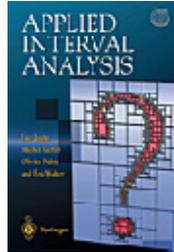


Calcul par intervalles : état de l'art et applications industrielles



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Jeudi 20 novembre 2009,
Journée «Mathématiques, Entreprises et Innovations»
UHA- Mulhouse.

1 Intervalles et propagation

1.1 Calcul par intervalles

Si $\diamond \in \{+, -, \cdot, /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3] \cdot [2, 5] &= [-5, 15], \\ [-1, 3] / [2, 5] &= \left[-\frac{1}{2}, \frac{3}{2}\right], \\ [-1, 3] \vee [2, 5] &= [2, 5]. \end{aligned}$$

$$\begin{aligned}
[x^-, x^+] + [y^-, y^+] &= [x^- + y^-, x^+ + y^+], \\
[x^-, x^+] \cdot [y^-, y^+] &= [x^- y^- \wedge x^+ y^- \wedge x^- y^+ \wedge x^+ y^+, \\
&\quad x^- y^- \vee x^+ y^- \vee x^- y^+ \vee x^+ y^+], \\
[x^-, x^+] \vee [y^-, y^+] &= [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = \{f(x) \mid x \in [x]\}.$$

Par exemple,

$$\begin{aligned}\sin([0, \pi]) &= [0, 1], \\ \text{sqr}([-1, 3]) &= [-1, 3]^2 = [0, 9], \\ \text{abs}([-7, 1]) &= [0, 7], \\ \text{sqrt}([-10, 4]) &= \sqrt{[-10, 4]} = [0, 2], \\ \log([-2, -1]) &= \emptyset.\end{aligned}$$

1.2 Projection de contraintes

Soient x, y, z trois variables telles que

$$x \in [-\infty, 5],$$

$$y \in [-\infty, 4],$$

$$z \in [6, \infty],$$

$$z = x + y.$$

Les valeurs < 2 pour x , < 1 pour y et > 9 pour z sont inconsistantes.

1.3 Méthode numérique de projection

En effet, puisque $x \in [-\infty, 5]$, $y \in [-\infty, 4]$, $z \in [6, \infty]$ et $z = x + y$, nous avons

$$z = x + y \Rightarrow z \in [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ = [6, \infty] \cap [-\infty, 9] = [6, 9].$$

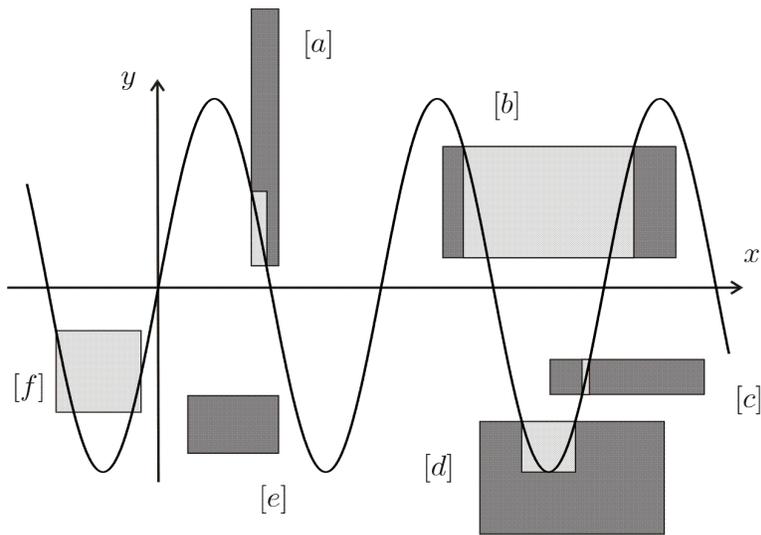
$$x = z - y \Rightarrow x \in [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ = [-\infty, 5] \cap [2, \infty] = [2, 5].$$

$$y = z - x \Rightarrow y \in [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ = [-\infty, 4] \cap [1, \infty] = [1, 4].$$

Pour la contrainte

$$y = \sin x, \quad x \in [x], y \in [y]$$

le problème est un peu plus difficile.



1.4 Algorithme de propagation-bissection

Exemple. Cherchons à résoudre.

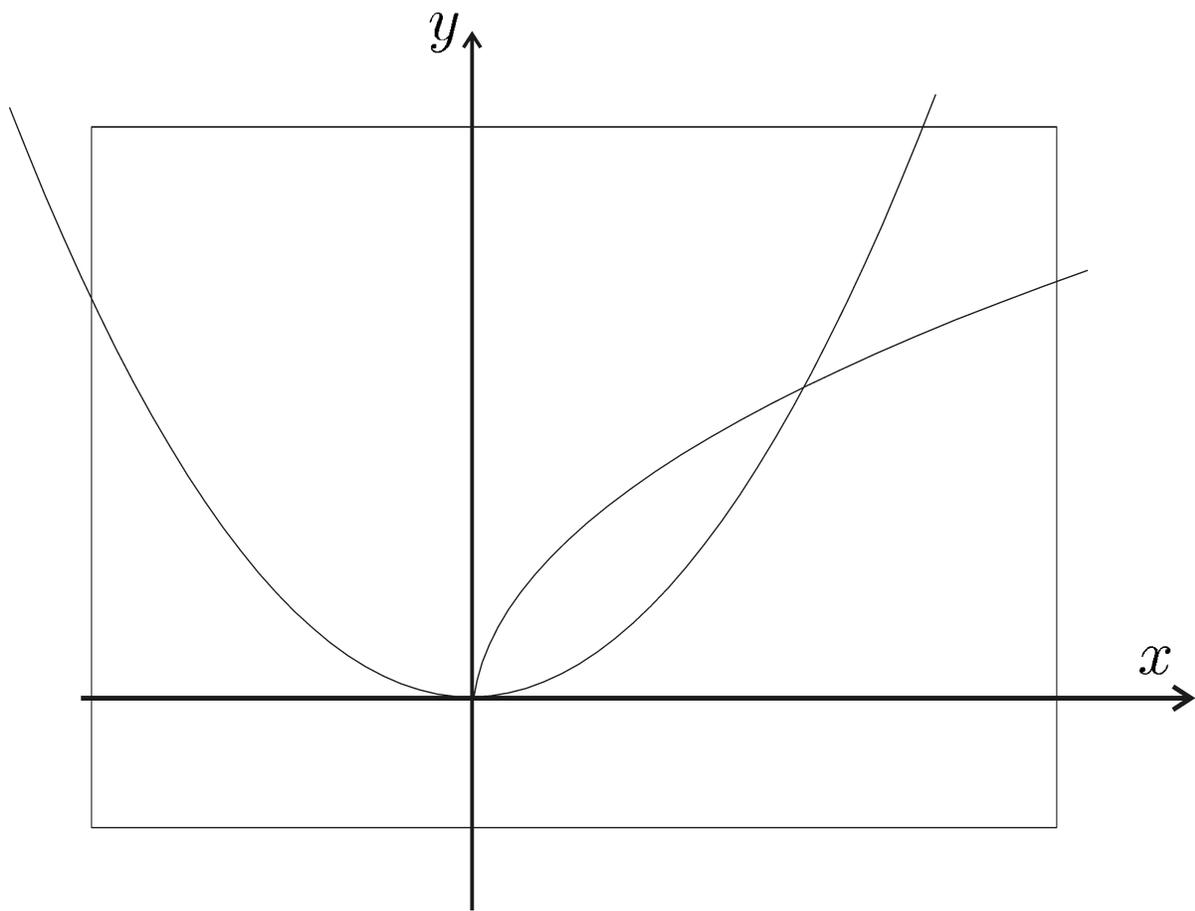
$$y = x^2$$

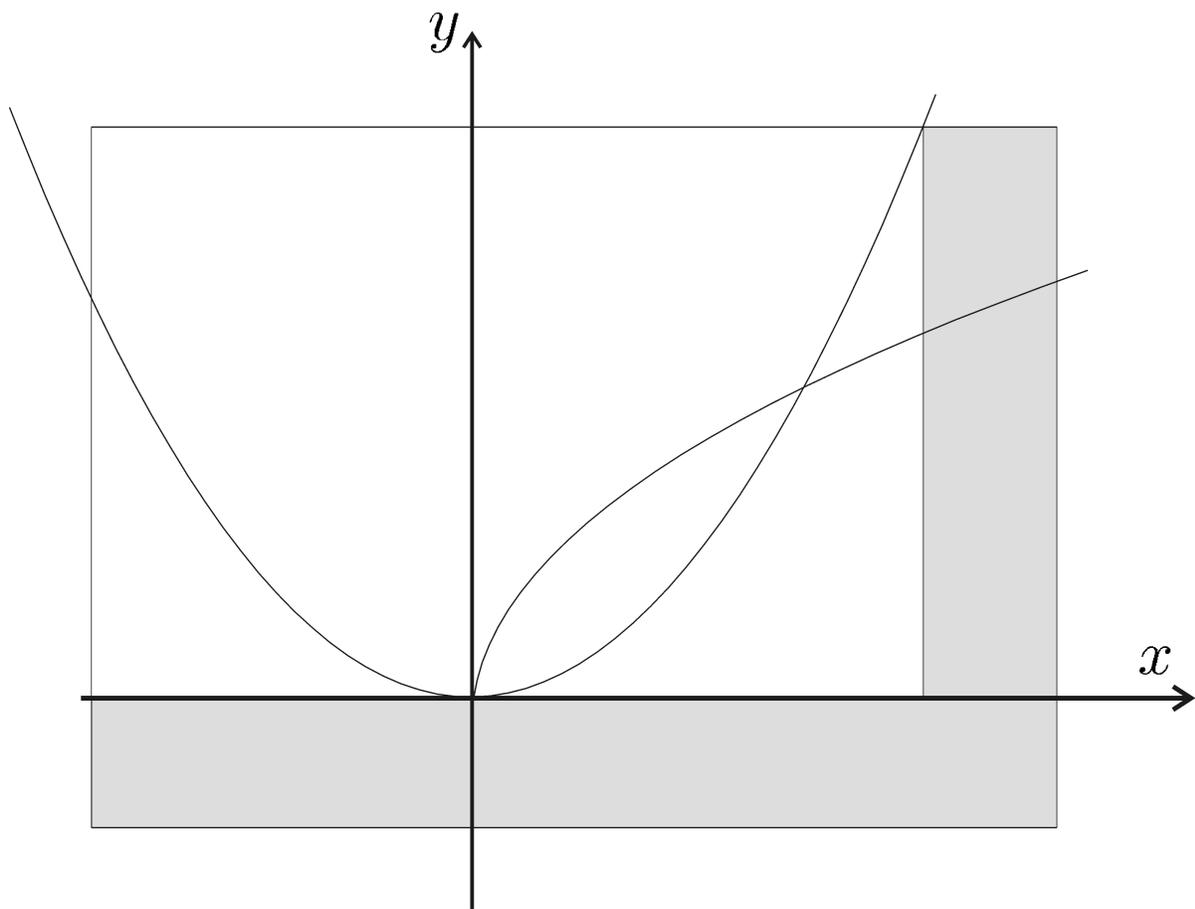
$$y = \sqrt{x}.$$

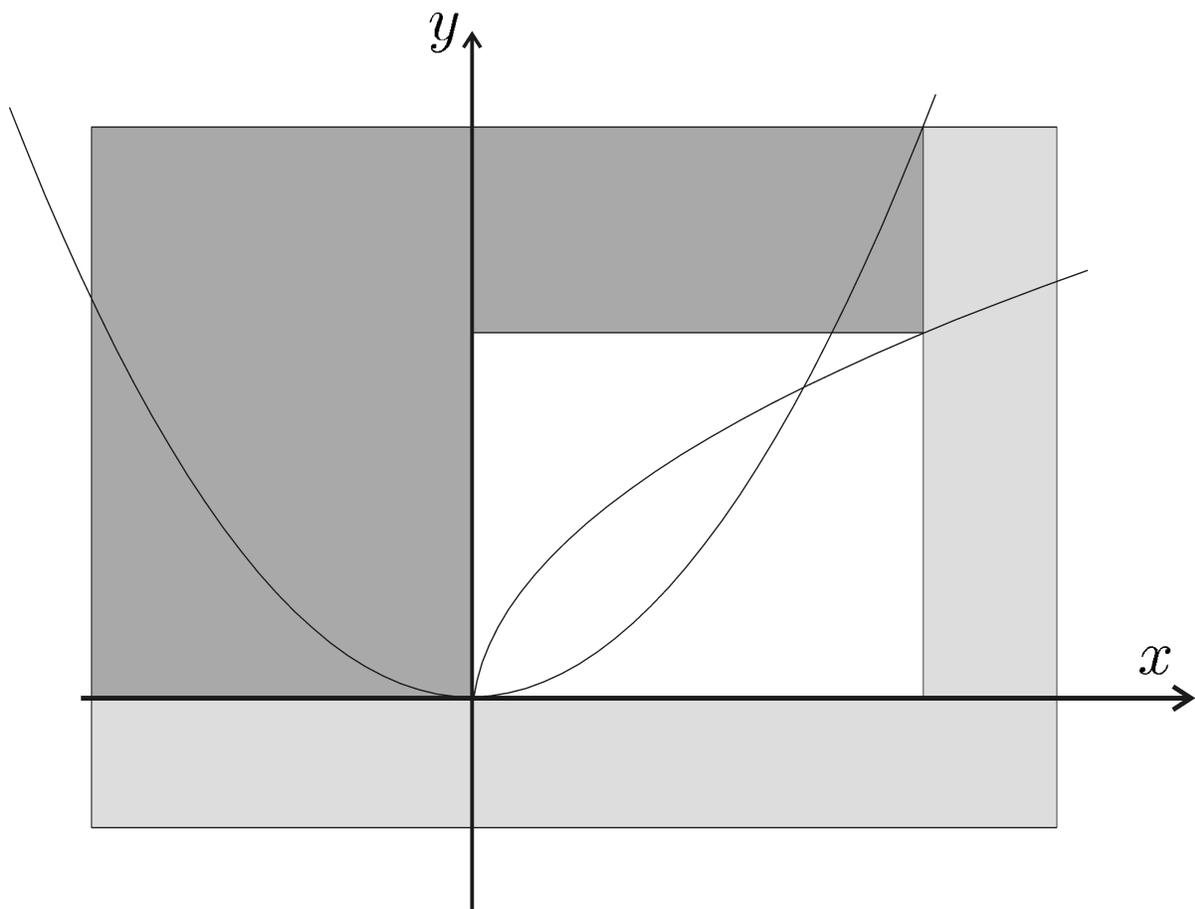
On a deux contracteurs

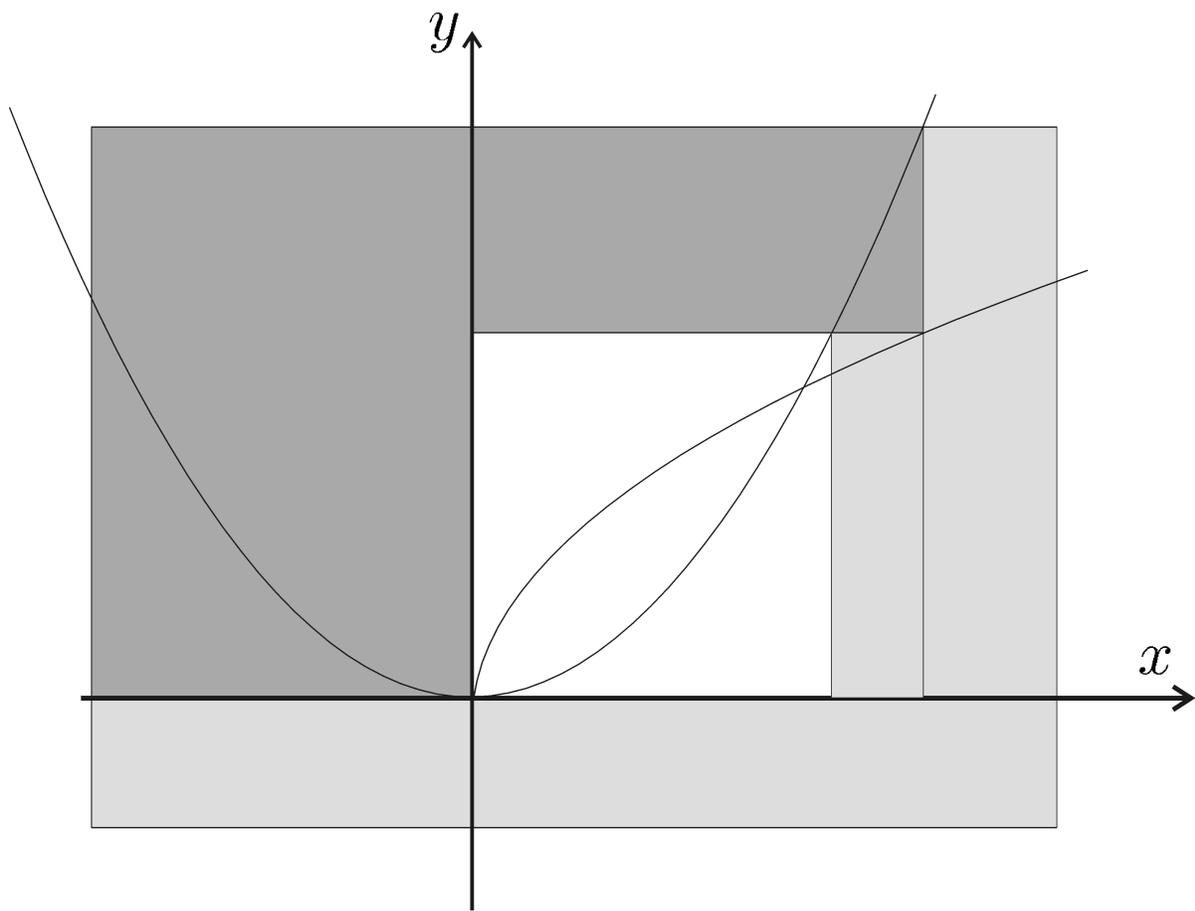
$$C_1 : \begin{cases} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{cases} \text{ associé à } y = x^2$$

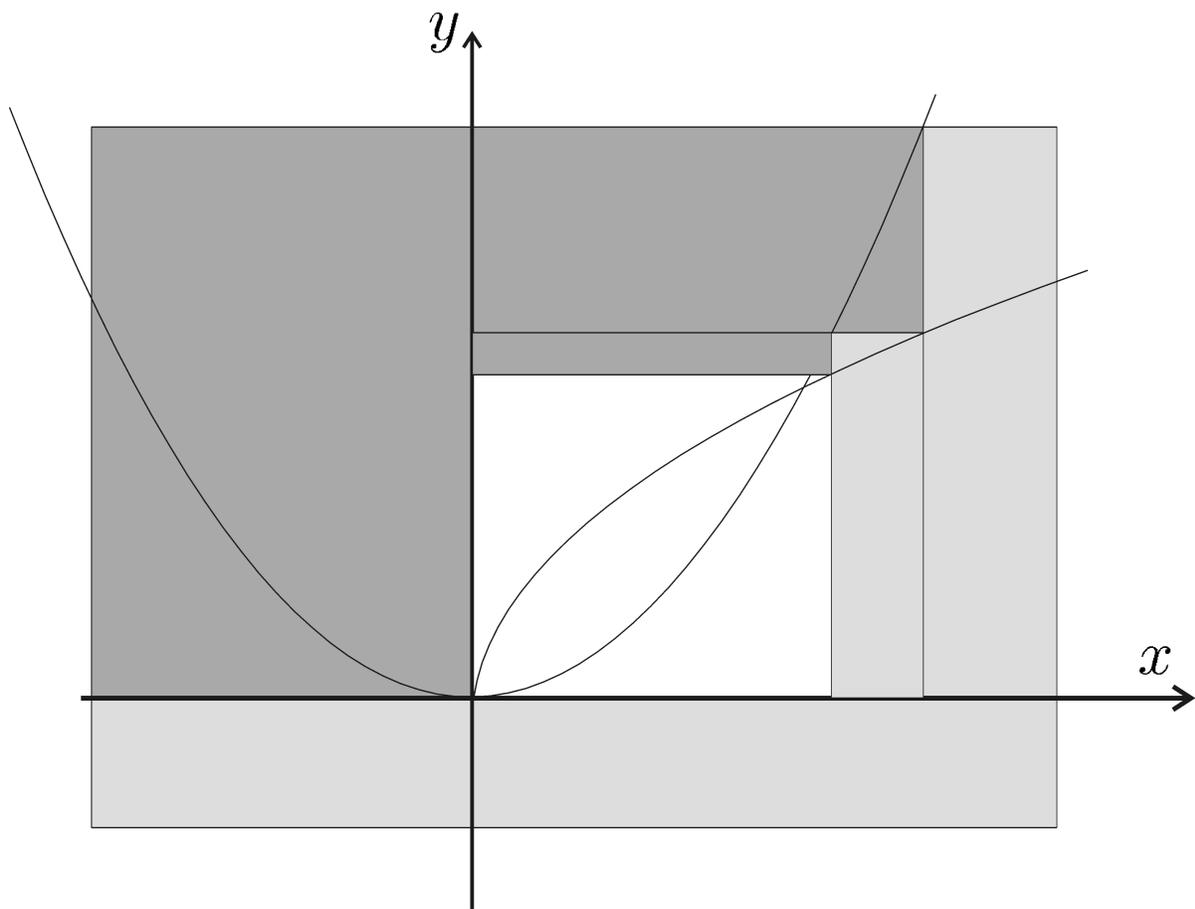
$$C_2 : \begin{cases} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{cases} \text{ associé à } y = \sqrt{x}$$

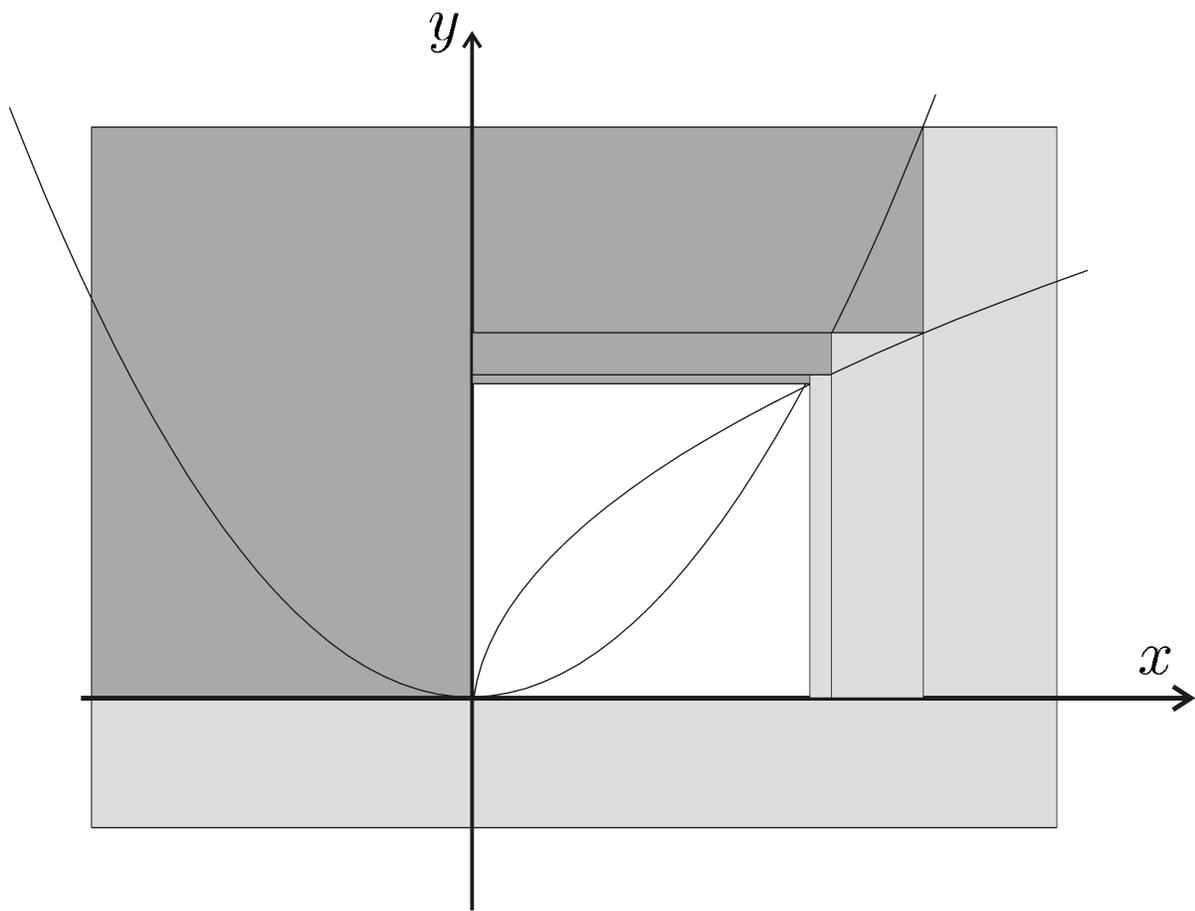


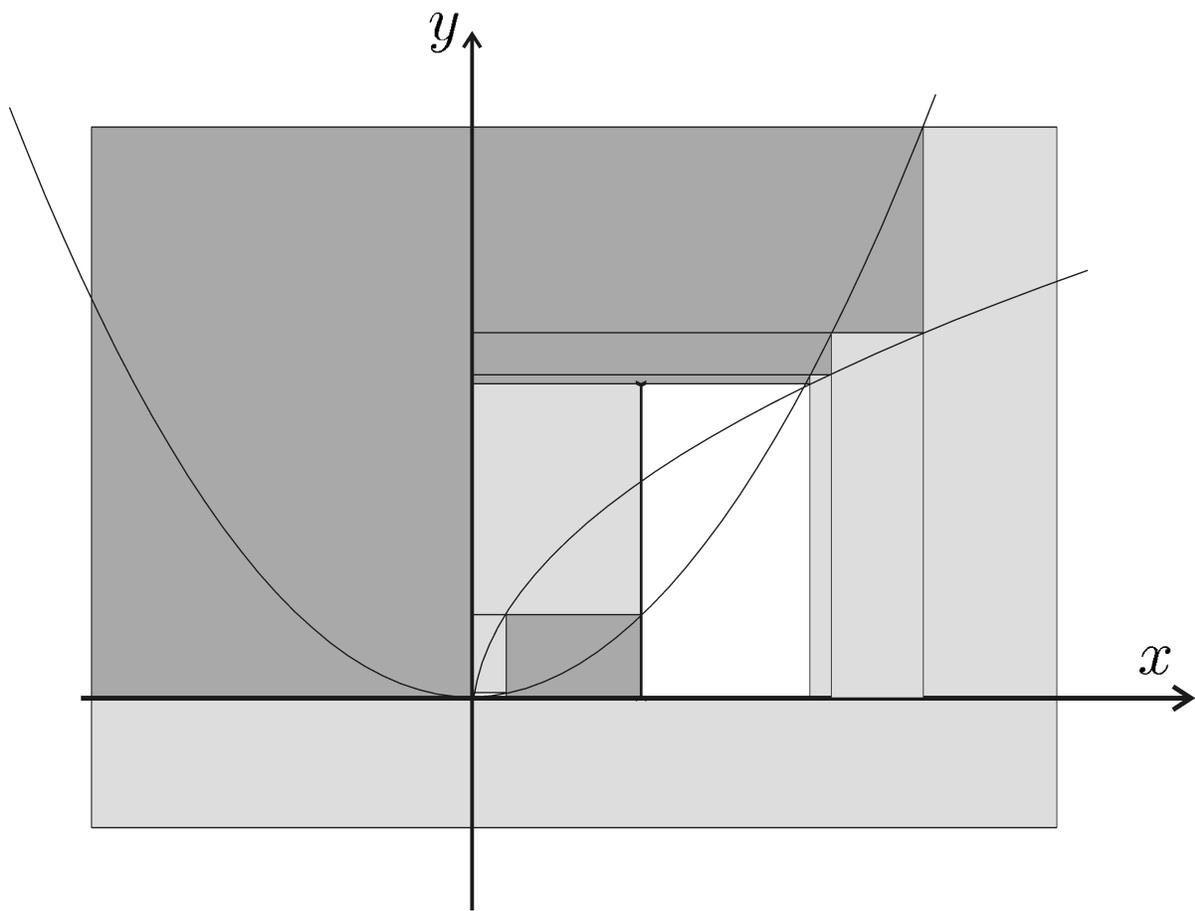


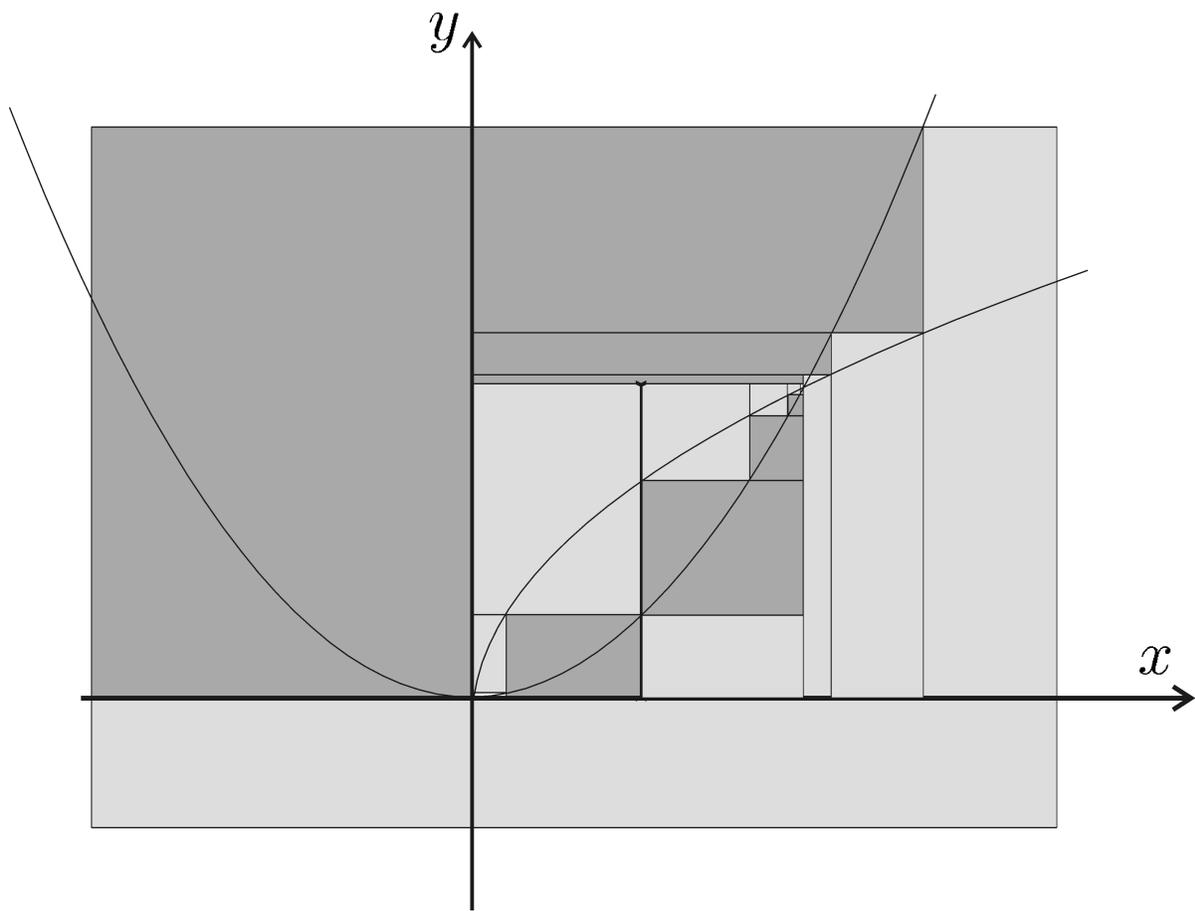












1.5 Décomposition

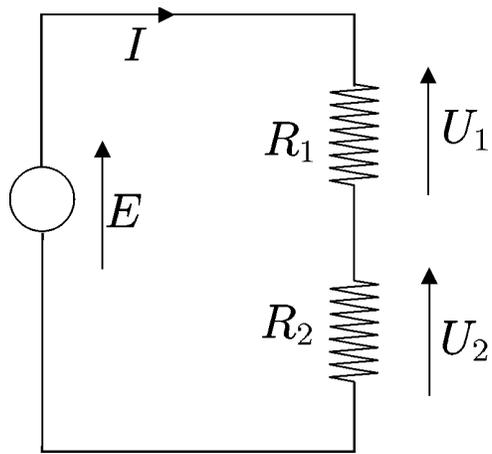
Pour les contraintes plus complexes, il nous faut effectuer une décomposition

$$\begin{aligned}x + \sin(y) - xz &\leq 0, \\x \in [-1, 1], y \in [-1, 1], z \in [-1, 1]\end{aligned}$$

se décompose en

$$\left\{ \begin{array}{l} a = \sin(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array} \right. , \quad \begin{array}{l} x \in [-1, 1] \\ y \in [-1, 1] \\ z \in [-1, 1] \end{array} \quad \begin{array}{l} a \in] - \infty, \infty [\\ b \in] - \infty, \infty [\\ c \in] - \infty, \infty [\\ d \in] - \infty, 0] \end{array}$$

1.6 Un exemple académique



Contraintes

$$\begin{aligned}P &= EI; E = (R_1 + R_2) I; \\U_1 &= R_1 I; U_2 = R_2 I; E = U_1 + U_2.\end{aligned}$$

Domaines initiaux

$$\begin{aligned}R_1 &\in [0, \infty]\Omega, & R_2 &\in [0, \infty]\Omega, \\E &\in [23, 26]\text{V}, & I &\in [4, 8]\text{A}, \\U_1 &\in [10, 11]\text{V}, & U_2 &\in [14, 17]\text{V}, \\P &\in [124, 130]\text{W},\end{aligned}$$

La propagation renvoie le pavé

$$[24; 26] \times [1.846; 2.307] \times [2.584; 3.355] \\ \times [4.769; 5.417] \times [10; 11] \times [14; 16] \times [124; 130].$$

c'est-à-dire

$$\begin{aligned} E &\in [24; 26], & R_1 &\in [1.846; 2.307], \\ R_2 &\in [2.584; 3.355], & I &\in [4.769; 5.417], \\ U_1 &\in [10; 11], & U_2 &\in [14; 16], \\ P &\in [124; 130]. \end{aligned}$$

1.7 Contractors

L'opérateur $\mathcal{C}_{\mathbb{X}} : \mathbb{R}^n \rightarrow \mathbb{R}^n$ est un *contractor* pour l'ensemble \mathbb{X} de \mathbb{R}^n si

$$\forall [\mathbf{x}] \in \mathbb{R}^n, \begin{cases} \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance),} \\ \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \cap \mathbb{X} = [\mathbf{x}] \cap \mathbb{X} & \text{(complétude).} \end{cases}$$

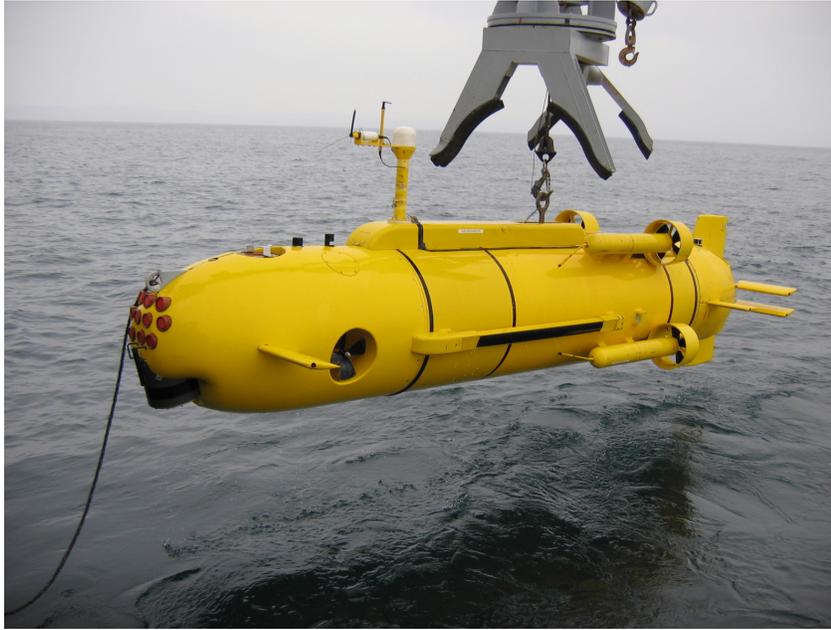
Nous dirons que

$\mathcal{C}_{\mathbb{X}}$ est <i>monotone</i> si	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) \subset \mathcal{C}_{\mathbb{X}}([\mathbf{y}])$
$\mathcal{C}_{\mathbb{X}}$ est <i>minimal</i> si	$\forall [\mathbf{x}] \in \mathbb{I}\mathbb{R}^n, \mathcal{C}_{\mathbb{X}}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathbb{X}]$
$\mathcal{C}_{\mathbb{X}}$ est <i>thin</i> si	$\forall \mathbf{x} \in \mathbb{R}^n, \mathcal{C}_{\mathbb{X}}(\{\mathbf{x}\}) = \{\mathbf{x}\} \cap \mathbb{X}$
$\mathcal{C}_{\mathbb{X}}$ est <i>idempotent</i> si	$\forall [\mathbf{x}] \in \mathbb{I}\mathbb{R}^n, \mathcal{C}_{\mathbb{X}}(\mathcal{C}_{\mathbb{X}}([\mathbf{x}])) = \mathcal{C}_{\mathbb{X}}([\mathbf{x}])$.

$\mathcal{C}_{\mathbb{X}}$ est *convergent* si pour presque tout \mathbf{x} , et pour toute suite de pavé $[\mathbf{x}](k)$,

$$[\mathbf{x}](k) \rightarrow \mathbf{x} \Rightarrow \mathcal{C}_{\mathbb{X}}([\mathbf{x}](k)) \rightarrow \{\mathbf{x}\} \cap \mathbb{X}.$$

2 SLAM



Le *Redermor*, fabriqué par le GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Montrer la simulation

Pourquoi une approche par intervalles ?

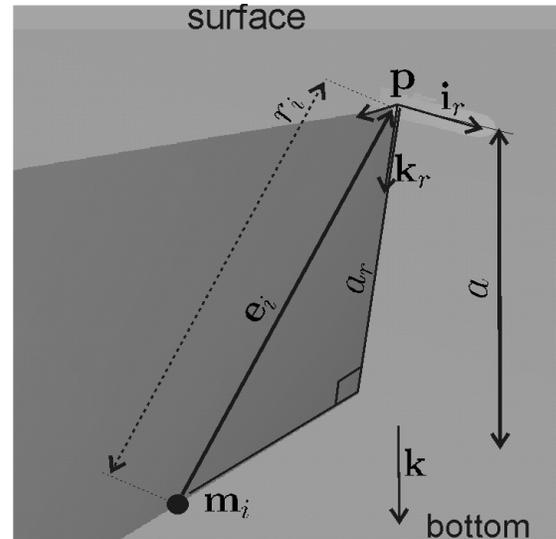
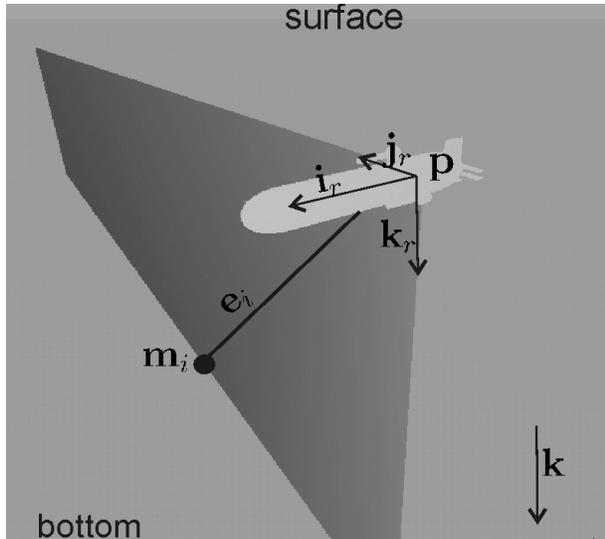
- 1) Besoin d'une approche fiable.
- 2) Les équations du robot sont non linéaires.
- 3) Les bruits de mesure sont non gaussiens.
- 4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
- 5) Beaucoup de mesures redondantes sont disponibles.

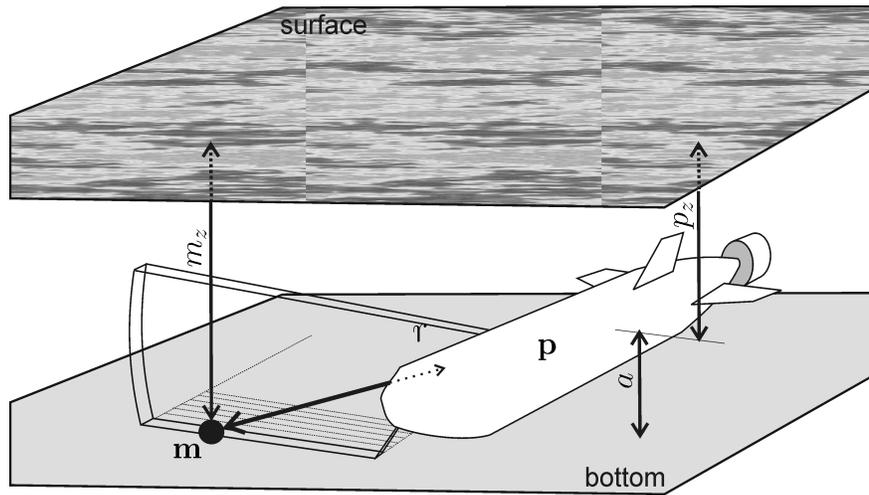
2.1 Capteurs

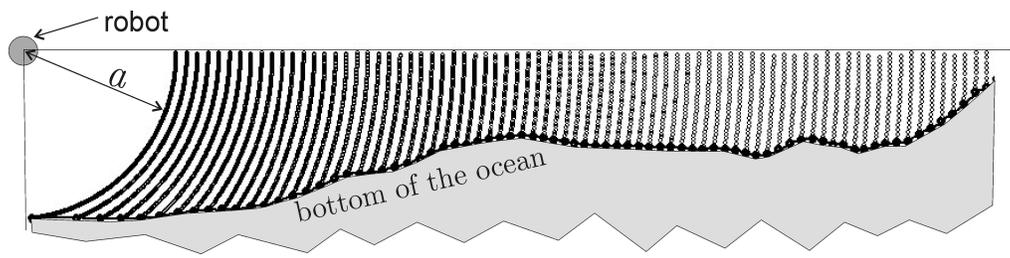
Un GPS (Global positioning system), disponible à la surface.

$$t_0 = 6000 \text{ s}, \quad \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m$$
$$t_f = 12000 \text{ s}, \quad \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m$$

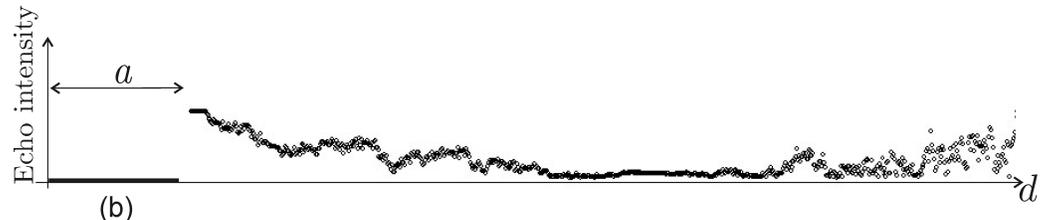
Un sonar (KLEIN 5400 side scan sonar). Donne la distance r entre le robot et la mine



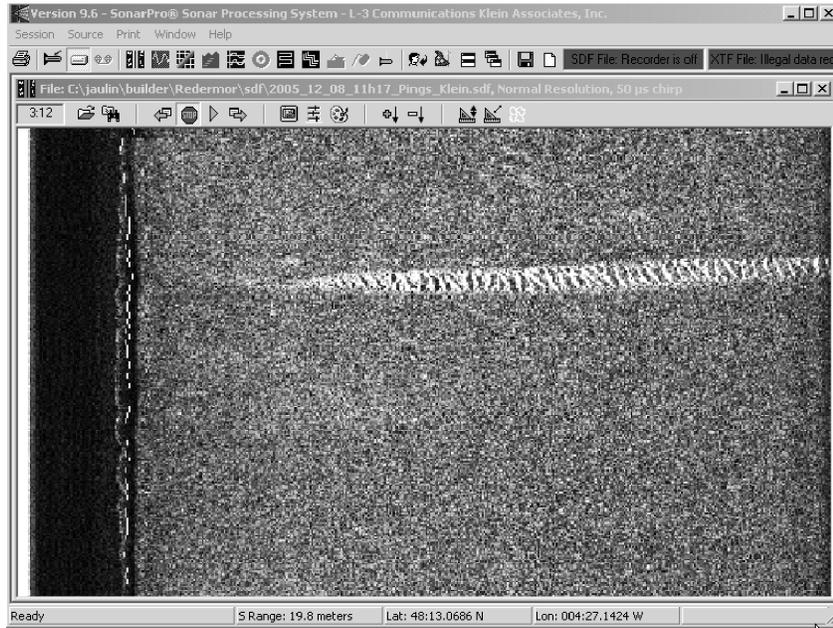




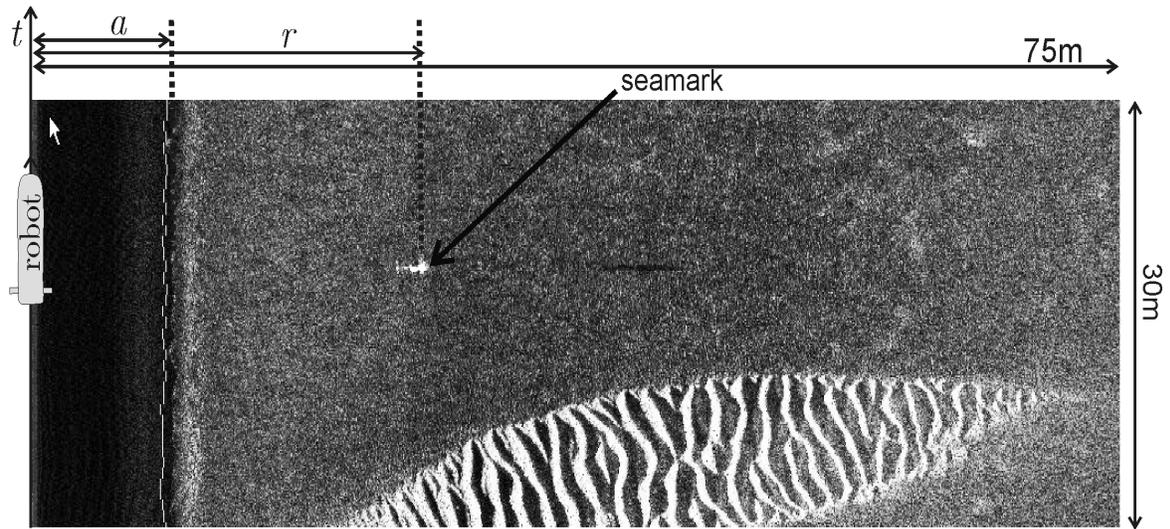
(a)



(b)



Screenshot du logiciel SonarPro



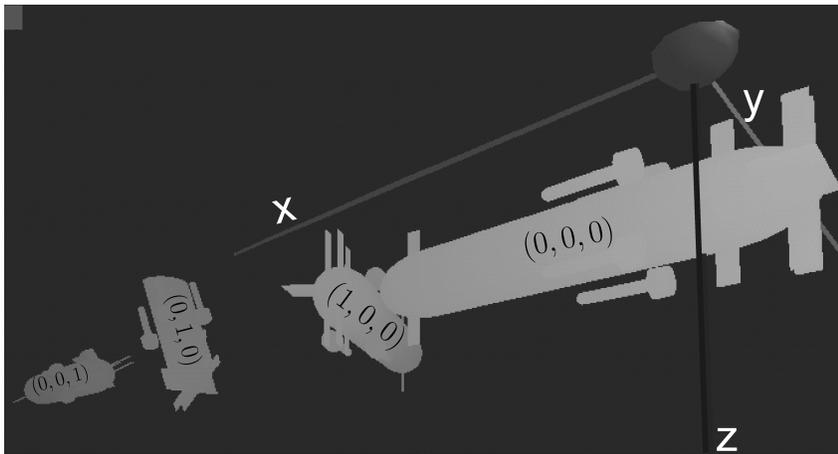
Détection d'une mine à l'aide de SonarPro

Le Loch-Doppler renvoie la vitesse du robot \mathbf{v}_r et son altitude a .

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1, 1] . \tilde{\mathbf{v}}_r + 0.004 * [-1, 1]$$

Une centrale inertielle (Octans III from IXSEA) renvoie le roulis ϕ , le tangage θ et le cap ψ du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$



2.2 Données

Pour chaque $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$, nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

2.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi(t) & -\sin \varphi(t) \\ 0 & \sin \varphi(t) & \cos \varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

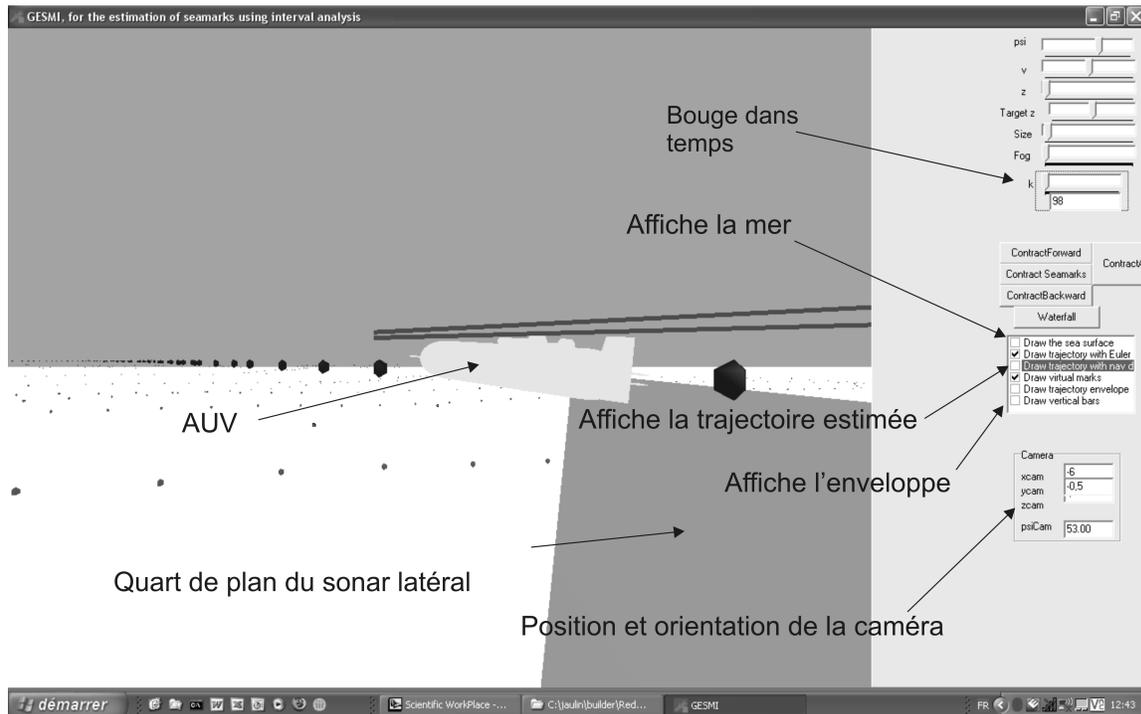
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t) \cdot \mathbf{v}_r(t),$$

$$\|\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))\| = r(i),$$

$$\mathbf{R}^\top(\tau(i)) (\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5]$$

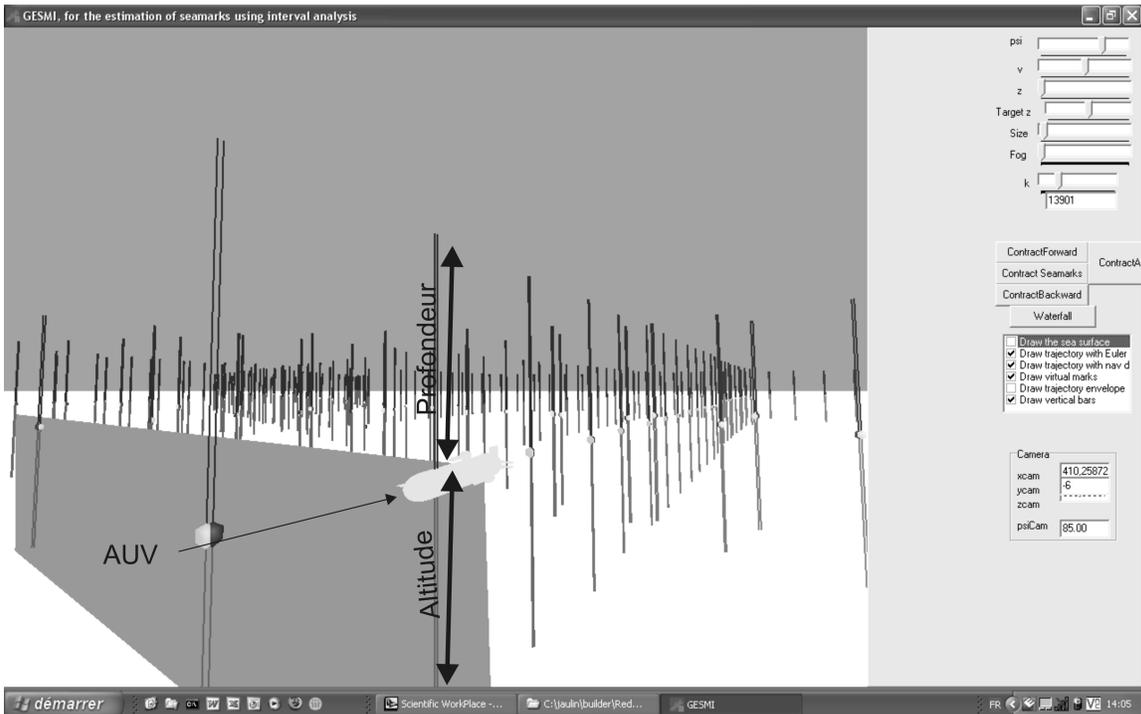
2.4 Logiciel GESMI

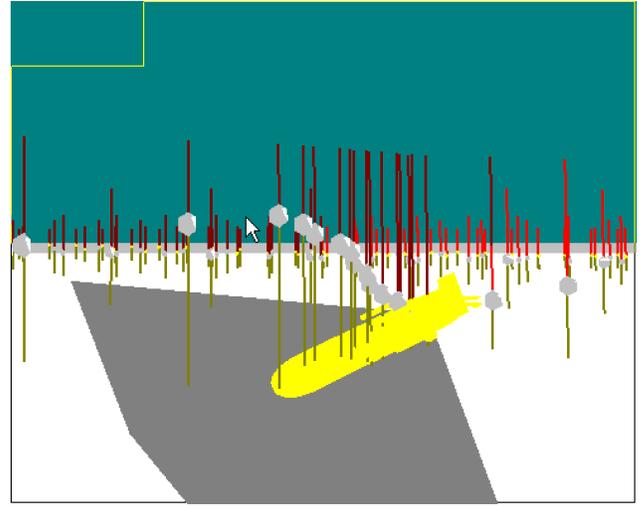
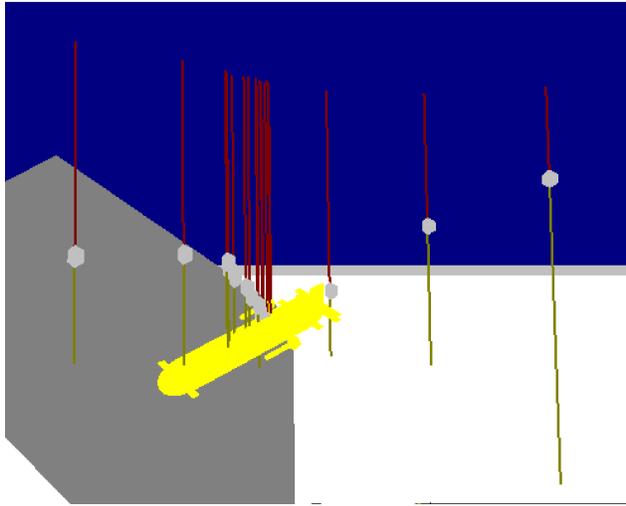


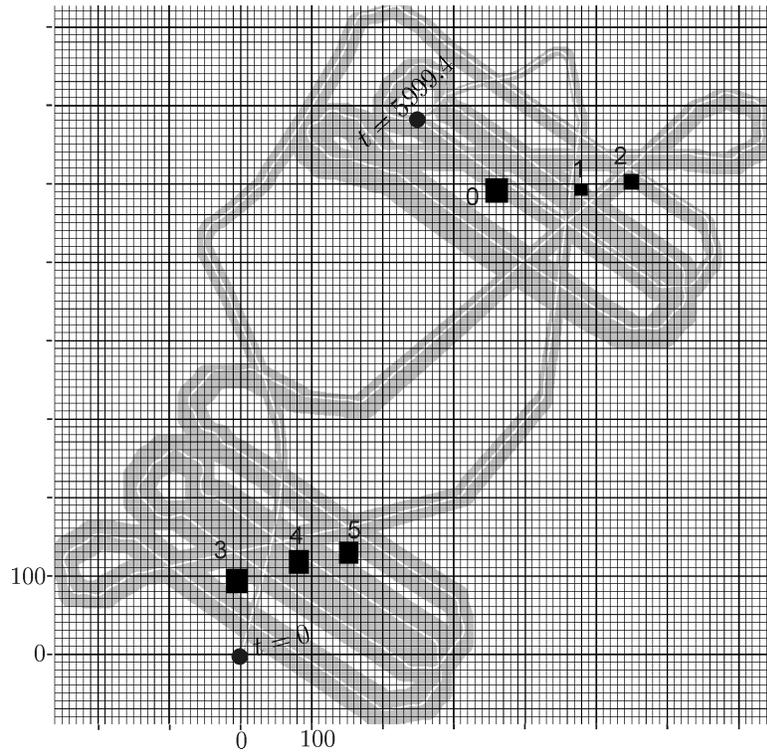
```

% This file has been generated by a generator
% and will be used by GESMI to solve a SLAM problem
% Note : every line starting by a '%' is considered as a comment by GESMI.
% -----
%
% This file contains
% 1) Some basic information about the scope of the sonar and the sampling time
% 2) The prior domains for the seamarks
% 3) The coordinates for some virtual marks
% 4) sensor data (angles, speeds, depth, altitude, position) with bounds
% 5) ping table : at the end of this file
%
% -----
% maximal distance of the lateral sonar and error error interval [min,max]
75.0 1.0
% Sampling time
0.1
% -----
% Domain for seamarks detected using the software SonarPro
% -----
% Initial domains for the mines
% xmin xmax ymin ymax zmin zmax
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
-10000 10000 -10000 10000 0 100
$
% -----
% Virtual marks (only for graphism, not used for computation)
% -----
% x y z colorRed ColorGreen ColorBlue LocalFrame
594.0533723 374.7293035 19 0.1 0 0 0 % seamark 0
599.6093723 484.5528643 19 0.1 0.1 0 0 % seamark 1
601.4613723 557.3573086 19 0.1 0 0.1 0 % seamark 2
94.01337232 -2.868189501 19 0 0.1 0 0 % seamark 3
119.9413723 84.75155672 20 0 0 0.1 0 % seamark 4
127.3493723 156.3280366 20 0 0 0 0 % seamark 5
0 0 0 1 0 0 1 % origin of the local frame
689.96 264.10 0 0 0 1 1 % final GPS
$
% -----
% -----
%t phi dphi theta dtheta psi dpsi vx dvx
0 -0.011505 0.0001745329252 0.033556 0.0001745329252 0.415613 0.0052679
0.1 -0.012272 0.0001745329252 0.034994 0.0001745329252 0.416284 0.00526
0.2 -0.012847 0.0001745329252 0.037583 0.0001745329252 0.417051 0.00526
0.3 -0.013422 0.0001745329252 0.040938 0.0001745329252 0.418202 0.00526
0.4 -0.014093 0.0001745329252 0.044773 0.0001745329252 0.419831 0.00526
0.5 -0.014285 0.0001745329252 0.0488 0.0001745329252 0.422228 0.0052679
0.6 -0.013998 0.0001745329252 0.052539 0.0001745329252 0.425488 0.00526
0.7 -0.012943 0.0001745329252 0.055703 0.0001745329252 0.42961 0.005267
0.8 -0.011121 0.0001745329252 0.0581 0.0001745329252 0.434308 0.0052679
0.9 -0.008725 0.0001745329252 0.059825 0.0001745329252 0.439773 0.00526
1 -0.005944 0.0001745329252 0.060496 0.0001745329252 0.445334 0.0052679
1.1 -0.003547 0.0001745329252 0.0604 0.0001745329252 0.450511 0.0052679
1.2 -0.001917 0.0001745329252 0.059729 0.0001745329252 0.455017 0.00526
1.3 -0.001438 0.0001745329252 0.058867 0.0001745329252 0.458564 0.00526
1.4 -0.002013 0.0001745329252 0.058387 0.0001745329252 0.461057 0.00526
1.5 -0.003356 0.0001745329252 0.058483 0.0001745329252 0.462303 0.00526
1.6 -0.005561 0.0001745329252 0.059346 0.0001745329252 0.462591 0.00526
1.7 -0.007958 0.0001745329252 0.060976 0.0001745329252 0.462112 0.00526

```







3 Inversion ensembliste

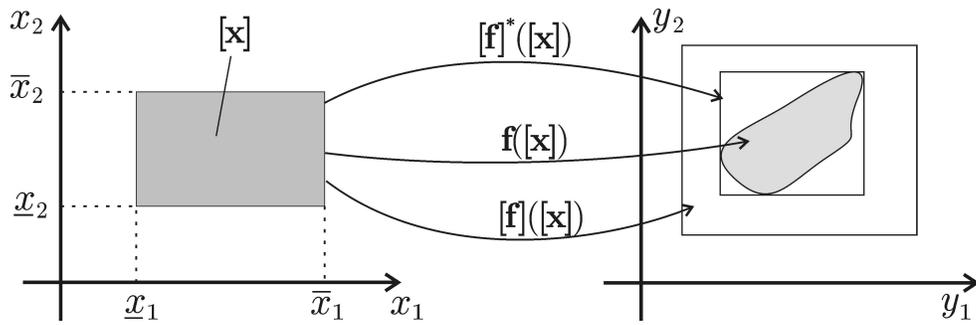
3.1 Fonction d'inclusion

Soit $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$. La fonction $[\mathbf{f}] : \mathbb{IR}^n \rightarrow \mathbb{IR}^m$, est une *fonction d'inclusion* de \mathbf{f} si

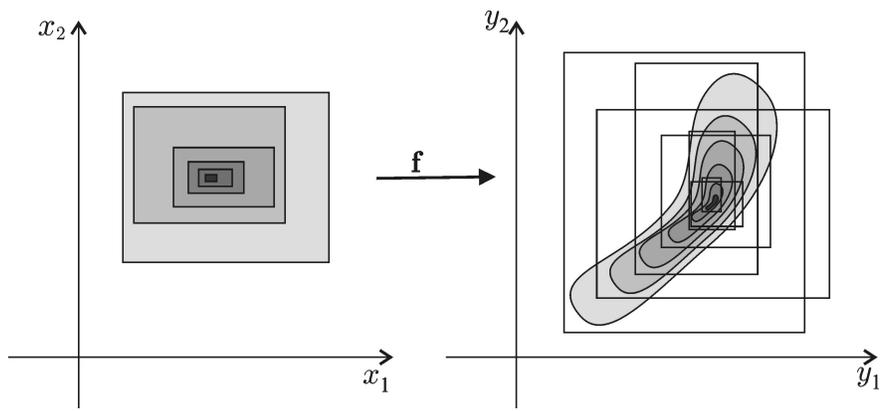
$$\forall [\mathbf{x}] \in \mathbb{IR}^n, \quad \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}]).$$

On dira que $[\mathbf{f}]$ est

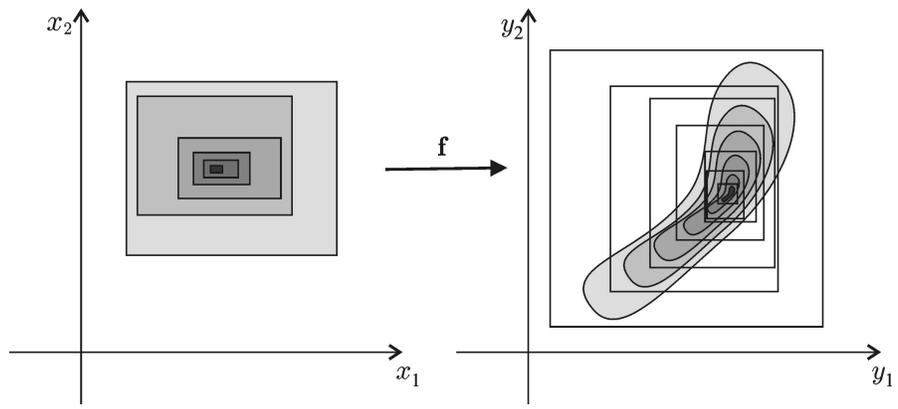
<i>monotone</i>	si	$([\mathbf{x}] \subset [\mathbf{y}]) \Rightarrow ([\mathbf{f}]([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{y}]))$
<i>minimale</i>	si	$\forall [\mathbf{x}] \in \mathbb{IR}^n, [\mathbf{f}]([\mathbf{x}]) = [\mathbf{f}([\mathbf{x}])]$
<i>convergente</i>	si	$w([\mathbf{x}](k)) \rightarrow 0 \Rightarrow w([\mathbf{f}]([\mathbf{x}](k))) \rightarrow 0$



Fonctions d'inclusion $[f]$ et $[f]^*$. Ici, $[f]^*$ est minimale.



Fonction d'inclusion convergente mais pas monotone



Fonction d'inclusion convergente et monotone

3.2 Fonction d'inclusion naturelle

Il suffit de remplacer dans l'expression de f , tous les x_i 's par $[x_i]$.

Exemple 1. Si $f(x) = x^2 + 2x + 4$ alors, sa fonction d'inclusion naturelle est

$$[f]([x]) = [x]^2 + 2[x] + 4.$$

Pour $[x] = [-3, 4]$, nous avons

$$\begin{aligned} [f]([-3, 4]) &= [-3, 4]^2 + 2[-3, 4] + 4 \\ &= [0, 16] + [-6, 8] + 4 = [-2, 28]. \end{aligned}$$

Notons que

$$f([-3, 4]) = [3, 28] \subset [f]([-3, 4]) = [-2, 28].$$

Exemple 2. Soit

$$\mathbf{f} : \begin{array}{l} \mathbb{R}^2 \\ (x_1, x_2) \end{array} \begin{array}{l} \rightarrow \mathbb{R}^3 \\ \mapsto (x_1x_2, x_1^2, x_1 - x_2) \end{array} .$$

Sa fonction d'inclusion naturelle $[f]$ est

$$\begin{array}{ccc} \mathbb{R}^2 & \rightarrow & \mathbb{R}^3 \\ ([x_1], [x_2]) & \rightarrow & ([x_1] * [x_2], [x_1]^2, [x_1] - [x_2]) . \end{array}$$

Puisque dans les f_i , les x_i n'apparaissent qu'une seule fois, la fonction d'inclusion est minimale.

Exemple 3 : Si f est donnée par

Algo $f(\text{in: } \mathbf{x} = (x_1, x_2, x_3), \text{out: } \mathbf{y} = (y_1, y_2))$

```
1   $z := x_1;$   
2  for  $k := 0$  to 100  
3       $z := x_2(z + kx_3);$   
4  next;  
5   $y_1 := z;$   
6   $y_2 := \sin(zx_1);$ 
```

Quelle est sa fonction d'inclusion naturelle ?

Sa fonction d'inclusion naturelle est

Algo $[f]$ (in: $[x] = ([x_1], [x_2], [x_3])$, out: $[y] = ([y_1], [y_2])$)
1 $[z] := [x_1]$;
2 for $k := 0$ to 100
3 $[z] := [x_2] * ([z] + k * [x_3])$;
4 next;
5 $[y_1] := [z]$;
6 $[y_2] := \sin([z] * [x_1])$;

Ici, $[f]$ est convergente et monotone.

3.3 Sous-pavages

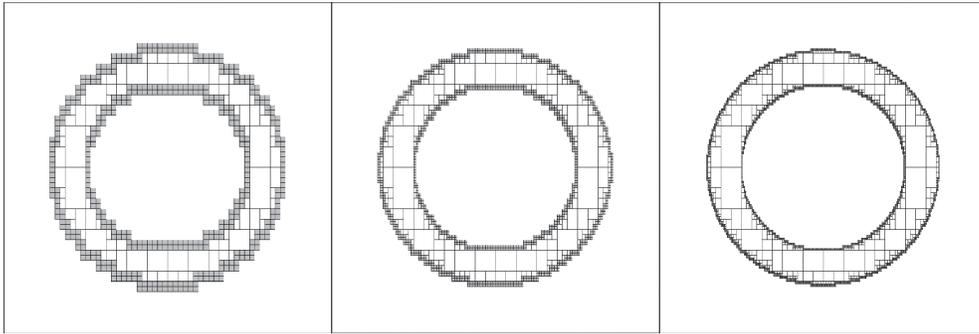
Les ensembles \mathbb{X} peuvent être encadrés par deux sous-pavages \mathbb{X}^- et \mathbb{X}^+ :

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Par exemple, l'ensemble

$$\mathbb{X} = \{(x_1, x_2) \mid x_1^2 + x_2^2 \in [1, 2]\}$$

peut être encadré comme ci-dessous



3.4 Algorithme

Il nous faut encadrer l'ensemble

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}),$$

où $\mathbb{Y} \subset \mathbb{R}^m$. et $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$, par deux sous-pavages

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Tests pour montrer si $[\mathbf{x}]$ à l'intérieur ou à l'extérieur de \mathbb{X} :

- (i) $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y} \Rightarrow [\mathbf{x}] \subset \mathbb{X}$
- (ii) $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset \Rightarrow [\mathbf{x}] \cap \mathbb{X} = \emptyset.$

Algo Sivia(in: $[x]$; out: $\mathcal{L}^-, \mathcal{L}^+$)

- 1 $\mathcal{L} := \{[x]\}; \mathcal{L}^- = \emptyset; \mathcal{L}^+ := \emptyset;$
- 2 if $\mathcal{L} \neq \emptyset$, $[x] := \text{pop}(\mathcal{L})$, else end;
- 3 if $[f]([x]) \subset \mathbb{Y}$, $\text{stack}(\mathcal{L}^-, \mathcal{L}^+, [x]);$ goto 2;
- 4 if $[f]([x]) \cap \mathbb{Y} = \emptyset$, goto 2;
- 5 if $w([x]) < \varepsilon$, $\text{stack}(\mathcal{L}^+, [x]);$ goto 2;
- 6 $\text{stack}(\mathcal{L}, \text{left}([x]), \text{right}([x]));$ goto 2.

3.5 Estimation de paramètres

Modèle : $\phi(\mathbf{p}, t) = p_1 e^{-p_2 t}$,

Paramètres : p_1, p_2 ,

Pavé initial pour les paramètres : $[\mathbf{p}] \subset \mathbb{R}^2$,

Temps de mesure : t_1, t_2, \dots, t_m ,

Barres des données : $[y_1^-, y_1^+], [y_2^-, y_2^+], \dots, [y_m^-, y_m^+]$,

Ensemble de vraisemblance :

$$\mathbb{S} = \{\mathbf{p} \in [\mathbf{p}], \phi(\mathbf{p}, t_1) \in [y_1^-, y_1^+], \dots, \phi(\mathbf{p}, t_m) \in [y_m^-, y_m^+]\}.$$

Posons

$$\phi(\mathbf{p}) = \begin{pmatrix} \phi(\mathbf{p}, t_1) \\ \phi(\mathbf{p}, t_m) \end{pmatrix}$$

et

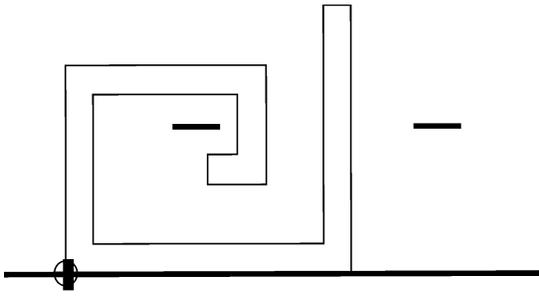
$$[\mathbf{y}] = [y_1^-, y_1^+] \times \cdots \times [y_m^-, y_m^+]$$

alors

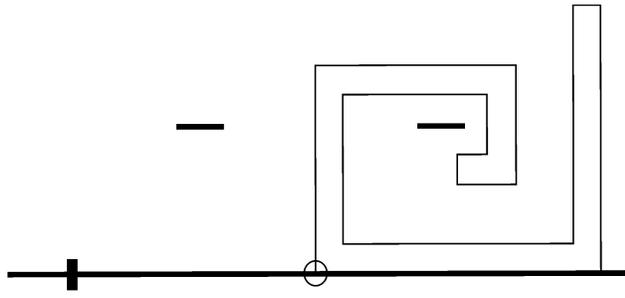
$$S = [\mathbf{p}] \cap \phi^{-1}([\mathbf{y}]).$$

Lancer Setdemo (fait par Guillaume Baffet)

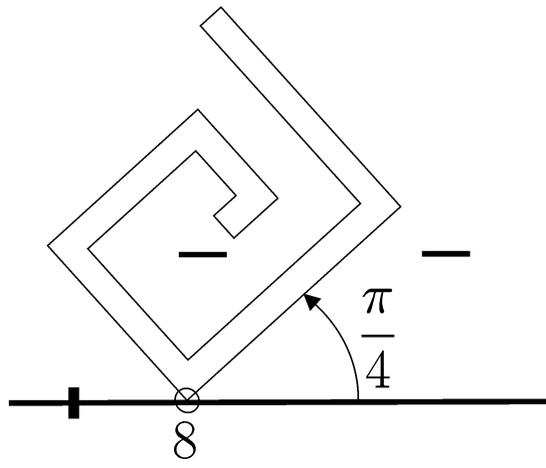
3.6 Planification de chemin



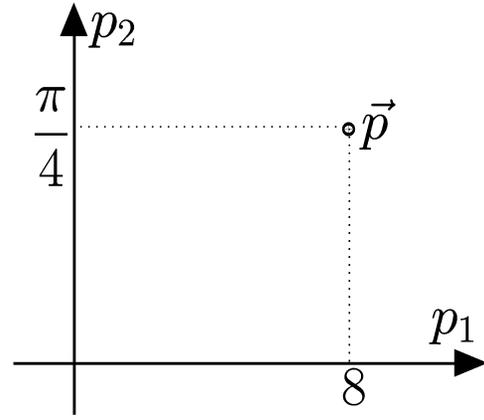
Initial configuration: $\vec{p} = (0\ 0)^T$



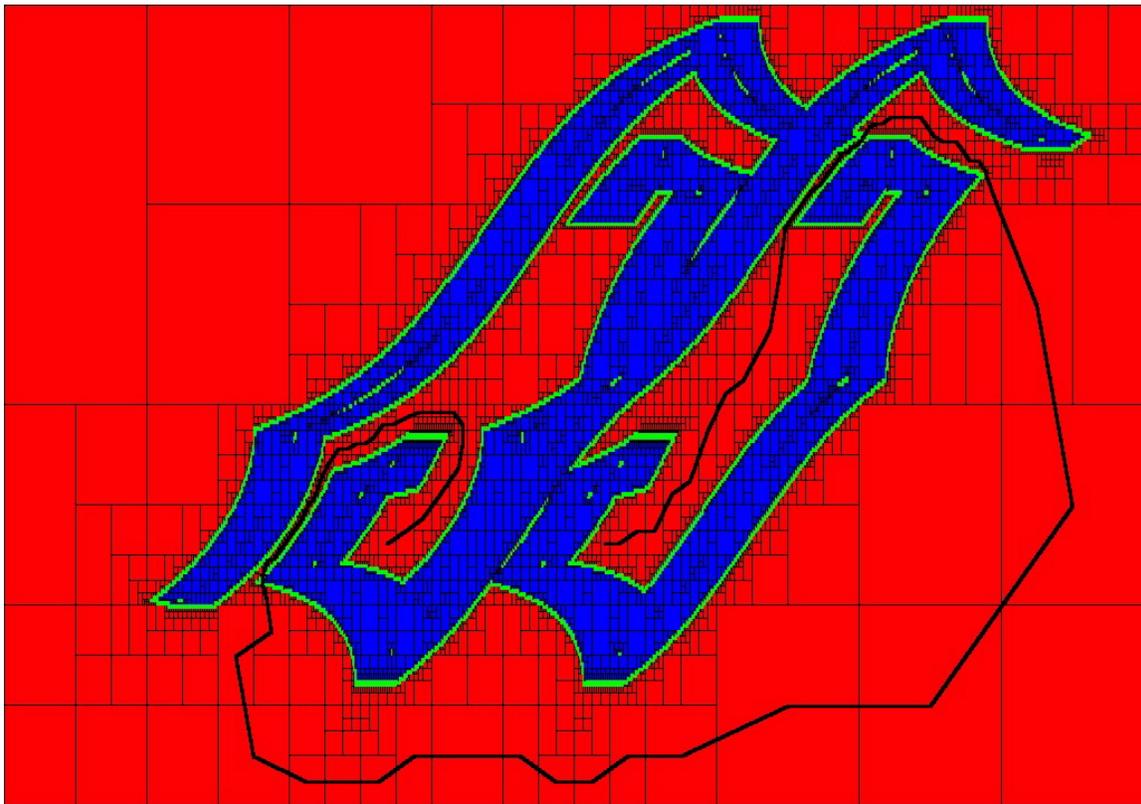
Goal configuration: $\vec{p} = (17\ 0)^T$

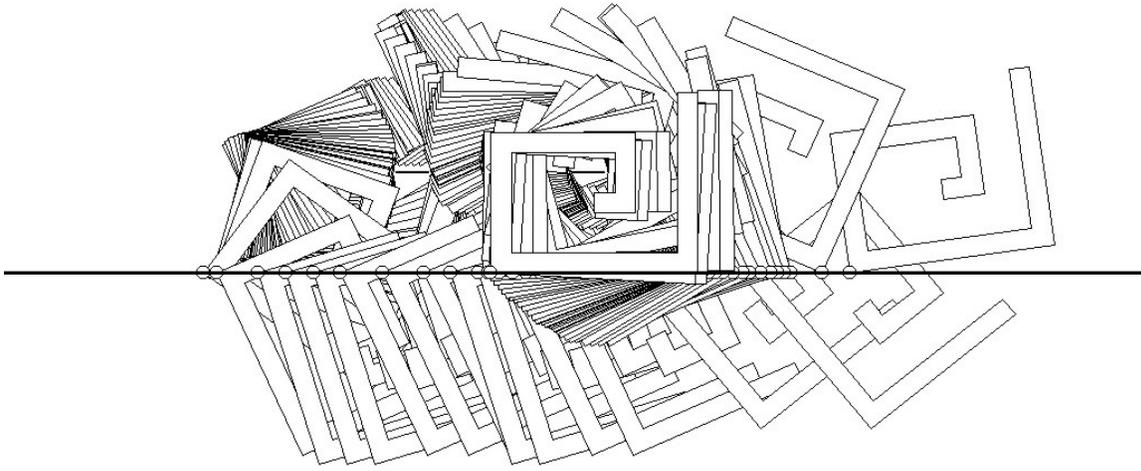


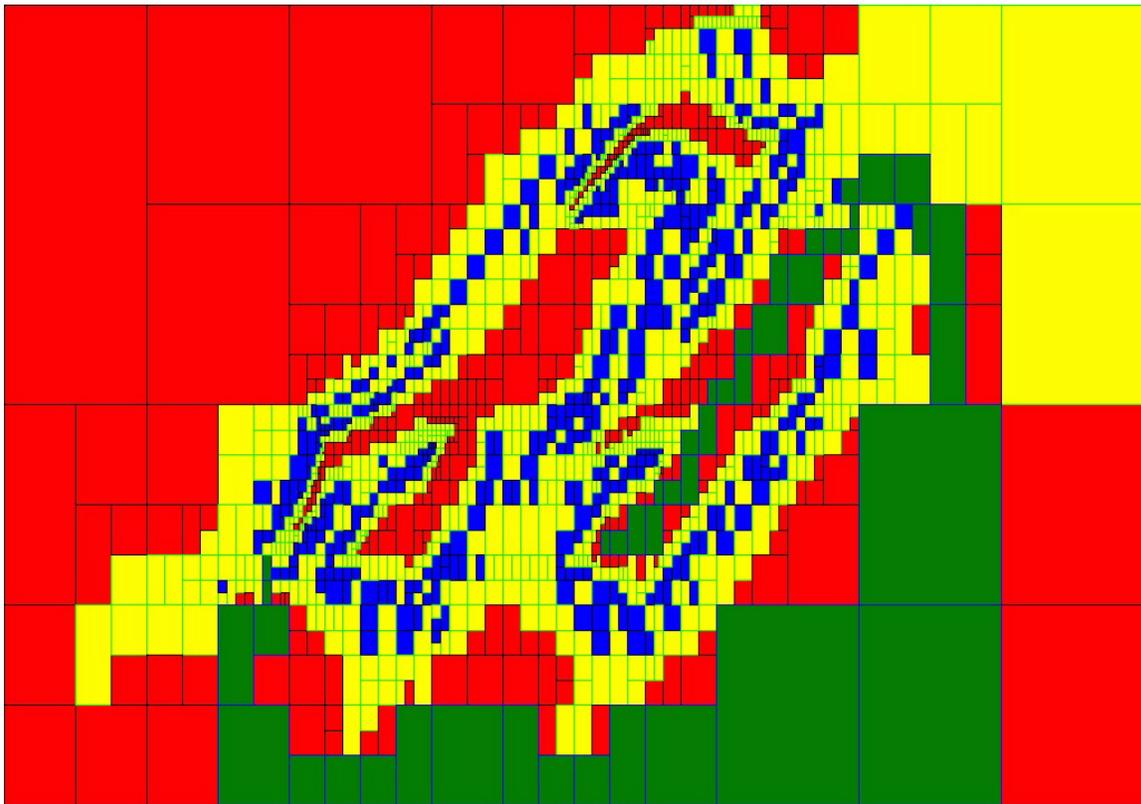
Room

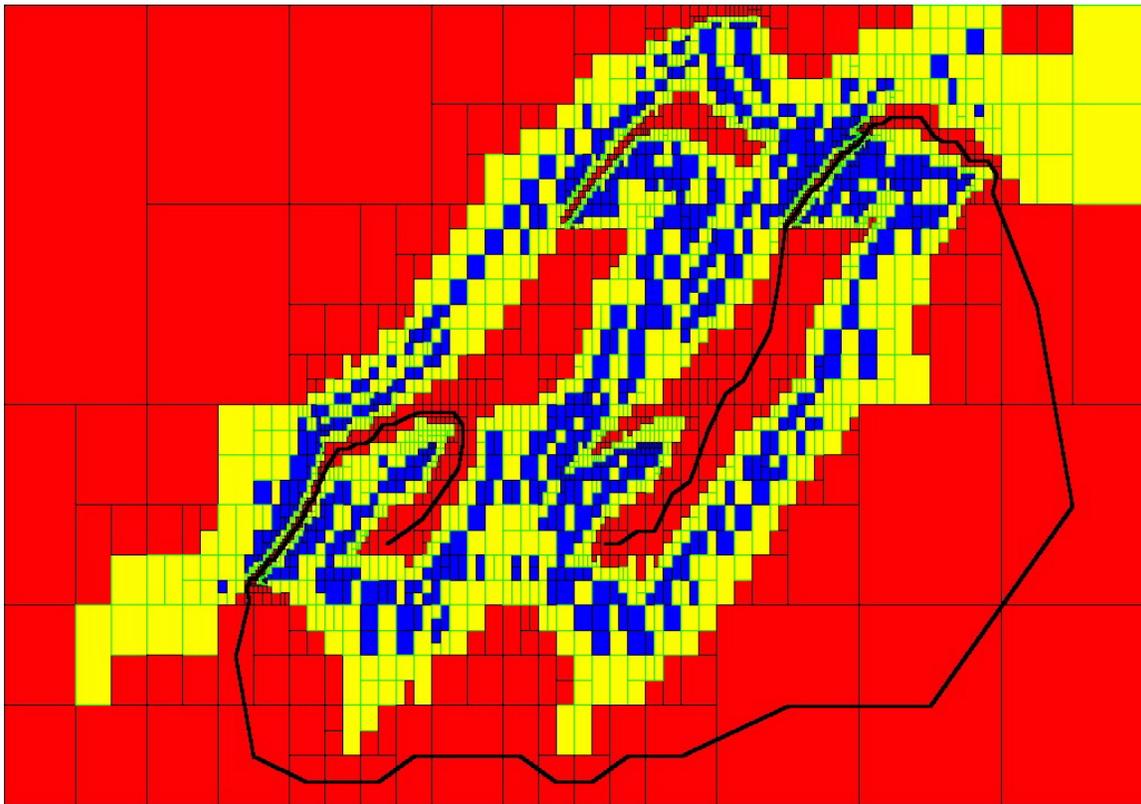


Configuration space









3.7 Commande d'un voilier

$$\left\{ \begin{array}{l}
 \dot{x} = v \cos \theta \\
 \dot{y} = v \sin \theta - \beta V \\
 \dot{\theta} = \omega \\
 \dot{\delta}_s = u_1 \\
 \dot{\delta}_r = u_2 \\
 \dot{v} = \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\
 \dot{\omega} = \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\
 f_s = \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\
 f_r = \alpha_r v \sin \delta_r.
 \end{array} \right.$$

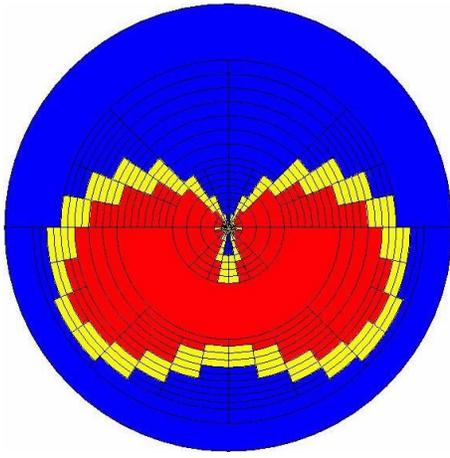
Est-il possible de l'immobiliser ?

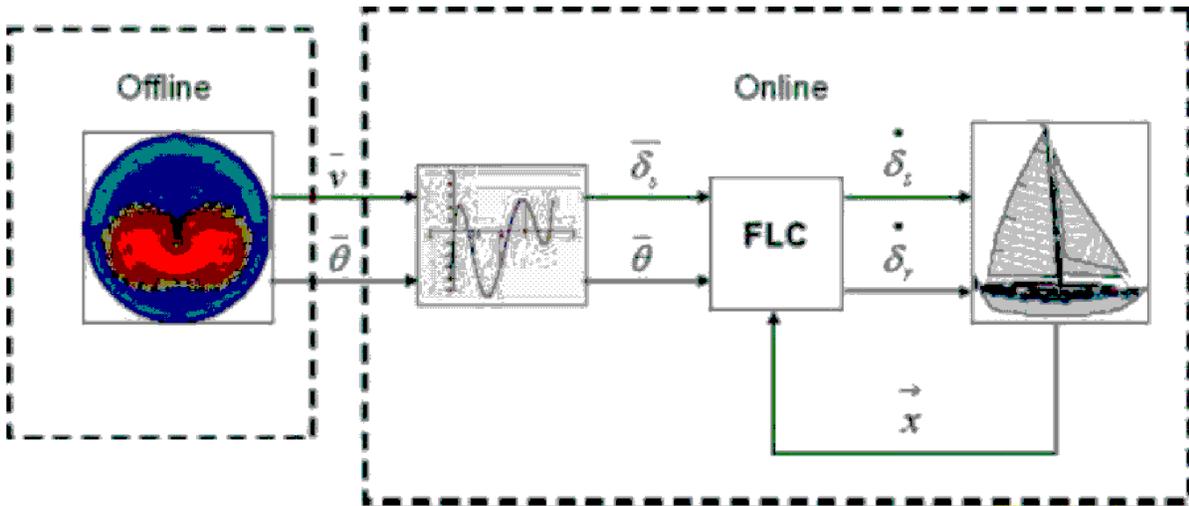
$$\left\{ \begin{array}{l} 0 = v \cos \theta \\ 0 = v \sin \theta - \beta V \\ 0 = \omega \\ 0 = u_1 \\ 0 = u_2 \\ 0 = \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{m} \\ 0 = \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r - \alpha_\theta \omega}{J} \\ f_s = \alpha_s (V \cos (\theta + \delta_s) - v \sin \delta_s) \\ f_r = \alpha_r v \sin \delta_r. \end{array} \right.$$

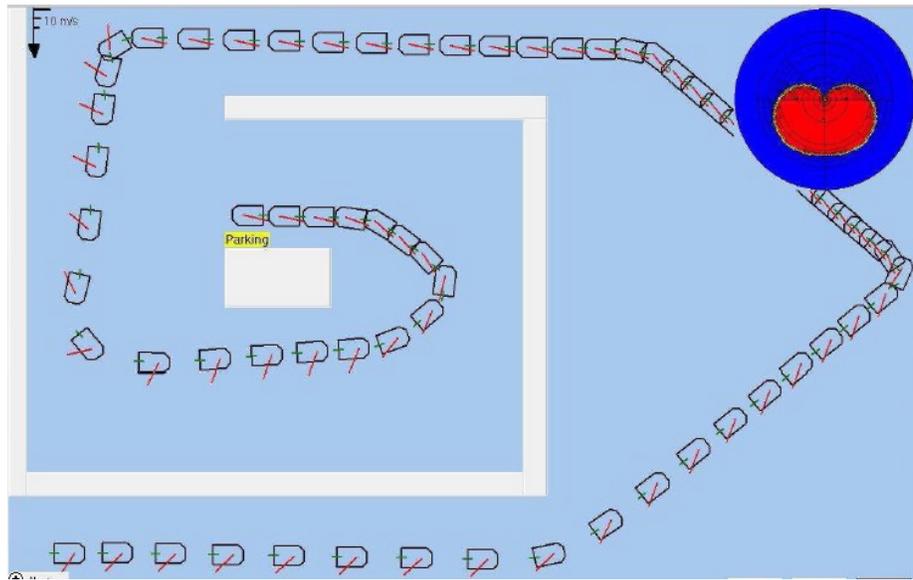
Les techniques intervalles montrent qu'aucune solution existe si $V \neq 0$.

Polaire des vitesses

$$\mathbb{W} = \{ (\theta, v) \mid \begin{array}{l} \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\ \omega = 0, u_1 = 0, u_2 = 0 \\ \frac{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v}{f_s \sin \delta_s - f_r \sin \delta_r - \alpha_f v} = 0 \\ \frac{(\ell - r_s \cos \delta_s) f_s - r_r \cos \delta_r f_r}{J} = 0 \\ f_s = \alpha_s (V \cos(\theta + \delta_s) - v \sin \delta_s) \\ f_r = \alpha_r v \sin \delta_r \end{array} \}.$$







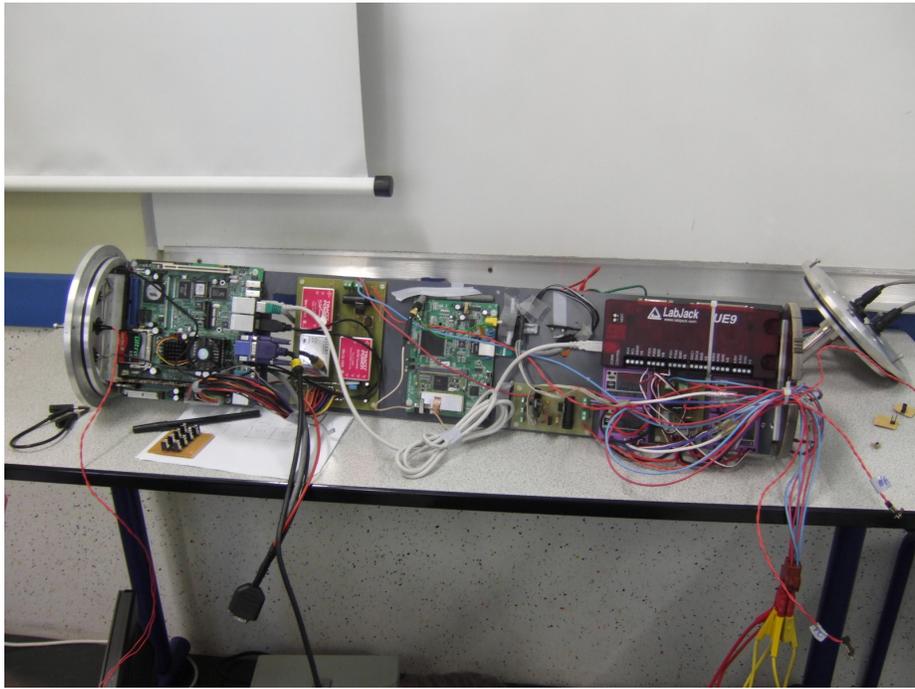
4 Robustesse

Montrer le logiciel de démo de Jan Sliwka



Portsmouth, July 12-15, 2007.

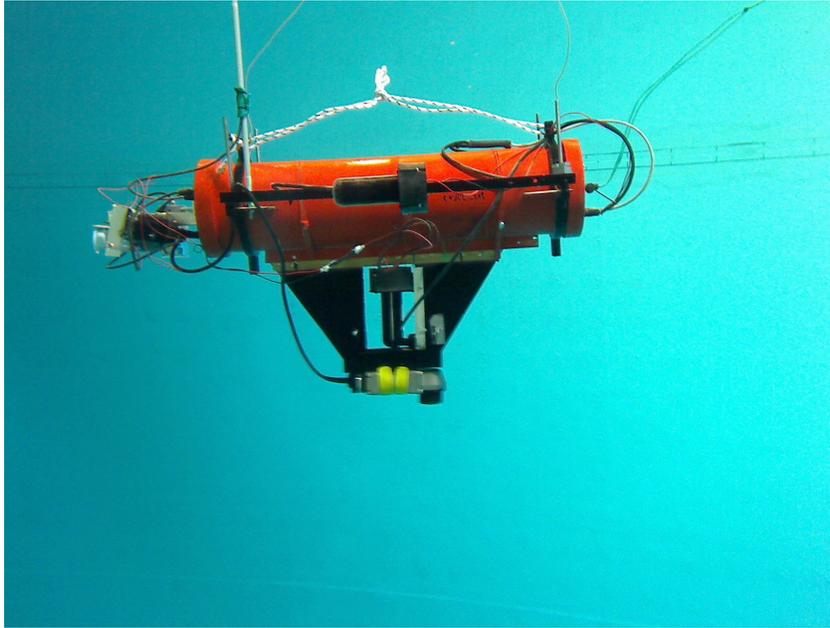












Robot Sauc'isse dans une piscine

