Some applications of interval analysis to sea robotics

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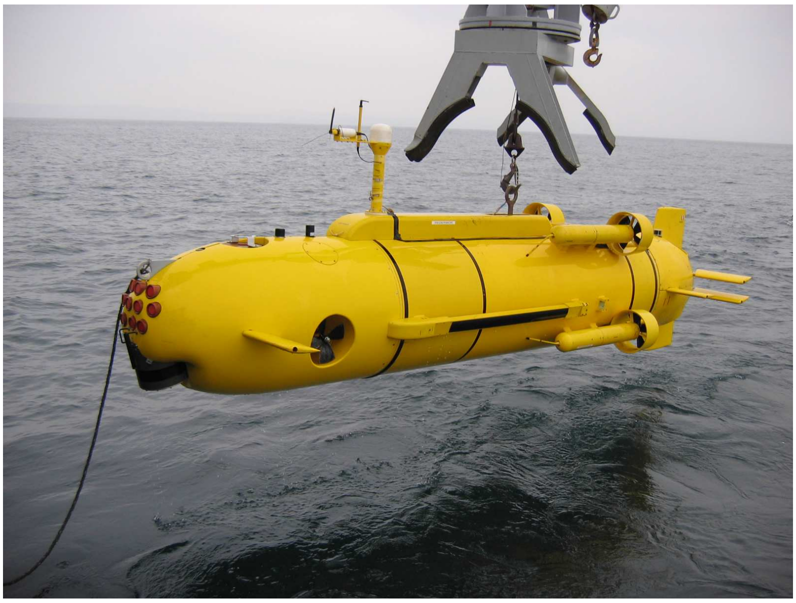
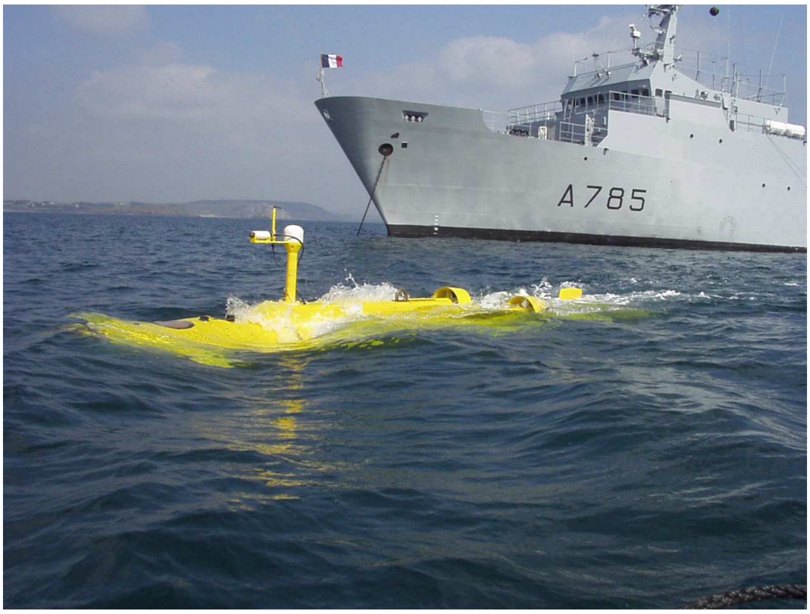
1 Redermor



The *Redermor*, GESMA



The *Redermor* at the surface

Show simulation

Why choosing an interval constraint approach for SLAM ?

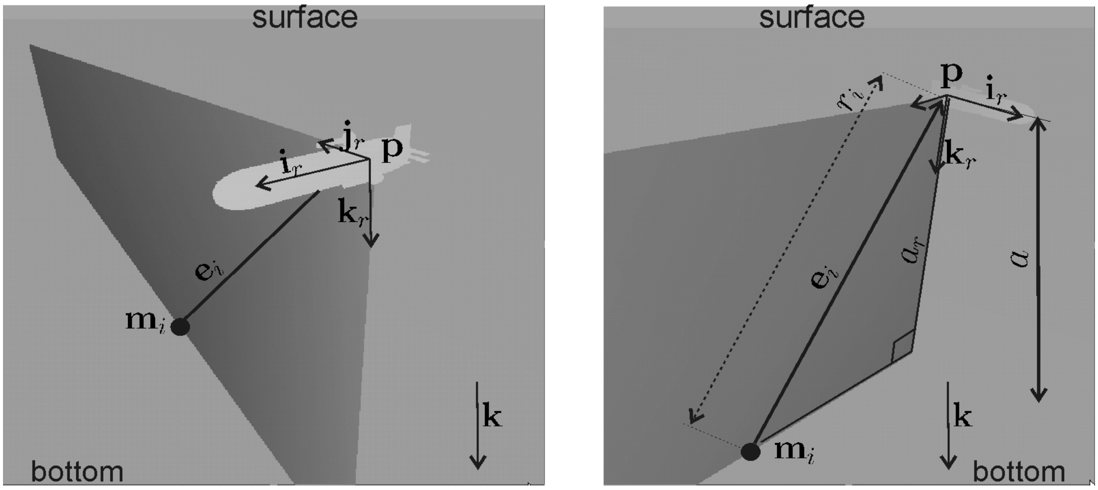
1) A reliable method is needed.
2) The model is nonlinear.
3) The pdf of the noises are unknown.
4) Reliable error bounds are provided by the sensors.
5) A huge number of redundant data are available.

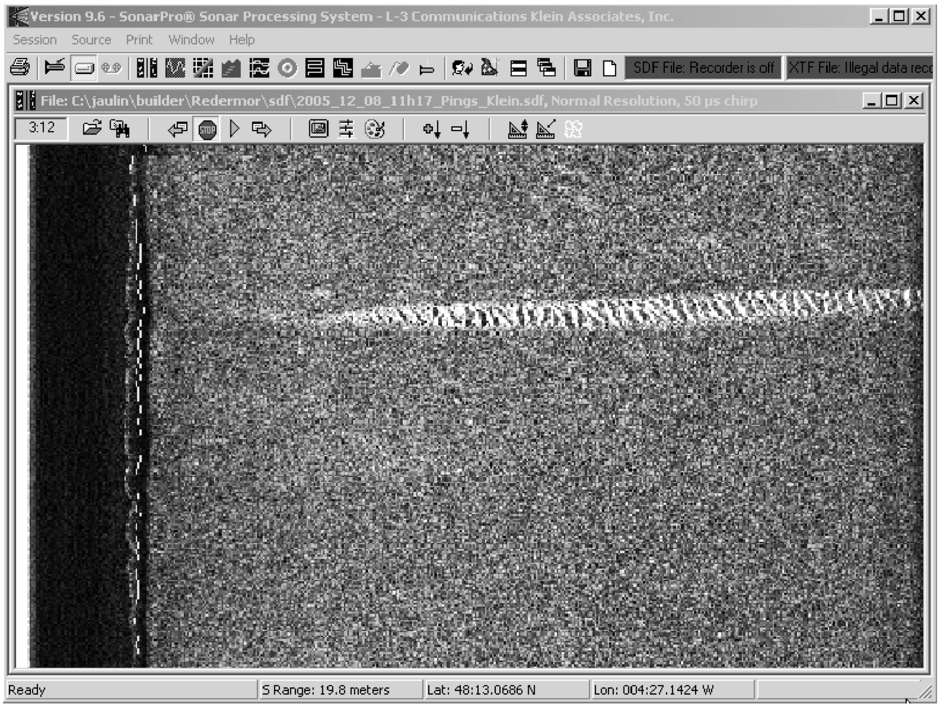
1.1 Sensors

A GPS (Global positioning system) at the surface only.

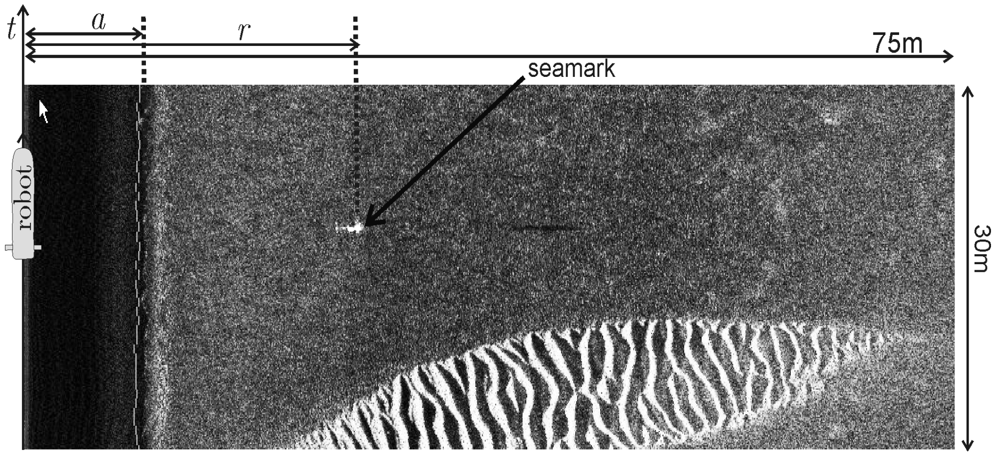
$$
\begin{array}{ll}
t_0 = 6000 \text{ s}, & \ell^0 = (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\
t_f = 12000 \text{ s}, & \ell^f = (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m
\end{array}
$$

A sonar (KLEIN 5400 side scan sonar). Gives the distance r between the robot to the detected object.





Screenshot of SonarPro



Detection of a mine using SonarPro

A Loch-Doppler. Returns the speed of the robot $\mathbf{v}_r$ and the altitude a of the robot $\pm$ 10cm.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ, the pitch θ and the head ψ.

$$\left(\begin{array}{c}\phi\\ \theta\\ \psi\end{array}\right) \in \left(\begin{array}{c}\tilde{\phi}\\ \tilde{\theta}\\ \tilde{\psi}\end{array}\right) + \left(\begin{array}{c}1.75 \times 10^{-4}.[-1,1]\\ 1.75 \times 10^{-4}.[-1,1]\\ 5.27 \times 10^{-3}.[-1,1]\end{array}\right).$$

1.2 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines have been detected by the sonar:

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

1.3 Constraints satisfaction problem

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t).\mathbf{R}_{\theta}(t).\mathbf{R}_{\varphi}(t),$$

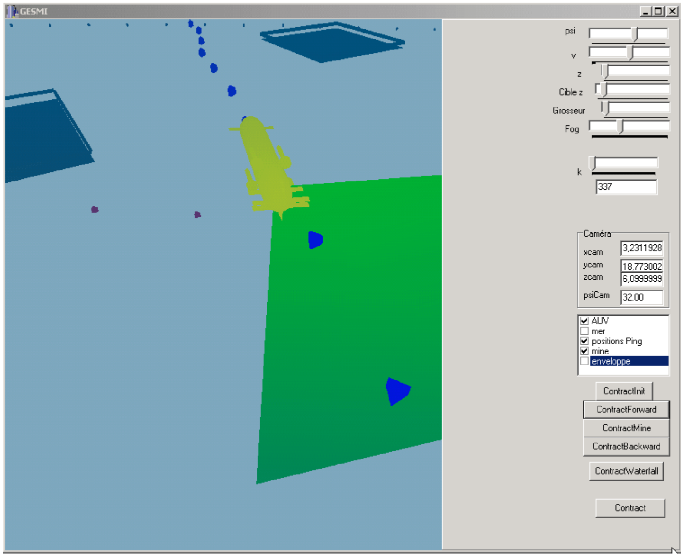
$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t)$$

$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \; = r(i),$$

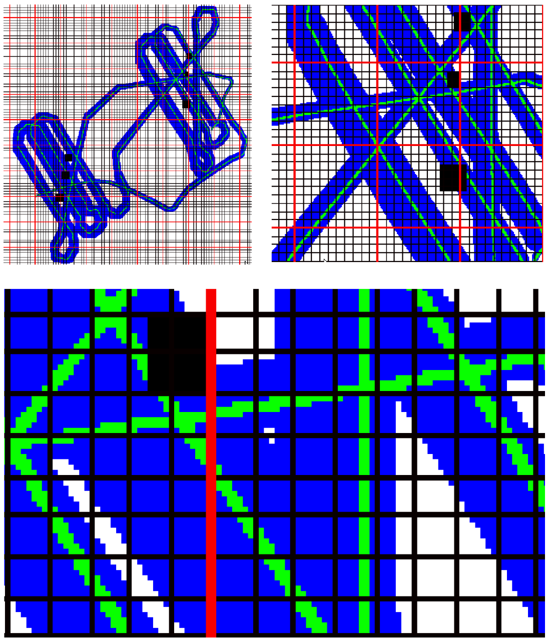
$$\mathbf{R}^{\mathsf{T}}(\tau(i))\,(\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2},$$

$$m_z(\sigma(i)) - p_z(\tau(i)) - a(\tau(i)) \in [-0.5, 0.5].$$

1.4 GESMI

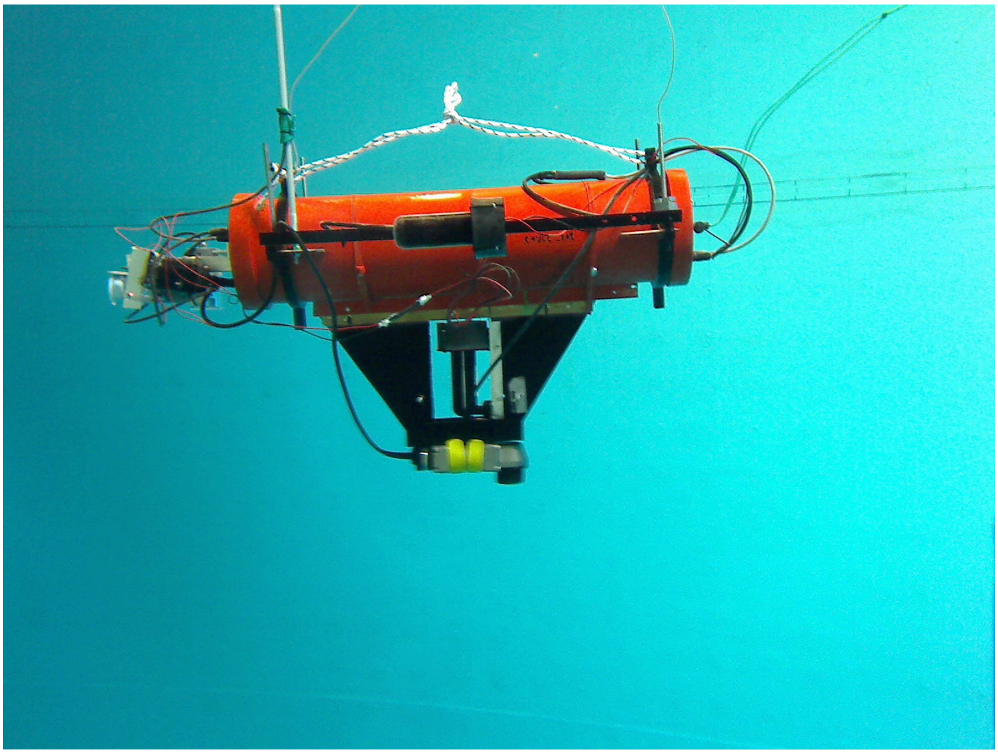


GESMI (Guaranteed Estimation of Sea Mines with Intervals)



Trajectory reconstructed by GESMI

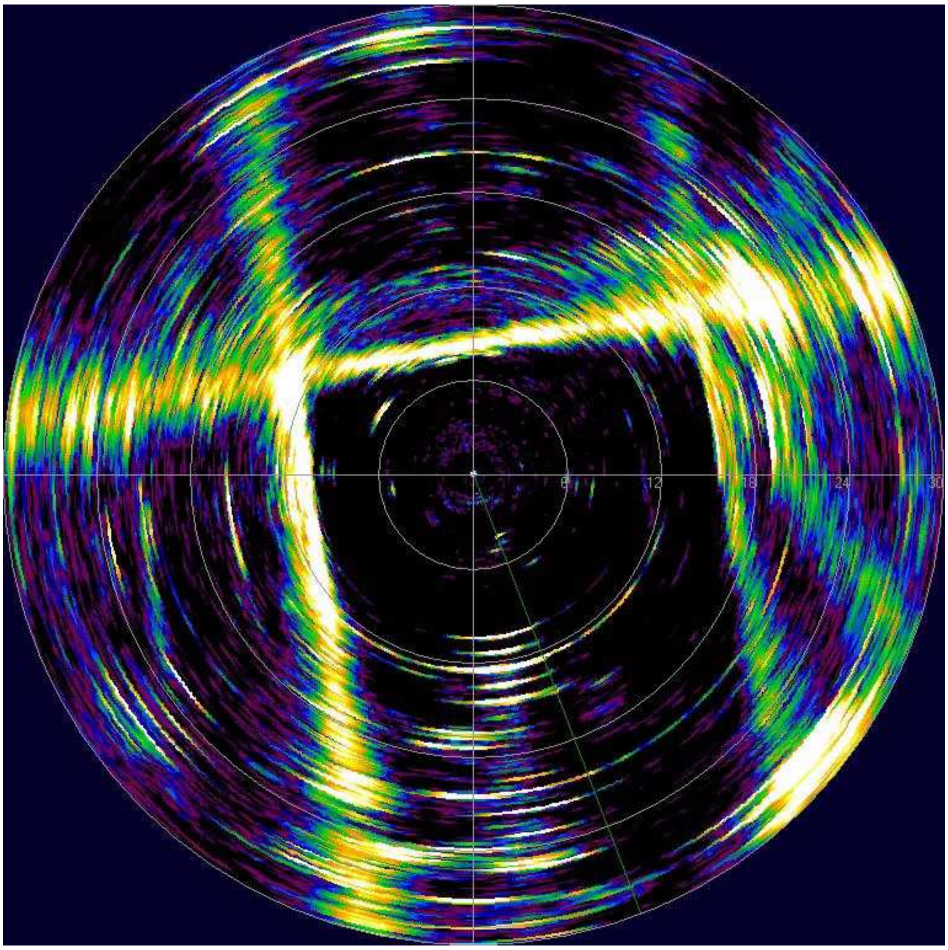
2 SAUC'ISSE



Robot SAUC'ISSE

Montrer une vidéo

2.1 Localisation with sonar


6
12
18
24
30

2.2 Set-membership approach

$$\mathbf{y} = \boldsymbol{\psi}(\mathbf{p}) + \mathbf{e},$$

where

- $\mathbf{e} \in \mathbb{E} \subset \mathbb{R}^m$ is the error vector,
- $\mathbf{y} \in \mathbb{R}^m$ is the data vector,
- $\mathbf{p} \in \mathbb{R}^n$ is the parameter vector.

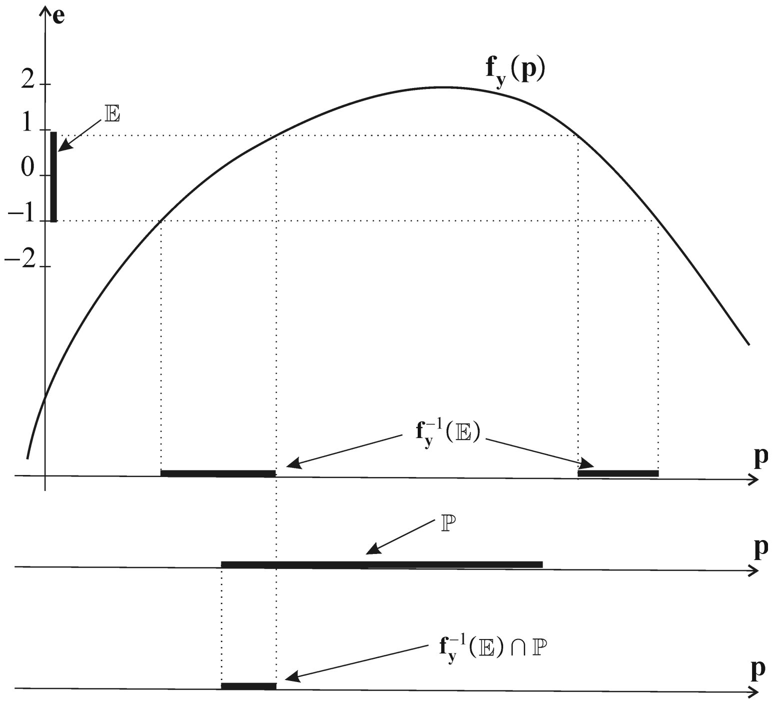
Show setdemo

The equation $\mathbf{y} = \boldsymbol{\psi}(\mathbf{p}) + \mathbf{e}$, can be rewritten

$$\mathbf{e} = \underbrace{\mathbf{y} - \boldsymbol{\psi}(\mathbf{p})}_{\mathbf{f}_{\mathbf{y}}(\mathbf{p})}.$$

The *posterior feasible set* is

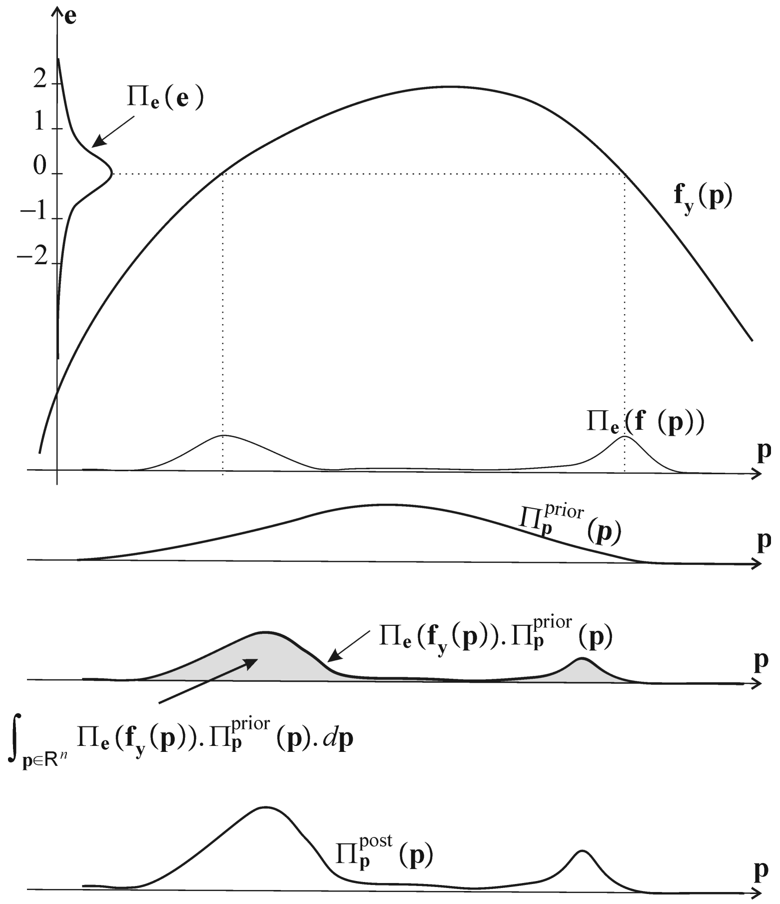
$$\widehat{\mathbb{P}} = \mathbf{f}_{\mathbf{y}}^{-1}(\mathbb{E}) \cap \mathbb{P}.$$



In a Bayesian approach, prior pdf $\Pi_{\mathbf{e}}, \Pi_{\mathbf{p}}^{\text{prior}}$ are available for $\mathbf{e}, \mathbf{p}$.

The Bayes rule gives is the posterior pdf for $\mathbf{p}$

$$\Pi_{\mathbf{p}}^{\text{post}}(\mathbf{p}) = \frac{\Pi_{\mathbf{e}}(\mathbf{f_y}(\mathbf{p})).\Pi_{\mathbf{p}}^{\text{prior}}(\mathbf{p})}{\int_{\mathbf{p}\in\mathbb{R}^n} \Pi_{\mathbf{e}}(\mathbf{f_y}(\mathbf{p})).\Pi_{\mathbf{p}}^{\text{prior}}(\mathbf{p}).d\mathbf{p}}.$$



e
2
1
0
−1
−2
Πe(e)
fy(p)
Πe(f (p))
p
Πp^prior(p)
p
Πe(fy(p)). Πp^prior(p)
p
∫p∈R^n Πe(fy(p)). Πp^prior(p). dp
Πp^post(p)
p

2.3 Computing with sets is easy !

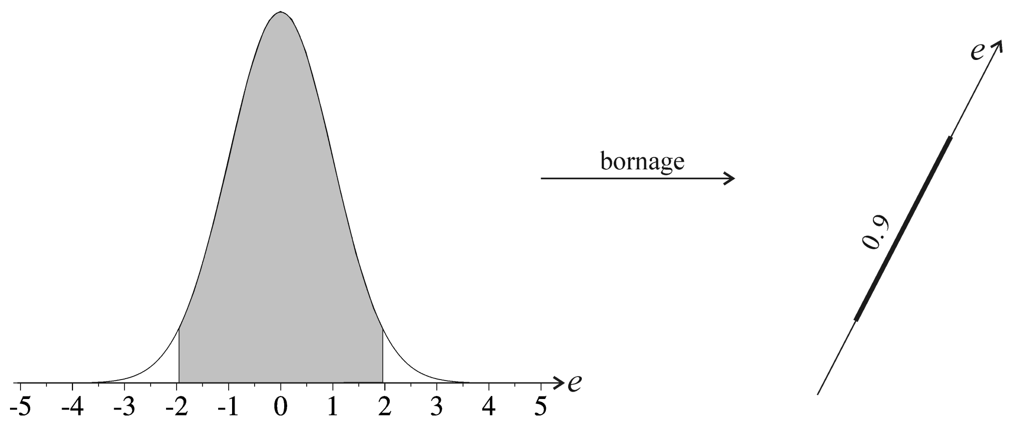
$$
\begin{aligned}
[-1,3]+[3,5] &= [2,8] \\
[-1,3]*[3,5] &= [-5,15] \\
\sin([-1,3]) &= [\sin(-1),1]
\end{aligned}
$$

We can also solve nonlinear equations such as

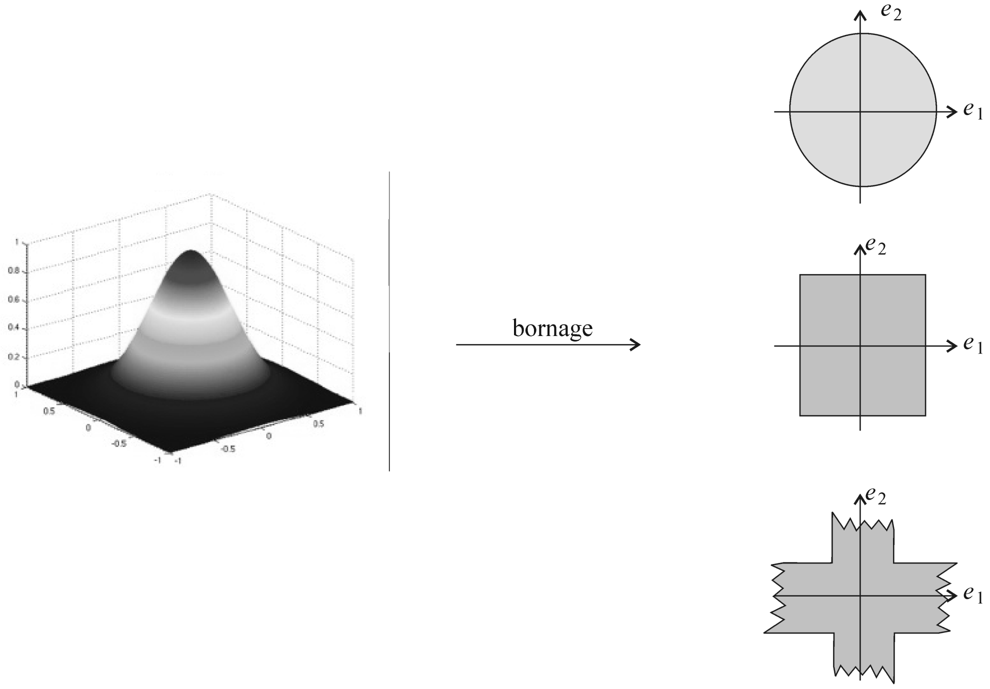
$$x.y.\sin(y\sqrt{x-y}+x)=1$$

2.4 Error bounding

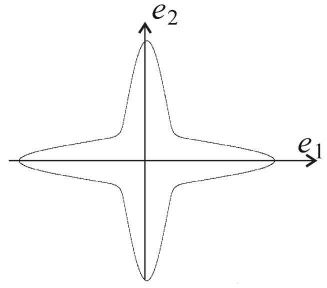
From $\Pi_{\mathbf{e}}$, we can find some bounds for $\mathbf{e}$..

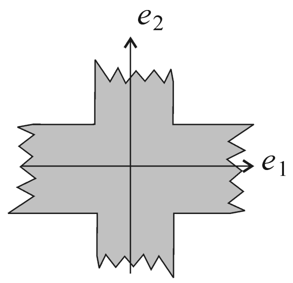


-5 -4 -3 -2 -1 0 1 2 3 4 5
e
bornage
0.9
e


bornage
e_2
e_1
e_2
e_1
e_2
e_1

$$\Pi(\mathbf{e}) \propto \left(\exp(-e_1^2) + \exp\left(-\frac{e_1^2}{10}\right)\right) * \left(\exp(-e_2^2) + \exp\left(-\frac{e_2^2}{10}\right)\right) \xrightarrow{\text{bornage}}$$





For $m = 100$ data, with $\pi = \Pr(e_k \in [e_k]) = 0.6$, the probability to have less that $q = 60$ outliers, is

$$\sum_{k=0}^{m-q-1} \frac{m!}{k!\,(m-k)!} \pi^k . (1-\pi)^{m-k} = 0.99998.$$

2.5 State observer

$$\begin{cases} \mathbf{x}(k+1) & = & \mathbf{f}_k(\mathbf{x}(k), \mathbf{n}\,(k)) \\ \mathbf{y}(k) & = & \mathbf{g}_k(\mathbf{x}(k)), \end{cases}$$

where $\mathbf{n}\,(k) \in \mathbb{N}\,(k)$ and $\mathbf{y}\,(k) \in \mathbb{Y}\,(k)$.

If $\mathbb{X}(k)$ is the set of all $\mathbf{x}$ consistent with the following assumptions

(i) within all past time windows of length m, there is less than q outliers

(ii) $\mathbb{X}(0)$ contains $\mathbf{x}(0)$

Theorem.

$$\Pr\left(\mathbf{x}\left(k\right) \in \mathbb{X}(k)\right) \geq \alpha \; * \; \Pr\left(\mathbf{x}\left(k-1\right) \in \mathbb{X}(k-1)\right)$$

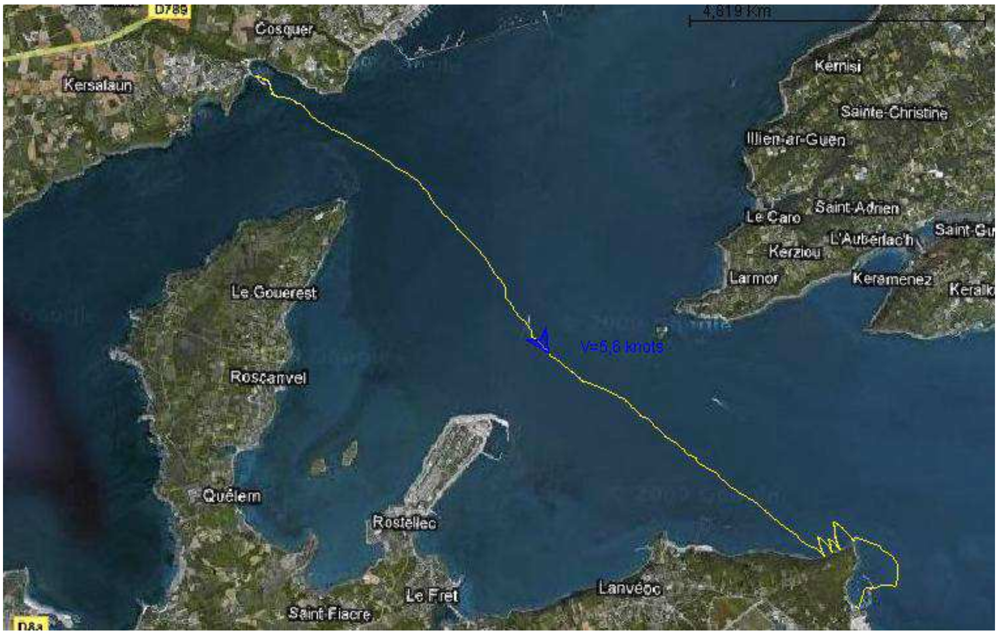
with

$$\alpha = \sqrt[m]{\sum_{i=m-q}^{m} \frac{m!\; \pi_y^i.\left(1-\pi_y\right)^{m-i}}{i!\left(m-i\right)!}}$$

Montrer la simu et la vidéo du concours

3 Breizh Spririt

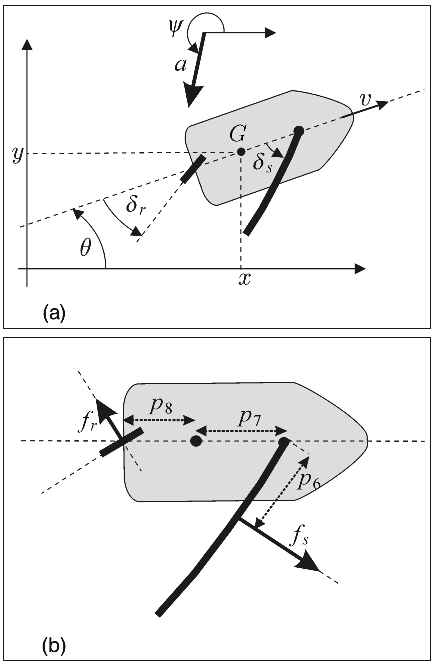

BREIZH
SPIRIT
ENSIETA


D789
Cosquer
Kersalaun
4,819 Km
Kernisi
Sainte-Christine
Illien-ar-Guen
Saint-Adrien
Le Caro
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
V=5,6 knots
Roscanvel
Quélern
Rostellec
Le Fret
Lanvéoc
Saint-Fiacre
D8a

Montrer une vidéo

3.1 Normalized State equations

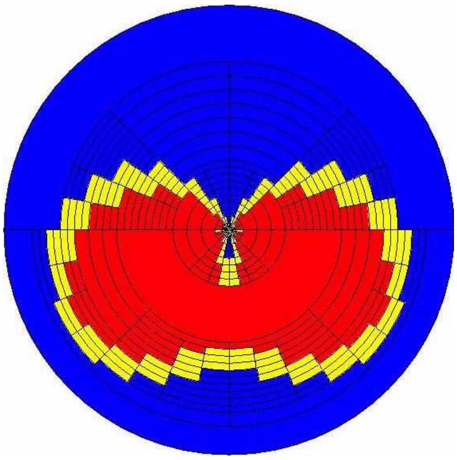
$$
\begin{cases}
\dot{x} & = & v\cos\theta + a\cos\psi \\
\dot{y} & = & v\sin\theta + a\sin\psi \\
\dot{\theta} & = & \omega \\
\dot{v} & = & f_s.\sin\delta_s - f_r.\sin u_1 - v \\
\dot{\omega} & = & f_s.\left(1-\cos\delta_s\right) - f_r.\cos u_1 - \omega \\
f_s & = & a\sin\left(\theta-\psi+\delta_s\right) \\
f_r & = & v\sin u_1 \\
\gamma & = & \cos\left(\theta-\psi\right)+\cos\left(u_2\right) \\
\delta_s & = & \begin{cases} \pi-\theta+\psi & \text{if } \gamma\le 0 \\ sign\left(\sin\left(\theta-\psi\right)\right).u_2 & \text{otherwise.} \end{cases}
\end{cases}
$$



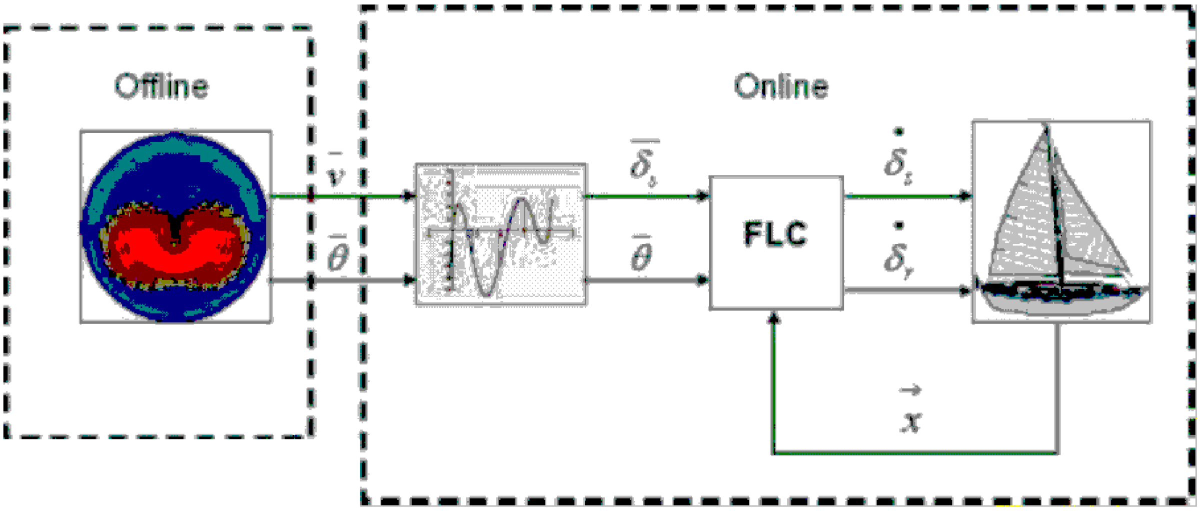
ψ
a
G
v
y
δ_s
δ_r
θ
x
(a)
f_r
p_8
p_7
p_6
f_s
(b)

3.2 Polar speed diagram

$$
\begin{array}{llll}
\mathbb{W} = \{ & (\theta, v) \mid & \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) & \\
& & \omega = 0, u_1 = 0, u_2 = 0 & \\
& & f_s \sin\delta_s - f_r \sin\delta_r - v = 0 & \\
& & (1 - \cos\delta_s)\, f_s - \cos\delta_r f_r = 0 & \\
& & f_s = a\cos(\theta + \delta_s) - v\sin\delta_s & \\
& & f_r = v\sin\delta_r & \}.
\end{array}
$$



3.3 Control



From the state equations of the sailboat, it is easy to check that

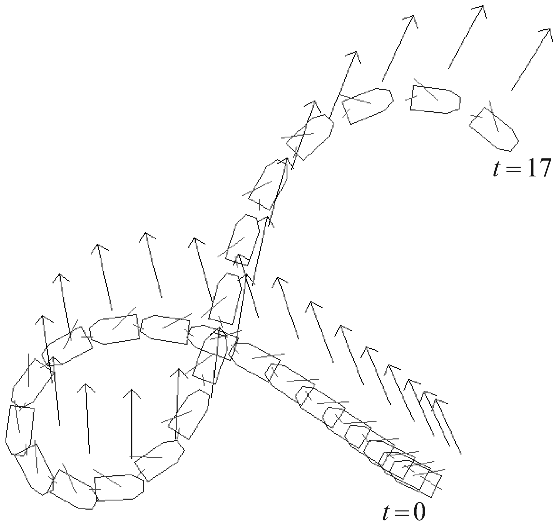
$$\underbrace{\begin{pmatrix} \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{pmatrix}}_{\mathbf{z}} = \mathbf{\Psi}_t \underbrace{\begin{pmatrix} \theta \\ v \\ \omega \\ a \\ \psi \end{pmatrix}}_{\mathbf{x}}$$

with

$$\mathbf{\Psi}_t\left(\mathbf{x}\right)=\left(\begin{array}{c}\theta\\ v\sin\theta+a\sin\psi\\ v\sin\theta+a\sin\psi\\ \omega\\ \left(f_s.\sin\delta_s-f_r\sin u_1-v\right)\cos\theta-\omega v\sin\theta\\ \left(f_s\sin\delta_s-f_r\sin u_1-v\right)\sin\theta+\omega v\cos\theta\\ f_s\left(1-\cos\delta_s\right)-f_r\cos u_1-\omega\end{array}\right)$$

and

$$\begin{cases} f_s(\mathbf{x}) & = & a \sin(\theta - \psi + \delta_s) \\ f_r(\mathbf{x}, t) & = & v \sin u_1 \\ \delta_s(\mathbf{x}, t) & = & \begin{cases} \pi - \theta + \psi & \text{if } \gamma(\mathbf{x}, t) \leq 0 \\ sign(\sin(\theta - \psi)) .u_2 & \text{otherwise} \end{cases} \\ \gamma(\mathbf{x}, t) & = & \cos(\theta - \psi) + \cos(u_2) . \end{cases}$$



Simulated experiment