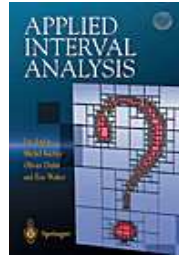


Méthodes ensemblistes pour la localisation et la cartographie dans un contexte de robotique sous-marine



Luc Jaulin, ENSIETA
Mercredi 22 Octobre 2008,
workshop du PEA action, LAAS, Toulouse.

1 Approche ensembliste

1.1 Calcul par intervalles

Si $\diamond \in \{+, -, ., /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3] \cdot [2, 5] &= [-5, 15], \\ [-1, 3] / [2, 5] &= [-\frac{1}{2}, \frac{3}{2}], \\ [-1, 3] \vee [2, 5] &= [2, 5]. \end{aligned}$$

$$\begin{aligned}
[x^-, x^+] + [y^-, y^+] &= [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] &= [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
&\quad x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+], \\
[x^-, x^+] \vee [y^-, y^+] &= [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\sin([0, \pi]) = [0, 1],$$

$$\text{sqr}([-1, 3]) = [-1, 3]^2 = [0, 9],$$

$$\text{abs}([-7, 1]) = [0, 7],$$

$$\text{sqrt}([-10, 4]) = \sqrt{[-10, 4]} = [0, 2],$$

$$\log([-2, -1]) = \emptyset.$$

1.2 Projection de contraintes

Soient x, y, z trois variables telles que

$$\begin{aligned}x &\in [-\infty, 5], \\y &\in [-\infty, 4], \\z &\in [6, \infty], \\z &= x + y.\end{aligned}$$

Les valeurs < 2 pour x , < 1 pour y et > 9 pour z sont inconsistantes.

1.3 Méthode numérique de projection

En effet, puisque $x \in [-\infty, 5]$, $y \in [-\infty, 4]$, $z \in [6, \infty]$ et $z = x + y$, nous avons

$$z = x + y \Rightarrow z \in [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ = [6, \infty] \cap [-\infty, 9] = [6, 9].$$

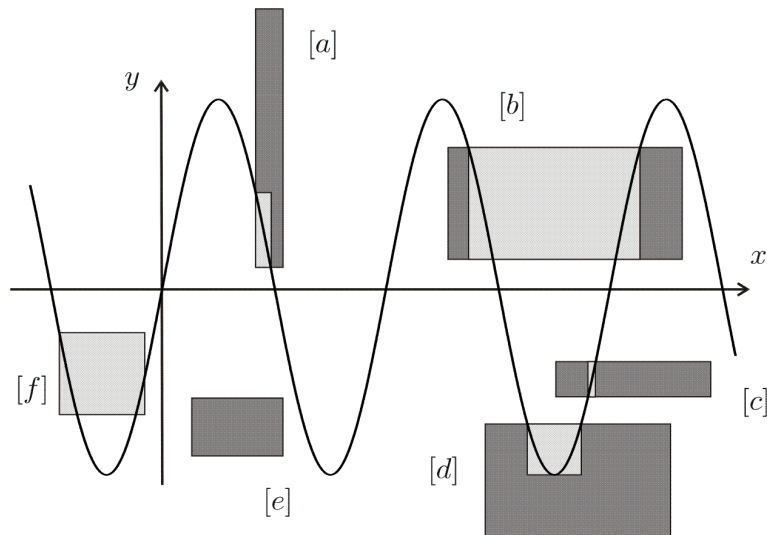
$$x = z - y \Rightarrow x \in [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ = [-\infty, 5] \cap [2, \infty] = [2, 5].$$

$$y = z - x \Rightarrow y \in [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ = [-\infty, 4] \cap [1, \infty] = [1, 4].$$

Pour la contrainte

$$y = \sin x, \quad x \in [x], y \in [y]$$

le problème est un peu plus difficile.



1.4 Algorithme de propagation-bissection

Exemple. Cherchons à résoudre.

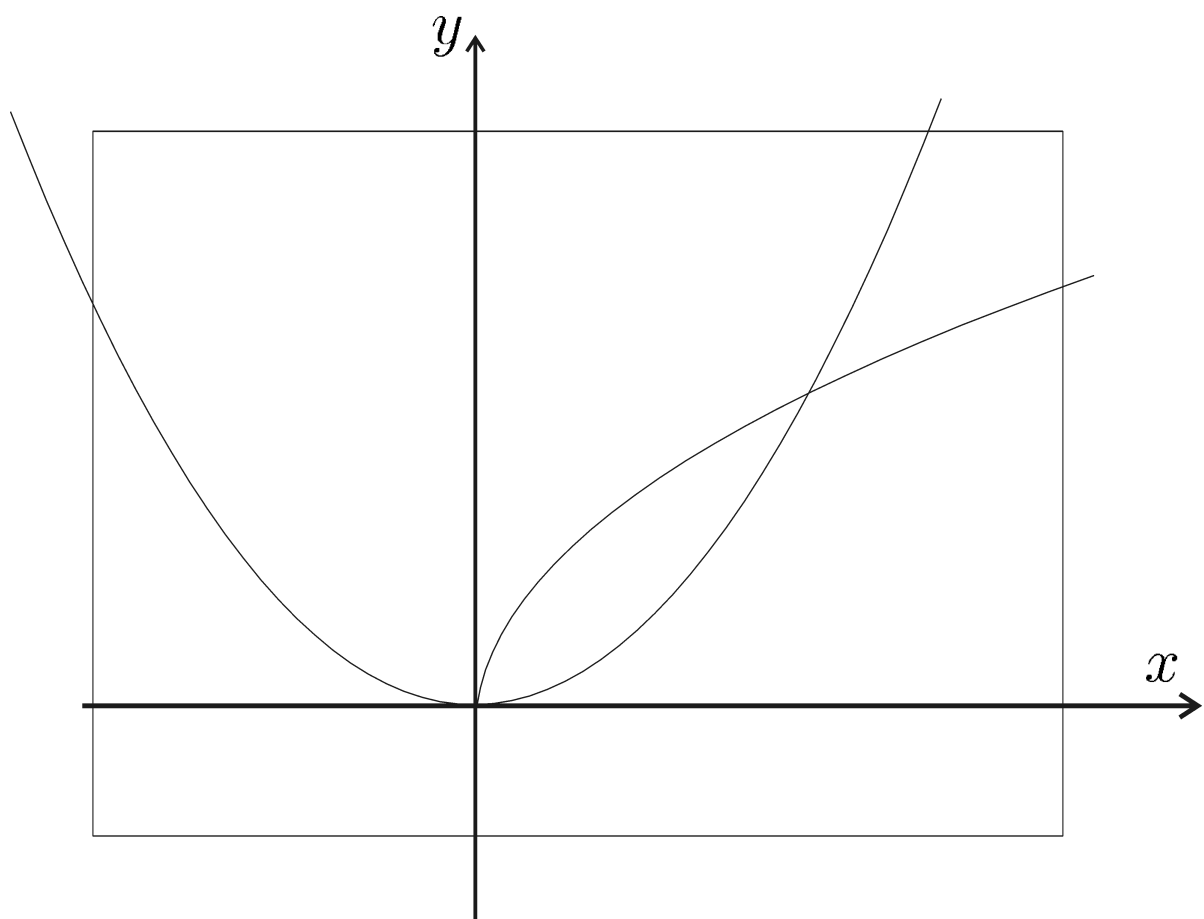
$$y = x^2$$

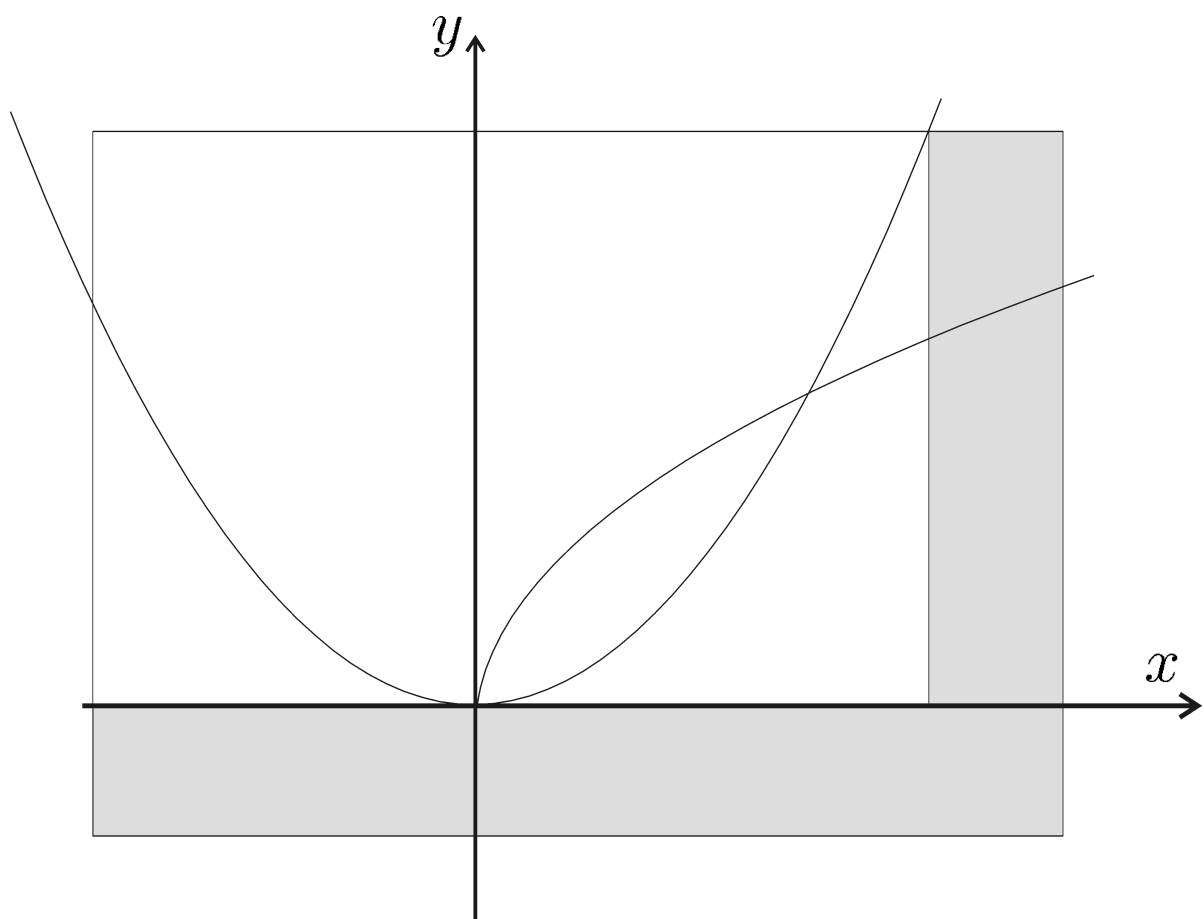
$$y = \sqrt{x}.$$

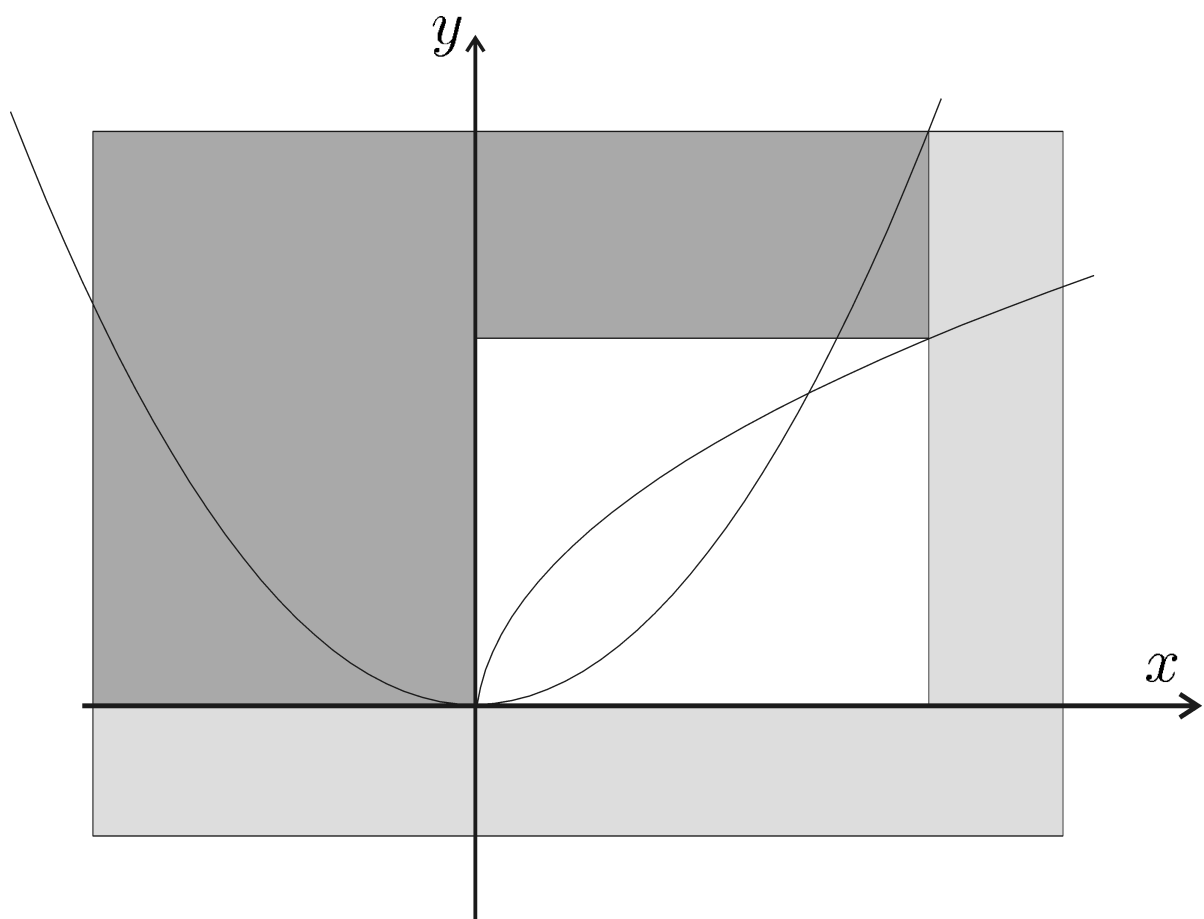
On a deux contracteurs

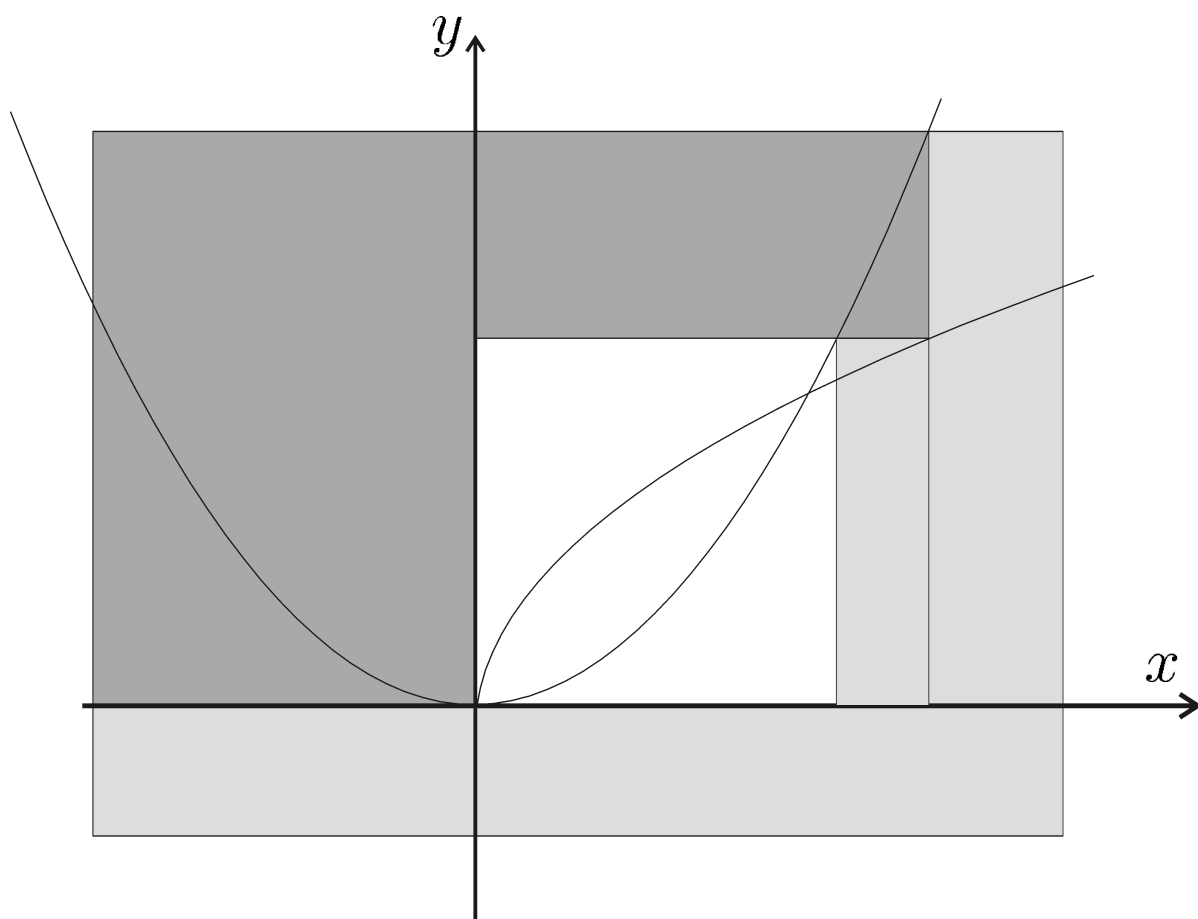
$$\mathcal{C}_1 : \begin{cases} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{cases} \quad \text{associé à } y = x^2$$

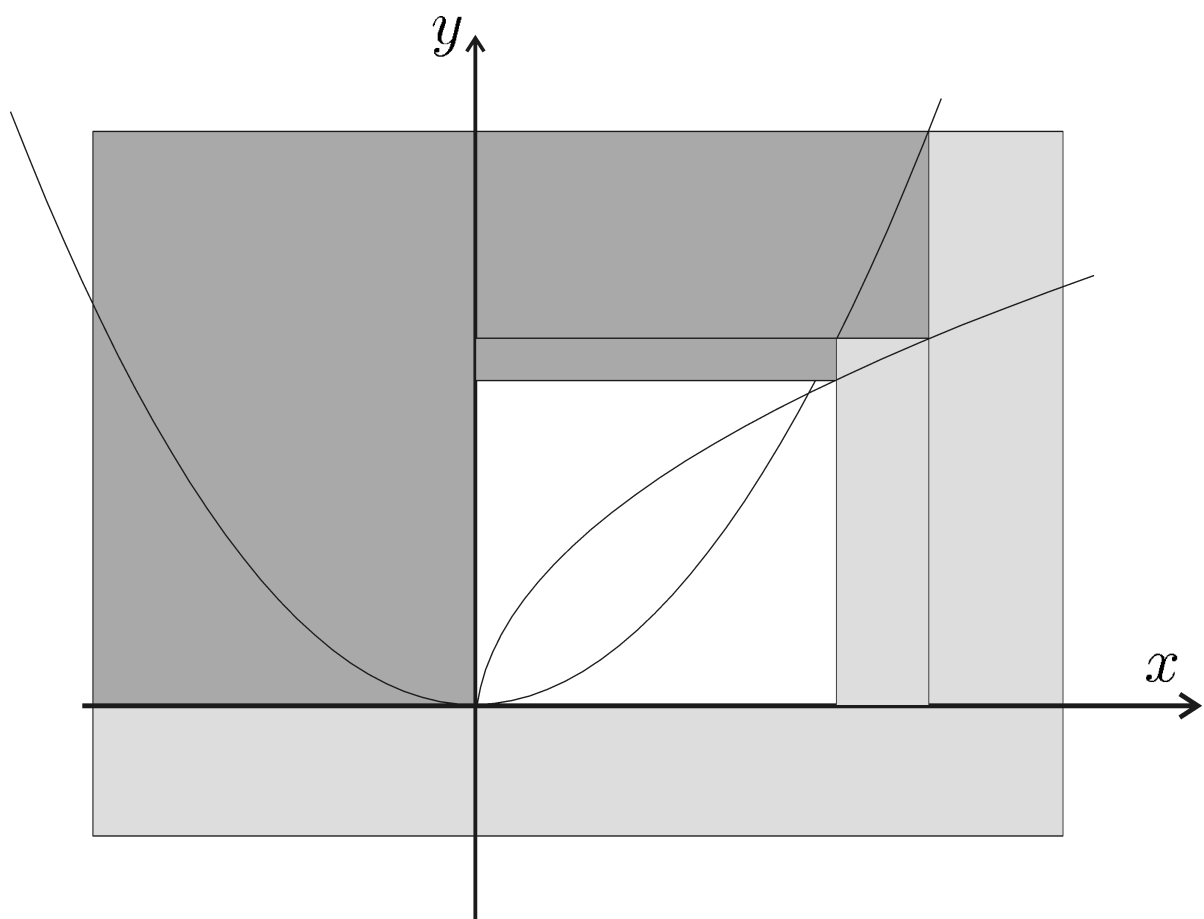
$$\mathcal{C}_2 : \begin{cases} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{cases} \quad \text{associé à } y = \sqrt{x}$$

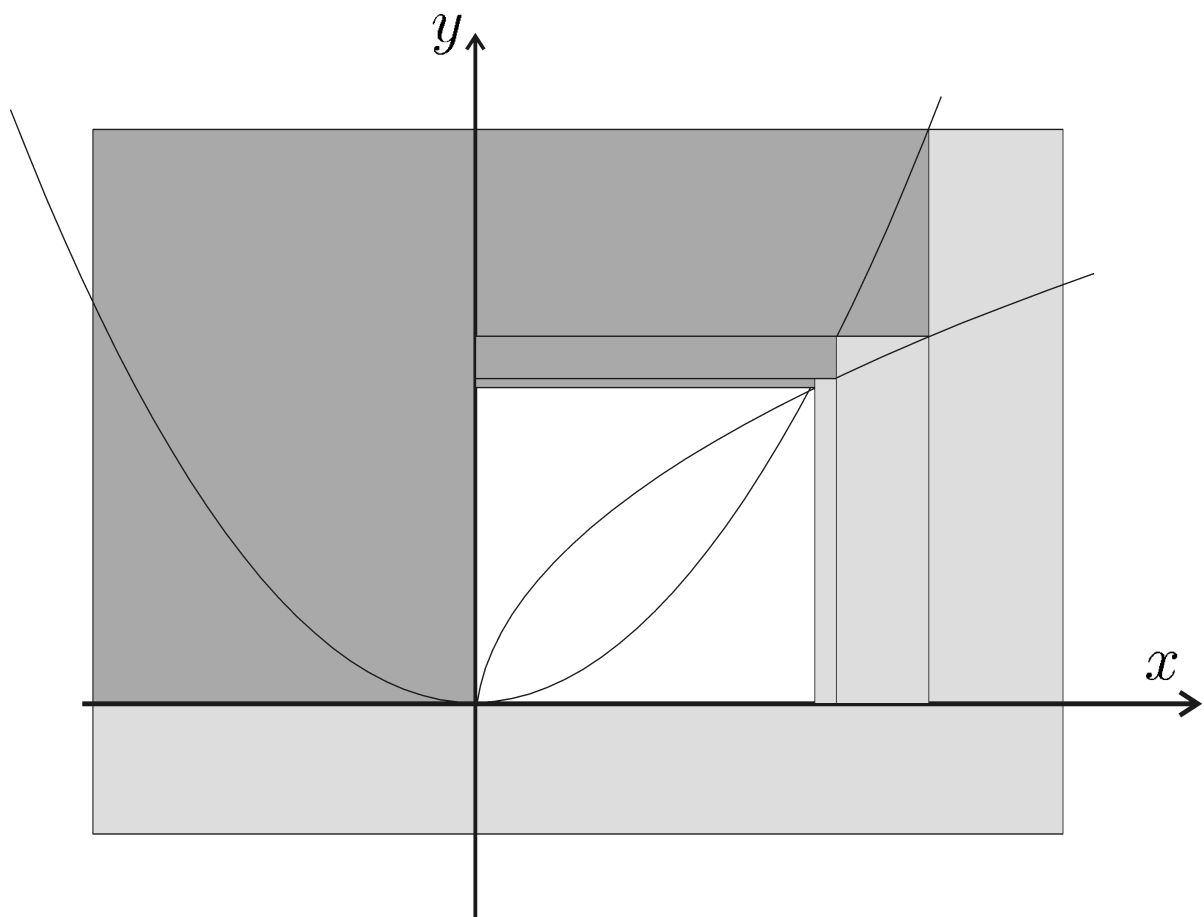


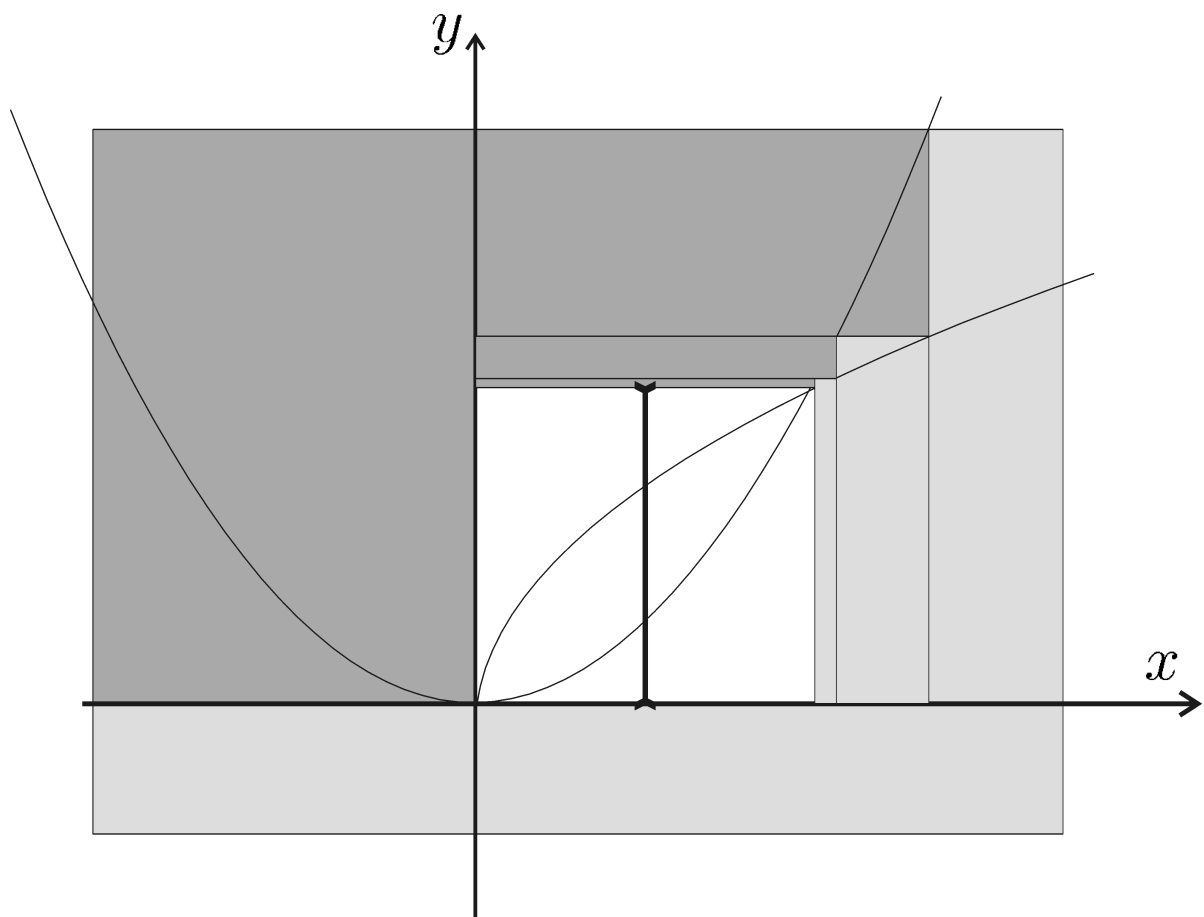


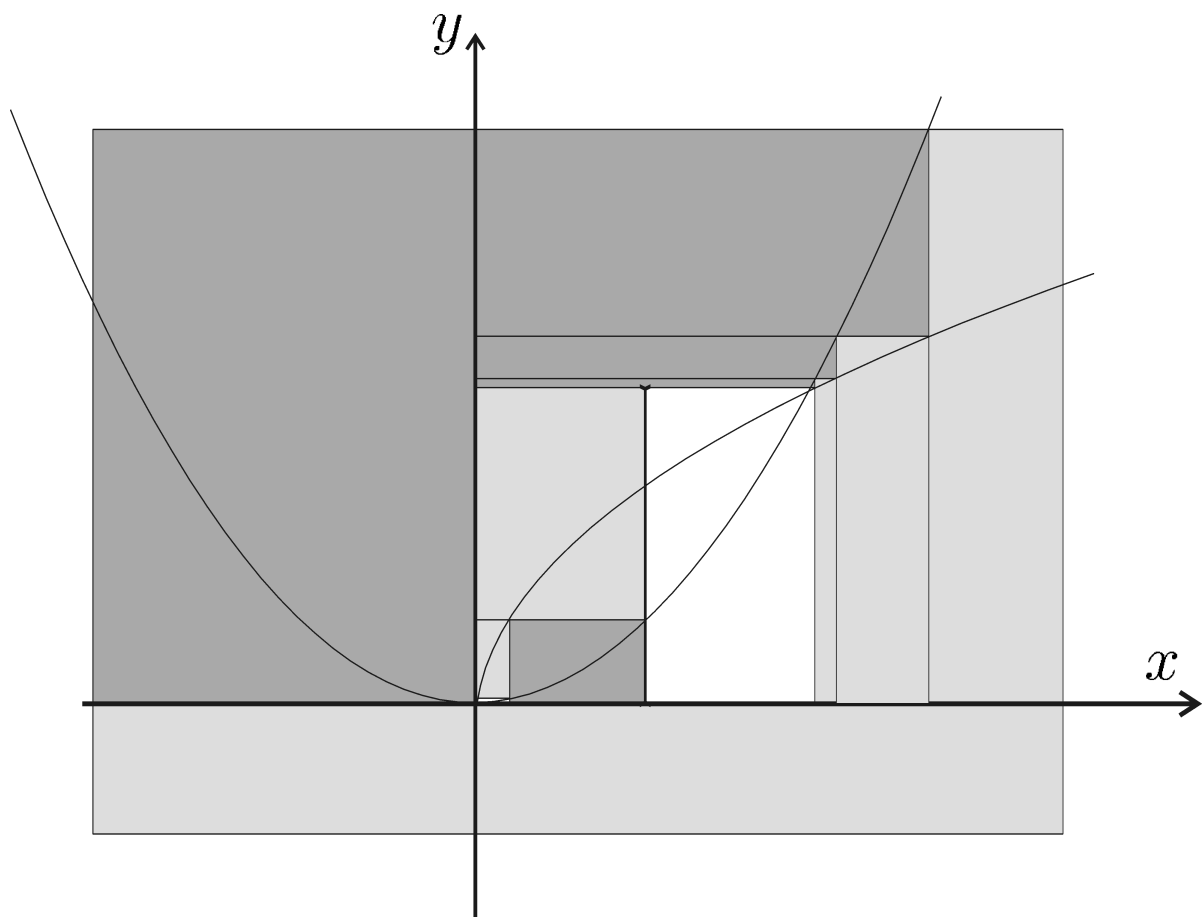


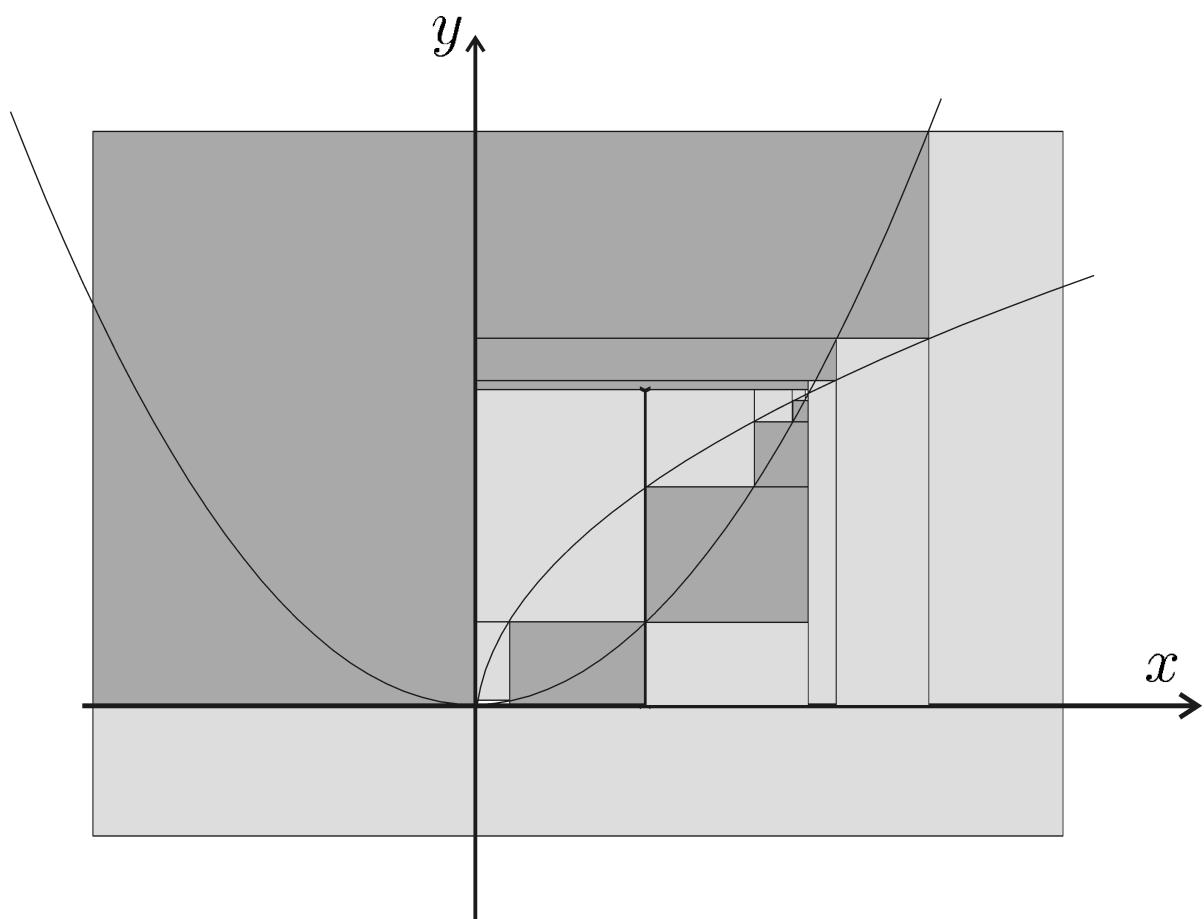












1.5 Décomposition

Pour les contraintes plus complexes, il nous faut effectuer une décomposition

$$\begin{aligned} x + \sin(y) - xz &\leq 0, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{aligned}$$

se décompose en

$$\left\{ \begin{array}{l} a = \sin(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array} \right., \quad \begin{array}{ll} x \in [-1, 1] & a \in]-\infty, \infty[\\ y \in [-1, 1] & b \in]-\infty, \infty[\\ z \in [-1, 1] & c \in]-\infty, \infty[\\ & d \in]-\infty, 0] \end{array}$$

1.6 QUIMPER

Quimper is a high-level language for QUick Interval Modeling and Programming in a bounded-ERror context.

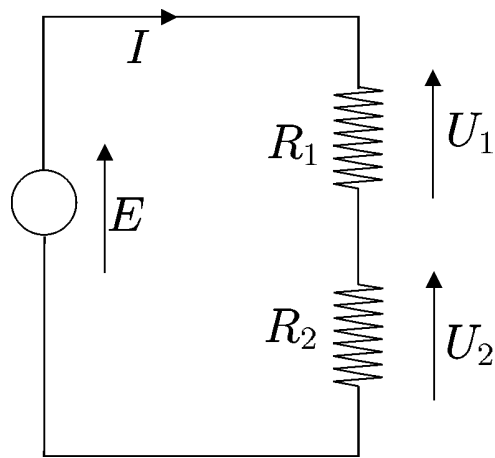
Quimper est un langage interprété pour le calcul ensembliste.

Un programme Quimper se décrit par un ensemble de contracteurs.

Logiciel libre disponible sur

<http://ibex-lib.org/>

2 Un exemple académique



Contraintes

$$\begin{aligned}P &= EI; E = (R_1 + R_2) I; \\U_1 &= R_1 I; U_2 = R_2 I; E = U_1 + U_2.\end{aligned}$$

Domaines initiaux

$$\begin{aligned}R_1 &\in [0, \infty]\Omega, & R_2 &\in [0, \infty]\Omega, \\E &\in [23, 26]\text{V}, & I &\in [4, 8]\text{A}, \\U_1 &\in [10, 11]\text{V}, & U_2 &\in [14, 17]\text{V}, \\P &\in [124, 130]\text{W},\end{aligned}$$

```
variables
  E in [23,26];
  I in [4,8];
  U1 in [10,11];
  U2 in [14,17];
  P in [124,130];
  R1 in [0,1e08];
  R2 in [0,1e08];
contractor_list L
  P=E*I;
  E=(R1+R2)*I;
  U1=R1*I;
  U2=R2*I;
  E=U1+U2;
end
```

```
contractor C
  compose(L);
end
contractor epsilon
  precision(1);
end
```

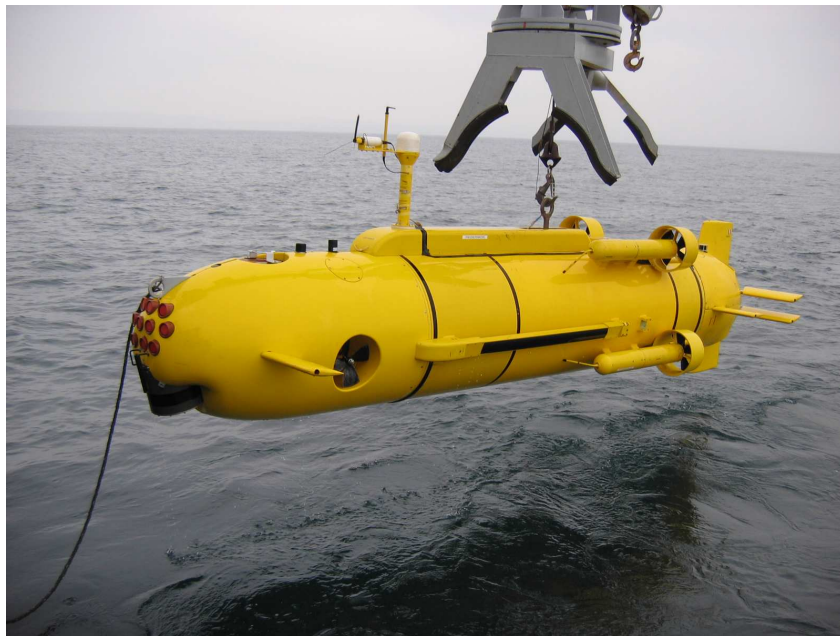
Quimper renvoie le pavé

$$[24; 26] \times [1.846; 2.307] \times [2.584; 3.355] \\ \times [4.769; 5.417] \times [10; 11] \times [14; 16] \times [124; 130] .$$

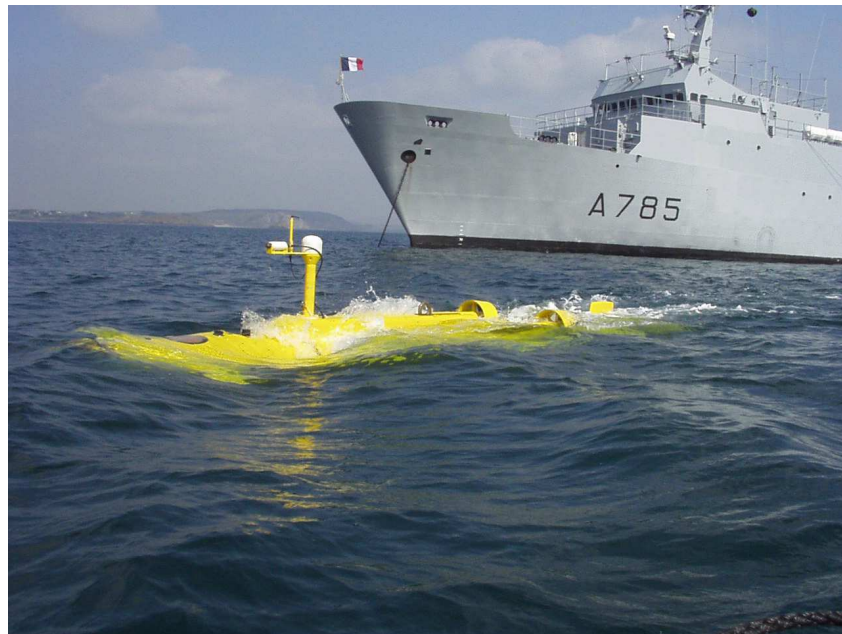
c'est-à-dire

$$\begin{array}{ll} E \in [24; 26] , & R_1 \in [1.846; 2.307] , \\ R_2 \in [2.584; 3.355] , & I \in [4.769; 5.417] , \\ U_1 \in [10; 11] , & U_2 \in [14; 16] , \\ P \in [124; 130] . & \end{array}$$

3 SLAM



Le *Redermor*, fabriqué par le GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Montrer la simulation

Pourquoi une approche par intervalles ?

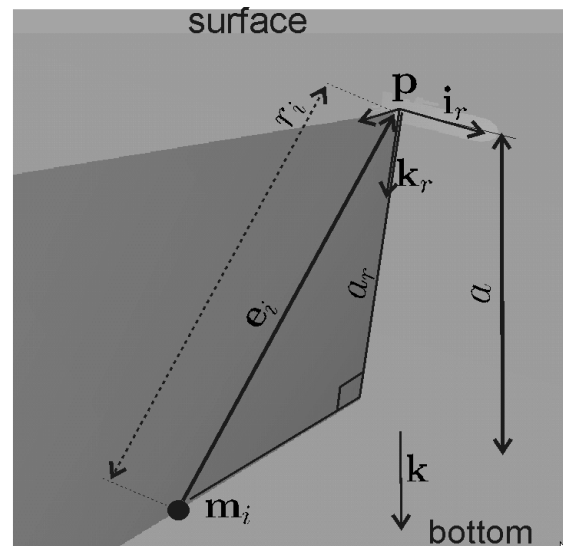
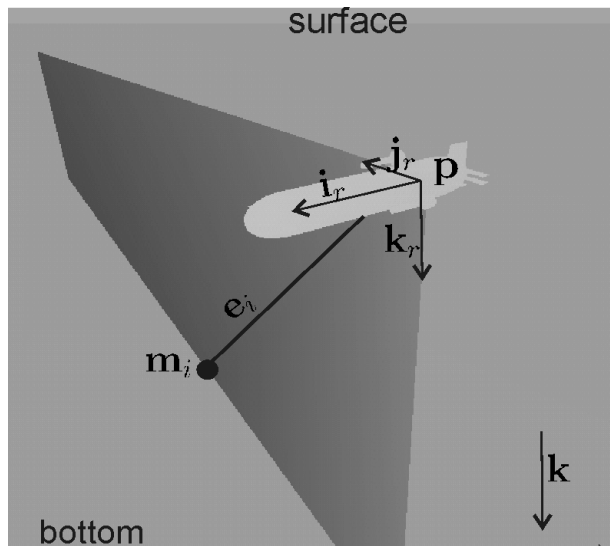
- 1) Besoin d'une approche fiable.
- 2) Les équations du robot sont non linéaires.
- 3) Les bruits de mesure sont non gaussiens.
- 4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
- 5) Beaucoup de mesures redondantes sont disponibles.

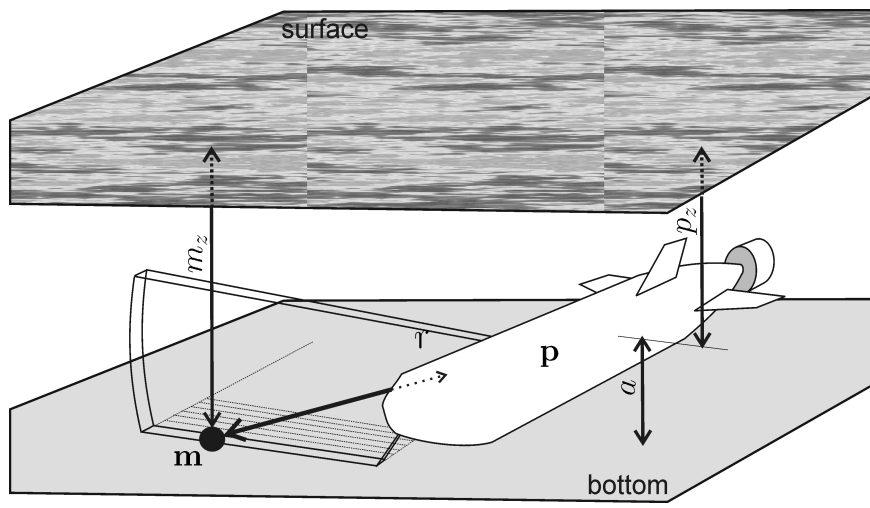
3.1 Capteurs

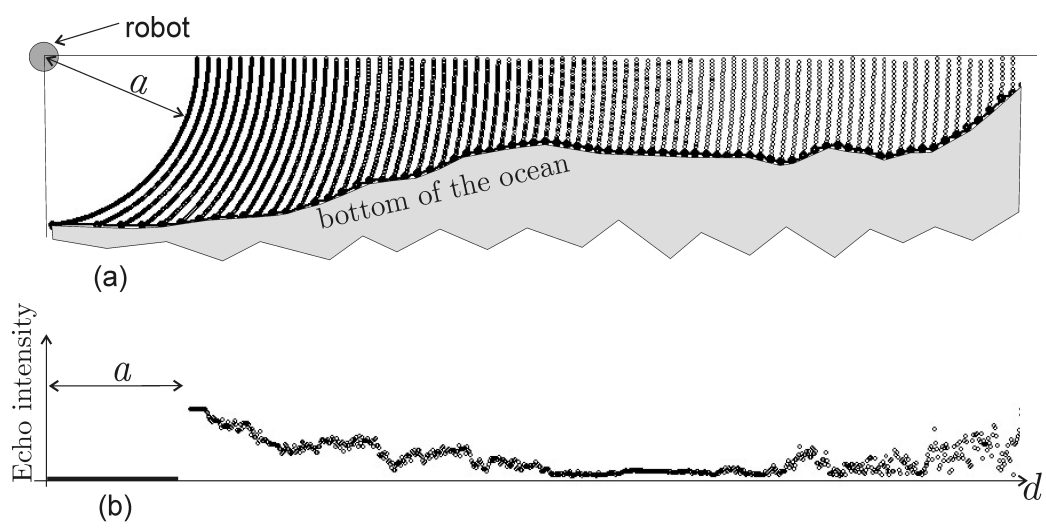
Un GPS (Global positioning system), disponible à la surface.

$$\begin{aligned} t_0 &= 6000 \text{ s}, & \ell^0 &= (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\ t_f &= 12000 \text{ s}, & \ell^f &= (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m \end{aligned}$$

Un sonar (KLEIN 5400 side scan sonar). Donne la distance r entre le robot et la mine

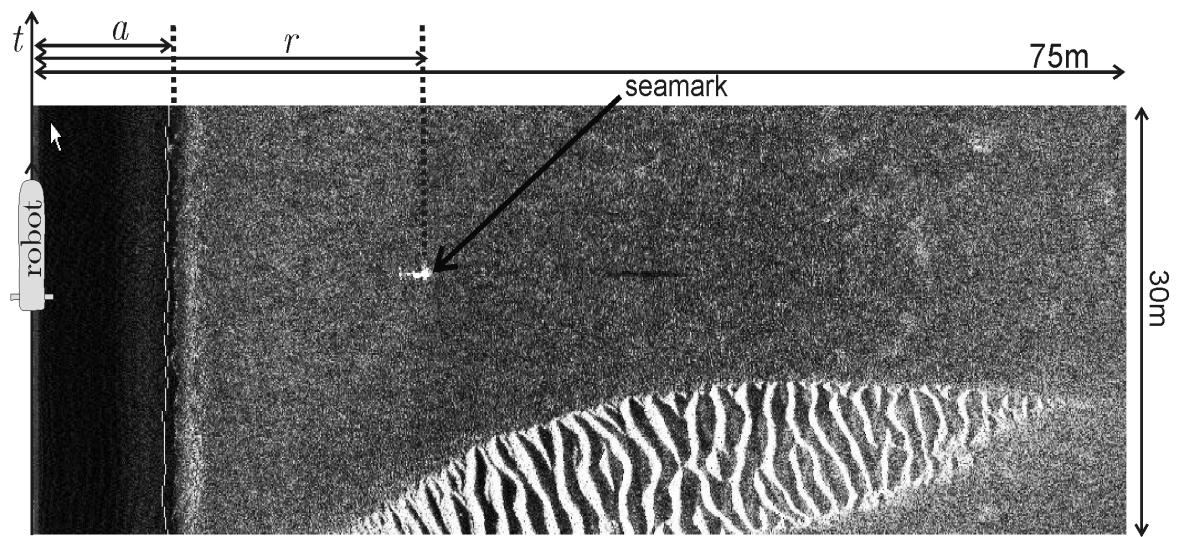








Screenshot du logiciel SonarPro



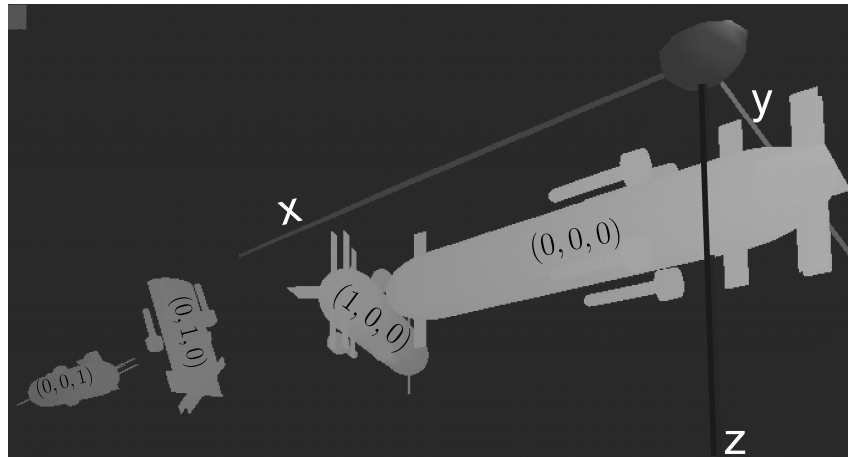
Détection d'une mine à l'aide de SonarPro

Le Loch-Doppler renvoie la vitesse du robot $\mathbf{v}_r$ et son altitude a .

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] . \tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

Une centrale inertielle (Octans III from IXSEA) renvoie le roulis ϕ , le tangage θ et le cap ψ du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$



3.2 Données

Pour chaque $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$, nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

3.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi(t) & -\sin \varphi(t) \\ 0 & \sin \varphi(t) & \cos \varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))||\;=\;r(i),$$

$$\mathbf{R}^\top(\tau(i))\left(\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))\right)\in [0]\times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i))-p_z(\tau(i))-a(\tau(i))\in [-0.5,0.5]$$

```
//-----  
Constants  
N = 59996; // Number of time steps  
Variables  
  R[N-1][3][3], // rotation matrices  
  p[N][3], // positions  
  v[N-1][3], // speed vectors  
  phi[N-1],theta[N-1],psi[N-1]; // Euler angles  
  px[N],py[N]; // for display only  
//-----
```

```
function R[3][3]=euler(phi,theta,psi)
    cphi = cos(phi);
    sphi = sin(phi);
    ctheta = cos(theta);
    stheta = sin(theta);
    cpsi = cos(psi);
    spsi = sin(psi);
    R[1][1]=ctheta*cpsi;
    R[1][2]=-cphi*spsi+stheta*cpsi*sphi;
    R[1][3]=spsi*sphi+stheta*cpsi*cphi;
    R[2][1]=ctheta*spsi;
    R[2][2]=cpsi*cphi+stheta*spsi*sphi;
    R[2][3]=-cpsi*sphi+stheta*cphi*spsi;
    R[3][1]=-stheta;
    R[3][2]=ctheta*sphi;
    R[3][3]=ctheta*cphi;
end
```

```

contractor-list rotation
    for k=1:N-1;
        R[k]=euler(phi[k],theta[k],psi[k]);
    end
end
//-----
contractor-list statequ
    for k=1:N-1;
        p[k+1]=p[k]+0.1*R[k]*v[k];
    end
end
//-----
contractor init
    inter k=1:N-1;
        rotation(k)
    end
end

```

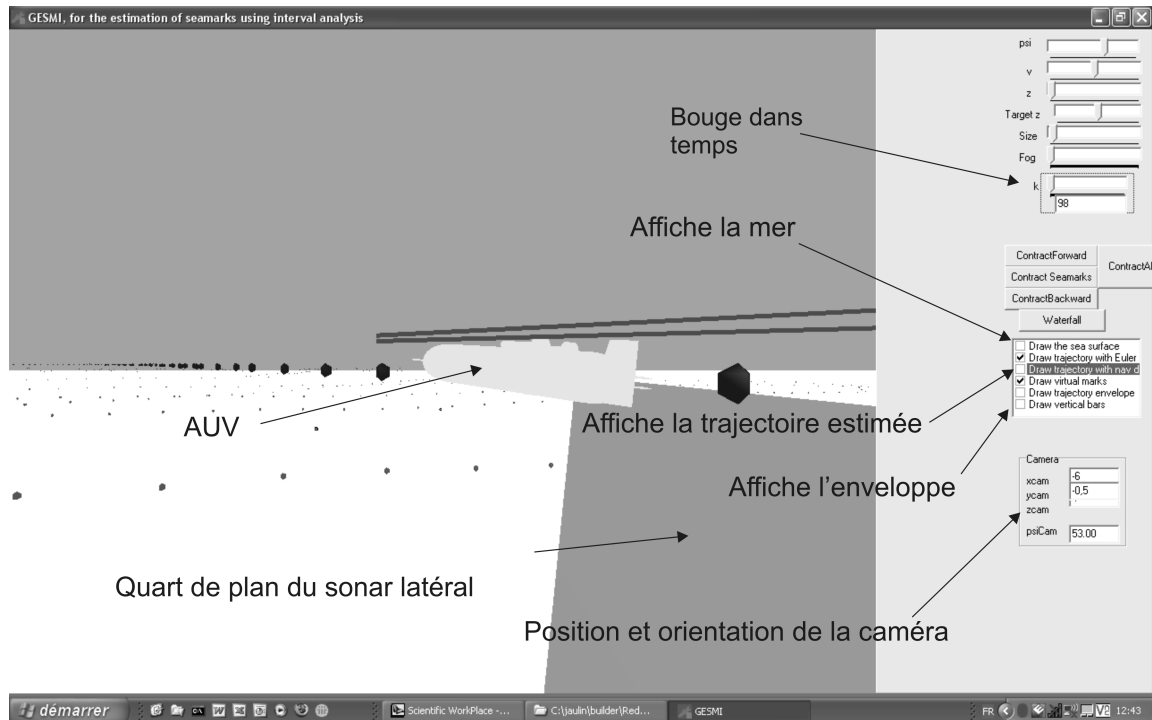
```
contractor fwd
  inter k=1:N-1;
    statequ(k)
  end
end
```

```
//-----
```

```
contractor bwd
  inter k=1:N-1;
    statequ(N-k)
  end
end
```

```
main
  p[1] :=read("gps_init.dat");
  v :=read("Quimper_v.dat");
  phi :=read("Quimper_phi.dat");
  theta :=read("Quimper_theta.dat");
  psi :=read("Quimper_psi.dat");
  init;
  fwd;
  bwd;
  column(p,px,1);
  column(p,py,2);
  print("--- Robot positions: ---");
  newplot("gesmi.dat");
  plot(px,py,color(rgb(1,1,1),rgb(0,0,0)));
end
```

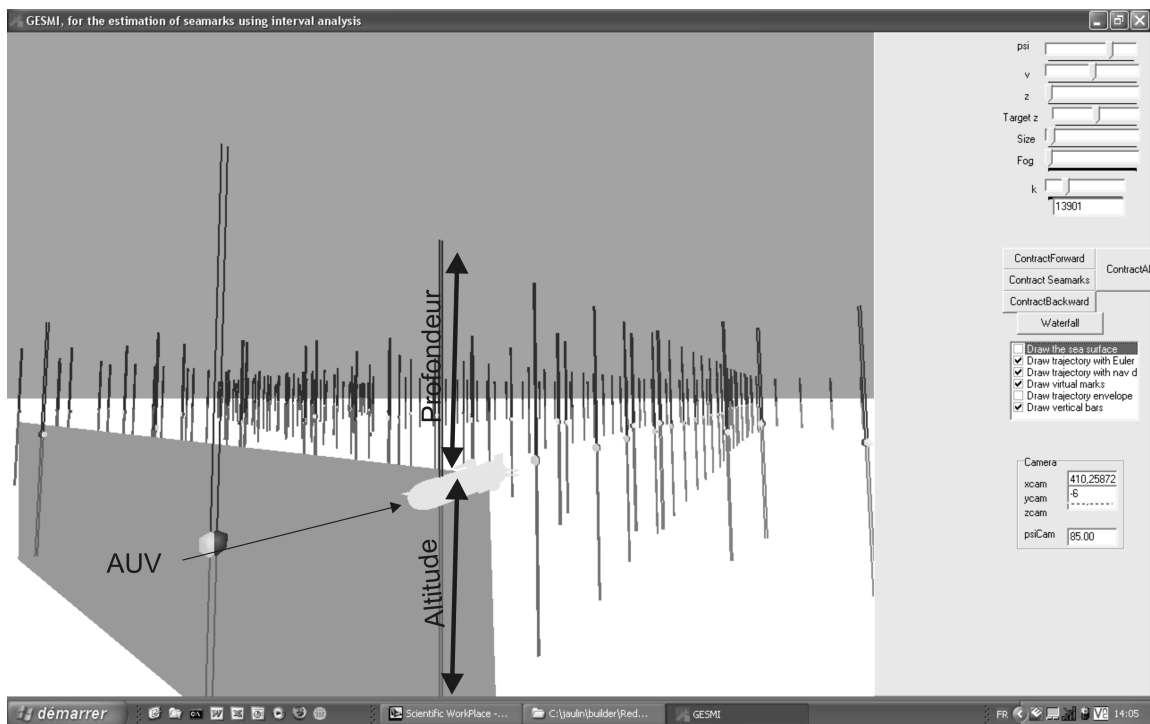
3.4 Logiciel GESMI

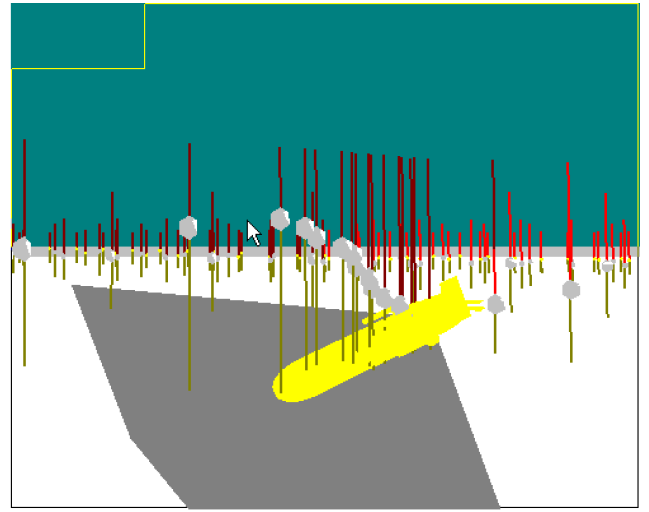
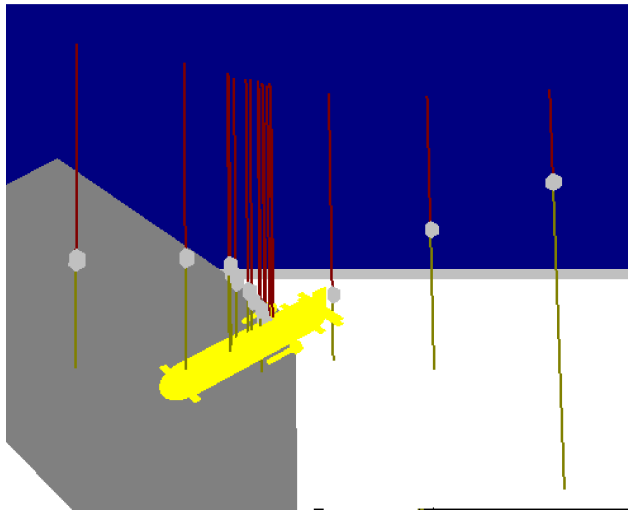


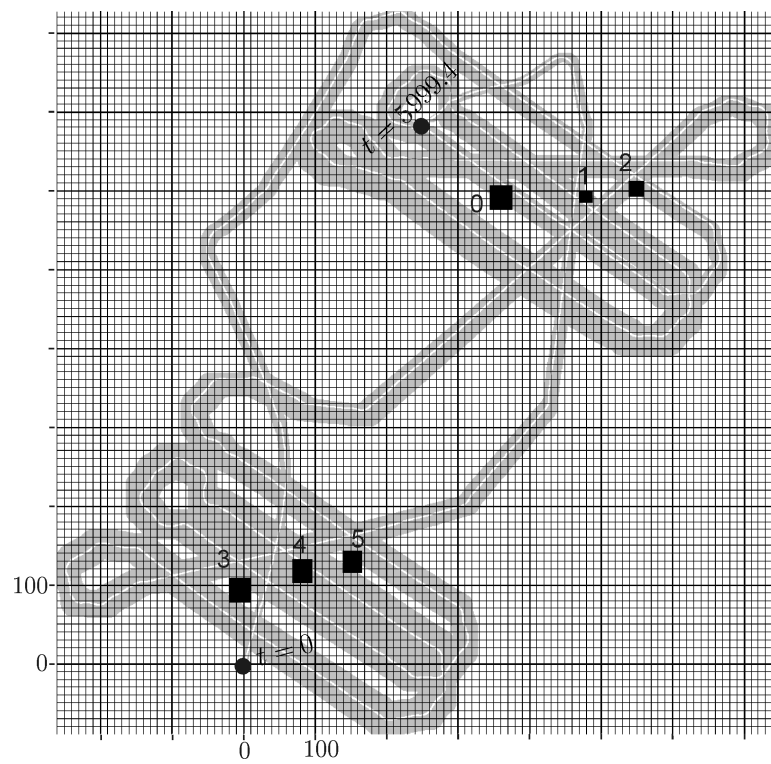
```

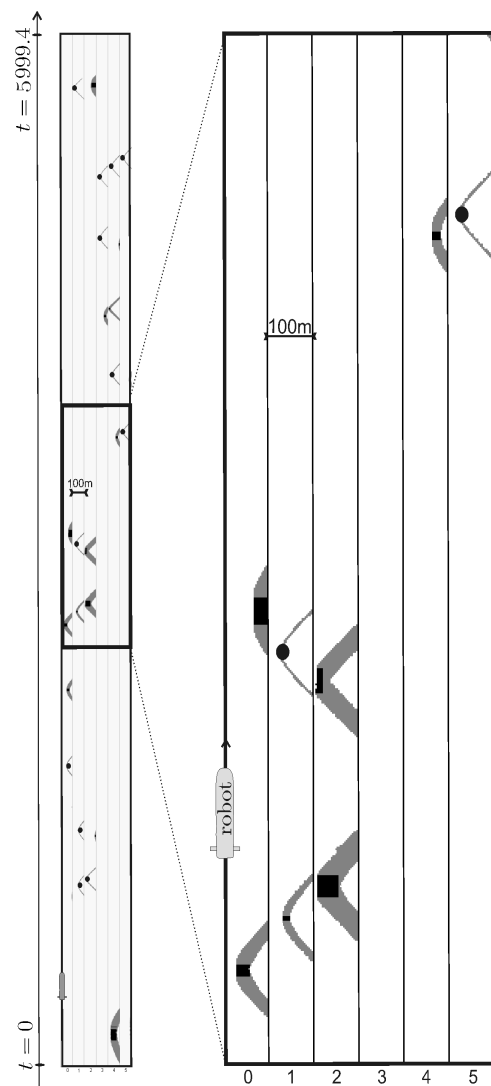
% This file has been generated by a generator
% and will be used by GESMI to solve a SLAM problem
% Note : every line starting by a '%' is considered as a comment by GESMI.
% -----
%
% This file contains
% 1) Some basic information about the scope of the sonar and the sampling time
% 2) The prior domains for the seamounts
% 3) The coordinates for some virtual marks
% 4) sensor data (angles, speeds, depth, altitude, position) with bounds
% 5) ping table : at the end of this file
%
% -----
% maximal distance of the lateral sonar and error interval [min,max]
75.0 1.0
% Sampling time
0.1
% -----
% Domain for seamounts detected using the software SonarPro
% -----
% Initial domains for the mines
% xmin xmax ymin ymax zmin zmax
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
$
% -----
% Virtual marks (only for graphism, not used for computation)
% -----
% x y z colorRed ColorGreen ColorBlue LocalFrame
594.0533723 374.7293035 19 0.1 0 0 0 % seamount 0
599.6093723 484.5528643 19 0.1 0.1 0 0 % seamount 1
601.4613723 557.3573086 19 0.1 0 0.1 0 % seamount 2
94.01337232 -2.868189501 19 0 0.1 0 0 % seamount 3
119.9413723 84.75155672 20 0 0 0.1 0 % seamount 4
127.3493723 156.3280366 20 0 0 0 0 % seamount 5
0 0 0 1 0 0 1 % origin of the local frame
689.96 264.10 0 0 0 1 1 % final GPS
$
% -----
% -----
%t phi dphi theta dtheta psi dpsi vx dvx
0 -0.011505 0.0001745329252 0.033556 0.0001745329252 0.415613 0.0052679
0.1 -0.012272 0.0001745329252 0.034994 0.0001745329252 0.416284 0.00526
0.2 -0.012847 0.0001745329252 0.037583 0.0001745329252 0.417051 0.00526
0.3 -0.013422 0.0001745329252 0.040938 0.0001745329252 0.418202 0.00526
0.4 -0.014093 0.0001745329252 0.044773 0.0001745329252 0.419831 0.00526
0.5 -0.014285 0.0001745329252 0.0488 0.0001745329252 0.422228 0.0052679
0.6 -0.013998 0.0001745329252 0.052539 0.0001745329252 0.425488 0.00526
0.7 -0.012943 0.0001745329252 0.055703 0.0001745329252 0.42961 0.005267
0.8 -0.011121 0.0001745329252 0.0581 0.0001745329252 0.434308 0.0052679
0.9 -0.008725 0.0001745329252 0.059825 0.0001745329252 0.439773 0.00526
1 -0.005944 0.0001745329252 0.060496 0.0001745329252 0.445334 0.0052679
1.1 -0.003547 0.0001745329252 0.0604 0.0001745329252 0.450511 0.0052679
1.2 -0.001917 0.0001745329252 0.059729 0.0001745329252 0.455017 0.00526
1.3 -0.001438 0.0001745329252 0.058867 0.0001745329252 0.458564 0.00526
1.4 -0.002013 0.0001745329252 0.058387 0.0001745329252 0.461057 0.00526
1.5 -0.003356 0.0001745329252 0.058483 0.0001745329252 0.462303 0.00526
1.6 -0.005561 0.0001745329252 0.059346 0.0001745329252 0.462591 0.00526
1.7 -0.007958 0.0001745329252 0.060976 0.0001745329252 0.462112 0.00526

```









Reconstruction de la waterfall.

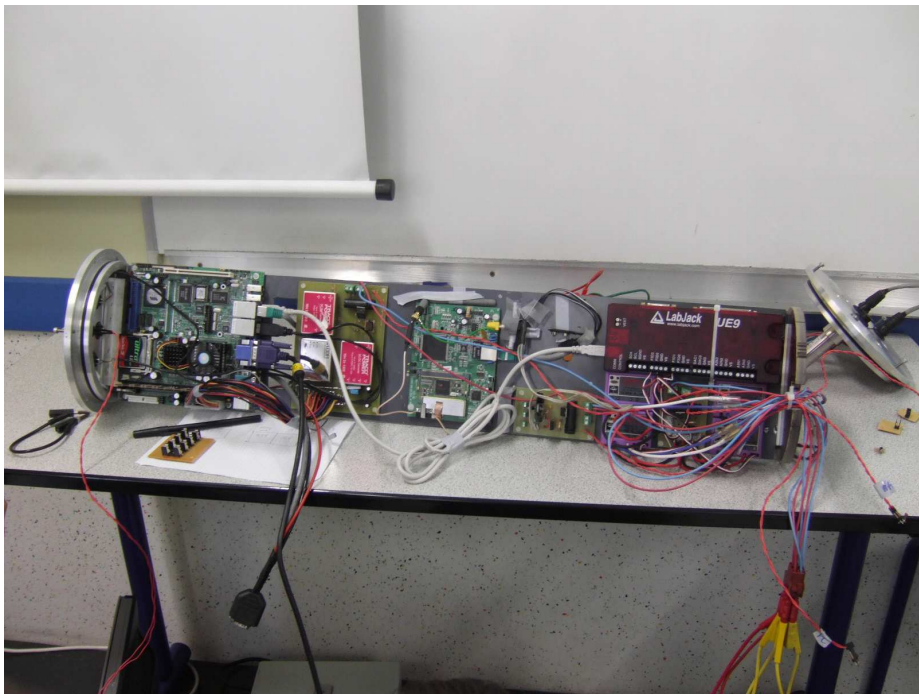
4 Robustesse

Logiciel de démo de Jan Sliwka



Portsmouth, July 12-15, 2007.

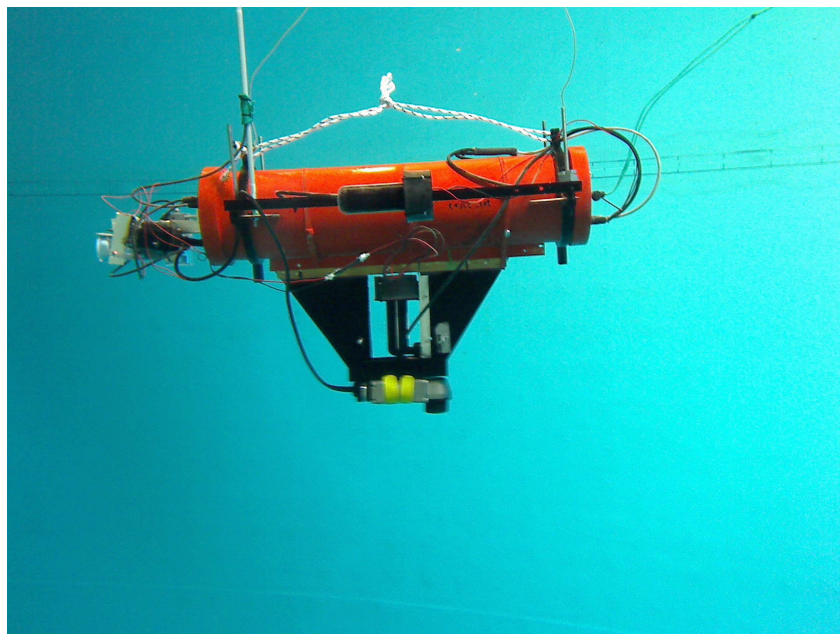




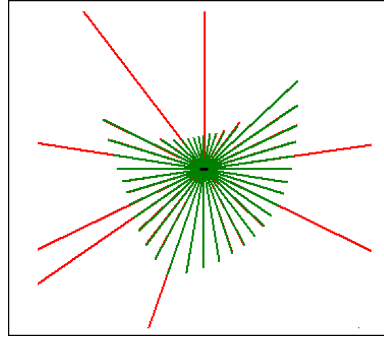
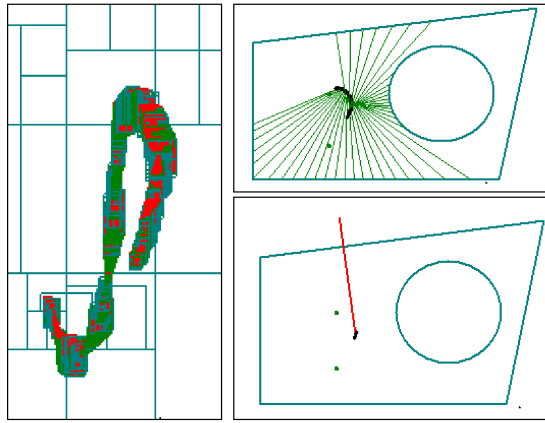








Robot Sauc'isse dans une piscine



5 CSPs

Un CSP est constitué.

- d'un ensemble de variables $\mathcal{V} = \{x_1, \dots, x_n\}$,
- d'un ensemble de contraintes $\mathcal{C} = \{c_1, \dots, c_m\}$
- d'un ensemble d'intervalles $\{[x_1], \dots, [x_n]\}$.

Les informations sur la localisation d'un robot et sur sa carte peuvent être représentées par un CSP.

Les contraintes c_i représentent les relations entre les variables, comme les matrices de covariances. On peut leur associer un contracteur.

Les CSP se distribuent facilement entre différents robots.