A constraint approach for the data association problem with application to underwater localization

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La Cordelière


DGA

Boatbot


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HONDA
caribe
ENSTA


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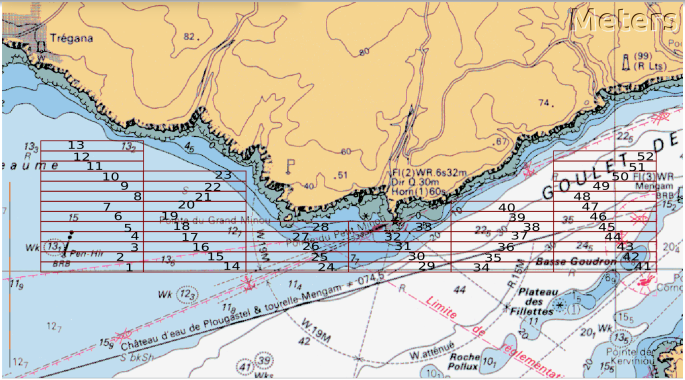
Meters
Trégana
GOULET DE
Pointe du Grand Minou
Château d'eau de Plougastel & tourelle Mengam
Plateau des Fillettes
Basse Goudron
Roche Pollux
Limite de réglementati
W.atténué
Fl(2)WR.6s32m
Dir Q.30m
Horn(1)60s
Fl(3)WR
Mengam
Pointe de Kerviniou
Pen-Hir
W.19M
R.15M
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Robots

Folaga with Kopadia

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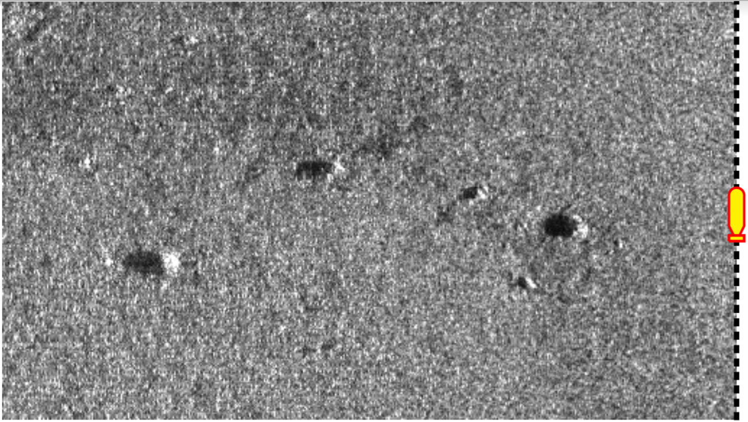


Formalization

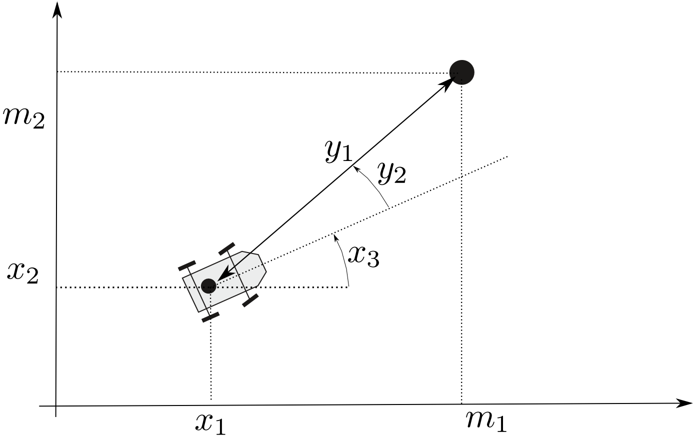
State estimation problem

$$\left\{\begin{array}{rr} \dot{\mathbf{x}}(t)=\mathbf{f}\left(\mathbf{x}(t)\right),\mathbf{u}(t)) & \text{(evolution equation)} \\ \mathbf{g}(\mathbf{x}(t_i)) \in [\mathbf{y}](t_i) & \text{(observation constraint)} \\ \mathbf{x}(0) \in \mathbb{X}_0 & \text{(initial state)} \end{array}\right.$$

Implicit form

$$\left\{\begin{array}{r}\dot{\mathbf{x}}(t)=\mathbf{f}\left(\mathbf{x}(t)\right),\mathbf{u}(t))\\ \mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i))=\mathbf{0},\\ \mathbf{y}(t_i)\in[\mathbf{y}](t_i)\\ \mathbf{x}(0)\in\mathbb{X}_0\end{array}\right.$$

Example. Assume that we have a robot at position (x_1,x_2) with a heading x_3. It measures a landmark located at $(4,5)$.



We have

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = \begin{pmatrix} x_1 + y_1 \cdot \cos(x_3 + y_2) - 4 \\ x_2 + y_1 \cdot \sin(x_3 + y_2) - 5 \end{pmatrix}$$

If now, the robot only measures the distance to the landmark, we get

$$g(\mathbf{x}, y) = (x_1 - 4)^2 + (x_2 - 5)^2 - y^2.$$

In the more general case, the observation constraint is

$$\begin{gathered}\mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i),\mathbf{m}(t_i))=\mathbf{0}\\ \mathbf{y}(t_i)\in[\mathbf{y}](t_i)\\ \mathbf{m}(t_i)\in[\mathbf{m}](t_i)\end{gathered}$$

In our context, the data are not associated. The observation constraint has the form

$$\begin{array}{c}\mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i),\mathbf{m}(t_i))=\mathbf{0}\\ \mathbf{m}(t_1)\in[\mathbf{m}_1]\vee\cdots\vee\mathbf{m}(t_\ell)\in[\mathbf{m}_\ell]\end{array}$$

or equivalently

$$\begin{array}{c}\mathbf{g}(\mathbf{x}(t_i),\mathbf{y}(t_i),\mathbf{m}(t_i))=\mathbf{0}\\ \mathbf{m}(t_i)\in\mathbb{M}=[\mathbf{m}_1]\cup\cdots\cup[\mathbf{m}_\ell]\end{array}$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t)), \mathbf{u}(t)\right) \\ \mathbf{g}(\mathbf{x}(t_i), \mathbf{y}(t_i), \mathbf{m}(t_i)) = \mathbf{0} \\ \mathbf{y}(t_i) \in [\mathbf{y}](t_i) \\ \mathbf{m}(t_i) \in \mathbb{M} \\ \mathbf{x}(0) \in \mathbb{X}_0 \end{cases}$$

Contractors

The operator $\mathscr{C} : \mathbb{IR}^n \to \mathbb{IR}^n$ is a *contractor* for $f(\mathbf{x}) = 0$, if

$$\begin{cases} \mathscr{C}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathbf{x} \in [\mathbf{x}] \text{ and } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathscr{C}([\mathbf{x}]) & \text{(consistence)} \end{cases}$$

Building contractors

Consider the equation

$$x_1 + x_2 - x_3 = 0$$

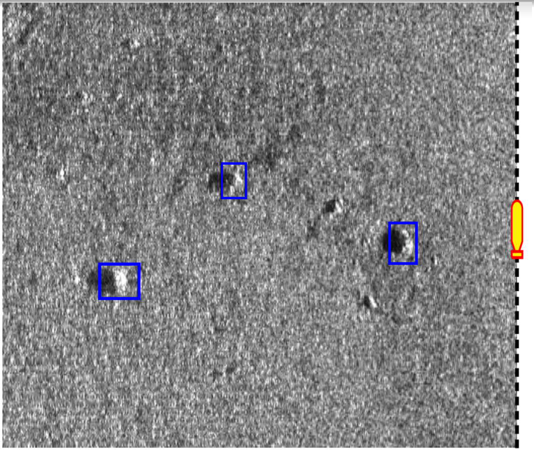
with $x_1 \in [x_1]$, $x_2 \in [x_2]$, $x_3 \in [x_3]$.

We have

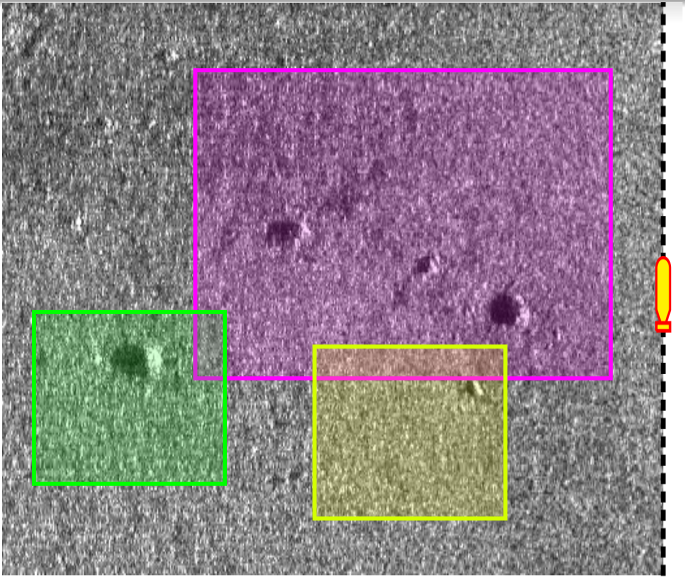
$$\begin{aligned} x_3 = x_1 + x_2 \Rightarrow \quad & x_3 \in \quad [x_3] \cap ([x_1] + [x_2]) \\ x_1 = x_3 - x_2 \Rightarrow \quad & x_1 \in \quad [x_1] \cap ([x_3] - [x_2]) \\ x_2 = x_3 - x_1 \Rightarrow \quad & x_2 \in \quad [x_2] \cap ([x_3] - [x_1]) \end{aligned}$$

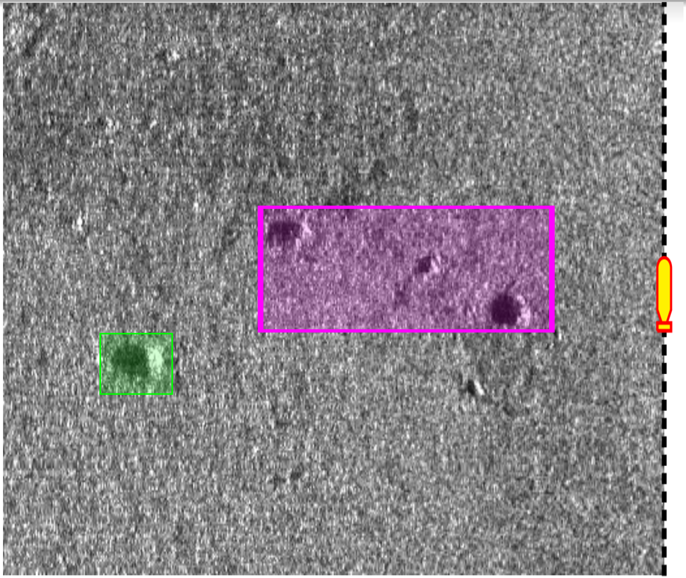
The contractor associated with $x_1 + x_2 = x_3$ is thus

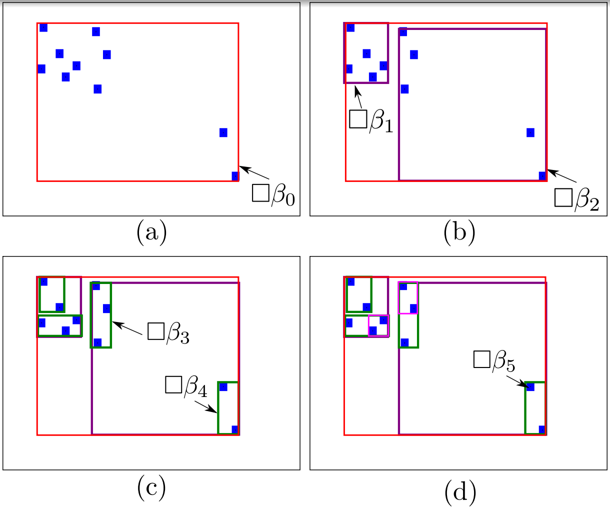
$$\mathscr{C}\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} = \begin{pmatrix} [x_1] \cap ([x_3] - [x_2]) \\ [x_2] \cap ([x_3] - [x_1]) \\ [x_3] \cap ([x_1] + [x_2]) \end{pmatrix}$$



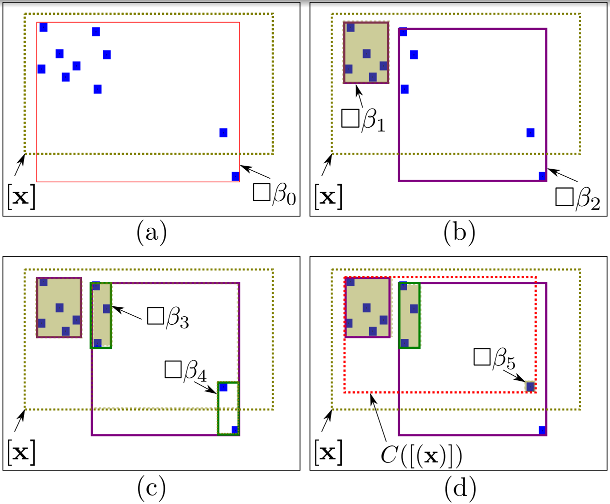
Our constraint : $\mathbf{m} \in \mathbb{M}$





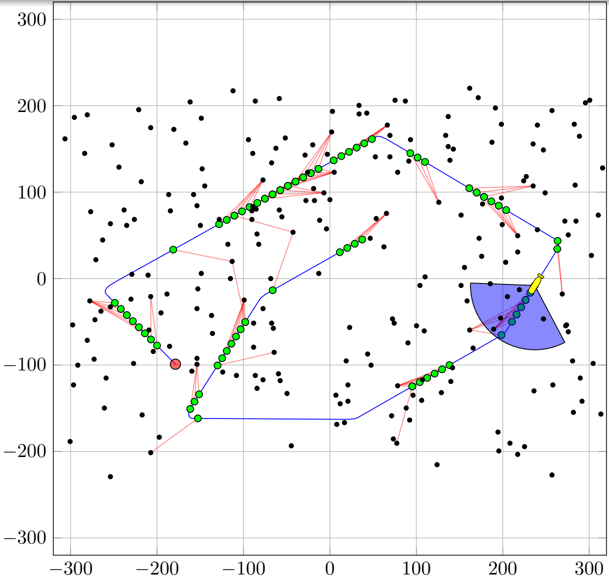


For efficiency, a balanced quadtree is created first



The contractor has now a logarithmic complexity

Test-case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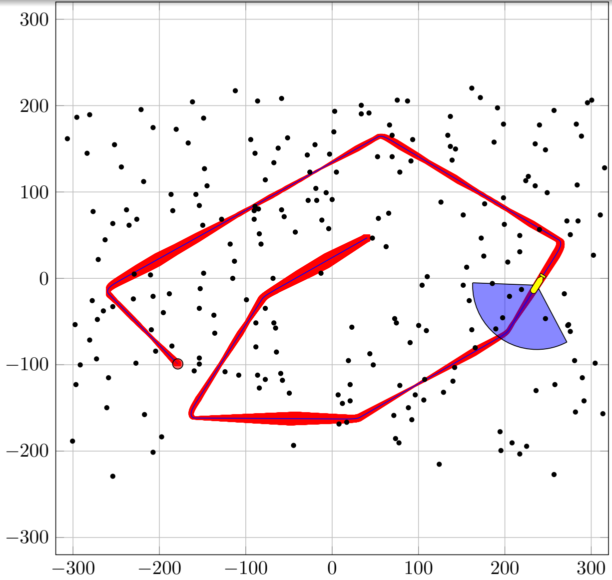
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