Construction of a mosaic from an underwater video; an interval analysis approach

M. Laranjeira, L. Jaulin, S. Tauvry, C. Aubry
Universidade Federal da Bahia, Lab STICC, IHSEV, OSM,
ENSTA Bretagne, ECA Robotics, Ecole Navale

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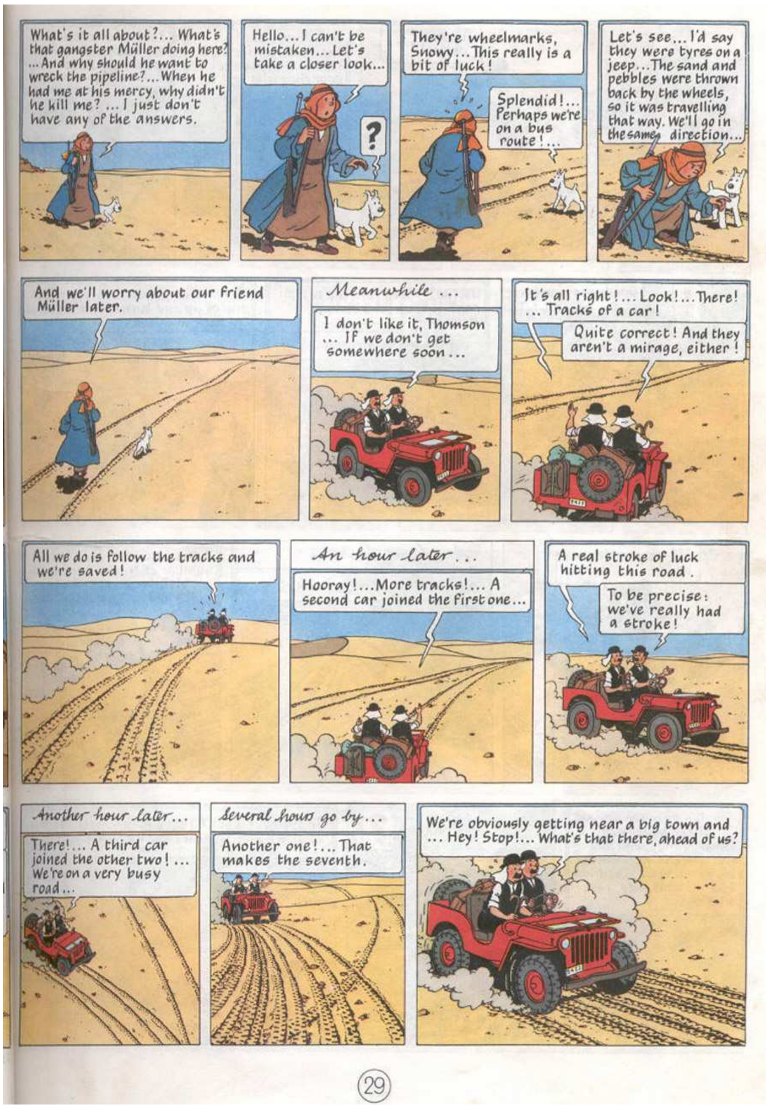
Question: it is possible to perform a localization in an unknown environment without building a map (i.e. without SLAM) ?

1 Loop detection problem

Example. We are driving a car in the desert. We measure the speed of the car and its orientation. We have no GPS, no camera.

Problem. Count the number of loops we made.



Land of Black Gold

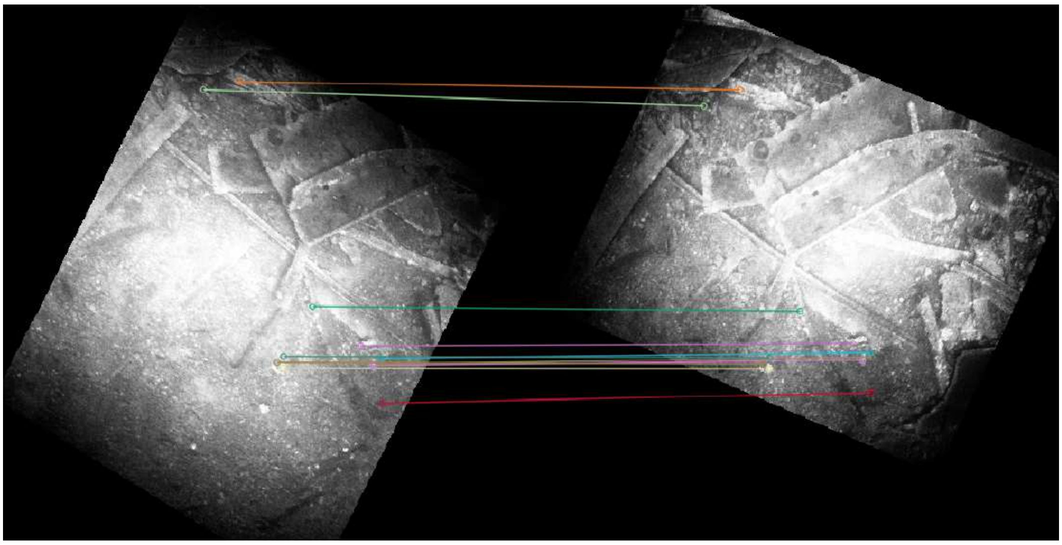
Robot:

$$\left\{\begin{array}{lcl}\dot{\mathbf{x}} & = & \mathbf{f}\left(\mathbf{x},\mathbf{u}\right)\\ \mathbf{y} & = & \mathbf{g}\left(\mathbf{x}\right)\end{array}\right.$$

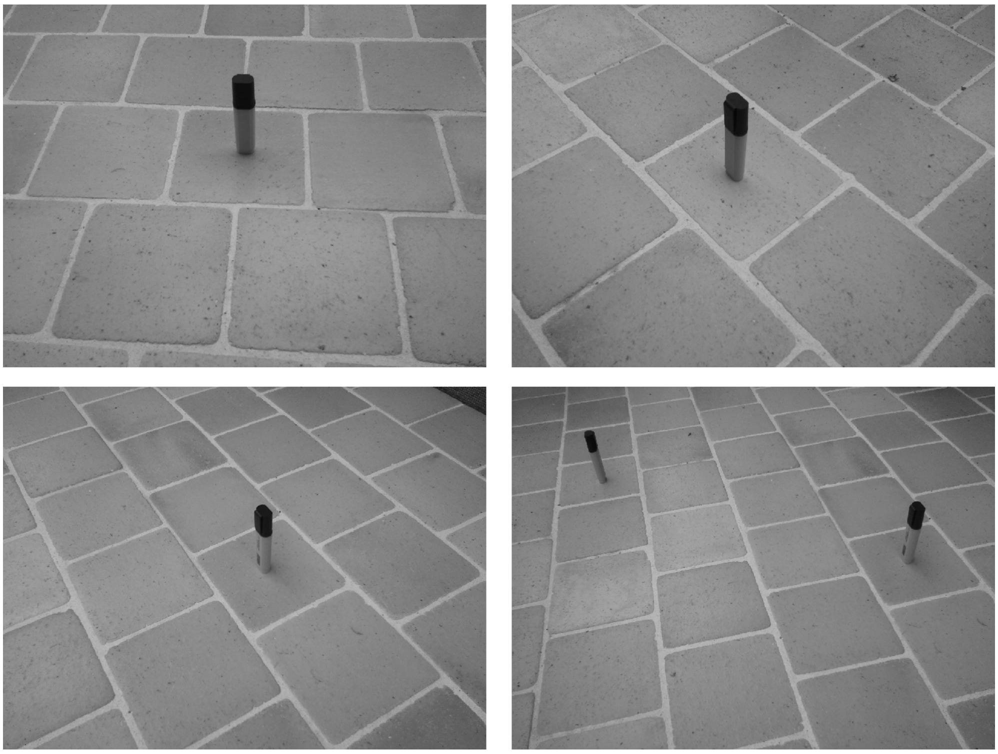
$\mathbf{u}$: proprioceptive sensors

$\mathbf{y}$: exteroceptive sensors

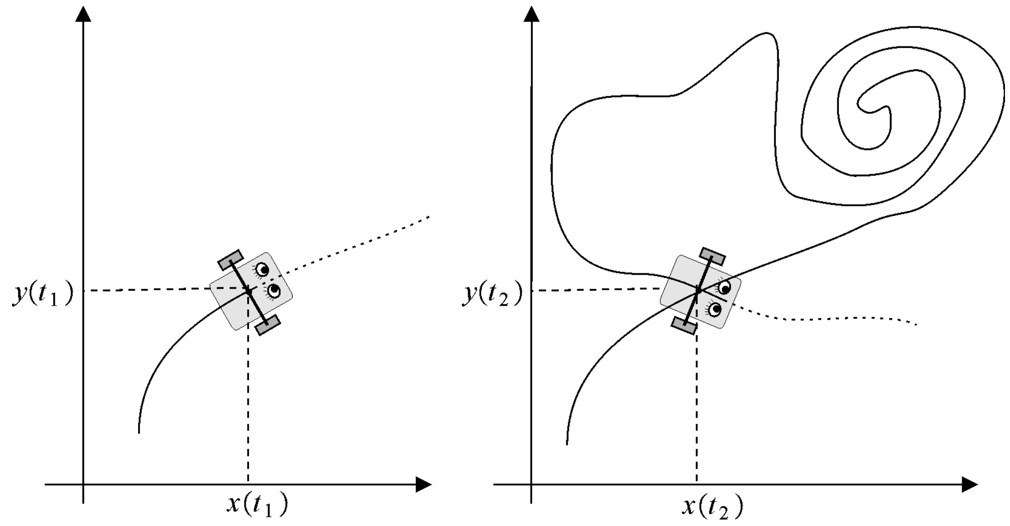
Problem: detect loops with proprioceptive sensors only.



Are you sure we made a loop ?



2 T-plane



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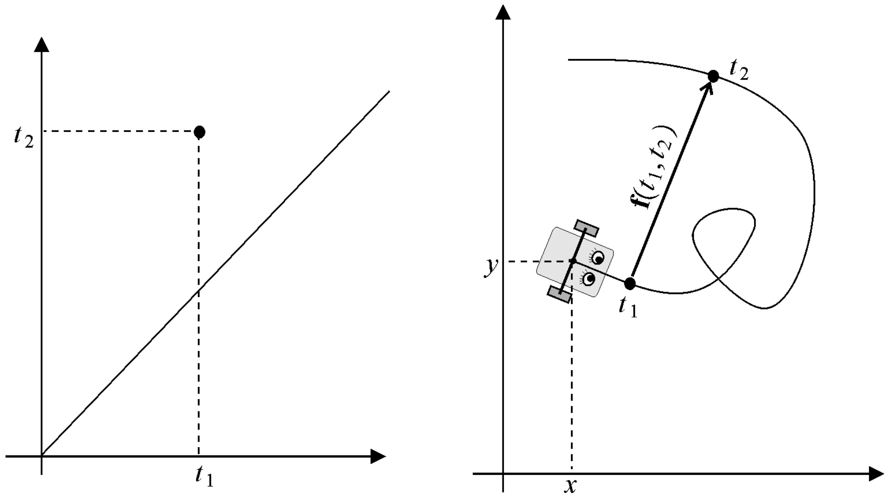
A robot trajectory with one single loop

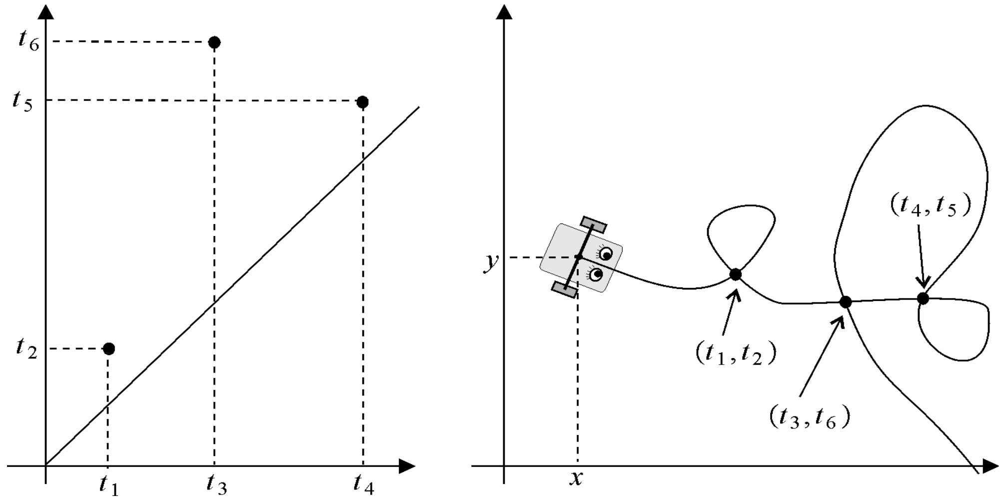
Define the shift function

$$\mathbf{f}(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}(\tau)\, d\tau.$$

The loop set is

$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\mathsf{max}}]^2 \mid \mathbf{f}(t_1, t_2) = \mathbf{0}, t_2 > t_1 \right\}$$

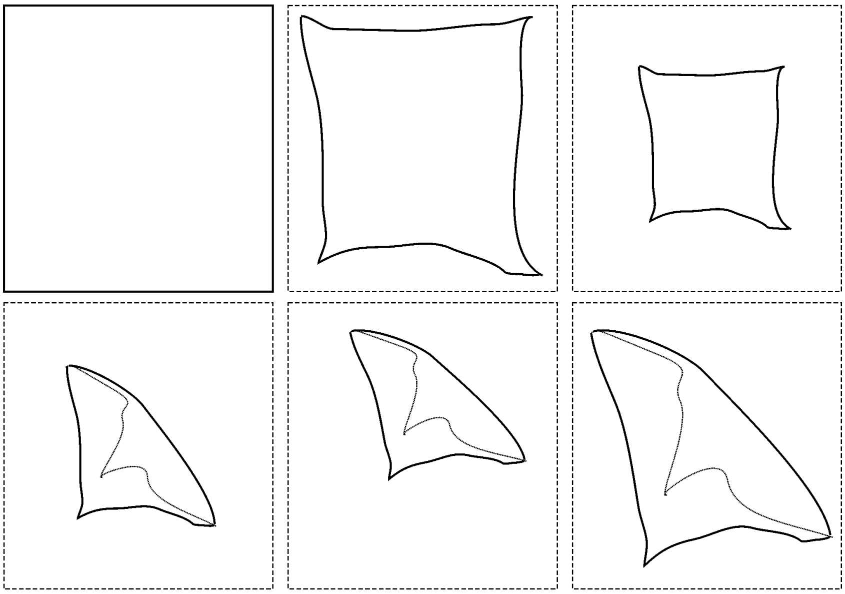

t_2
t_1
t_2
$\mathbf{f}(t_1, t_2)$
y
t_1
x



Left: t-plane; Right: trajectory

3 Brouwer fixed point theorem

Brouwer fixed point theorem (1909). Any continuous function $\mathbf{n}$ from bounded convex subset of $\mathbb{R}^n$ to itself has a fixed point; i.e., a point such that $\mathbf{n}(\mathbf{x}) = \mathbf{x}$.



Distortion; narrowing; folding; shifting; enlargement : at least one point has not moved

Example. If

$$\mathbf{n}(t_1,t_2)=\begin{pmatrix}\cos\left(t_1-t_2^2\right)\\ \sin(t_1t_2)\end{pmatrix}$$

Since

$$\mathbf{n}([-1,1],[-1,1])\subset[-1,1]\times[-1,1]$$

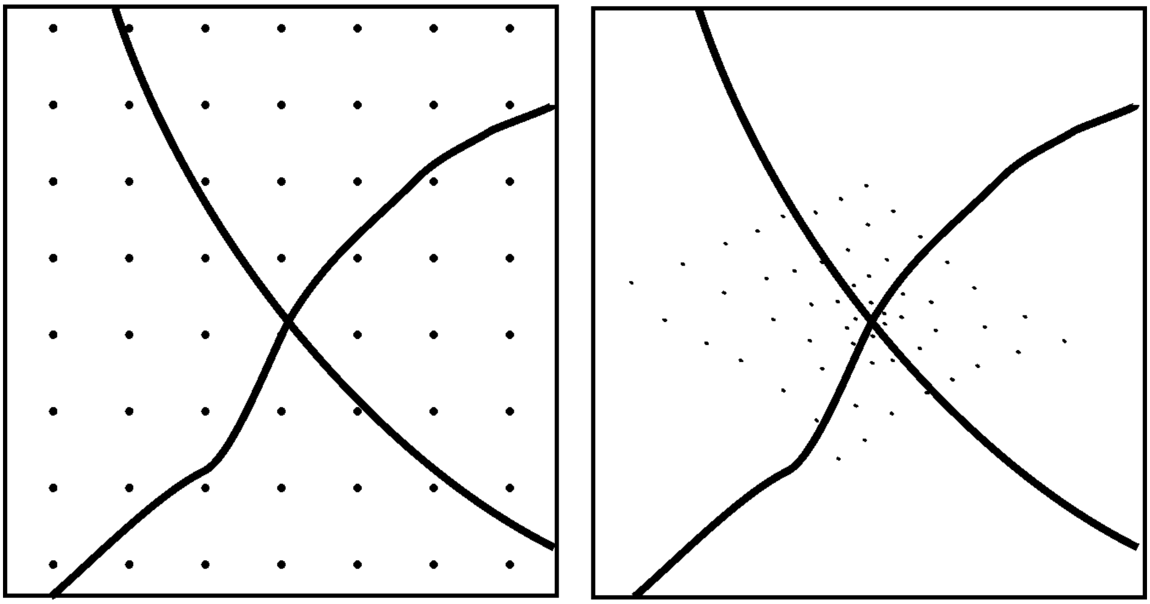
we conclude

$$\exists\,(t_1,t_2)\in[-1,1]^2 \ \mid \mathbf{n}(t_1,t_2)=(t_1,t_2)\,.$$

If we have a function $\mathbf{n}$ (such as the Newton operator) such that

$$\mathbf{n}(\mathbf{x}) \quad = \mathbf{x} \Rightarrow \mathbf{f}(\mathbf{x}) = \mathbf{0},$$

then using Brouwer theorem we can detect loops.



4 Interval analysis

Problem. Given $f : \mathbb{R}^n \to \mathbb{R}$ and a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Example. Is the function

$$f(\mathbf{x}) = x_1 x_2 - (x_1 + x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2$$

always positive for $x_1, x_2 \in [-1, 1]$?

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= [1,8] \\ [-1,3]\cdot[2,5] &= [-5,15] \\ \sin\left([0,2]\right) &= [0,1] \end{aligned}$$

The interval extension of

$$\begin{aligned} f(x_1, x_2) &= x_1 \cdot x_2 - (x_1 + x_2) \cdot \cos x_2 \\ &\quad + \sin x_1 \cdot \sin x_2 + 2 \end{aligned}$$

is

$$\begin{aligned} [f]([x_1], [x_2]) &= [x_1] \cdot [x_2] - ([x_1] + [x_2]) \cdot \cos [x_2] \\ &\quad + \sin [x_1] \cdot \sin [x_2] + 2. \end{aligned}$$

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0$$

Theorem (Moore-Brouwer)

For $\mathbf{f} : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, we have

$$[\mathbf{f}]\left([\mathbf{x}]\right) \subset [\mathbf{x}] \Rightarrow \exists \mathbf{x} \in [\mathbf{x}]\,, \mathbf{f}\left(\mathbf{x}\right) = \mathbf{x}.$$

5 Characterizing sets

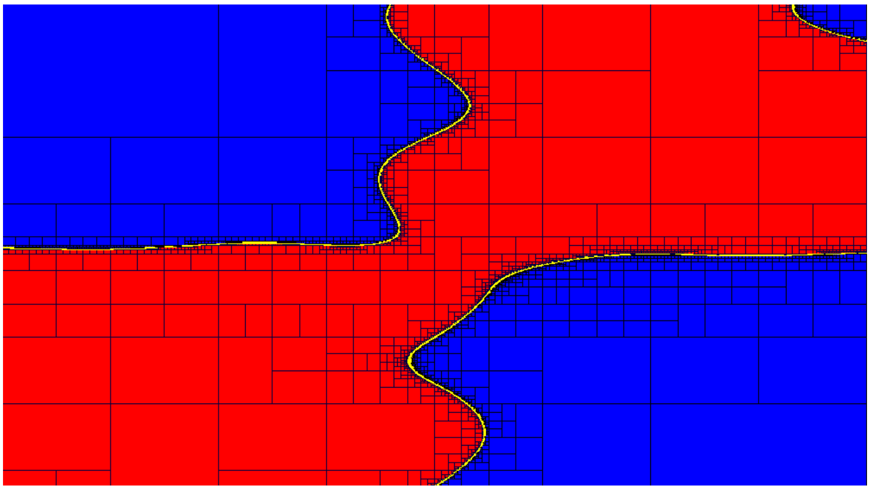
Subsets $\mathbb{X} \subset \mathbb{R}^n$ can be bracketed by subpavings :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

which can be obtained using interval calculus

Example.

$$\mathbb{X} = \{\mathbf{x} \mid x_1x_2 - (x_1+x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2 \geq 0\}.$$



6 Uncertain trajectories

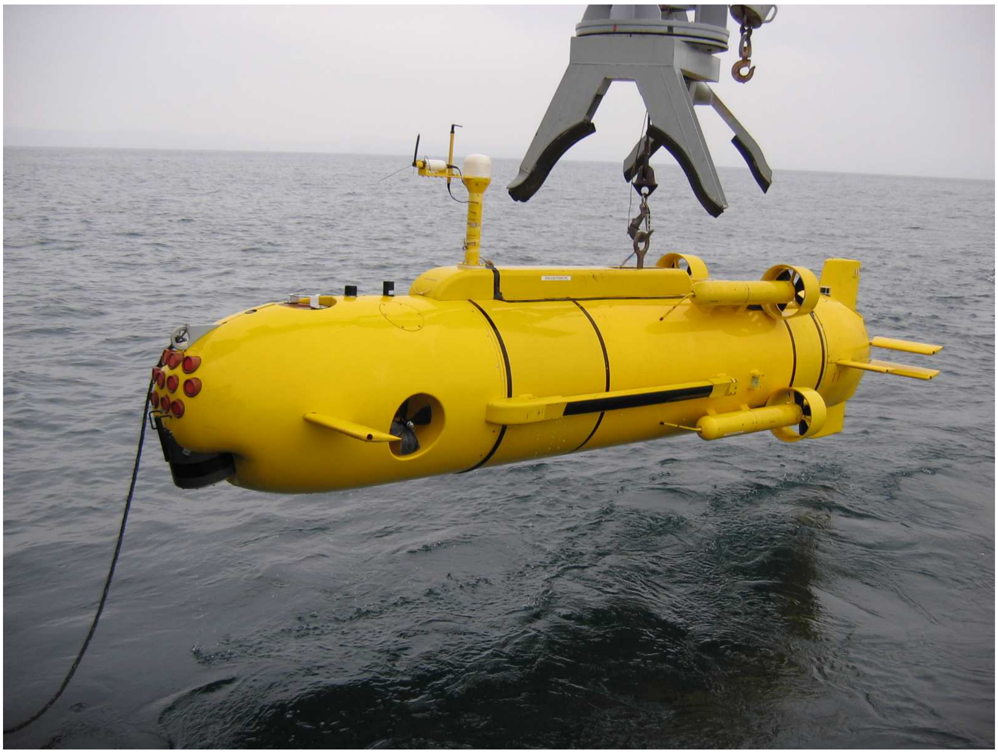
The robot knows a box $[\mathbf{v}](t)$ for $\mathbf{v}(t)$. We have

$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\mathsf{max}}]^2 \mid \exists \mathbf{v} \in [\mathbf{v}], \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0}, t_1 < t_2 \right\}$$

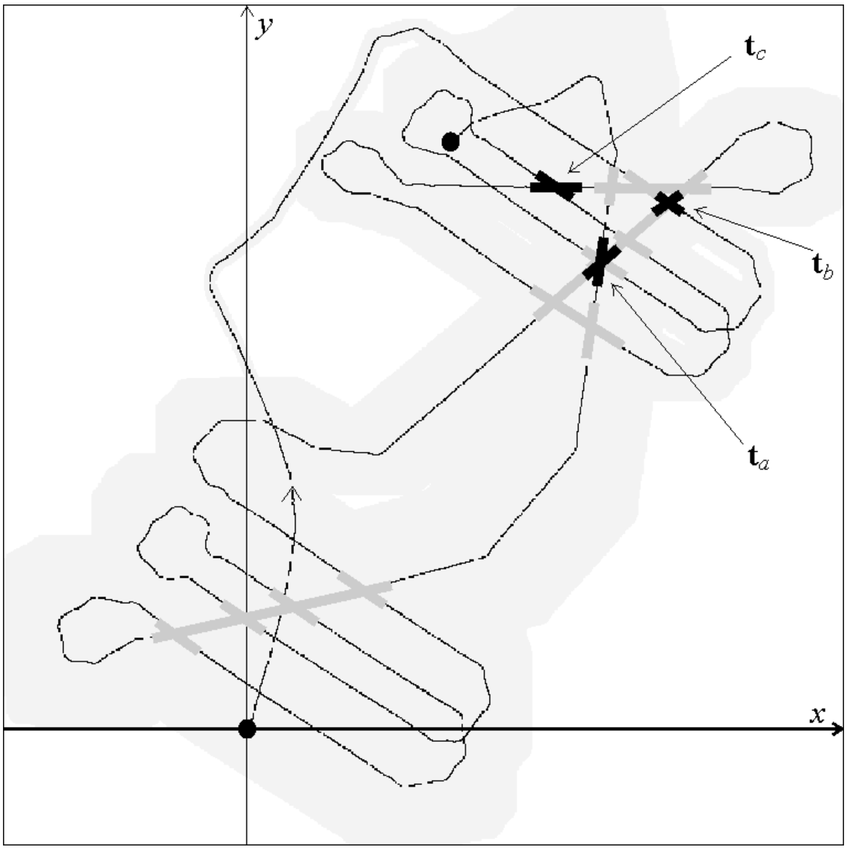
Thus $\mathbb{T}$ is defined by

$$\mathbf{h}\left(t_1, t_2\right) = \left(\begin{array}{c} \int_{t_1}^{t_2} \mathbf{v}^{-}(\tau) d\tau \\ -\int_{t_1}^{t_2} \mathbf{v}^{+}(\tau) d\tau \\ t_1 - t_2 \end{array} \right) < \mathbf{0}.$$

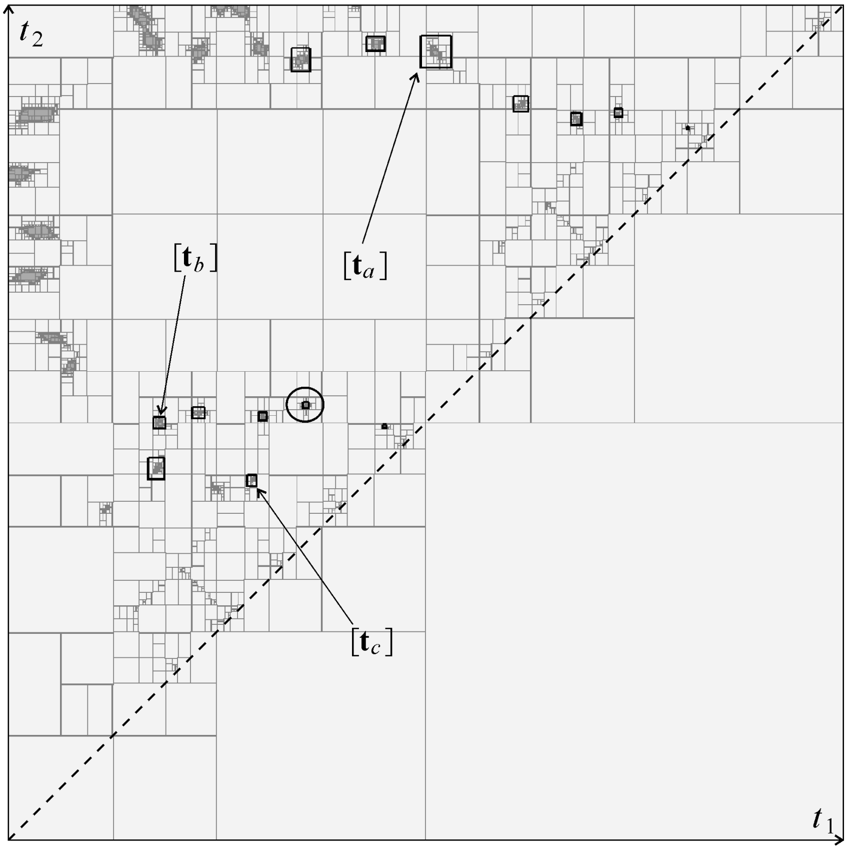
7 Testcase



Redermor, DGA-TN

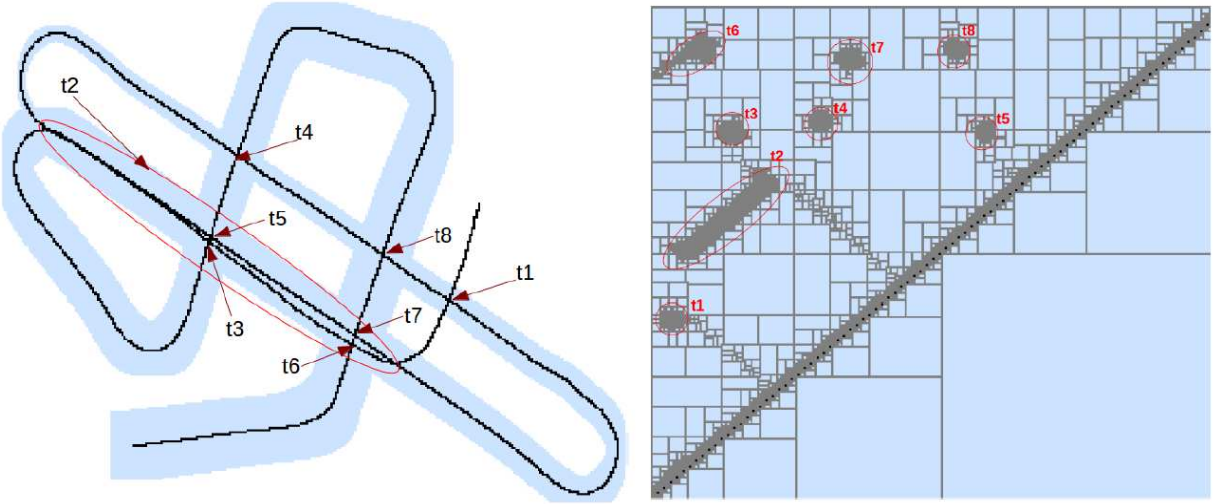


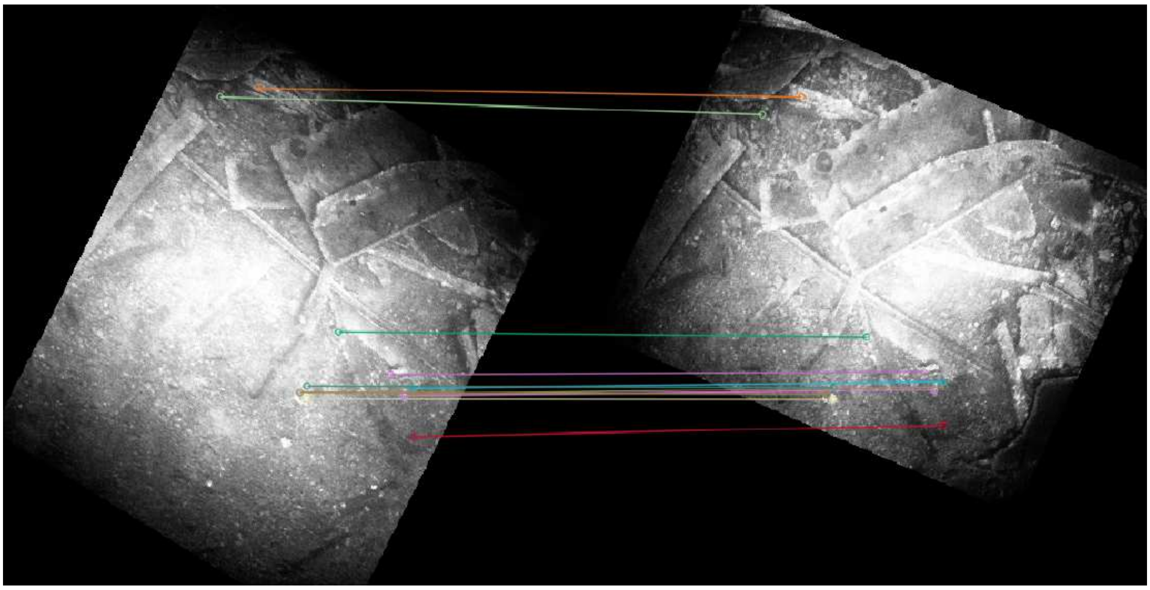
y
x
$\mathbf{t}_c$
$\mathbf{t}_b$
$\mathbf{t}_a$



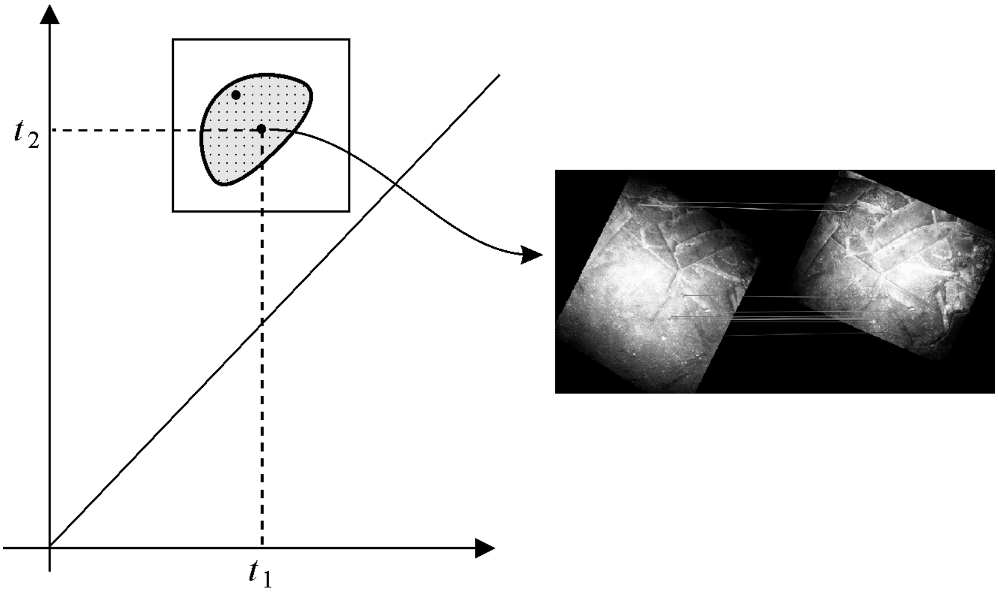
t_2
$[\mathbf{t}_b]$
$[\mathbf{t}_a]$
$[\mathbf{t}_c]$
t_1

8 Mosaic

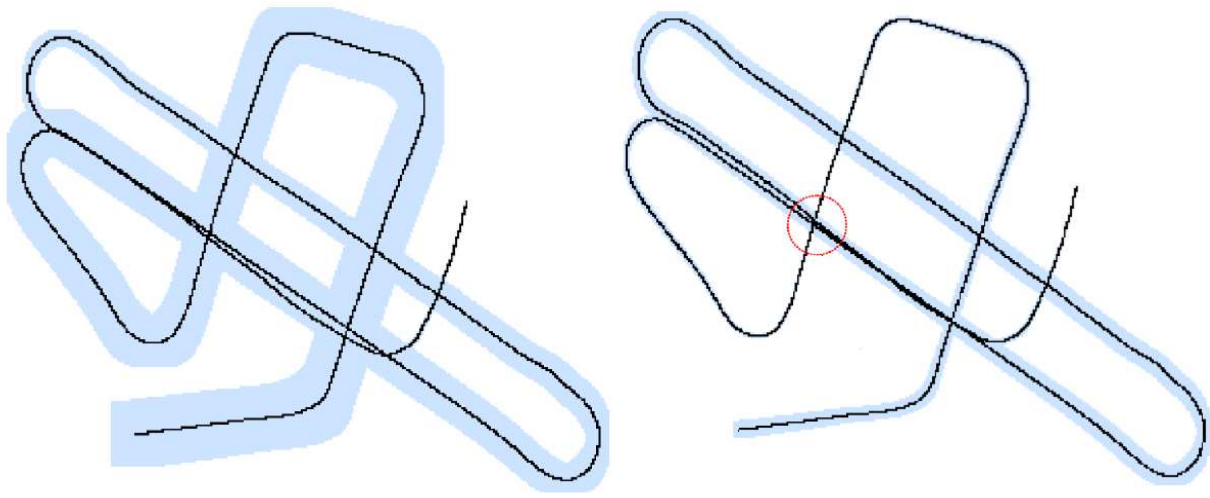

t2
t4
t5
t8
t1
t3
t7
t6
t6
t7
t8
t3
t4
t5
t2
t1



Compatible or incompatible ?

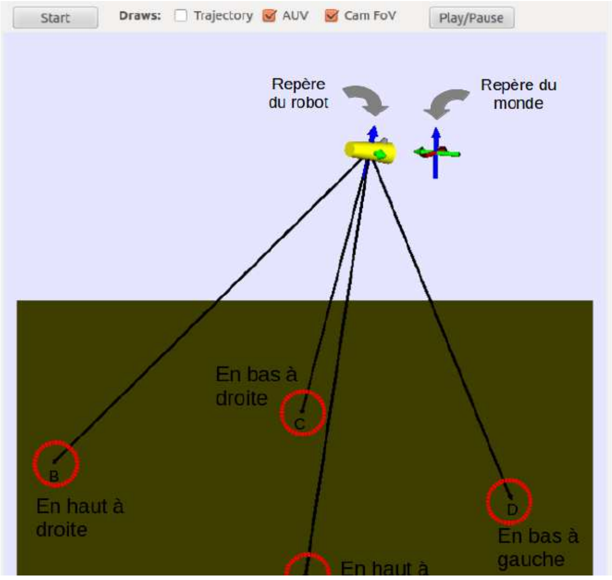


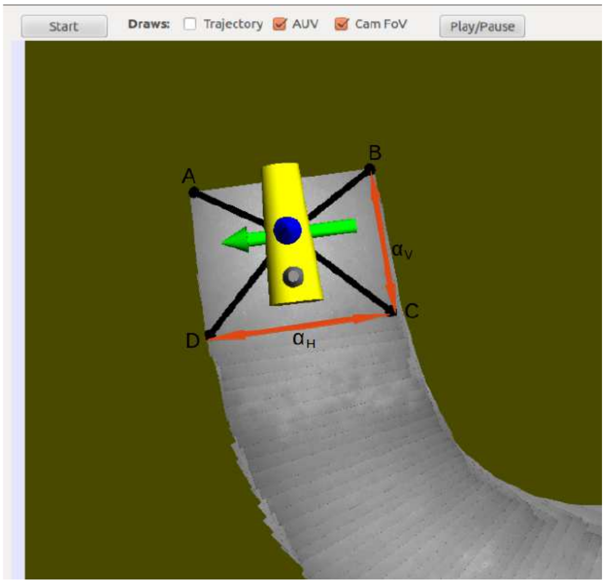
t_2
t_1



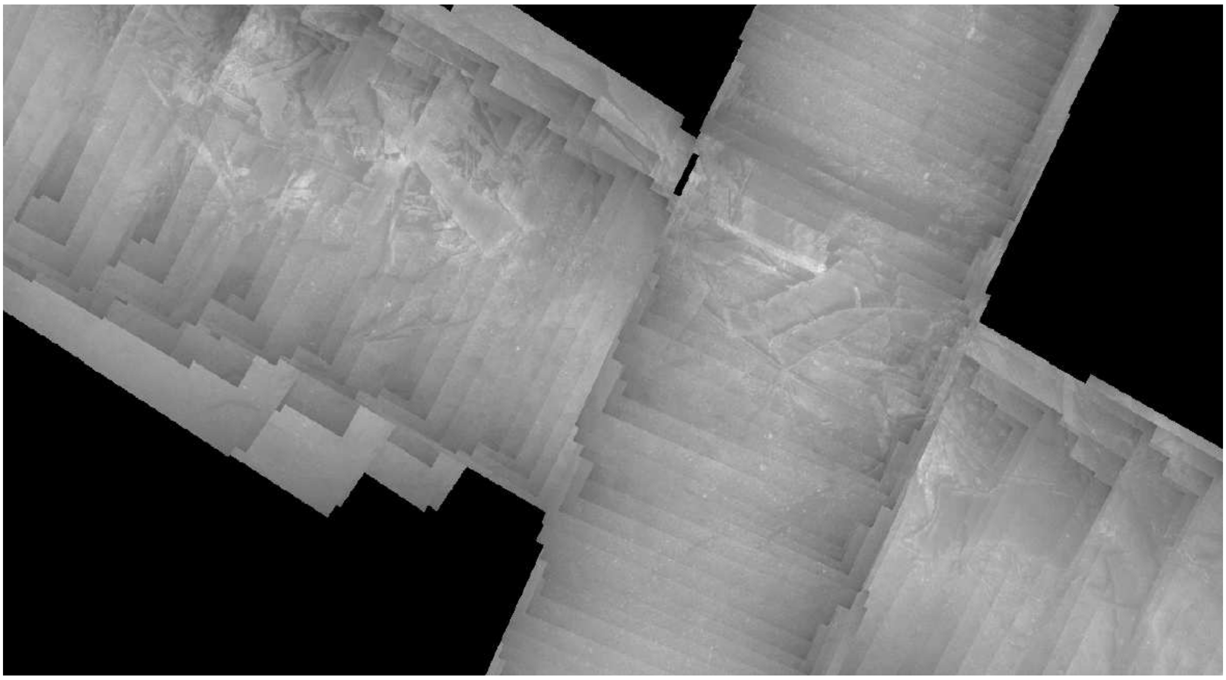
Contraction of the interval trajectory

8.1 Projection

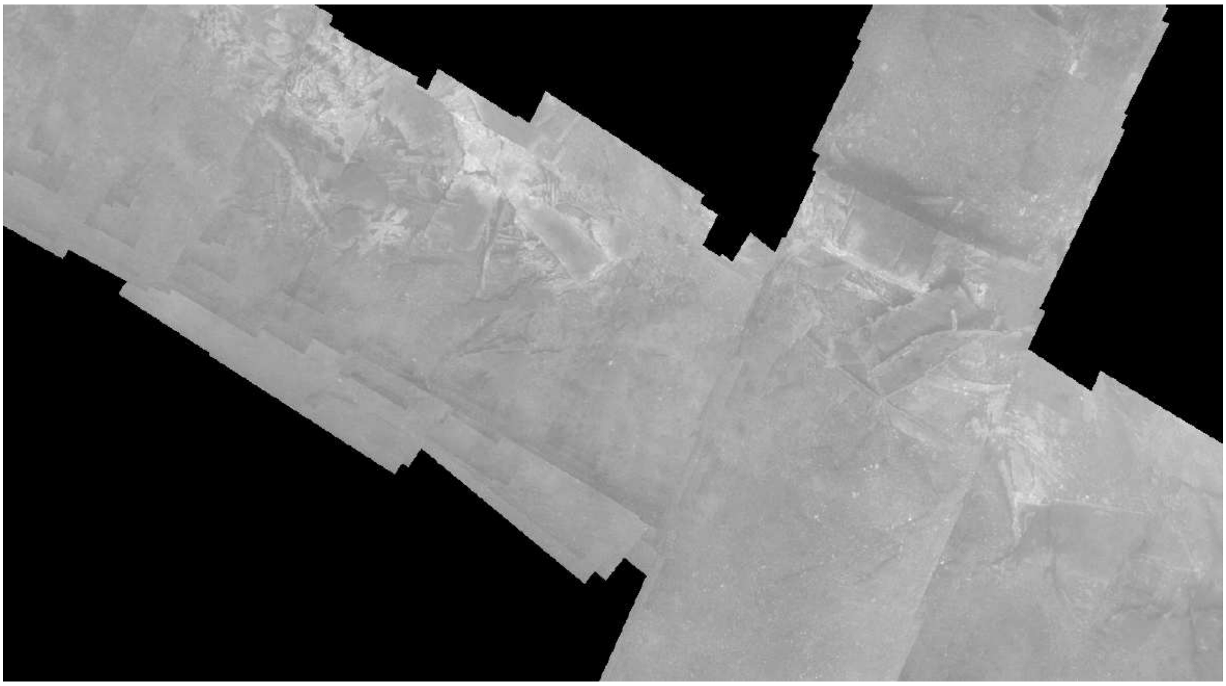

Start
Draws:
Trajectory
AUV
Cam FoV
Play/Pause
Repère du robot
Repère du monde
En bas à droite
C
B
En haut à droite
D
En bas à gauche
En haut à


Start
Draws:
Trajectory
AUV
Cam FoV
Play/Pause
A
B
α_V
C
D
α_H

8.2 Illumination equalization



Before illumination equalization



After illumination equalization

Video of the presention available at
http://youtu.be/sPKOBunlBEM