Optimal control tuning of a redundant robot

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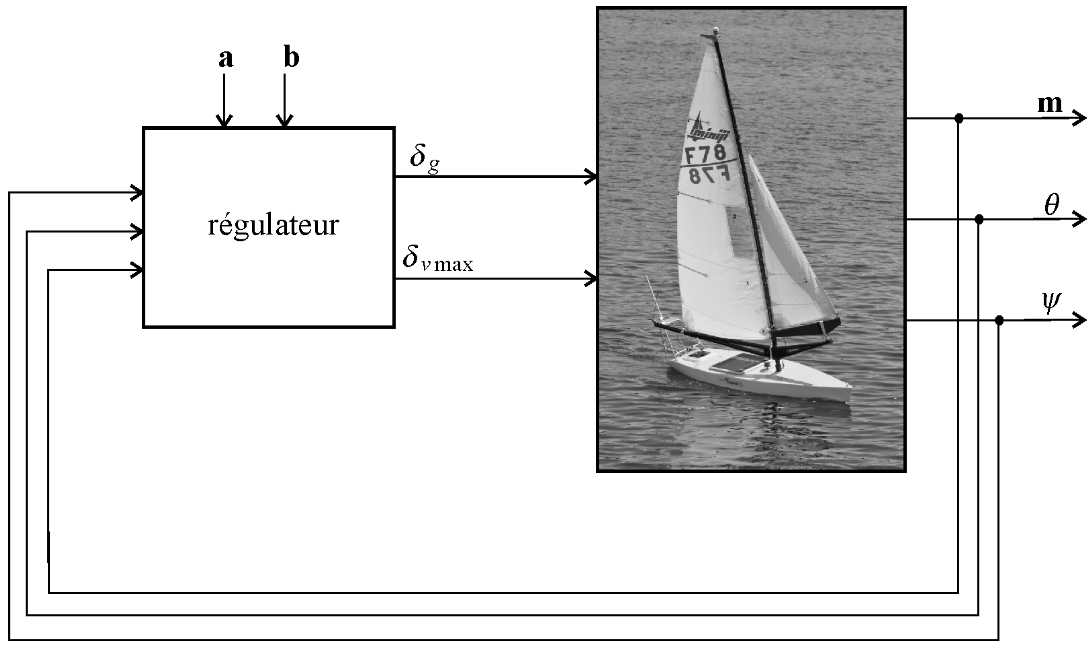
1 Motivations



a
b
régulateur
δ_g
$\delta_{v\max}$
m
θ
ψ

2 Formalism

A mobile robot is described by

$$\left\{ \begin{array}{lcl} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) \end{array} \right.$$

where $\mathbf{u} \in \mathbb{R}^m$ are the inputs, $\mathbf{x} \in \mathbb{R}^n$ the state, $\mathbf{y} \in \mathbb{R}^p$ is the variables to be controlled.

If $m > p$ the robot is overactuated.

We then want to maximize a performance criterion $h(\mathbf{x})$.

In operating conditions ($\dot{\mathbf{x}} = \mathbf{0}$), the optimization problem is

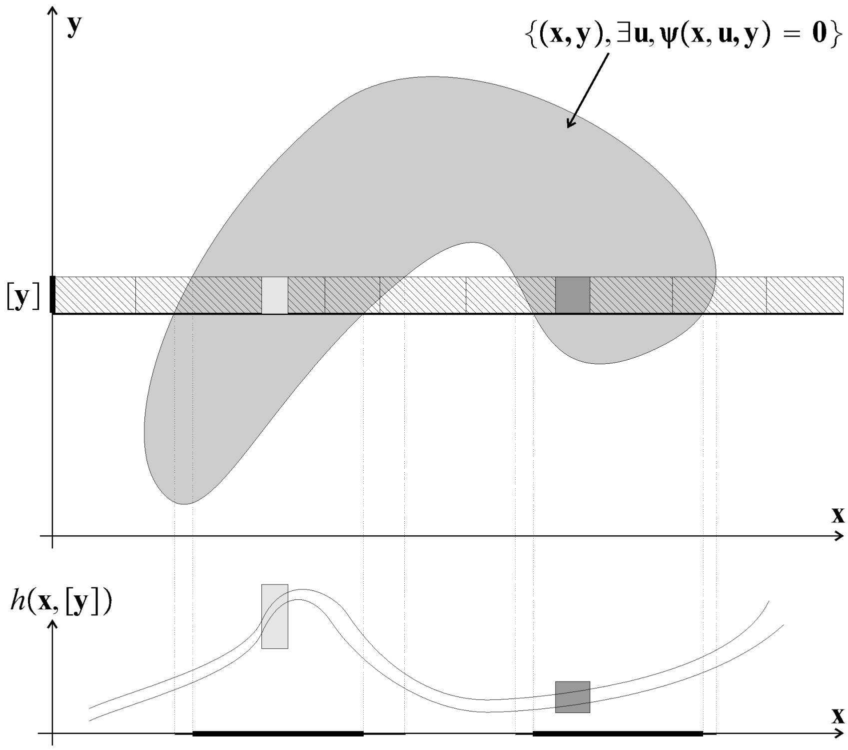
$$\hat{h}\left(\bar{\mathbf{y}}\right) = \max_{\bar{\mathbf{u}} \in \mathbb{R}^m, \bar{\mathbf{x}} \in \mathbb{R}^n} h\left(\bar{\mathbf{x}}\right) \qquad \text{s.t.} \; \left\{ \begin{array}{rcl} \mathbf{0} & = & \mathbf{f}\left(\bar{\mathbf{x}}, \bar{\mathbf{u}}\right) \\ \bar{\mathbf{y}} & = & \mathbf{g}\left(\bar{\mathbf{x}}\right). \end{array} \right.$$

We have $n + p$ equations for $n + p + m$ variables.

3 Parametric optimization

$$\begin{aligned}\hat{h}(\mathbf{y}) = &\max_{\mathbf{u}\in\mathbb{R}^m,\mathbf{x}\in\mathbb{R}^n} h(\mathbf{x},\mathbf{u},\mathbf{y}) \\ &\text{s.t. } \boldsymbol{\psi}(\mathbf{x},\mathbf{u},\mathbf{y}) = \mathbf{0}\end{aligned}$$

with $\dim \boldsymbol{\psi} = \dim \mathbf{u} = m$.



y
{(x, y), ∃u, ψ(x, u, y) = 0}
[y]
x
h(x, [y])
x

We need an inner test to prove that

$$[\mathbf{x}] \times [\mathbf{y}] \subset \underbrace{\{(\mathbf{x}, \mathbf{y}), \exists \mathbf{u}, \boldsymbol{\psi}(\mathbf{x}, \mathbf{u}, \mathbf{y}) = \mathbf{0}\}}_{\mathbb{S}_{\mathbf{xy}}}.$$

4 Newton inner test

Given a box $[\mathbf{p}]$, we need to be able to prove that

$$\forall \mathbf{p} \in [\mathbf{p}], \exists \mathbf{u} \in [\mathbf{u}], \boldsymbol{\psi}(\mathbf{u}, \mathbf{p}) = \mathbf{0}.$$

with $\dim \boldsymbol{\psi} = \dim \mathbf{u}$.

Parametric interval Newton method

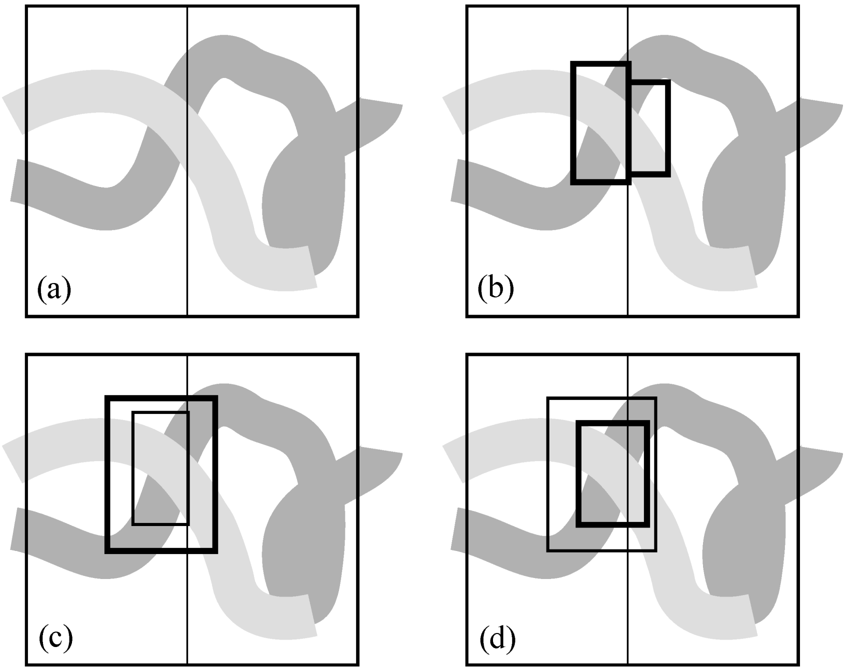
Define

$$\mathcal{N}_{\psi}\left([\mathbf{u}], [\mathbf{p}]\right) = \hat{\mathbf{u}} - \left[\frac{\partial \boldsymbol{\psi}}{\partial \mathbf{u}}\left([\mathbf{u}], [\mathbf{p}]\right)\right]^{-1} \cdot [\boldsymbol{\psi}]\left(\hat{\mathbf{u}}, [\mathbf{p}]\right).$$

We have

$$\mathcal{N}_{\psi}\left([\mathbf{u}], [\mathbf{p}]\right) \subset [\mathbf{u}] \;\Rightarrow\; \forall \mathbf{p} \in [\mathbf{p}],\ \exists! \mathbf{u} \in [\mathbf{u}], \boldsymbol{\psi}(\mathbf{u}, \mathbf{p}) = \mathbf{0}.$$

Epsilon inflation



5 Sailboat

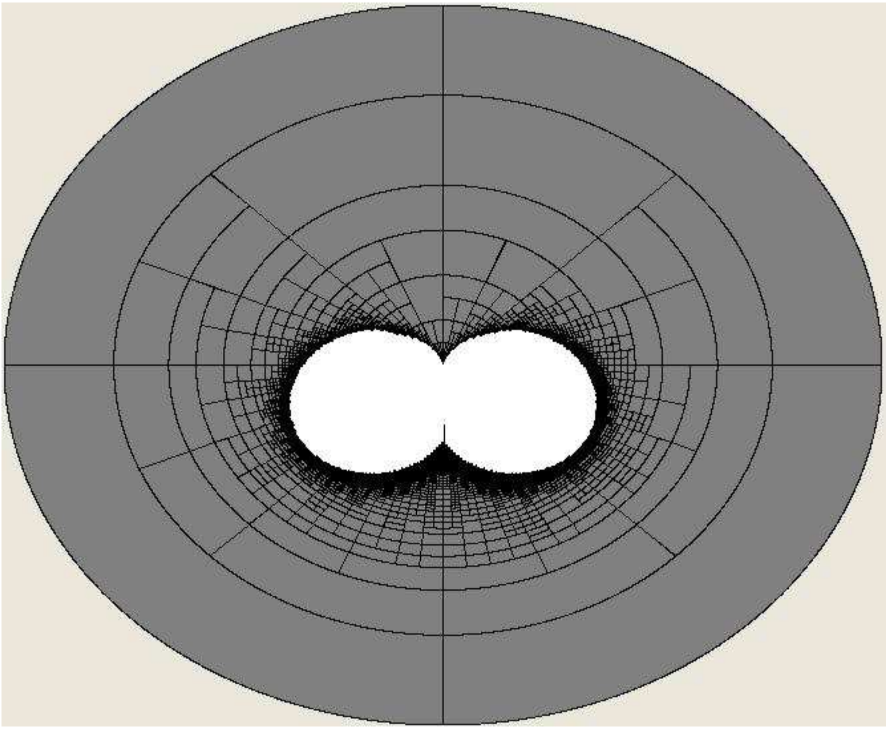
Two inputs: the sail angle u_1 and the rudder angle u_2.

The output is the heading θ.

The variable to be maximized is v.

The optimization problem is

$$\hat{v}(\theta) = \max_{\mathbf{u}\in\mathbb{R}^2, v\in\mathbb{R}} \quad v$$
$$\text{s.t.} \quad \begin{cases} 0 &= \sin u_1 \left(\cos(\theta + u_1) - v \sin u_1\right) - v \sin^2 u_2 - v \\ 0 &= (1 - \cos u_1)\left(\cos(\theta + u_1) - v \sin u_1\right) - v \frac{\sin 2u_2}{2}. \end{cases}$$



5th edition of the Small Workshop on Interval Methods
SWIM 2012
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http://hs.informatik.uni-oldenburg.de/swim2012