Modelisation, control and guidance of a quadrotor

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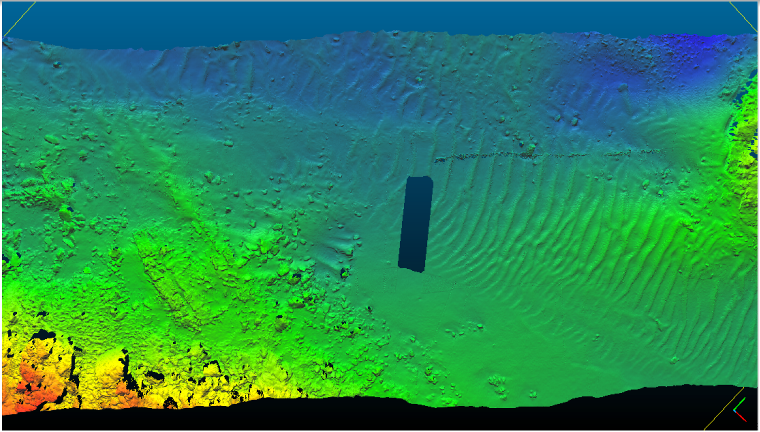
Video available at: youtu.be/MlnagaOBSiY

Guerlédan

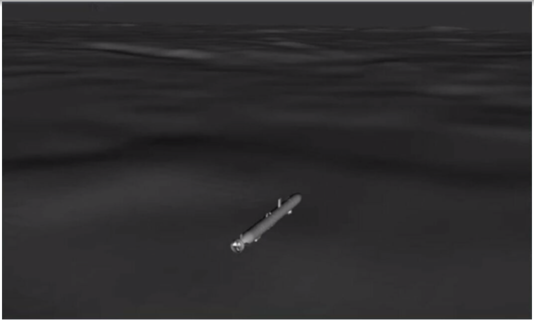
La Cordelière

Reconstitution de la bataille

youtu.be/yP4cM1UGrqY



Pris au R2Sonic à 700kHz.



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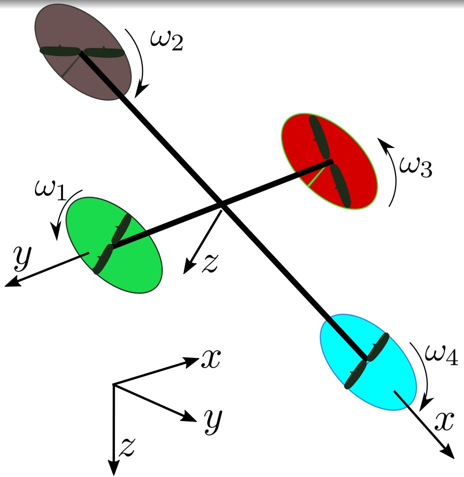
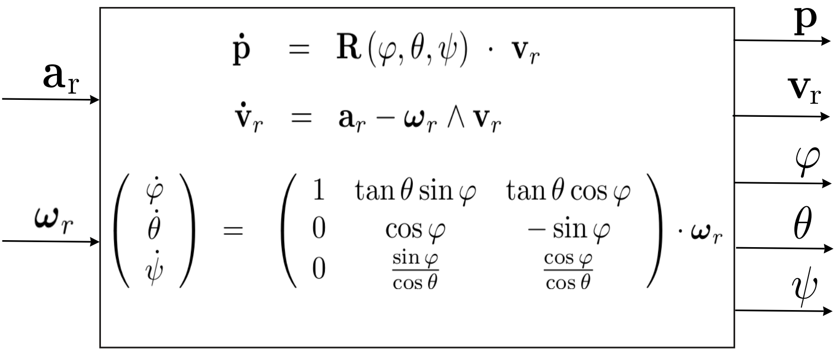
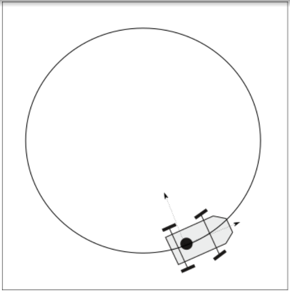
Quadrotor

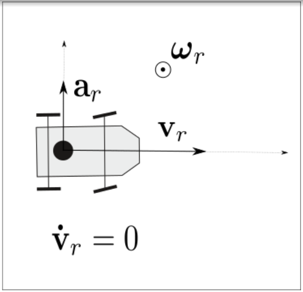

ω_2
ω_3
ω_1
y
z
ω_4
x
x
y
z

Inertial unit



Inertial unit [1][2]





$$\dot{\mathbf{v}}_r = \mathbf{a}_r - \omega_r \wedge \mathbf{v}_r$$

$$\left\{\begin{array}{ccc} \dot{\mathbf{p}} & = & \mathbf{R}(\varphi,\theta,\psi)\cdot\mathbf{v}_r \\ \begin{pmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix} & = & \begin{pmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta}\end{pmatrix}\cdot\omega_r \\ \dot{\mathbf{v}}_r & = & \mathbf{R}^T(\varphi,\theta,\psi)\cdot\begin{pmatrix}0\\0\\g\end{pmatrix}+\begin{pmatrix}0\\0\\-\frac{\tau_0}{m}\end{pmatrix}-\omega_r\wedge\mathbf{v}_r \\ \dot{\omega}_r & = & \mathbf{I}^{-1}\cdot\left(\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix}-\omega_r\wedge(\mathbf{I}\cdot\omega_r)\right) \end{array}\right.$$

where

$$\begin{pmatrix} \tau_0 \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \beta & \beta & \beta & \beta \\ -\beta\ell & 0 & \beta\ell & 0 \\ 0 & -\beta\ell & 0 & \beta\ell \\ -\delta & \delta & -\delta & \delta \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \cdot |\omega_1| \\ \omega_2 \cdot |\omega_2| \\ \omega_3 \cdot |\omega_3| \\ \omega_4 \cdot |\omega_4| \end{pmatrix}$$

τ_0 is the total thrustτ_1, τ_2, τ_3 are the torques , β is the thrust factor, δ is the drag factor and ℓ the distance between any rotor to the center.

Newton-Euler laws

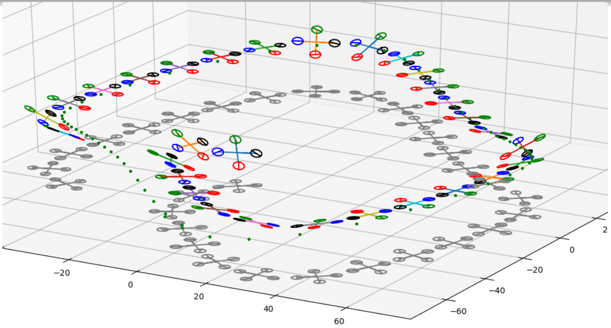
The Newton's second law for translation is

$$m\mathbf{a}_r = \mathbf{R}^\mathsf{T}(\varphi, \theta, \psi) \cdot \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\tau_0 \end{pmatrix}$$

The *Euler's rotation equation*,

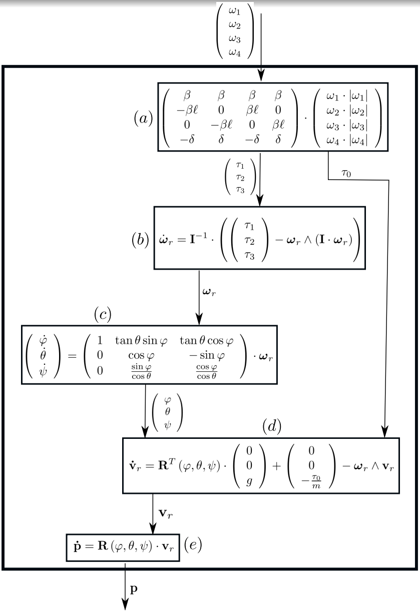
$$\mathbf{I}\dot{\omega}_r + \omega_r \wedge (\mathbf{I}\omega_r) = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix}.$$

Control



Quadrotor which follows the Van der Pol cycle

Causal chain for backstepping conctrol


$\begin{pmatrix}\omega_1\\\omega_2\\\omega_3\\\omega_4\end{pmatrix}$
(a) $\begin{pmatrix}\beta & \beta & \beta & \beta\\ -\beta\ell & 0 & \beta\ell & 0\\ 0 & -\beta\ell & 0 & \beta\ell\\ -\delta & \delta & -\delta & \delta\end{pmatrix} \cdot \begin{pmatrix}\omega_1\cdot|\omega_1|\\ \omega_2\cdot|\omega_2|\\ \omega_3\cdot|\omega_3|\\ \omega_4\cdot|\omega_4|\end{pmatrix}$
$\begin{pmatrix}\tau_1\\\tau_2\\\tau_3\end{pmatrix}$
τ_0
(b) $\dot{\boldsymbol{\omega}}_r = \mathbf{I}^{-1}\cdot\left(\begin{pmatrix}\tau_1\\\tau_2\\\tau_3\end{pmatrix} - \boldsymbol{\omega}_r \wedge (\mathbf{I}\cdot\boldsymbol{\omega}_r)\right)$
$\boldsymbol{\omega}_r$
(c)
$\begin{pmatrix}\dot\varphi\\\dot\theta\\\dot\psi\end{pmatrix} = \begin{pmatrix}1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi\\ 0 & \cos\varphi & -\sin\varphi\\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta}\end{pmatrix}\cdot\boldsymbol{\omega}_r$
$\begin{pmatrix}\varphi\\\theta\\\psi\end{pmatrix}$
(d)
$\dot{\mathbf{v}}_r = \mathbf{R}^T(\varphi,\theta,\psi)\cdot\begin{pmatrix}0\\0\\g\end{pmatrix} + \begin{pmatrix}0\\0\\-\frac{\tau_0}{m}\end{pmatrix} - \boldsymbol{\omega}_r\wedge\mathbf{v}_r$
$\mathbf{v}_r$
$\dot{\mathbf{p}} = \mathbf{R}(\varphi,\theta,\psi)\cdot\mathbf{v}_r$ (e)
$\mathbf{p}$

1) Feed forward controller with the new input $\left(\tau_0^d, \tau_1^d, \tau_2^d, \tau_2^d\right)$ with eliminates the effect of Block *(a)*:

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \sqrt[a]{\left(\begin{pmatrix} \beta & \beta & \beta & \beta \\ -\beta\ell & 0 & \beta\ell & 0 \\ 0 & -\beta\ell & 0 & \beta\ell \\ -\delta & \delta & -\delta & \delta \end{pmatrix}^{-1} \begin{pmatrix} \tau_0^d \\ \tau_1^d \\ \tau_2^d \\ \tau_3^d \end{pmatrix}\right)}$$

2) Using a feedback linearization approach, we take

$$\tau^d_{1:3} = \mathbf{I} \cdot k_\omega . \left(\omega^d_r - \omega_r\right) + \omega_r \wedge (\mathbf{I} \cdot \omega_r)$$

Therefore

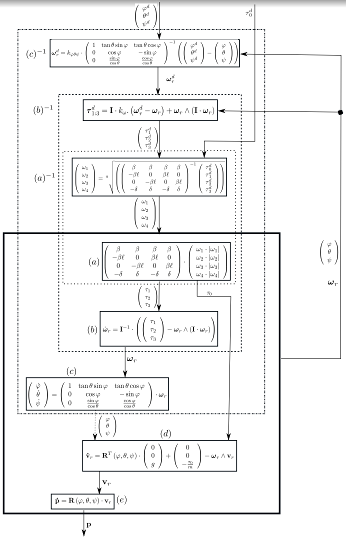
$$\begin{aligned} \dot{\omega}_r &= \mathbf{I}^{-1} \cdot \left(\tau^d_{1:3} - \omega_r \wedge (\mathbf{I} \cdot \omega_r)\right) \\ &= \mathbf{I}^{-1} \cdot \left(\mathbf{I} \cdot k_\omega . \left(\omega^d_r - \omega_r\right) + \omega_r \wedge (\mathbf{I} \cdot \omega_r) - \omega_r \wedge (\mathbf{I} \cdot \omega_r)\right) \\ &= k_\omega . \left(\omega^d_r - \omega_r\right) \end{aligned}$$

3) We now build a controller with a desired input $\left(\varphi^d, \theta^d, \psi^d\right)$ which and an output ω_r^d, such that the vector (φ, θ, ψ) converges to $\left(\varphi^d, \theta^d, \psi^d\right)$.

$$\omega_r^d = k_{\varphi\theta\psi} \cdot \underbrace{\begin{pmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta} \end{pmatrix}^{-1}}_{\mathbf{T}^{-1}} \left(\begin{pmatrix} \varphi^d \\ \theta^d \\ \psi^d \end{pmatrix} - \begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix} \right)$$

Thus, we have

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \mathbf{T} \cdot \omega_r^d = k_{\varphi\theta\psi} \cdot \left(\begin{pmatrix} \varphi^d \\ \theta^d \\ \psi^d \end{pmatrix} - \begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix} \right)$$


$(c)^{-1}$
$\boldsymbol{\omega}_r^d = k_{\varphi\theta\psi} \cdot \begin{pmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta} \end{pmatrix}^{-1} \left(\begin{pmatrix} \varphi^d \\ \theta^d \\ \psi^d \end{pmatrix} - \begin{pmatrix} \varphi \\ \theta \\ \psi \end{pmatrix} \right)$
τ_0^d
$\boldsymbol{\omega}_r^d$
$(b)^{-1}$
$\boldsymbol{\tau}_{1:3}^d = \mathbf{I} \cdot k_\omega \cdot (\boldsymbol{\omega}_r^d - \boldsymbol{\omega}_r) + \boldsymbol{\omega}_r \wedge (\mathbf{I} \cdot \boldsymbol{\omega}_r)$
$(a)^{-1}$
$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \sqrt[a]{\left(\begin{pmatrix} \beta & \beta & \beta & \beta \\ -\beta\ell & 0 & \beta\ell & 0 \\ 0 & -\beta\ell & 0 & \beta\ell \\ -\delta & \delta & -\delta & \delta \end{pmatrix}^{-1} \begin{pmatrix} \tau_0^d \\ \tau_1^d \\ \tau_2^d \\ \tau_3^d \end{pmatrix} \right)}$
(a)
$\begin{pmatrix} \beta & \beta & \beta & \beta \\ -\beta\ell & 0 & \beta\ell & 0 \\ 0 & -\beta\ell & 0 & \beta\ell \\ -\delta & \delta & -\delta & \delta \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \cdot |\omega_1| \\ \omega_2 \cdot |\omega_2| \\ \omega_3 \cdot |\omega_3| \\ \omega_4 \cdot |\omega_4| \end{pmatrix}$
τ_0
(b)
$\dot{\boldsymbol{\omega}}_r = \mathbf{I}^{-1} \cdot \left(\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} - \boldsymbol{\omega}_r \wedge (\mathbf{I} \cdot \boldsymbol{\omega}_r) \right)$
$\boldsymbol{\omega}_r$
(c)
$\begin{pmatrix} \dot\varphi \\ \dot\theta \\ \dot\psi \end{pmatrix} = \begin{pmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \frac{\sin\varphi}{\cos\theta} & \frac{\cos\varphi}{\cos\theta} \end{pmatrix} \cdot \boldsymbol{\omega}_r$
(d)
$\dot{\mathbf{v}}_r = \mathbf{R}^T(\varphi,\theta,\psi) \cdot \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\frac{\tau_0}{m} \end{pmatrix} - \boldsymbol{\omega}_r \wedge \mathbf{v}_r$
$\mathbf{v}_r$
$\dot{\mathbf{p}} = \mathbf{R}(\varphi,\theta,\psi) \cdot \mathbf{v}_r$ (e)
$\mathbf{p}$

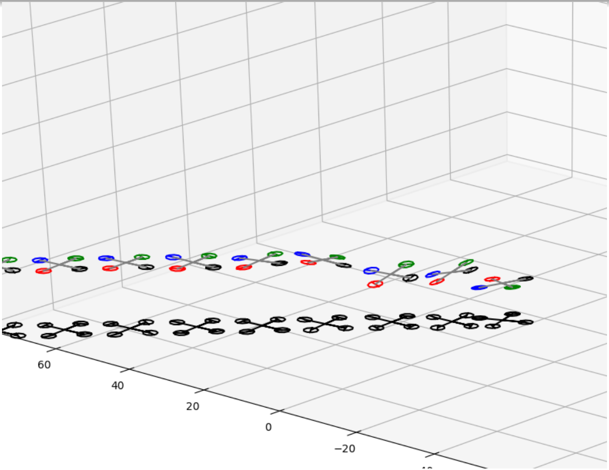
4) Last loop with artificial potential field method:

$$\begin{pmatrix} \varphi^d \\ \theta^d \\ \psi^d \\ \tau_0^d \end{pmatrix} = \dots$$

Altitude control

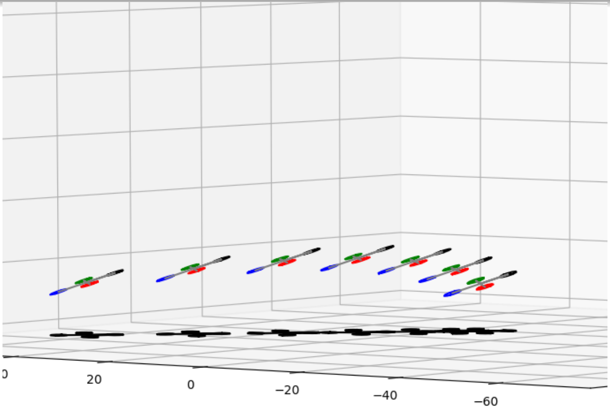
$$\tau_0^d = 300 \cdot \tanh(z - z_d) + 60 \cdot v_{r3}$$

Head control


60
40
20
0
−20

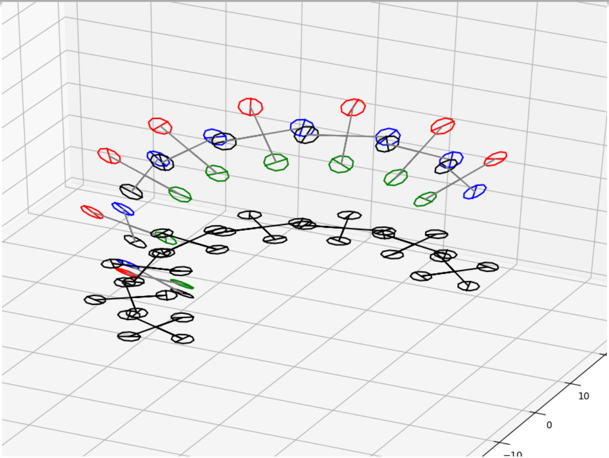
$$\psi^d = \text{angle}(\mathbf{R} \cdot \mathbf{v}_r)$$

Pitch control


20
0
−20
−40
−60

$$\theta^d = -0.3 \cdot \tanh(v_d - v_{r1})$$

Roll control


10
0
−10

$$\varphi^d = 0.5 \cdot \tanh(10 \cdot \text{sawtooth}(\text{angle}(\mathbf{f}_d) - \text{angle}(\mathbf{R} \cdot \mathbf{v}_r)))$$

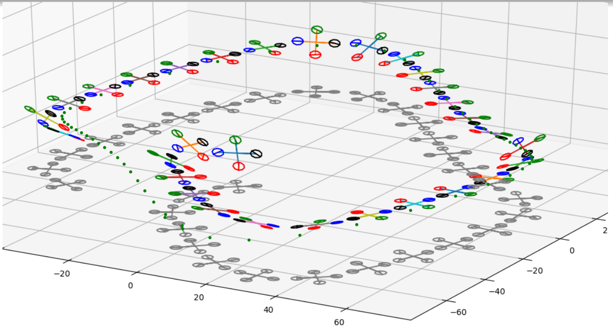
Van der Pol cycle

The quadrotor has to follow a path that obeys the Van der Pol equation:

$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -\left(0.001\ x_1^2 - 1\right) x_2 - x_1 \end{cases}$$

We add the following loop

$$
\begin{array}{rcl}
\varphi^d & = & 0.5 \cdot \tanh(10 \cdot \text{sawtooth}(\text{angle}\left(\mathbf{f}_{vdp}(x,y)\right) - \text{angle}(\mathbf{R} \cdot \mathbf{v}_r))) \\
\theta^d & = & -0.3 \cdot \tanh(v_d - v_{r1}) \\
\psi^d & = & \text{angle}(\mathbf{R} \cdot \mathbf{v}_r) \\
\tau_0^d & = & 300 \cdot \tanh(z - zd) + 60 \cdot v_{r3}
\end{array}
$$


−20
0
20
40
60
2
0
−20
−40
−60

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Automation for Robotics.
ISTE editions, 2015.

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