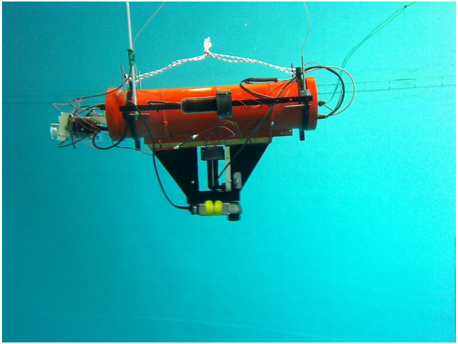
Interval methods and abstract interpretation for robotics

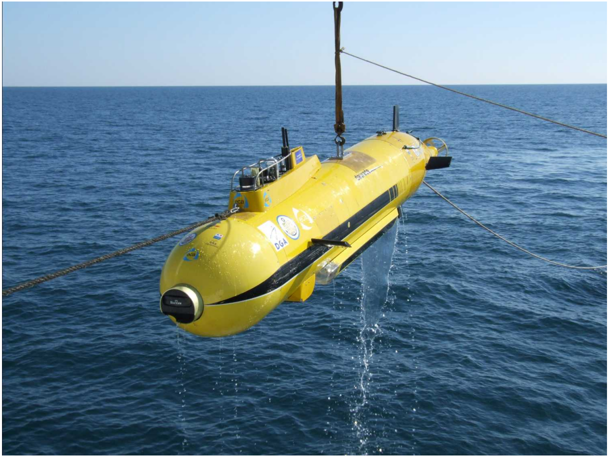
L. Jaulin, F. Le Bars, B. Zerr, S. Rohou, B. Desrochers, T. Le Mézo

Mobile robot

A mobile robot is a vehicle equipped with actuators, sensors and a brain.



Saucisse (ENSTA Bretagne)



Daurade, TGA TN, ECA.

Vaimos at the WRSC (ENSTA Bretagne-IFREMER),
F. Le Bars, O. Ménage, P. Rousseau (Vaimos in Angers).

A mobile robot is a vehicle with actuators, sensors and a brain

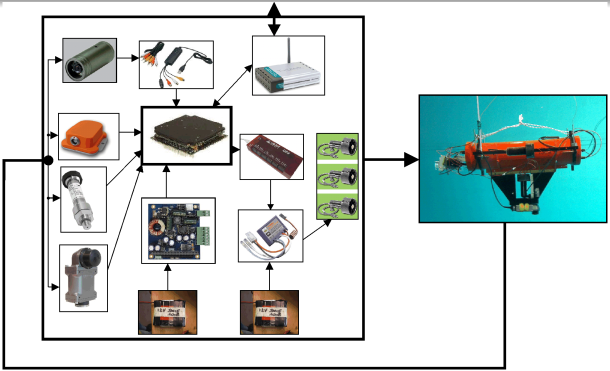
$$\begin{array}{rcll} \dot{\mathbf{x}} & = & \mathbf{f}(\mathbf{x},\mathbf{u}) & \text{(evolution)} \\ \mathbf{y} & = & \mathbf{g}(\mathbf{x}) & \text{(observation)} \\ \mathbf{u} & = & \mathbf{h}(\mathbf{y}). & \text{(control)} \end{array}$$

We have

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{h}(\mathbf{g}(\mathbf{x}))) = \phi(\mathbf{x}).$$

Thus a robot is a dynamical system. With uncertainty:

$$\dot{\mathbf{x}} \in \mathbf{\Phi}(\mathbf{x}).$$

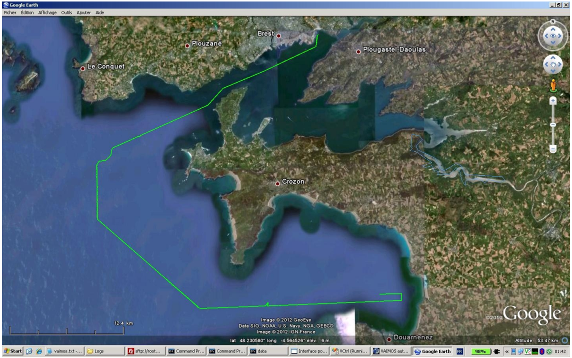


Why do we need robots ?

Vaimos in Angers 2013. https://youtu.be/tmfkKNM76Qg

Brest-Douarnenez. January 17, 2012


F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Plougastel-Daoulas
Le Conquet
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
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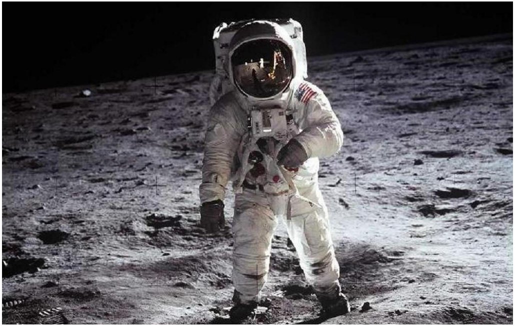
Ocean satellites ?



Robots are needed for dirty, dangerous and dull jobs

Curiosity

About 3,600 satellites in orbit (1,000 are operational).
In the ocean, we have gliders, drifting buoys.
They should be autonomous in energy, and should survive for years (persistent autonomy).

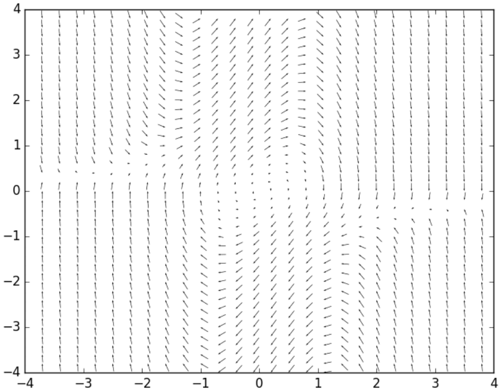
Vaimos in the middle of the Atlantique ocean
https://youtu.be/pb_KhcYZI_A

Security

A robot $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$.

Example: The Van der Pol system

$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \left(1 - x_1^2\right) \cdot x_2 - x_1 \end{cases}$$


4
3
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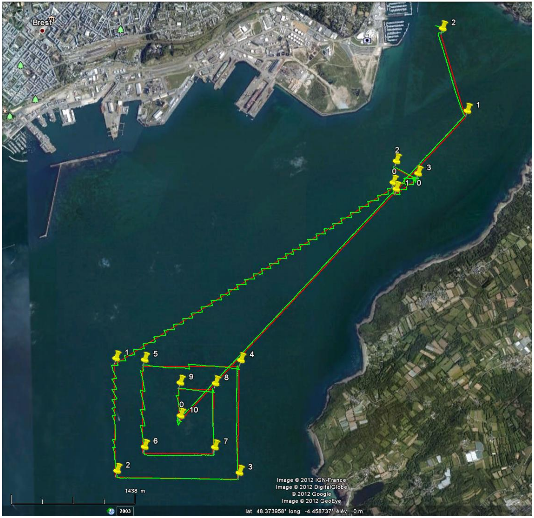
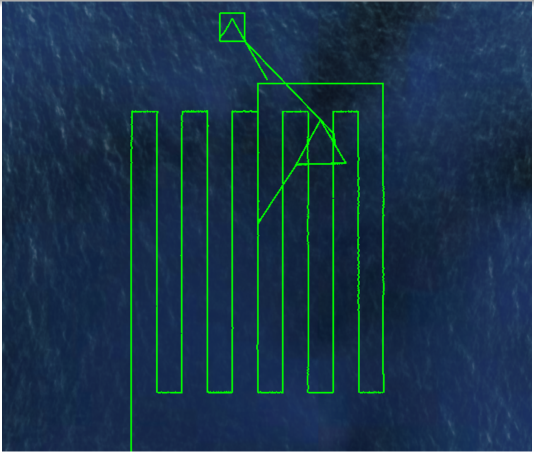


Square Spiral



Middle of Atlantique ocean, 350 km made by Vaimos, sept. 6-9, 2012.

Capture set

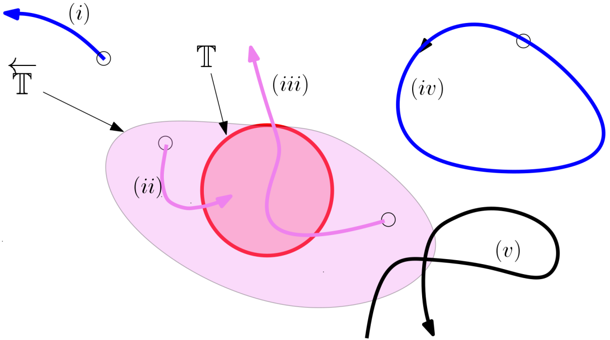
We consider a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$.

Let φ be the flow map.

The *capture* set of the *target* $\mathbb{T} \subset \mathbb{R}^n$ is:

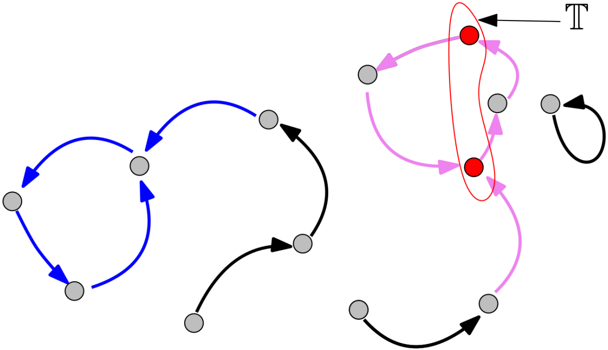
$$\overleftarrow{\mathbb{T}} = \{\mathbf{x}_0 \mid \exists t \geq 0, \varphi(t, \mathbf{x}_0) \in \mathbb{T}\}.$$

To each state, we associate a path.

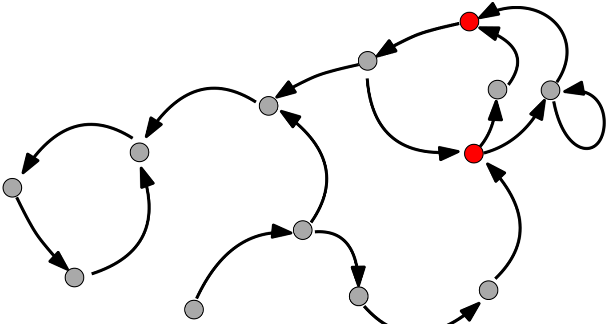


Graph analogy

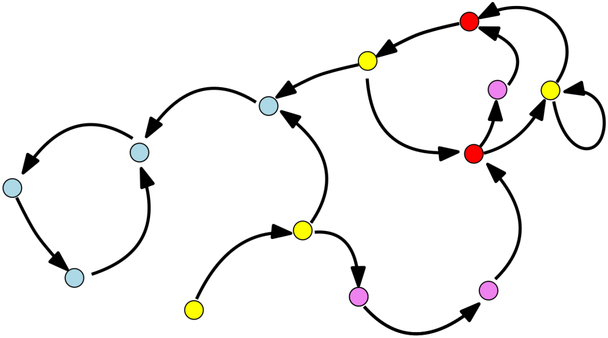
A deterministic graph $\mathscr{G}_1$ ($\sim \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$) with a target $\mathbb{T}$ (red), a dead path (blue).



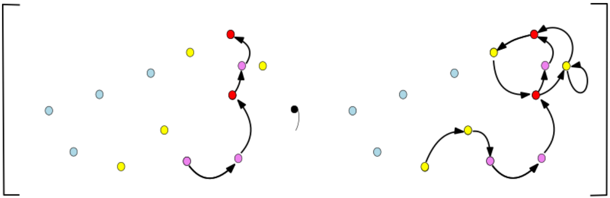
Approximated by a non deterministic graph $\mathscr{G}_2$ ($\sim \dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$)



Using a backward method, we can enclose $\overleftarrow{\mathbb{T}}$.



This corresponds to an interval of graphs:



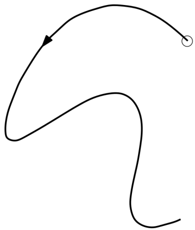
Maze

An ***interval*** is *a domain* which encloses a real number.

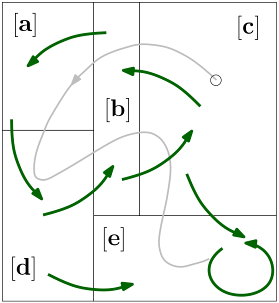
A ***polygon*** is *a domain* which encloses a vector of $\mathbb{R}^n$.

A ***maze*** is *a domain* which encloses a path.

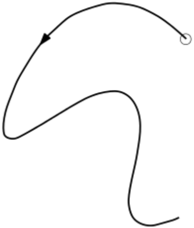
A maze is a set of paths.



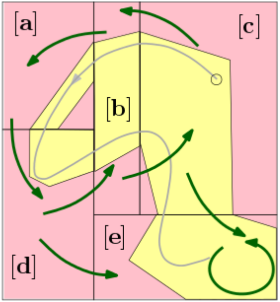
$\in$



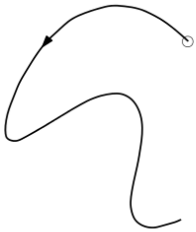
Mazes can be made more accurate by adding polygones.



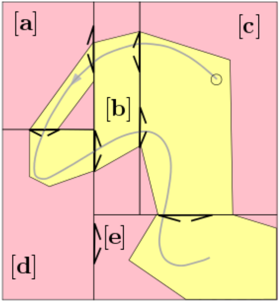
$\in$



Or using doors instead of a arcs

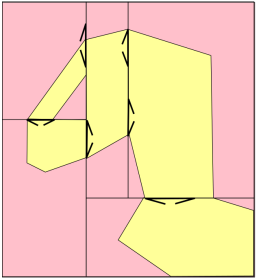


$\in$

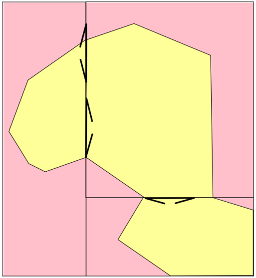


The set of mazes forms a lattice with respect to $\subset$.

The left maze contains less paths than the right maze.

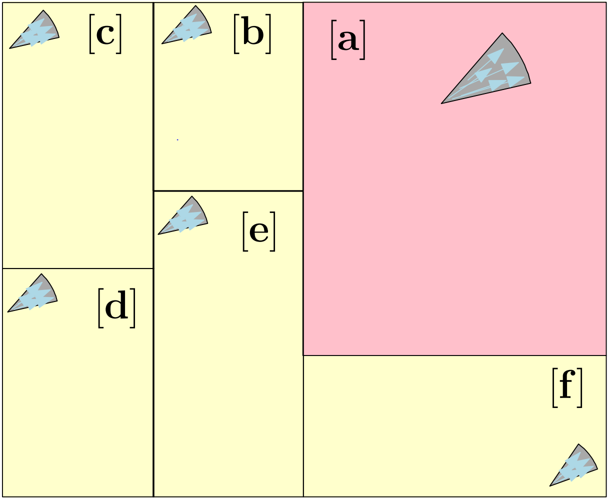


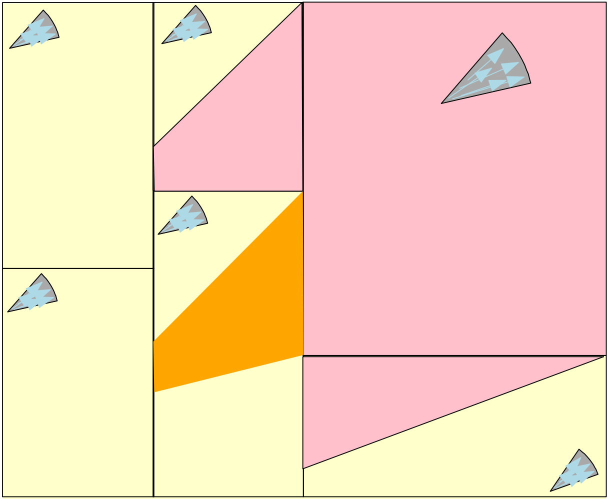
$\subset$

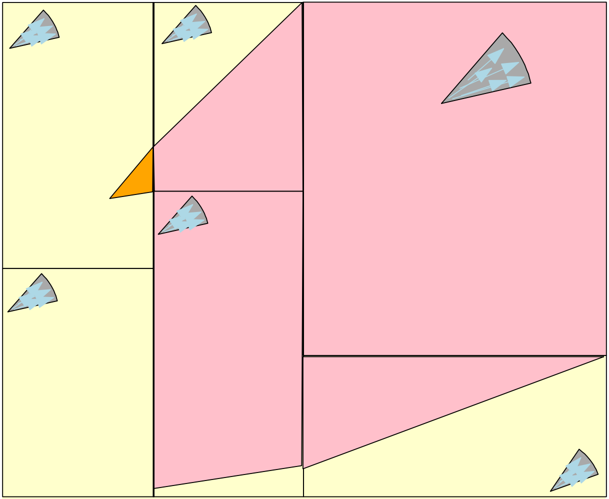


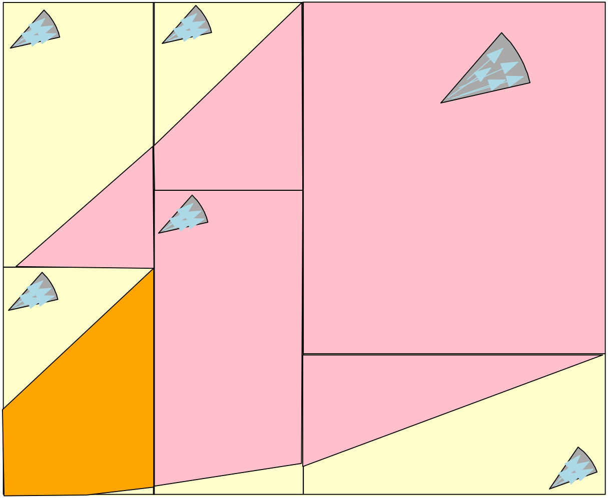
Inner approximation of $\overleftarrow{\mathbb{T}}$ (contractors)

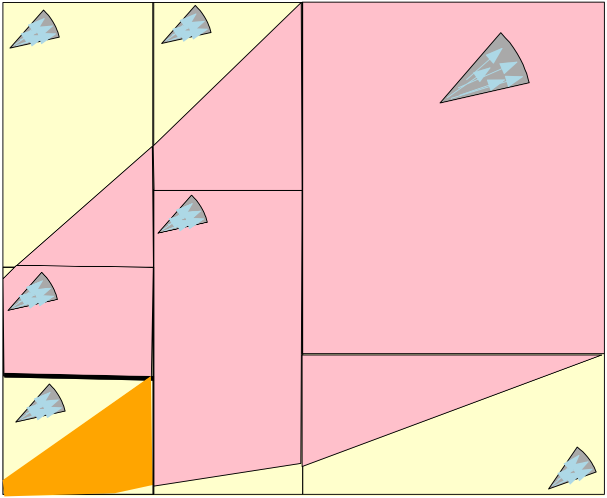
Thus, we search for a path that never reach $\mathbb{T}$.

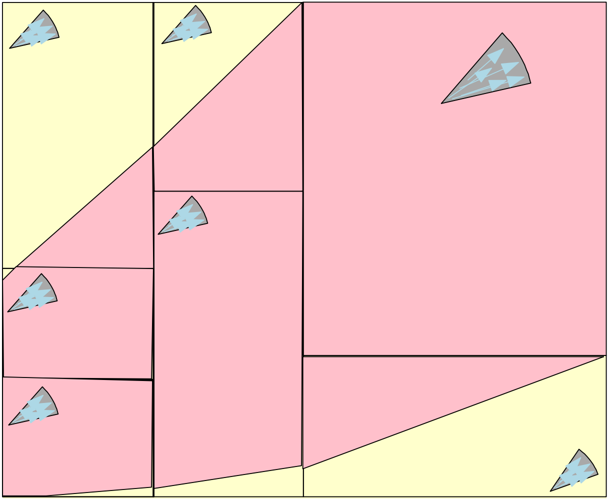

[c]
[b]
[a]
[e]
[d]
[f]



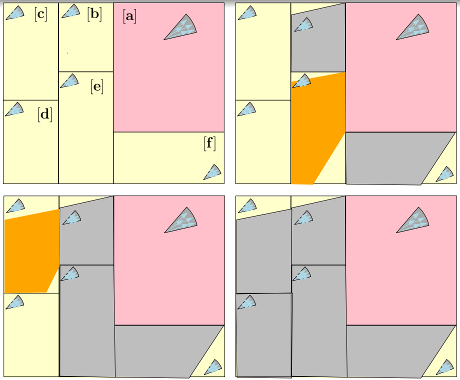








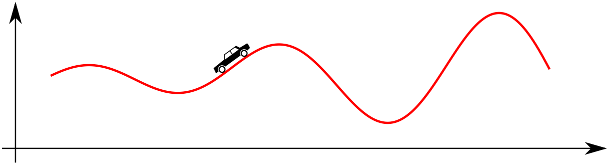
Outer propagation (abstract interpretation)

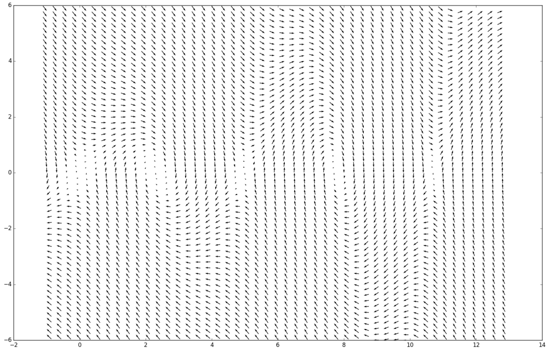


An interpretation can be given only when the fixed point is reached.

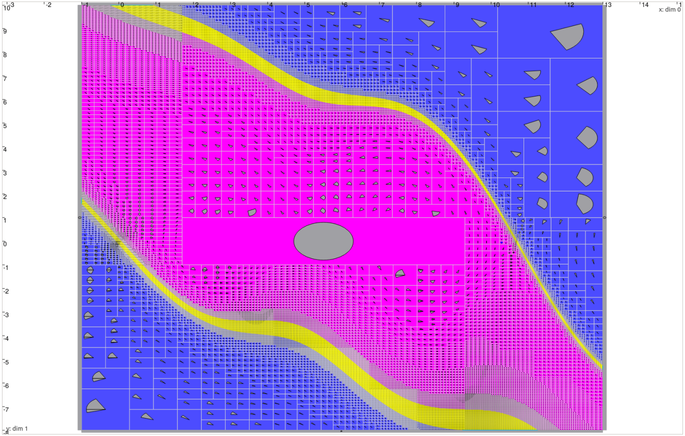
Car on the hill

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 9.81\sin\left(\frac{11}{24}\cdot\sin x_1 + 0.6\cdot\sin(1.1\cdot x_1)\right) - 0.7\cdot x_2 \end{cases}$$

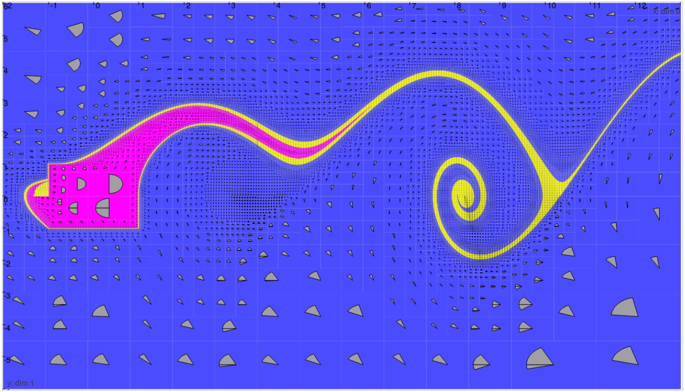




Vector field



Capture set



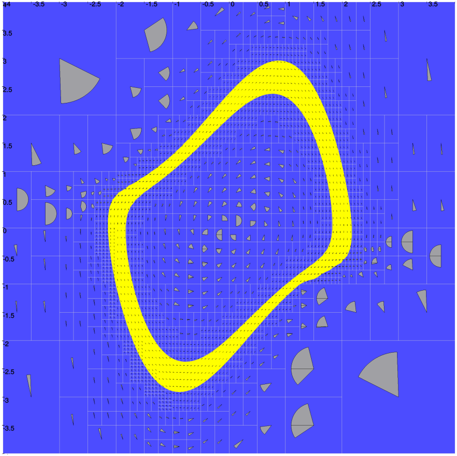
Capture set

Van der Pol system

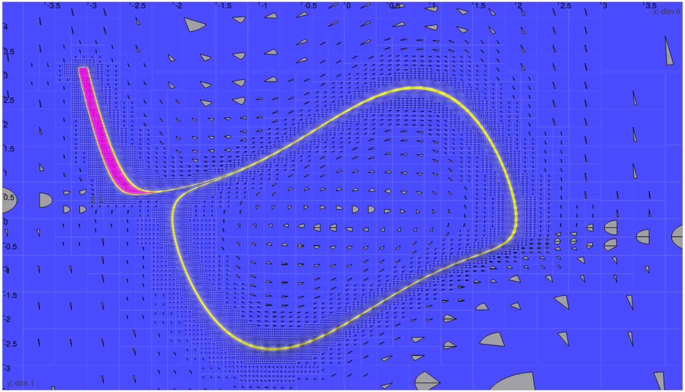
Consider the system

$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \left(1 - x_1^2\right) \cdot x_2 - x_1 \end{cases}$$

and the box $\mathbb{X}_0 = [-4, 4] \times [-4, 4]$.



Enclosure of the cycle



Guaranteed integration

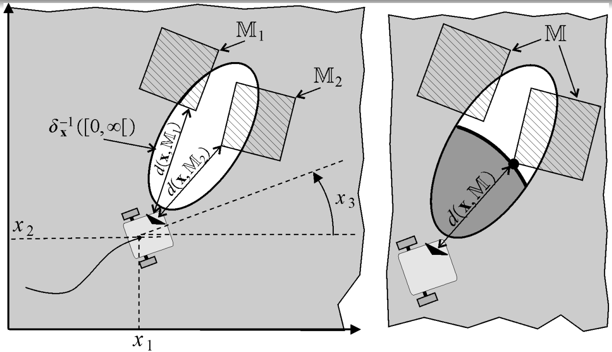
Range-only SLAM

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & \text{(evolution equation)} \\ z(t) & = & d(\mathbf{x}(t),\mathbb{M}) & \text{(map equation)} \end{cases}$$

where $t \in \mathbb{R}$, $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, $\mathbb{M} \in \mathscr{C}\left(\mathbb{R}^2\right)$ is the occupancy map.
Unknown: the map $\mathbb{M}$ and the trajectory $\mathbf{x}$.

Assumption. d corresponds to a *rangefinder*, i.e.,

$$\begin{cases} d(\mathbf{x}, \mathbb{M}_1 \cup \mathbb{M}_2) = \min\{d(\mathbf{x}, \mathbb{M}_1), d(\mathbf{x}, \mathbb{M}_2)\} \\ d(\mathbf{x}, \emptyset) = +\infty. \end{cases}$$



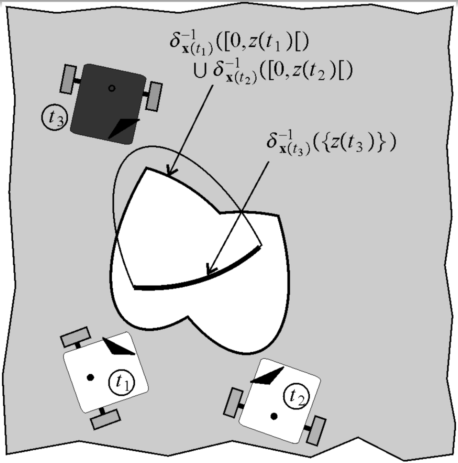
Impact, covering and dug zones

Define the function $\delta_{\mathbf{x}} : \mathbb{R}^2 \to \mathbb{R}$ as

$$\delta_{\mathbf{x}}(\mathbf{a}) = d(\mathbf{x}, \{\mathbf{a}\}).$$

For given $\mathbf{x}$ and z, we define

covering zone	$\delta_{\mathbf{x}}^{-1}([0,\infty[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < \infty\}$
impact zone	$\delta_{\mathbf{x}}^{-1}(\{z\}) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) = z\}$
dug zone	$\delta_{\mathbf{x}}^{-1}([0,z[) = \{\mathbf{a}, \delta_{\mathbf{x}}(\mathbf{a}) < z\}$



Incompatibility between configurations

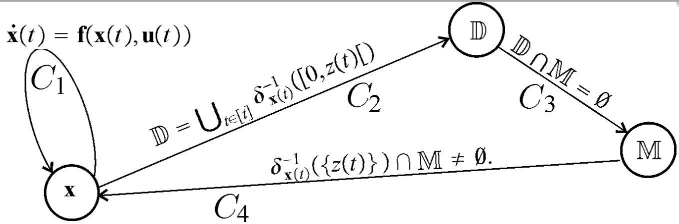
The range-only SLAM problem is a *hybrid CSP*.

Variables: $\mathbf{x}(t)$, $\mathbb{M}$ and $\mathbb{D}$.

Constraints:

$$\left.\begin{array}{ll}(1) & \dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t),\mathbf{u}(t))\\ (2) & \mathbb{D}=\bigcup_{t\in[t]}\delta_{\mathbf{x}(t)}^{-1}([0,z(t)[)\\ (3) & \mathbb{D}\cap\mathbb{M}=\emptyset\\ (4) & \delta_{\mathbf{x}(t)}^{-1}(\{z(t)\})\cap\mathbb{M}\neq\emptyset.\end{array}\right\}:z(t)=d(\mathbf{x}(t),\mathbb{M})$$

Domains: $[\mathbb{M}]=[\mathbb{D}]=[\emptyset,\mathbb{R}^q]$, $[\mathbf{x}](t)=\mathbb{R}^n$ for $t>0$ and $[\mathbf{x}](0)=\mathbf{x}(0)$.



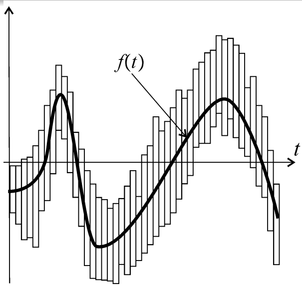
Constraint diagram of the range only SLAM problem

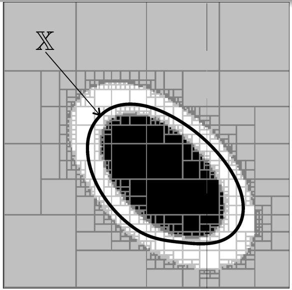
Intervals

A *closed interval* (or *interval* for short) $[x]$ of a complete lattice $\mathscr{E}$ is a subset of $\mathscr{E}$ which satisfies

$$[x] = \{x \in \mathscr{E} \mid \wedge [x] \leq x \leq \vee [x]\}$$

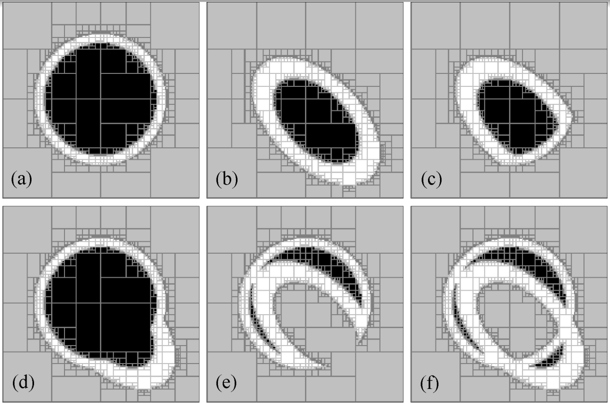
Both $\emptyset$ and $\mathscr{E}$ are intervals of $\mathscr{E}$.





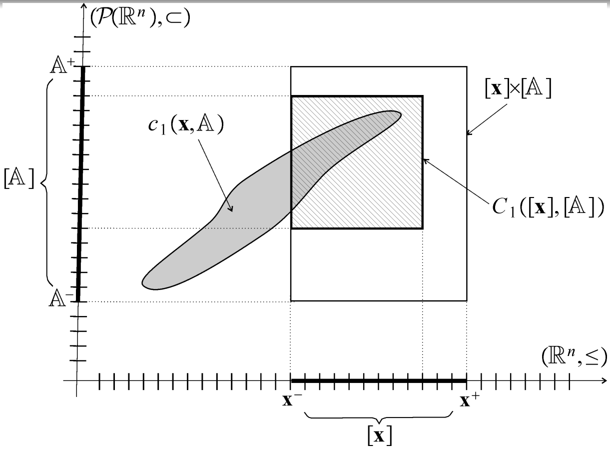
An interval function (or tube) and a set interval

Interval arithmetic



$$[\mathbb{A}],[\mathbb{B}],\ [\mathbb{A}]\cap[\mathbb{B}],\ [\mathbb{A}]\cup[\mathbb{B}],\ [\mathbb{A}]\setminus[\mathbb{B}],\ ([\mathbb{A}]\cup[\mathbb{B}])\setminus([\mathbb{A}]\cap[\mathbb{B}])$$

Contractors


$(\mathcal{P}(\mathbb{R}^n), \subset)$
$\mathbb{A}^+$
$[\mathbb{A}]$
$\mathbb{A}^-$
$c_1(\mathbf{x}, \mathbb{A})$
$[\mathbf{x}] \times [\mathbb{A}]$
$C_1([\mathbf{x}], [\mathbb{A}])$
$(\mathbb{R}^n, \leq)$
$\mathbf{x}^-$
$\mathbf{x}^+$
$[\mathbf{x}]$

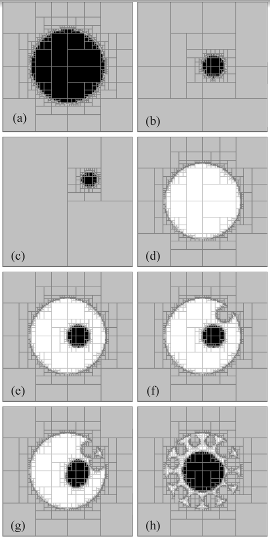
Propagation

Consider the following CSP

$$\begin{cases} \text{(i)} & \mathbb{X} \subset \mathbb{A} \\ \text{(ii)} & \mathbb{B} \subset \mathbb{X} \\ \text{(iii)} & \mathbb{X} \cap \mathbb{C} = \emptyset \\ \text{(iv)} & f(\mathbb{X}) = \mathbb{X}, \end{cases}$$

where $\mathbb{X}$ is an unknown subset of $\mathbb{R}^2$, f is a rotation of $-\frac{\pi}{6}$, and

$$\begin{cases} \mathbb{A} & = & \left\{(x_1, x_2), x_1^2 + x_2^2 \leq 3\right\} \\ \mathbb{B} & = & \left\{(x_1, x_2), (x_1 - 0.5)^2 + x_2^2 \leq 0.3\right\} \\ \mathbb{C} & = & \left\{(x_1, x_2), (x_1 - 1)^2 + (x_2 - 1)^2 \leq 0.15\right\} \end{cases}$$



(a) $[\mathbb{A}]$ (b) $[\mathbb{B}]$

(c) $[\mathbb{C}]$ (d) $\mathbb{X} \subset \mathbb{A}$

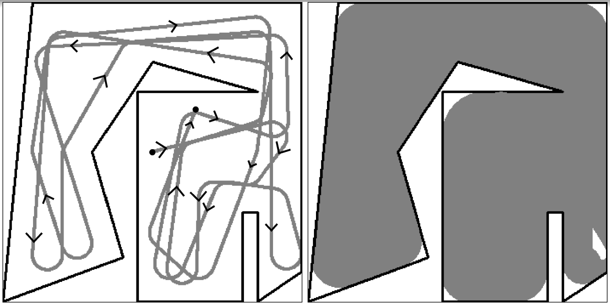
(e) $\mathbb{B} \subset \mathbb{X}$ (f) $\mathbb{X} \cap \mathbb{C} = \emptyset$

(g) $f(\mathbb{X}) = \mathbb{X}$ (h) $(f(\mathbb{X}) = \mathbb{X})^{\infty}$

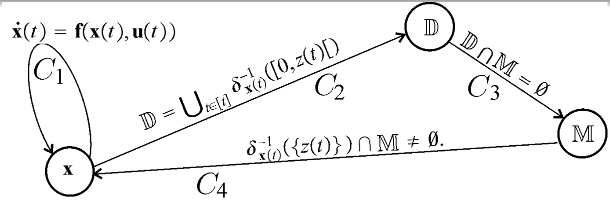
Range-only SLAM

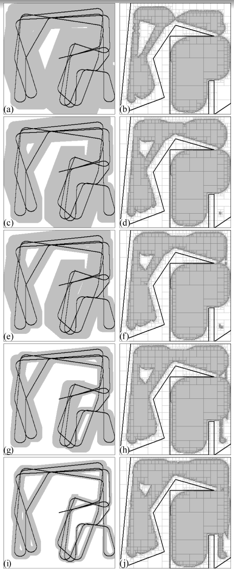
Range-only SLAM equations

$$\begin{cases} \dot{x}_1(t) & = & u_1(t)\cos(u_2(t)) \\ \dot{x}_2(t) & = & u_1(t)\sin(u_2(t)) \\ z(t) & = & d(\mathbf{x}(t),\mathbb{M}). \end{cases}$$



Actual trajectory and dug space


$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$
C_1
$\mathbb{D} = \bigcup_{t \in [t]} \delta^{-1}_{\mathbf{x}(t)}([0, z(t)[)$
C_2
$\mathbb{D}$
$\mathbb{D} \cap \mathbb{M} = \emptyset$
C_3
$\mathbb{M}$
$\delta^{-1}_{\mathbf{x}(t)}(\{z(t)\}) \cap \mathbb{M} \neq \emptyset.$
$\mathbf{x}$
C_4


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)

Some robots our robotics club : https://youtu.be/DFg3K09cMwU