Construction of a Mosaic from an Underwater Video

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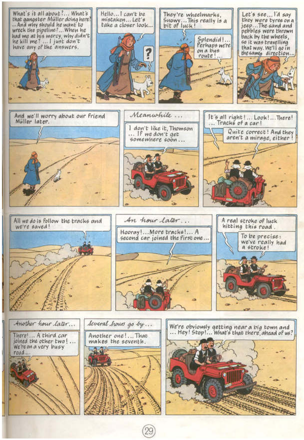
1. Loop detection problem

Example. We are driving a car in the desert. We measure the speed of the car and its orientation. We have no GPS, no camera.
Problem. Count the number of loops we made.

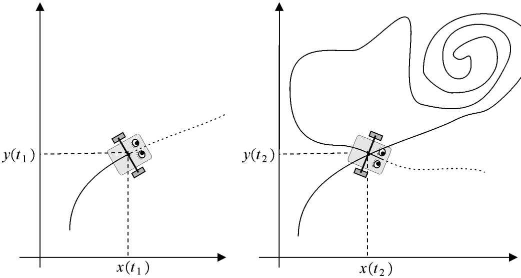

What's it all about?... What's that gangster Müller doing here? ...And why should he want to wreck the pipeline?...When he had me at his mercy, why didn't he kill me? ... I just don't have any of the answers.
Hello... I can't be mistaken... Let's take a closer look...
?
They're wheelmarks, Snowy... This really is a bit of luck!
Splendid!... Perhaps we're on a bus route!...
Let's see... I'd say they were tyres on a jeep... The sand and pebbles were thrown back by the wheels, so it was travelling that way. We'll go in the same direction...
And we'll worry about our friend Müller later.
Meanwhile ...
I don't like it, Thomson ... If we don't get somewhere soon ...
It's all right!... Look!... There! ... Tracks of a car!
Quite correct! And they aren't a mirage, either!
All we do is follow the tracks and we're saved!
An hour later ...
Hooray!...More tracks!... A second car joined the first one...
A real stroke of luck hitting this road.
To be precise: we've really had a stroke!
Another hour later...
There!... A third car joined the other two!... We're on a very busy road...
Several hours go by...
Another one!... That makes the seventh.
We're obviously getting near a big town and ... Hey! Stop!... What's that there, ahead of us?
29

Robot: We consider a state equation

$$\begin{cases} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x},\mathbf{u}) \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) \end{cases}$$

$\mathbf{u}$: proprioceptive sensors
$\mathbf{y}$: exteroceptive sensors
Problem: detect loops with proprioceptive (reliable) and exteroceptive (unreliable) sensors.

t-plane

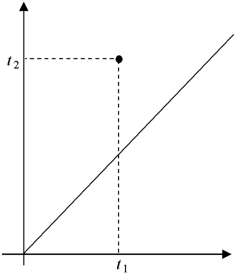

$y(t_1)$
$x(t_1)$
$y(t_2)$
$x(t_2)$

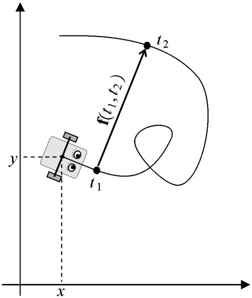
Define the shift function

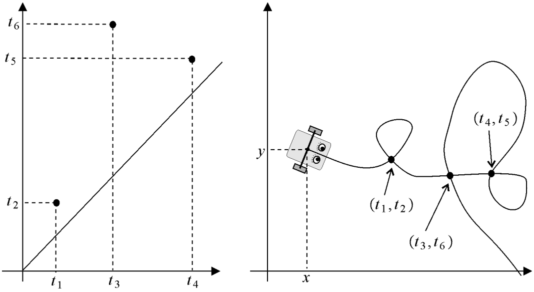
$$\mathbf{f}(t_1, t_2) = \int_{t_1}^{t_2} \mathbf{v}(\tau)\, d\tau.$$

The loop set is

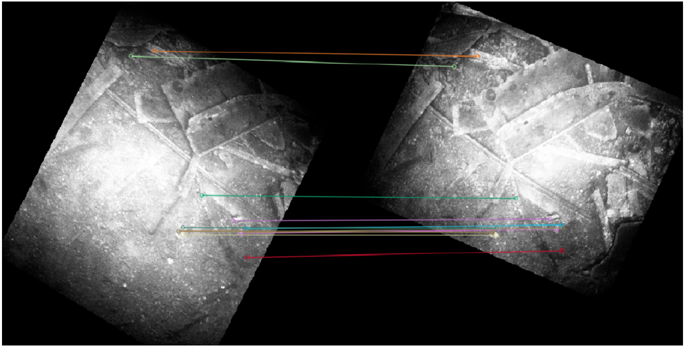
$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\max}]^2 \mid \mathbf{f}(t_1, t_2) = \mathbf{0}, t_2 > t_1 \right\}$$


t_2
t_1

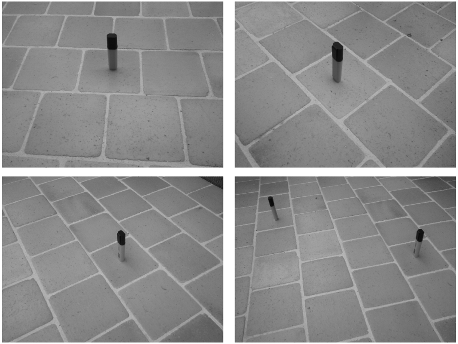

t_2
$\mathbf{f}(t_1, t_2)$
y
t_1
x


t_6
t_5
t_2
t_1
t_3
t_4
y
x
(t_4, t_5)
(t_1, t_2)
(t_3, t_6)

4. Reliability in perception



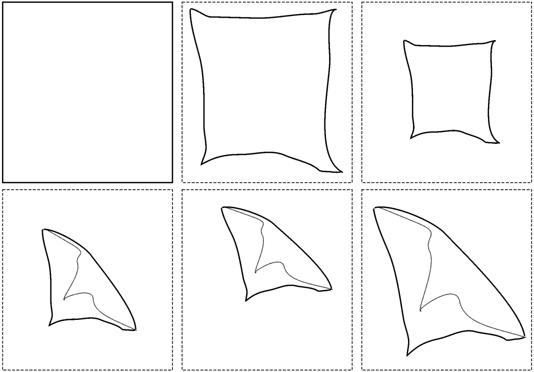
Are you sure we made a loop ?




Find 10
differences
dreamstime.com

2. Brouwer fixed point theorem

Brouwer fixed point theorem (1909). Any continuous function $\mathbf{n}$ from bounded convex subset of $\mathbb{R}^n$ to itself has a fixed point $\mathbf{x}$; *i.e.*, $\mathbf{n}(\mathbf{x}) = \mathbf{x}$.



Distortion; narrowing; folding; shifting; enlargement : at least one point has not moved

Example. If

$$\mathbf{n}(t_1, t_2) = \begin{pmatrix} \cos\left(t_1 - t_2^2\right) \\ \sin\left(t_1 t_2\right) \end{pmatrix}$$

Since

$$\mathbf{n}([-1,1],[-1,1]) \subset [-1,1] \times [-1,1]$$

we conclude

$$\exists (t_1, t_2) \in [-1,1]^2 \mid \mathbf{n}(t_1, t_2) = (t_1, t_2).$$

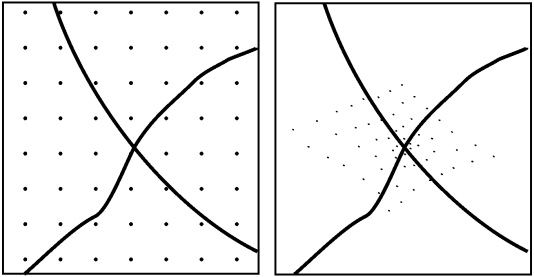
Define

$$\mathbf{n}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) + \mathbf{x}$$

where $\mathbf{x} = (t_1, t_2)$. We have

$$\mathbf{n}(\mathbf{x}) = \mathbf{x} \Leftrightarrow \mathbf{f}(\mathbf{x}) = \mathbf{0},$$

then using Brouwer theorem we can detect loops.



3. Interval analysis

Problem. Given $f:\mathbb{R}^n \to \mathbb{R}$ and a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Example. Is the function

$$f(\mathbf{x}) = x_1 x_2 - (x_1 + x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2$$

always positive for $x_1, x_2 \in [-1, 1]$?

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= [1,8] \\ [-1,3]\cdot[2,5] &= [-5,15] \\ \sin([0,2]) &= [0,1] \end{aligned}$$

The interval extension of

$$
\begin{aligned}
f(x_1, x_2) &= x_1 \cdot x_2 - (x_1 + x_2) \cdot \cos x_2 \\
&\quad + \sin x_1 \cdot \sin x_2 + 2
\end{aligned}
$$

is

$$
\begin{aligned}
[f]([x_1], [x_2]) &= [x_1] \cdot [x_2] - ([x_1] + [x_2]) \cdot \cos [x_2] \\
&\quad + \sin [x_1] \cdot \sin [x_2] + 2.
\end{aligned}
$$

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^+ \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0$$

Theorem (Moore-Brouwer)
For $\mathbf{n} : \mathbb{R}^n \to \mathbb{R}^n$, we have

$$[\mathbf{n}]([\mathbf{x}]) \subset [\mathbf{x}] \Rightarrow \exists \mathbf{x} \in [\mathbf{x}], \mathbf{n}(\mathbf{x}) = \mathbf{x}.$$

Bracketting sets

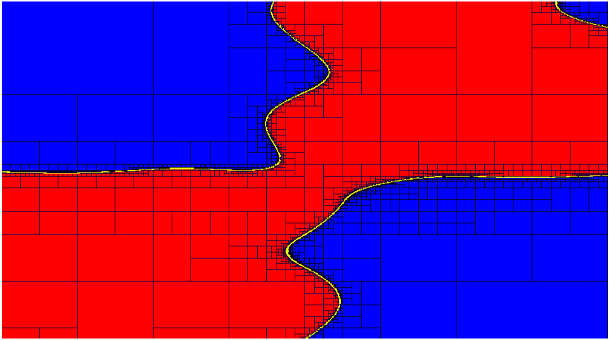
Subsets $\mathbb{X} \subset \mathbb{R}^n$ can be bracketed by subpavings :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

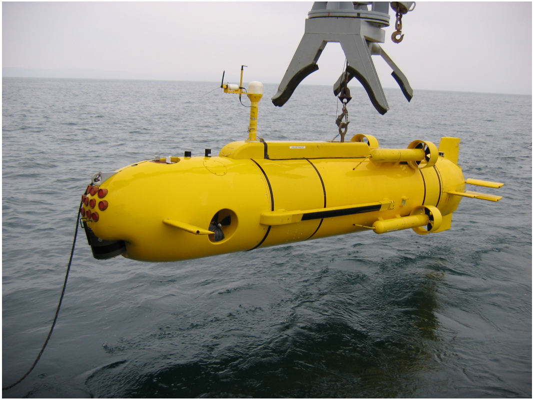
which can be obtained using interval calculus

Example.

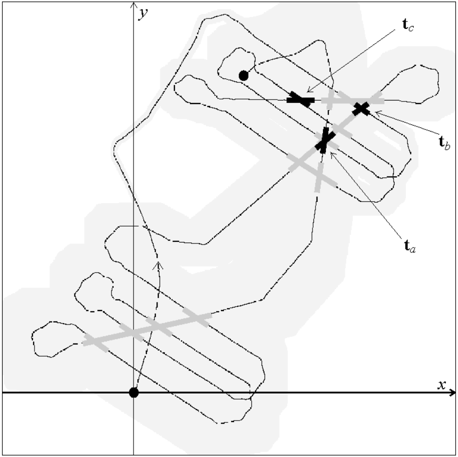
$$\mathbb{X} = \{\mathbf{x} \mid x_1 x_2 - (x_1 + x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2 \geq 0\}.$$



4. Test-case



Redermor, DGA-TN


y
x
$\mathbf{t}_c$
$\mathbf{t}_b$
$\mathbf{t}_a$

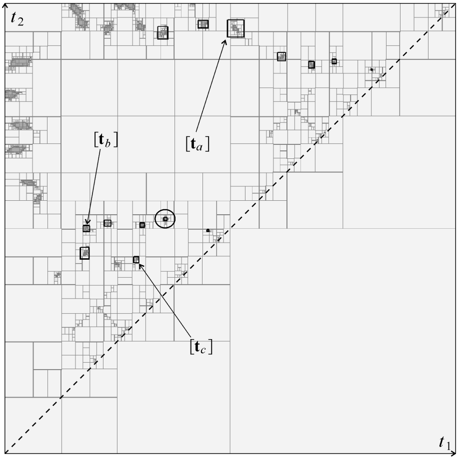
Loop set

The robot knows a box $[\mathbf{v}](t)$ for $\mathbf{v}(t)$. We have

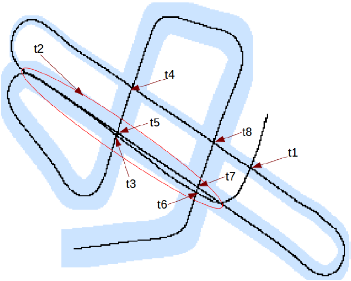
$$\mathbb{T} = \left\{ (t_1, t_2) \in [0, t_{\max}]^2 \mid \exists \mathbf{v} \in [\mathbf{v}], \int_{t_1}^{t_2} \mathbf{v}(\tau) d\tau = \mathbf{0}, t_1 < t_2 \right\}$$

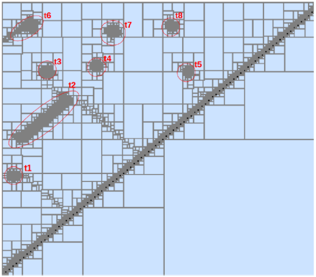
Thus $\mathbb{T}$ is defined by

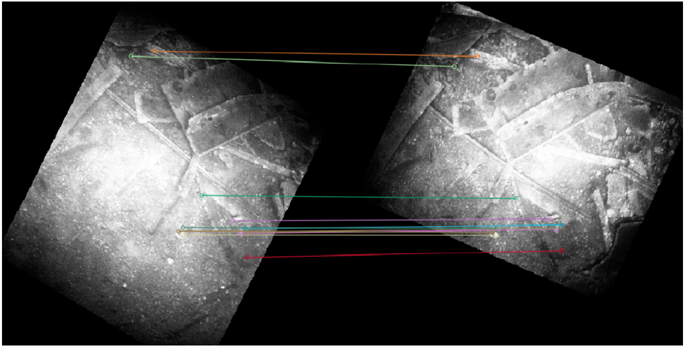
$$\mathbf{h}(t_1,t_2) = \begin{pmatrix} \int_{t_1}^{t_2} \mathbf{v}^-(\tau)d\tau \\ -\int_{t_1}^{t_2} \mathbf{v}^+(\tau)d\tau \\ t_1 - t_2 \end{pmatrix} < \mathbf{0}.$$


t_2
$[\mathbf{t}_b]$
$[\mathbf{t}_a]$
$[\mathbf{t}_c]$
t_1

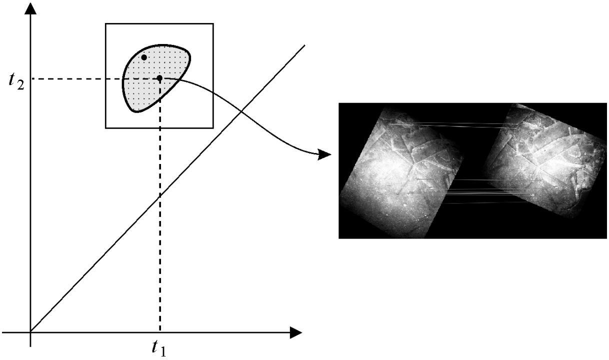
Mosaic


t2
t4
t5
t8
t1
t3
t7
t6

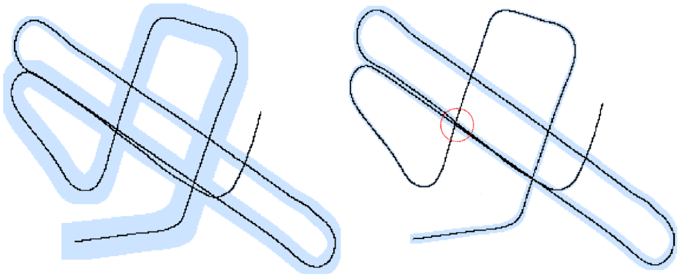

t6
t7
t8
t3
t4
t5
t2
t1



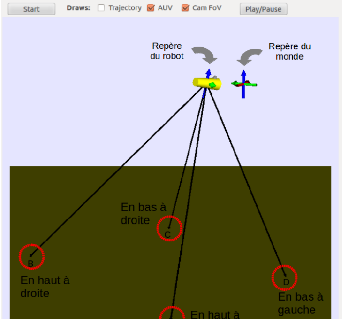
Compatible or incompatible ?

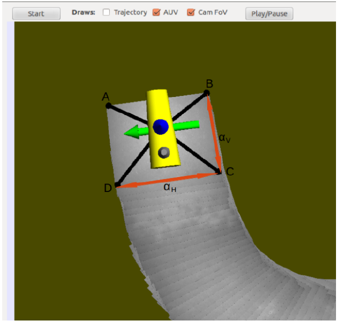

t_2
t_1

Contract the tube

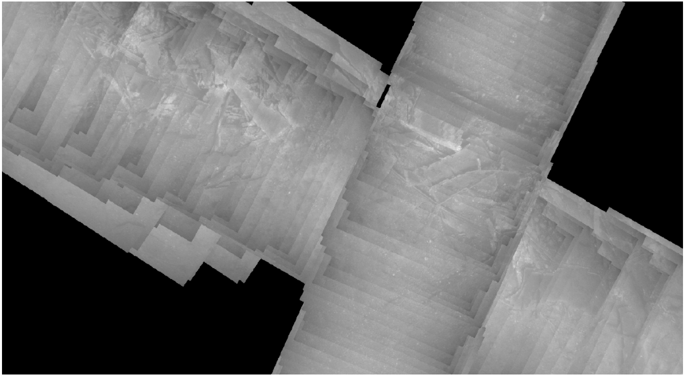


Projection

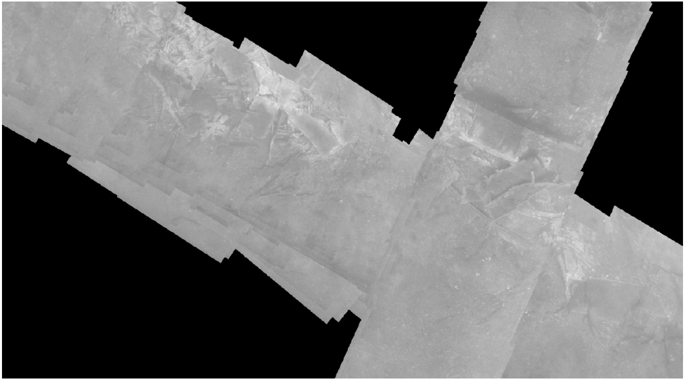

Start
Draws:
Trajectory
AUV
Cam FoV
Play/Pause
Repère du robot
Repère du monde
En bas à droite
C
B
En haut à droite
D
En bas à gauche
En haut à


Start
Draws:
Trajectory
AUV
Cam FoV
Play/Pause
A
B
C
D
α_V
α_H

Illumination Equalization



Before illumination equalization



After illumination equalization

References

1. Interval analysis [4]
2. Tubes [5][2]
3. Loop certification [1][6]
4. Underwater mosaic [3]

A video of the presentation is available at

http://youtu.be/sPKOBunlBEM

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