |v| c_x}{(1 + \beta b)\ell_z} \right) + 2(\dot{d}_0 - v) + d_0 - d \right).$$



# Perspectives

