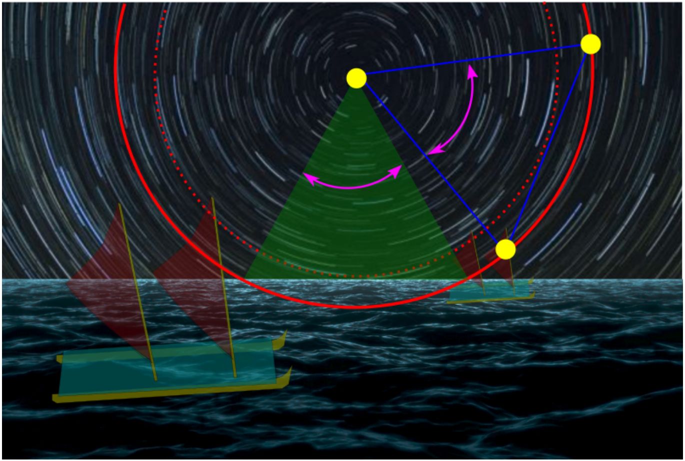
Computing the no-lost zone

T. Nico, L. Jaulin B. Zerr

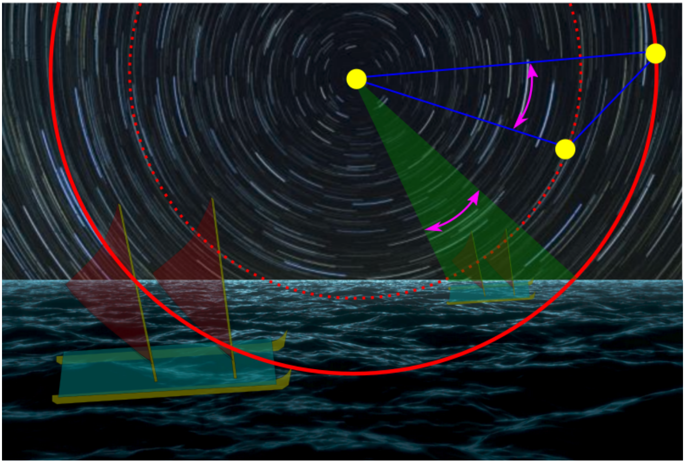
JAMACS, Nantes, 15-16 Nov 2018 Nantes

Polynesian navigation

Find the route without GPS, compass and clocks

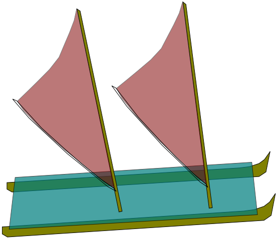


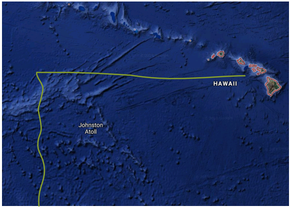
Pair of stars technique

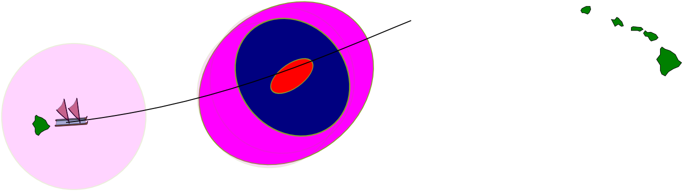


Pair of stars technique

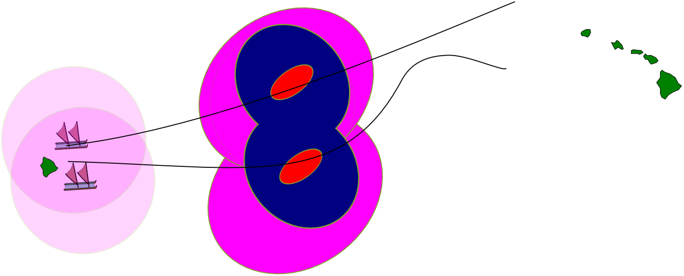

Arctic Ocean
Russia
Canada
Sea of Okhotsk
Bering Sea
Mongolia
Sea of Japan
China
South Korea
Japan
East China Sea
North Pacific Ocean
Myanmar (Burma)
Philippine Sea
Thailand
Vietnam
Malaysia
Indonesia
Banda Sea
Papua New Guinea
Arafura Sea
Coral Sea
Australia
Great Australian Bight
Tasman Sea
New Zealand
South Pacific Ocean
Google




HAWAII
Johnston Atoll

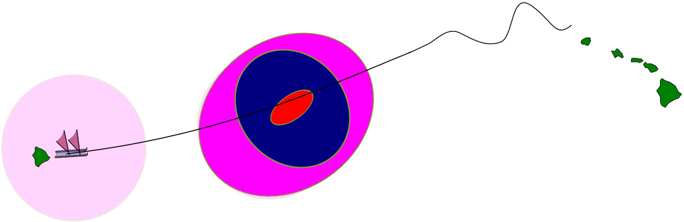


Prove that islands will be reached by one boat

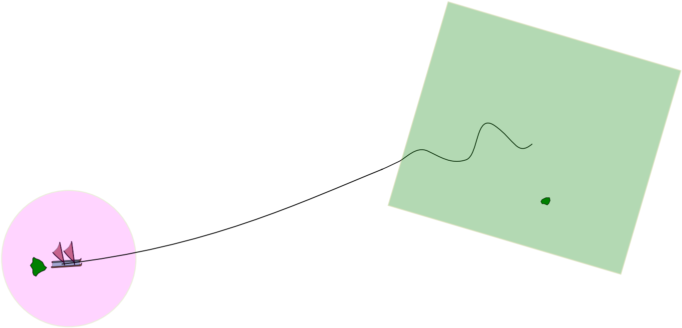


Prove that islands will be reached by the n boats

Alignment to keep the heading in case of clouds



Find a control to reach the geo-localized islands



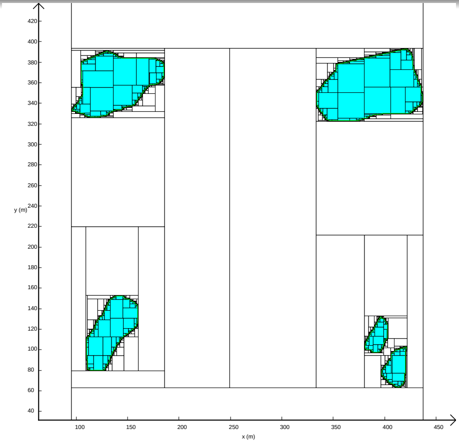
Explore a given area entirely to find new islands

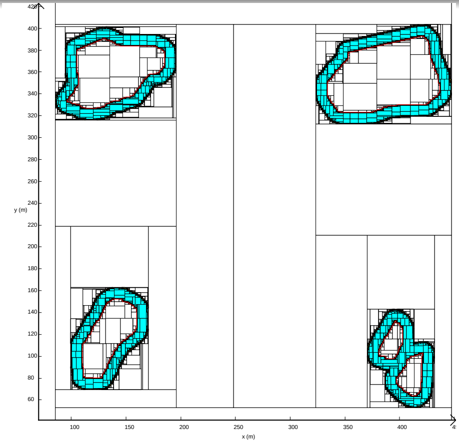
Our problem

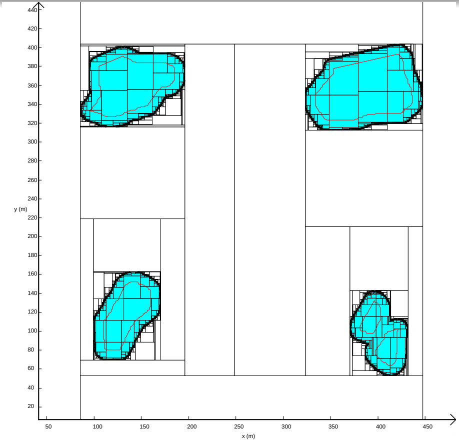
- Given a set of geo-localized islands $\mathbf{m}_i, i \geq 0$.
- The ith coastal area is:

$$\mathbb{C}_i = \{\mathbf{x} \,|\, c_i(\mathbf{x}) \leq 0\}.$$

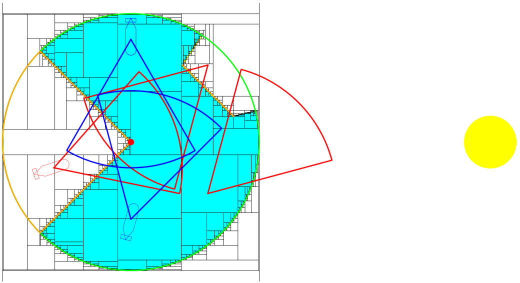
- A robot has to move in this environment without being lost.


y (m)
420
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
100
150
200
250
300
350
400
450
x (m)

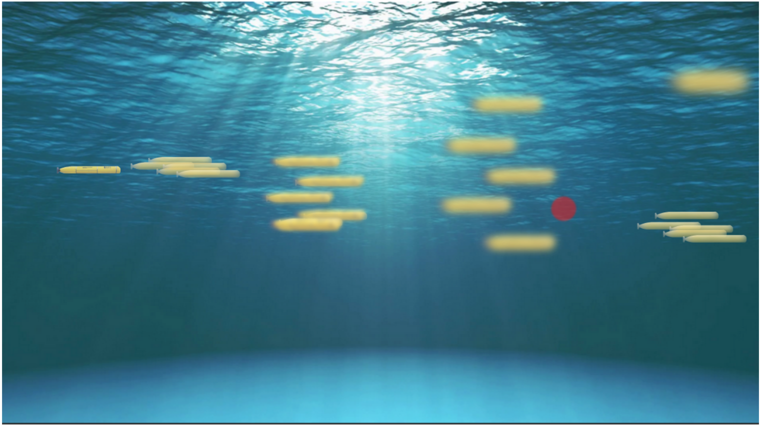

y (m)
x (m)
420
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
100
150
200
250
300
350
400



$$\mathbb{C}_1, \mathbb{C}_2, \mathbb{C}_3, \mathbb{C}_4$$



$$\mathbb{C}_1 = \{\mathbf{x} = (x, y, \theta) | (x, y) \in \text{blue and } \theta \text{ toward the buoy}\}$$



[3]

Exploration

Visible area

The robot has a state $\mathbf{x}$. The visible area is $\mathbb{V}(\mathbf{x})$

Example. The robot is able to see all up to 3 meters

$$\mathbb{V}(\mathbf{x}) = \left\{ (z_1, z_2) \,|\, (z_1 - x_1)^2 + (z_2 - x_2)^2 \leq 9 \right\}.$$

Blind observer

Given

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}), & \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}), & \mathbf{y}(t) \in [\mathbf{y}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 & \end{cases}$$

An observer is *blind* if $\dim \mathbf{y} = 0$.

Predict is a blind estimation problem

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t))+\mathbf{e}_y(t) & \mathbf{e}_y(t)\in[\mathbf{e}_y] \\ \mathbf{u}(t) &= \mathbf{r}(\mathbf{y}(t))+\mathbf{e}_u(t) & \mathbf{e}_u(t)\in[\mathbf{e}_u] \end{cases}$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{r}(\mathbf{g}(\mathbf{x}(t)) + \mathbf{e}_y(t)) + \mathbf{e}_u(t)) \\ \mathbf{e}_y(t) \in [\mathbf{e}_y], \mathbf{e}_u(t) \in [\mathbf{e}_u] \end{cases}$$

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{h}(\mathbf{x}(t), \mathbf{v}(t)) \\ & \quad \mathbf{v}(t) \in [\mathbf{v}] \end{aligned}$$

with

$$\mathbf{v} = (\mathbf{e}_y, \mathbf{e}_u)$$
$$\mathbf{h}(\mathbf{x}, \mathbf{v}) = \mathbf{f}\,(\mathbf{x}, \mathbf{r}(\mathbf{g}(\mathbf{x}) + \mathbf{e}_y) + \mathbf{e}_u)$$
$$[\mathbf{v}] = [\mathbf{e}_y] \times [\mathbf{e}_u]$$

Blind exploration

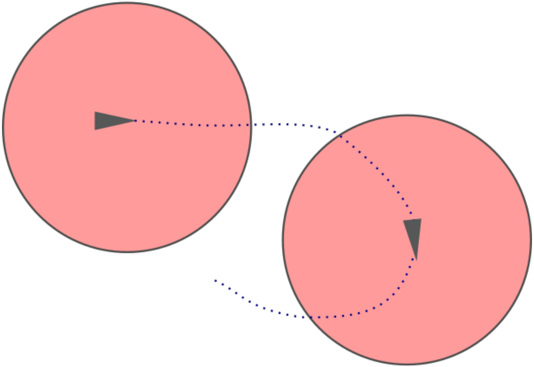
The explored zone $\mathbb{Z}$ is defined by [1]

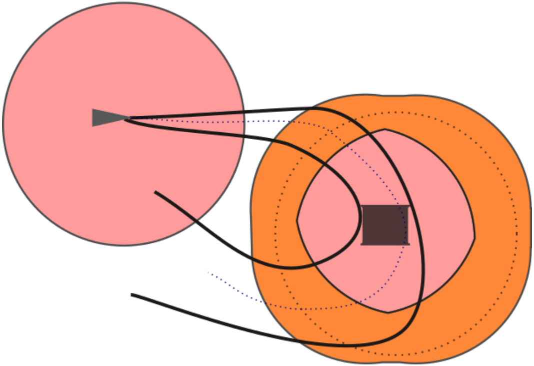
$$\left\{ \begin{array}{c} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}),\ \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbb{Z} = \bigcup_{t \geq 0} \mathbb{V}(\mathbf{x}(t)) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{array} \right.$$

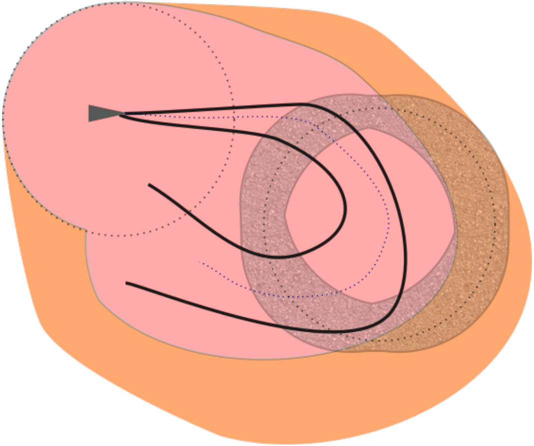
We have

$$\underbrace{\bigcap_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^-} \subset \mathbb{Z} \subset \underbrace{\bigcup_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^+}.$$

$\mathbb{Z}^-$ is the *certainly explored zone.*
$\mathbb{Z}^+$ is the *maybe explored zone*.
$\mathbb{Z}^+ \backslash \mathbb{Z}^-$ is the *penumbra.*



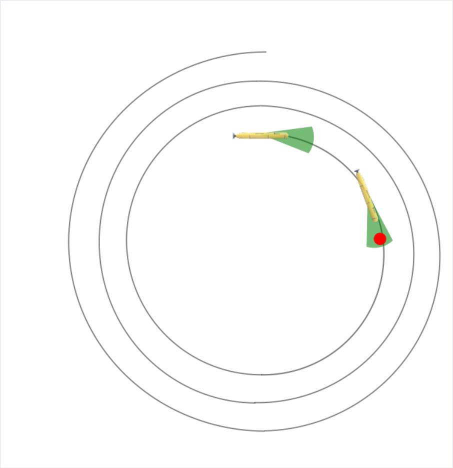


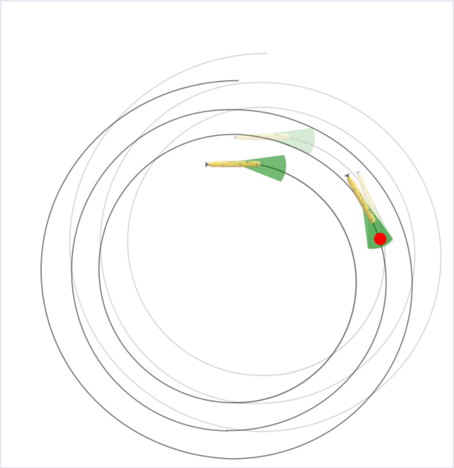


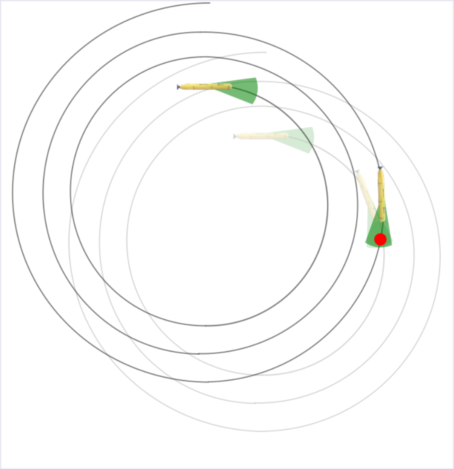
Spiral scan

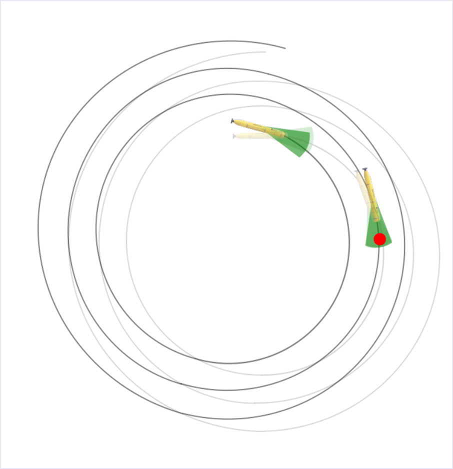
We have [2]

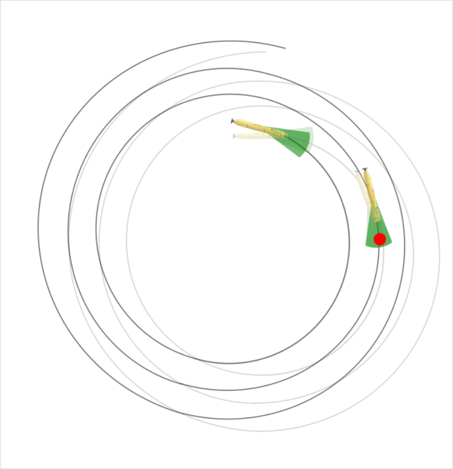
$$\underbrace{\bigcup_{t\geq 0}\ \bigcap_{\mathbf{x}\in\mathscr{X}(t)} \mathbb{V}(\mathbf{x})}_{\{\mathbf{z}\ \mid\ \exists t\ \forall \mathbf{x}\in\mathscr{X}(t),\ \mathbf{z}\in\mathbb{V}(\mathbf{x})\}} \subset \mathbb{Z}^- = \underbrace{\bigcap_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)}\ \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\{\mathbf{z}\ \mid\ \forall \mathbf{x}(\cdot)\in\mathscr{X}(\cdot),\exists t,\ \mathbf{z}\in\mathbb{V}(\mathbf{x})\}}$$

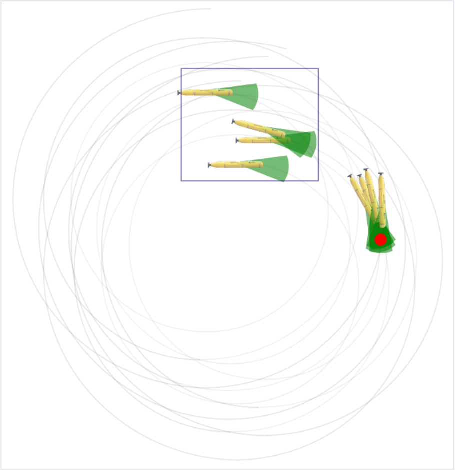






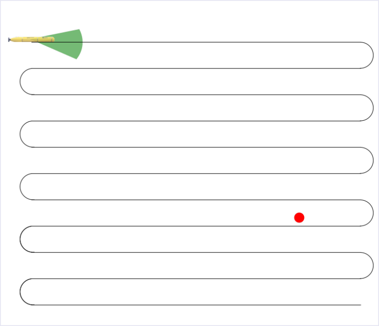


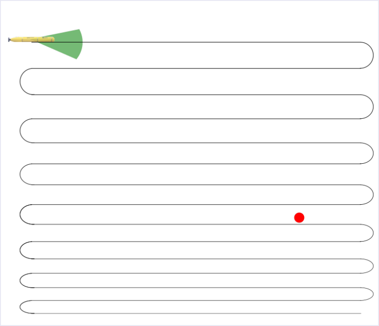




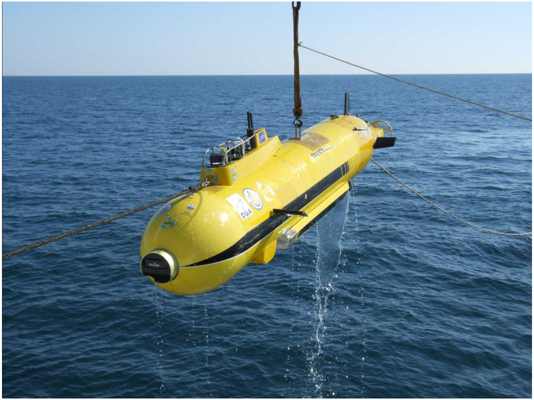
Boustrophedon

Which pattern is the best for exploration?

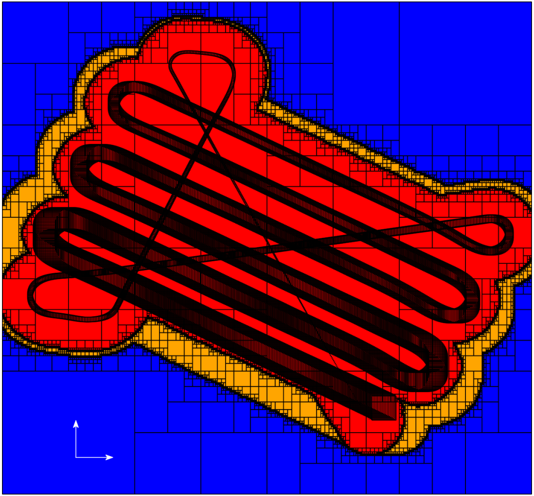




Applications



Daurade DGA-TN



During its boustrophedon Daurade explored $\mathbb{Z} \in [\![\mathbb{Z}]\!]$

Reach an island

Assumptions

- The coastal areas are small compare to the offshore area.
- In the coastal area, the robot knows its state
- Offshore, the robot is blind

Robot

The robot is described by

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), & \mathbf{u}(\cdot) \in [\mathbf{u}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$

Offshore, the robot is blind and an open loop strategy.

The set flow $\mathbf{\Phi}:\mathbb{R}\times\mathbb{R}^n\rightarrow\mathscr{P}(\mathbb{R})$ is defined as:

$$\mathbf{\Phi}(t_1,\mathbf{x}_0)=\{\mathbf{a}|\exists\mathbf{u}(\cdot)\in[\mathbf{u}](t),\mathbf{a}=\mathbf{x}(t_1),\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x},\mathbf{u}),\mathbf{x}(0)=\mathbf{x}_0\}$$

i.e., if at $t=0$, the robot is at $\mathbf{x}_0$, we have $\mathbf{x}(t_1)\in\mathbf{\Phi}(t_1,\mathbf{x}_0)$.

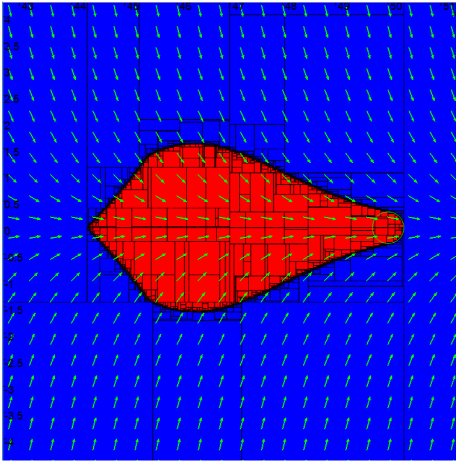
Backward reach set

Given the set $\mathbb{A}$, the backward reach set is defined by

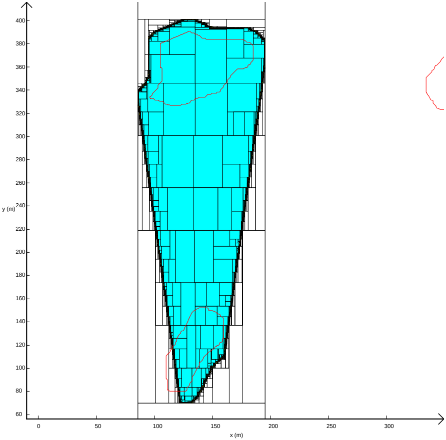
$$\text{Back}(\mathbb{A}) = \{\mathbf{x} \mid \forall\varphi \in \mathbf{\Phi}, \exists t \geq 0, \varphi(t,\mathbf{x}) \in \mathbb{A}\}.$$

Example.

$$\mathbf{\Phi}(t,\mathbf{x}) = \begin{pmatrix} x_1 + t \\ e^{-t} \cdot x_2 \end{pmatrix} + t\,(1 + |x_2|) \begin{pmatrix} [-1,1] \\ [-1,1] \end{pmatrix}$$



Safe start points with a East strategy to reach the circle


400
380
360
340
320
300
280
260
y (m)
240
220
200
180
160
140
120
100
80
60
0
50
100
150
200
250
300
x (m)

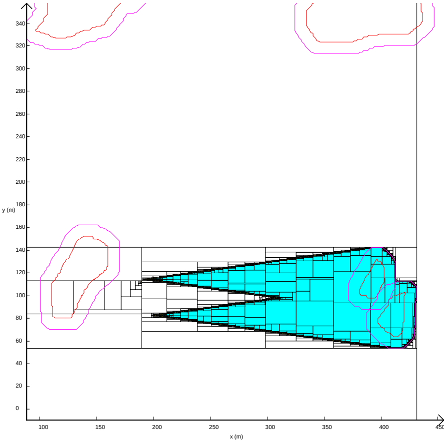
Archipelagic effect

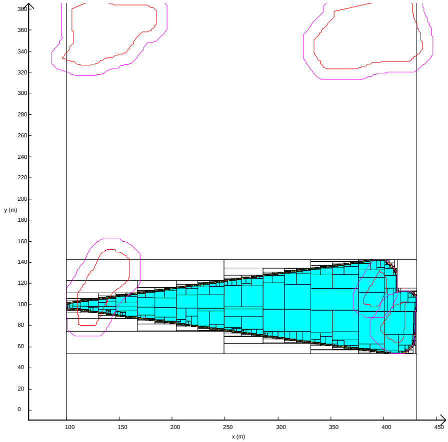
We have

$$\text{Back}(\mathbb{A}\cup\mathbb{B}) \quad \supset \quad \text{Back}(\mathbb{A})\cup\text{Back}(\mathbb{B})$$

Proof.

$$\begin{array}{cc} & \mathbf{x}\in\text{Back}(\mathbb{A})\cup\text{Back}(\mathbb{B}) \\ \Leftrightarrow & (\forall\varphi\in\mathbf{\Phi},\exists t\geq 0,\varphi(t,\mathbf{x})\in\mathbb{A})\vee(\forall\varphi\in\mathbf{\Phi},\exists t\geq 0,\varphi(t,\mathbf{x})\in\mathbb{B}) \\ \Leftrightarrow & \forall\varphi\in\mathbf{\Phi},(\exists t\geq 0,\varphi(t,\mathbf{x})\in\mathbb{A})\vee(\exists t\geq 0,\varphi(t,\mathbf{x})\in\mathbb{B}) \\ \Rightarrow & \forall\varphi\in\mathbf{\Phi},\exists t\geq 0,(\varphi(t,\mathbf{x})\in\mathbb{A}\vee\varphi(t,\mathbf{x})\in\mathbb{B}) \\ \Leftrightarrow & \forall\varphi\in\mathbf{\Phi},\exists t\geq 0,(\varphi(t,\mathbf{x})\in\mathbb{A}\cup\mathbb{B}) \\ \Leftrightarrow & \mathbf{x}\in\text{Back}(\mathbb{A}\cup\mathbb{B}) \end{array}$$


y (m)
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
100
150
200
250
300
350
400
x (m)


y (m)
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
100
150
200
250
300
350
400
450
x (m)

No-lost zone

Moving between coastal zones

- We have m coastal sets $\mathbb{C}_1, \mathbb{C}_2, \dots, i \in \{1, 2, \dots\}$
- We have open loop control strategies $\mathbf{u}_j, j \in \{1, 2, \dots\}$,
- Equivalently, we have set flows $\mathbf{\Phi}_j(t, \mathbf{x}_0)$.
- The control strategy cannot change offshore.

Graph

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2$ with the jth control strategy if

$$\mathbb{C}_1 \cap \mathrm{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2$ with at least one control strategy if

$$\mathbb{C}_1 \cap \bigcup_j \mathrm{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2 \cup \mathbb{C}_3$ with at least one control strategy if

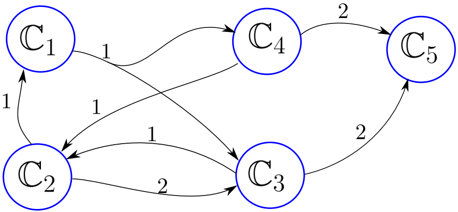
$$\mathbb{C}_1 \cap \bigcup_j \mathrm{Back}(j, \mathbb{C}_2 \cup \mathbb{C}_3) \neq \emptyset.$$

We define $\hookrightarrow$ as:
$\mathbb{C}_a \hookrightarrow \mathbb{C}_b$ if from $\mathbb{C}_a$ we can reach $\mathbb{C}_b$ with at least one control strategy j.

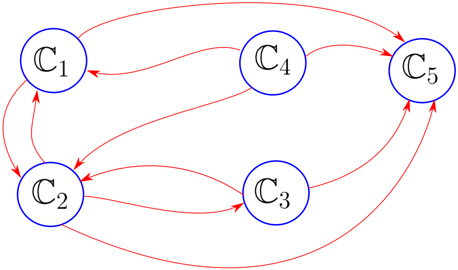
$\hookrightarrow$ is the smallest transitive relation which satisfies

$$\left\{ \begin{array}{c} \forall k \in \mathbb{K}, \mathbb{C}_{i_k} \hookrightarrow \mathbb{C}_b \\ \exists j, \mathbb{C}_a \cap \mathrm{Back}(j, \bigcup_{k \in \mathbb{K}} \mathbb{C}_{i_k}) \neq \emptyset \end{array} \right. \quad \Rightarrow \quad \mathbb{C}_a \hookrightarrow \mathbb{C}_b$$

Graph



$$\mathbb{C}_1 \cap \text{Back}(1, \mathbb{C}_3 \cup \mathbb{C}_4) \neq \emptyset \Rightarrow \mathbb{C}_1 \rightarrow (\mathbb{C}_3, \mathbb{C}_4)$$



Graph of $\hookrightarrow$

Cycle

If

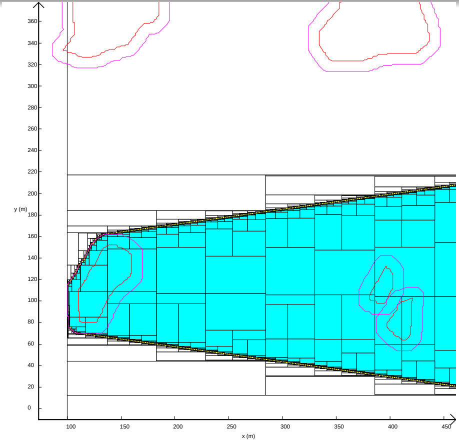
$$\begin{cases} \mathbb{C}_{i_1} \cap \mathbb{C}_{i_2} = \emptyset \\ \mathbb{C}_{i_1} \hookrightarrow \mathbb{C}_{i_2} \\ \mathbb{C}_{i_2} \hookrightarrow \mathbb{C}_{i_1} \end{cases}$$

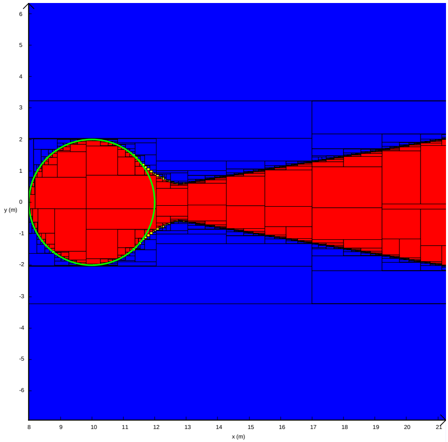
then we can revisit $\mathbb{C}_{i_1}$ and $\mathbb{C}_{i_2}$ for ever.

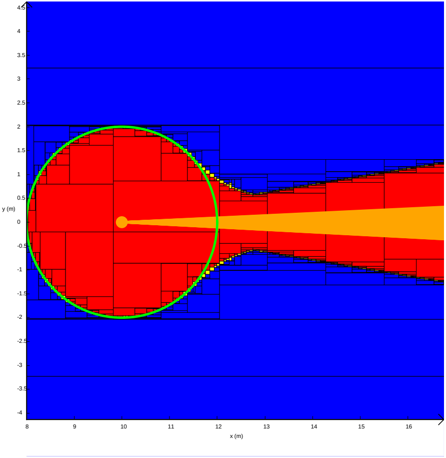
Refinement

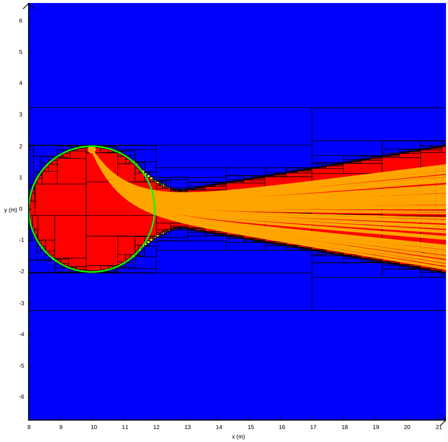
- A coastal area $\mathbb{C}_i$ can be into partitioned into several subareas.
- The relation can be made more accurate by such a decomposition.

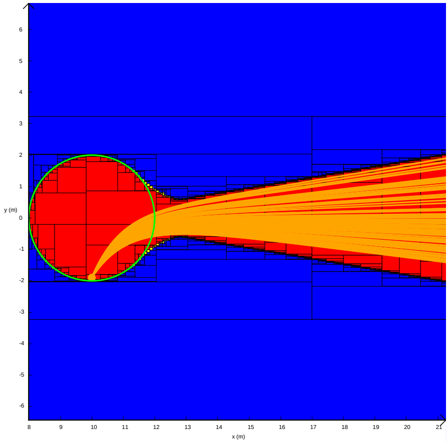
Forward


360
340
320
300
280
260
240
220
200
y (m)
180
160
140
120
100
80
60
40
20
0
100
150
200
250
300
350
400
450
x (m)


6
5
4
3
2
1
0
y (m)
-1
-2
-3
-4
-5
-6
8
9
10
11
12
13
14
15
16
17
18
19
20
21
x (m)


4.5
4
3.5
3
2.5
2
1.5
1
0.5
y (m)
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
8
9
10
11
12
13
14
15
16
x (m)


6
5
4
3
2
1
y (m) 0
-1
-2
-3
-4
-5
-6
8
9
10
11
12
13
14
15
16
17
18
19
20
21
x (m)


y (m)
x (m)
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
8
9
10
11
12
13
14
15
16
17
18
19
20
21

No lost zone

It is the set $\mathbb{S}$ of all states we may visit from a coastal area without being lost with the available control strategy.

Define

$$\mathscr{I}_j = \{k | \mathbb{C}_k \cap \text{Back}(j, \bigcup_{i \neq k} \mathbb{C}_i) \neq \emptyset\}.$$

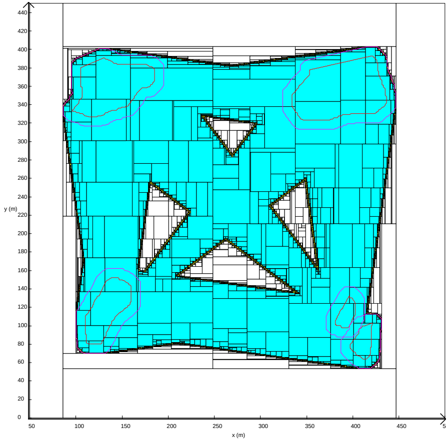
If we start from $\mathbb{C}_k, k \in \mathscr{I}_j$, then we will reach at least another coastal area with the control strategy j.

Thus

$$\left\{ \begin{array}{c} \mathbf{x} \in \mathrm{Back}(j, \bigcup_i \mathbb{C}_i) \\ \mathbf{x} \in \mathrm{Forw}(j, \mathbb{C}_k), k \in \mathscr{I}_j \end{array} \right. \Rightarrow \mathbf{x} \in \mathbb{S}$$

Thus

$$\mathbb{S} = \bigcup_{j} \bigcup_{k \in \mathscr{I}_j} \mathrm{Forw}(j, \mathbb{C}_k) \cap \mathrm{Back}(j, \bigcup_{i} \mathbb{C}_i)$$


440
420
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
y (m)
50
100
150
200
250
300
350
400
450
5
x (m)

Open question

The robot at position **a** is lost for all feasible control strategies the robot cannot guarantee that it will reach an island.
How to prove that the robot is lost?

B. Desrochers and L. Jaulin.
Computing a guaranteed approximation the zone explored by a robot.
IEEE Transaction on Automatic Control, 62(1):425–430, 2017.

V. Drevelle, L. Jaulin, and B. Zerr.
Guaranteed characterization of the explored space of a mobile robot by using subpavings.
In *Proc. Symp. Nonlinear Control Systems (NOLCOS'13)*, Toulouse, 2013.

S. Rohou.
Reliable robot localization: a constraint programming approach over dynamical systems.
PhD dissertation, Université de Bretagne Occidentale, ENSTA-Bretagne, France, december 2017.