No-lost radius problem

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Problem

Boat

A boat described by

$$\begin{cases} \dot{x}_1 = \cos x_3 \\ \dot{x}_2 = \sin x_3 \\ \dot{x}_3 = u(t) + n(t) \end{cases}$$

with a position (x_1, x_2) and heading x_3 explores the ocean with nothing inside except a buoy at point $(0, 0)$.

We assume that $\mathbf{x}(0) = (0, 0, 0)$.
The boat can choose its control $u(t) \in [-1, 1]$.
The noise satisfies $n(t) \in [-0.01, 0.01]$ and is not measured.
The boat has no GPS and no compass.
Its only way to estimate its position is through the buoy.

Visible area

The visible area $\mathbb{V}(\mathbf{x})$ is a disk of one meter in the neighborhood of the boat :

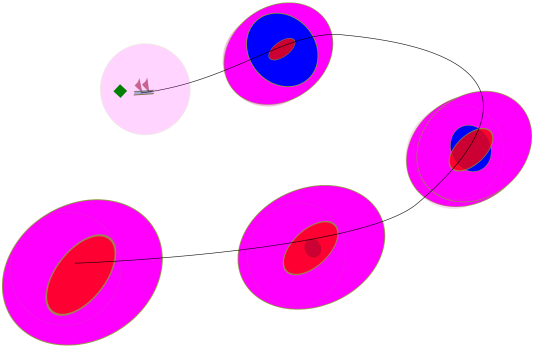
$$\mathbb{V}(\mathbf{x}) = \left\{ (z_1, z_2) \,|\, (z_1 - x_1)^2 + (z_2 - x_2)^2 \leq 1 \right\}.$$

When the boat sees the buoy, it is able to know exactly where the buoy is in its own frame.

No-lost radius

- When $\mathbf{0} \in \mathbb{V}(\mathbf{x})$, the boat sees the buoy and is not lost.
- When the boat knows a control strategy $u(t)$ such that it can guarantee that it will see the buoy, then the boat is not lost.

To which distance r from the buoy the boat can go without being lost?
Give an interval for r as precise as possible.



The boat starts near to the buoy (green)
It knows that it is inside the red set which become larger and larger.
The certainly visible zone (blue) deflates and become empty
The possibly visible zone (magenta) inflates.
At the last point, is the boat lost ? (*not necessary: a spiral search can be done*)