Set computation and ocean robotics

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1 Interval computation

Problem. Given $f : \mathbb{R}^n \to \mathbb{R}$, a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Interval arithmetic can solve efficiently this problem.

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= [1,8], \\ [-1,3]\cdot[2,5] &= [-5,15], \\ \mathsf{abs}\,([-7,1]) &= [0,7] \end{aligned}$$

If f is given by

Algorithm f(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: y)

```
z := |x_3| + x_1;
for k := 0 to 100
    z := (cos x_2) · (sin (z) + k x_3);
next;
y := sin(z · x_1);
```

Its interval extension is

Algorithm $[f]$(in: $[\mathbf{x}] = ([x_1], [x_2], [x_3])$, out: $[y]$)

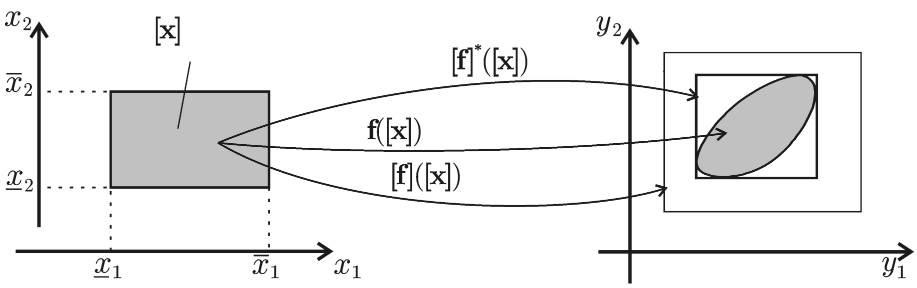
$[z] := |[x_3]| + [x_1]$;
for $k := 0$ to 100
  $[z] := (\cos[x_2]) \cdot (\sin([z]) + k * [x_3])$;
next;
$[y] := \sin([z] \cdot [x_1])$;

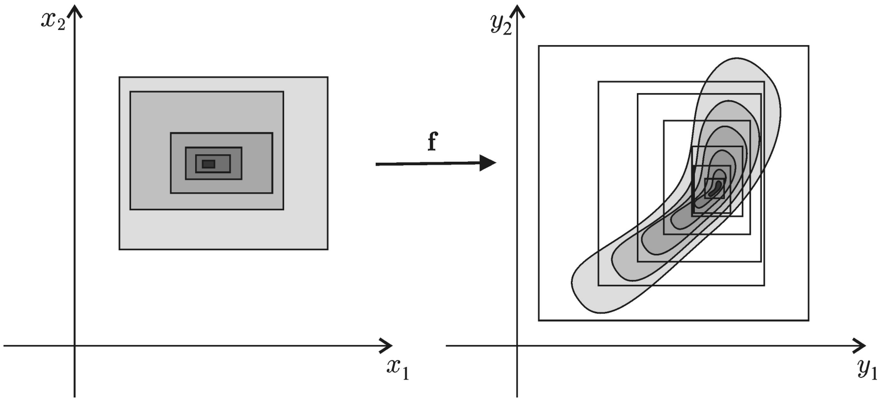
Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0$$

Inclusion function

$$\forall\, [\mathbf{x}] \in \mathbb{IR}^n, \quad \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]\,([\mathbf{x}]).$$





Convergent and monotonic

2 Set estimation

2.1 Subpavings

A subpaving of $\mathbb{R}^n$ is a set of non-overlapping boxes of $\mathbb{R}^n$.

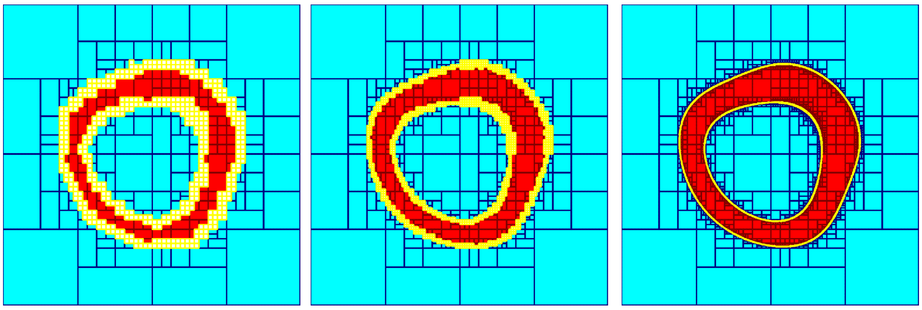
Compact sets $\mathbb{X} \subset \mathbb{R}^n$ can be bracketed between inner and outer subpavings:

$$\mathbb{X}^- \subset \mathbb{X} \subset \mathbb{X}^+.$$

Example. The set

$$\mathbb{X} = \{(x_1, x_2) \,\big|\, x_1^2 + x_2^2 + \sin(x_1 + x_2) \in [4, 9]\}.$$

can be approximated by subpavings.



2.2 Set inversion

If $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $\mathbb{Y} \subset \mathbb{R}^m$.

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}).$$

$$
\begin{array}{lllll}
\text{(i)} & [\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y} & \Rightarrow & [\mathbf{x}] \subset \mathbb{X} \\
\text{(ii)} & [\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset & \Rightarrow & [\mathbf{x}] \cap \mathbb{X} = \emptyset.
\end{array}
$$

.

Algorithm Sivia(in: $[\mathbf{x}](0), \mathbf{f}, \mathbb{Y}$)

1 $\mathcal{L} := \{[\mathbf{x}](0)\}$;
2 pull $[\mathbf{x}]$ from $\mathcal{L}$;
3 if $[\mathbf{f}]([\mathbf{x}]) \subset \mathbb{Y}$, draw($[\mathbf{x}]$, 'red');
4 elseif $[\mathbf{f}]([\mathbf{x}]) \cap \mathbb{Y} = \emptyset$, draw($[\mathbf{x}]$, 'blue');
5 elseif $w([\mathbf{x}]) < \varepsilon$, {draw ($[\mathbf{x}]$, 'yellow')};
6 else bisect $[\mathbf{x}]$ and push into $\mathcal{L}$;
7 if $\mathcal{L} \neq \emptyset$, go to 2

2.3 Bounded-error estimation

Model : $\phi(\mathbf{p}, t) = p_1 e^{-p_2 t}$.

Prior feasible box for the parameters : $[\mathbf{p}] \subset \mathbb{R}^2$

Measurement times : $t_1, t_2, \ldots, t_m$

Data bars : $[y_1^-, y_1^+], [y_2^-, y_2^+], \ldots, [y_m^-, y_m^+]$

$$\mathbb{S} = \{\mathbf{p} \in [\mathbf{p}], \phi(\mathbf{p}, t_1) \in [y_1^-, y_1^+], \ldots, \phi(\mathbf{p}, t_m) \in [y_m^-, y_m^+]\}.$$

If

$$\boldsymbol{\phi}(\mathbf{p}) = \begin{pmatrix} \phi(\mathbf{p}, t_1) \\ \\ \phi(\mathbf{p}, t_m) \end{pmatrix}$$

and

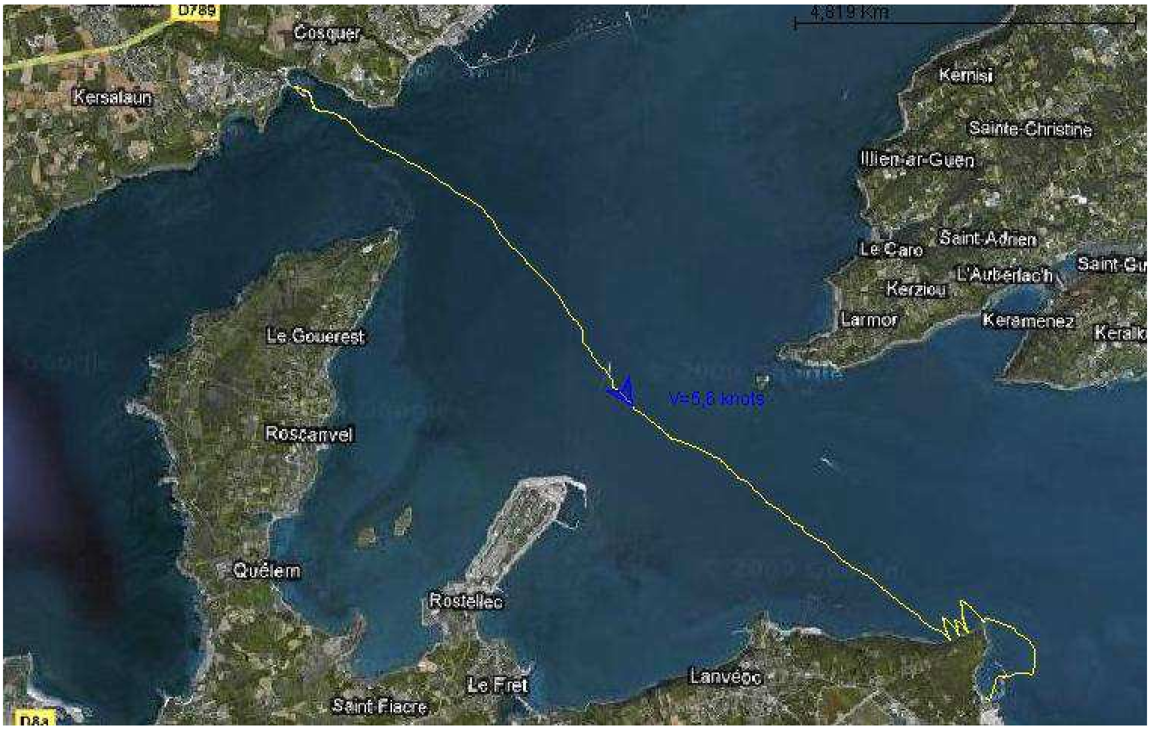
$$[\mathbf{y}] = [y_1^-, y_1^+] \times \cdots \times [y_m^-, y_m^+]$$

then

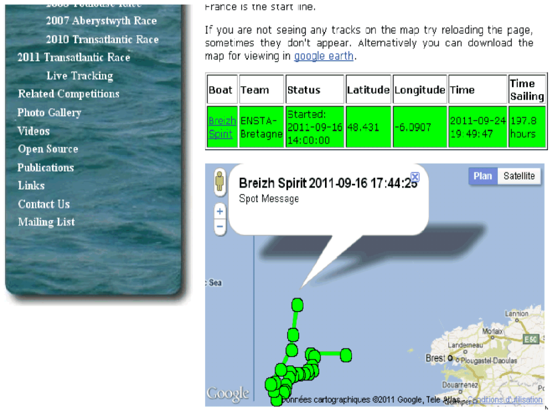
$$\mathbb{S} = [\mathbf{p}] \cap \boldsymbol{\phi}^{-1}([\mathbf{y}]).$$

3 Saiboat robotics

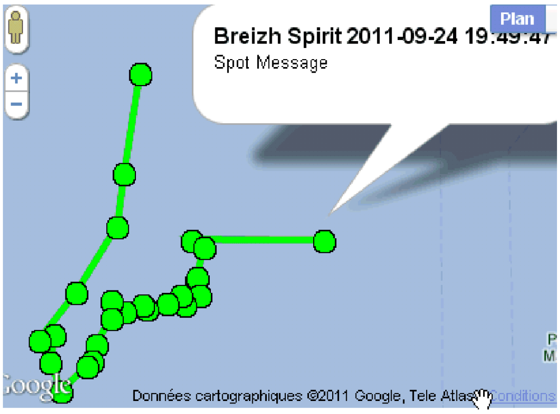

BREIZH
ENSIETA
SPIRIT



D789
Cosquer
Kersalaun
4,819 Km
Kernisi
Sainte-Christine
Illien-ar-Guen
Saint-Adrien
Le Caro
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
Roscanvel
Quélern
Rostellec
Le Fret
Lanvéoc
Saint-Fiacre



Boat	Team	Status	Latitude	Longitude	Time	Time Sailing
Breizh Spirit	ENSTA-Bretagne	Started: 2011-09-16 14:00:00	48.431	-6.0907	2011-09-24 19:49:47	197.8 hours



3.1 Vaimos

Vaimos (IFREMER and ENSTA)

The robot satisfies a state equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}).$$

With the controller $\mathbf{u} = \mathbf{g}(\mathbf{x})$, the robot satisfies an equation of the form

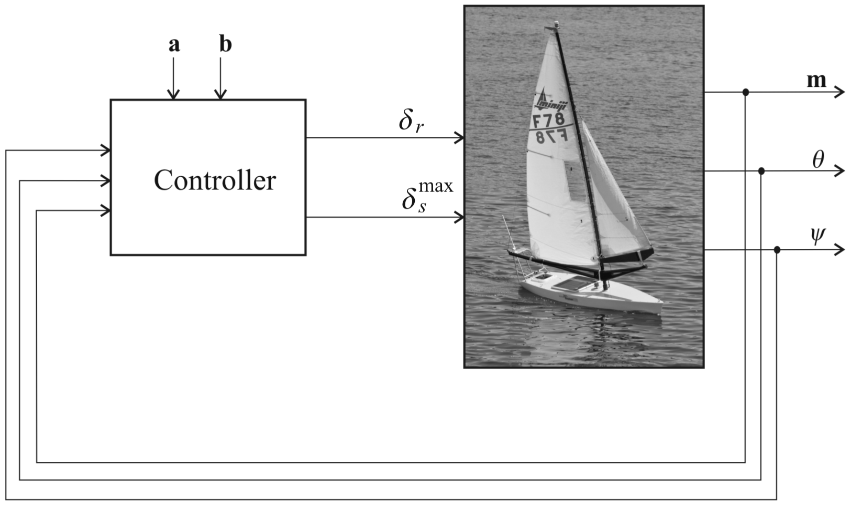
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

With all uncertainties, the robot satisfies.

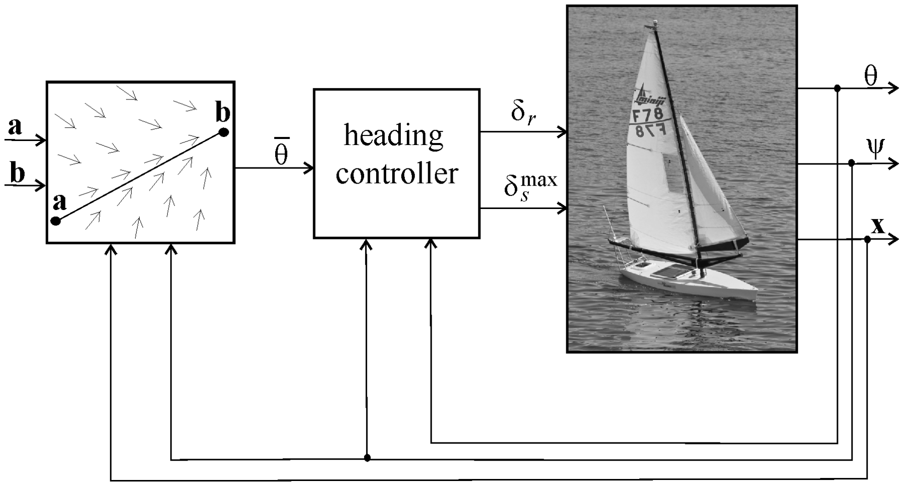
$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$$

which is *a differential inclusion*.

3.2 Line following

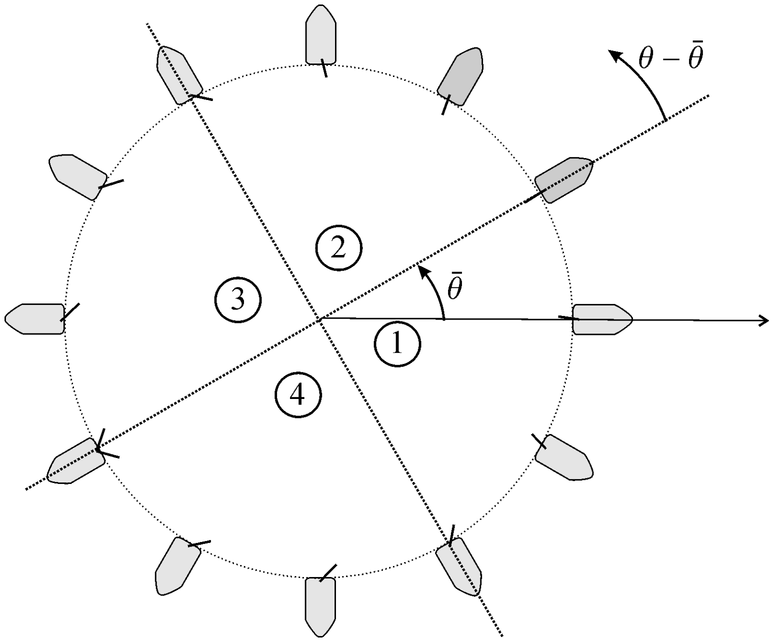


Controller of a sailboat robot


a
b
a
b
$\bar{\theta}$
heading controller
δ_r
δ_s^{max}
F78
θ
ψ
x

Heading controller

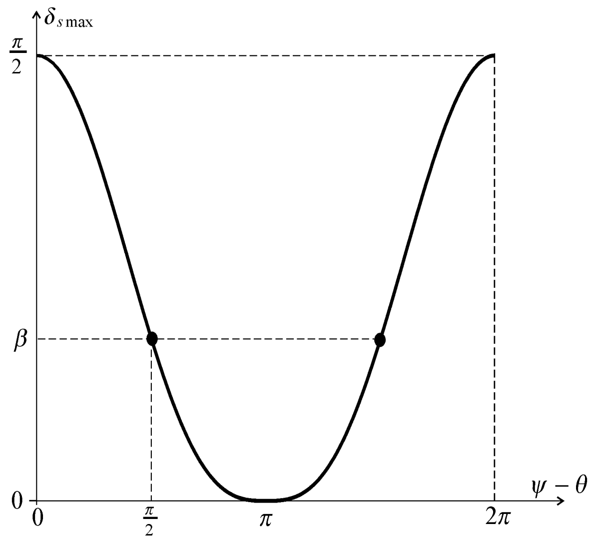
$$\left\{\begin{array}{lll} \delta_r & = & \frac{\delta_r^{\max}}{\pi}.\mathrm{atan}(\tan\frac{\theta-\bar{\theta}}{2}) \\ \delta_s^{\max} & = & \frac{\pi}{2}.\left(\frac{\cos(\psi-\bar{\theta})+1}{2}\right). \end{array}\right.$$

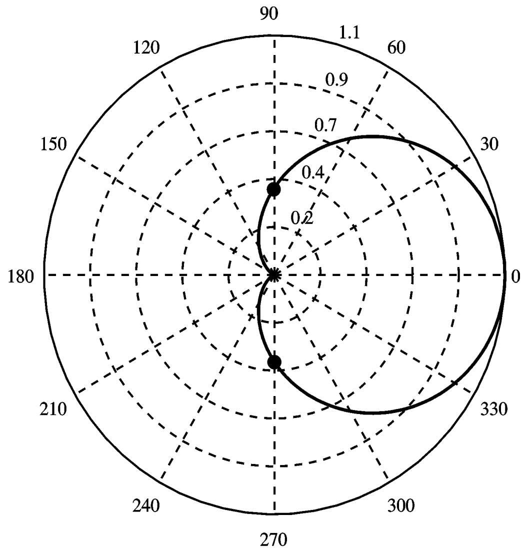


$\theta - \bar{\theta}$
$\bar{\theta}$
1
2
3
4

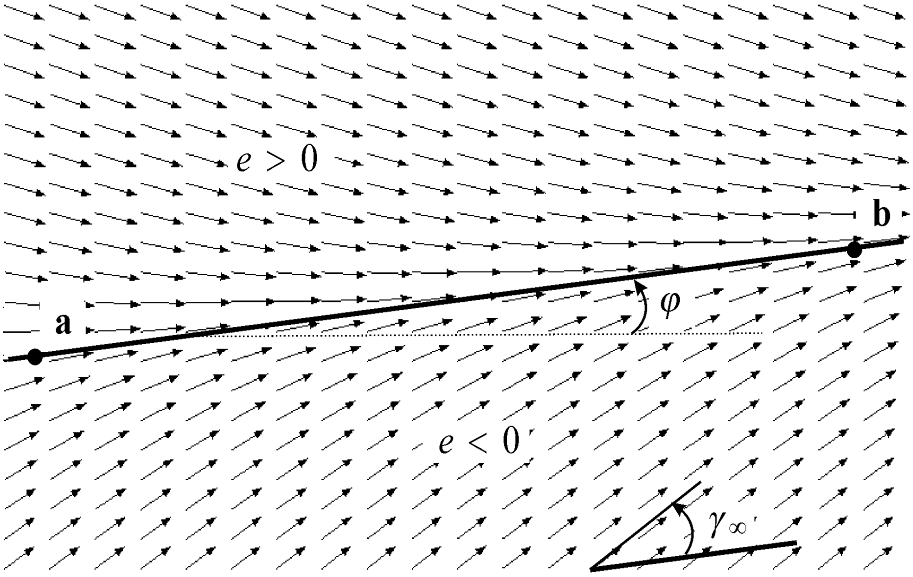
Sail

$$\delta_s^{\max} = \frac{\pi}{2} \cdot \left(\frac{\cos\left(\psi - \bar{\theta}\right) + 1}{2} \right)$$



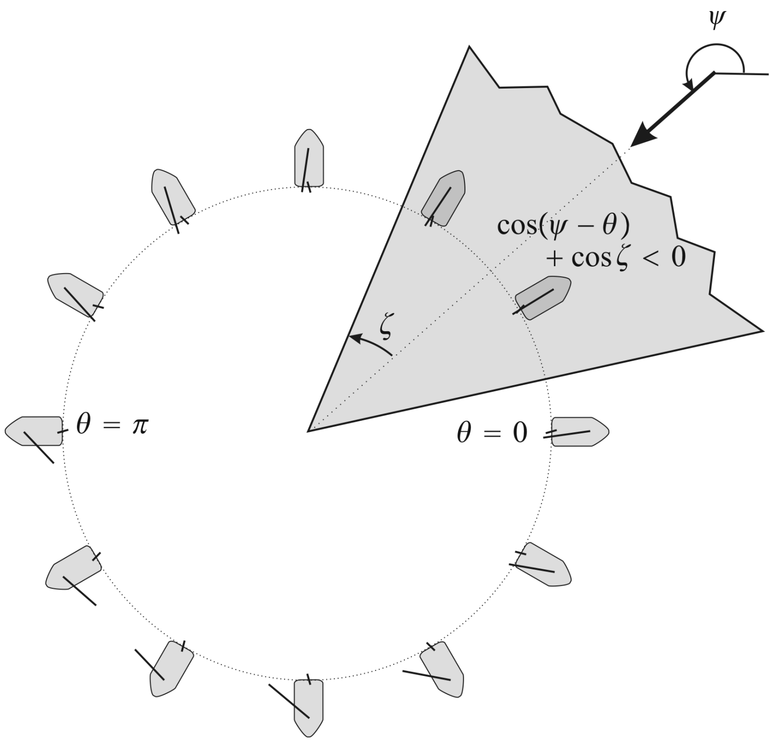


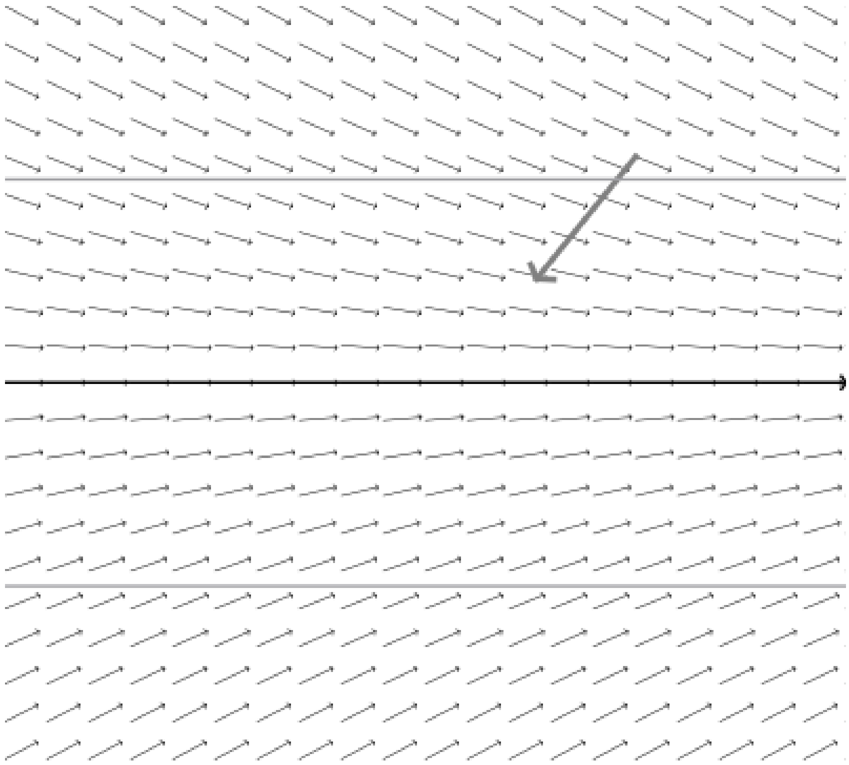
3.3 Vector field



Nominal vector field: $\theta^* = \varphi - \frac{1}{2}.\mathrm{atan}\left(\frac{e}{r}\right).$

A course θ^* may be unfeasible





$$\theta^* = -\frac{2.\gamma_\infty}{\pi}.\mathsf{atan}\left(\frac{e}{r}\right)$$

3.4 Controller

Function in: $\mathbf{m}, \theta, \psi, \mathbf{a}, \mathbf{b}$; out: $\delta_r, \delta_s^{\mathsf{max}}$; inout: q

$e = \frac{\mathsf{det}(\mathbf{b}-\mathbf{a}, \mathbf{m}-\mathbf{a})}{\|\mathbf{b}-\mathbf{a}\|}$

if $|e| > r$ then $q = \mathsf{sign}(e)$

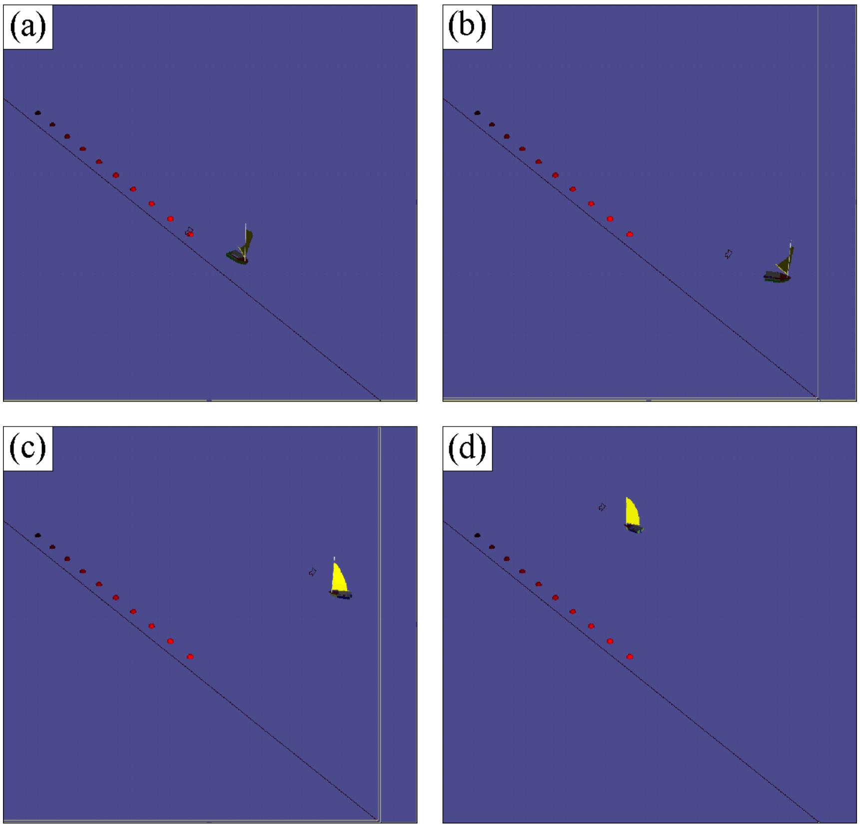
$\bar{\theta} = \mathsf{atan2}(\mathbf{b} - \mathbf{a}) - \frac{1}{2}.\mathsf{atan}\left(\frac{e}{r}\right)$

if $\cos\left(\psi - \bar{\theta}\right) + \cos\zeta < 0$ then $\bar{\theta} = \pi + \psi - q.\zeta.$

$\delta_r = \frac{\delta_r^{\mathsf{max}}}{\pi}.\mathsf{atan}(\tan\frac{\theta-\bar{\theta}}{2})$

$\delta_s^{\mathsf{max}} = \frac{\pi}{2}.\left(\frac{\cos(\psi-\bar{\theta})+1}{2}\right).$

3.5 Validation by simulation


(a)
(b)
(c)
(d)

3.6 Theoretical validation

When the wind is known, the sailboat with the heading controller is described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

The system

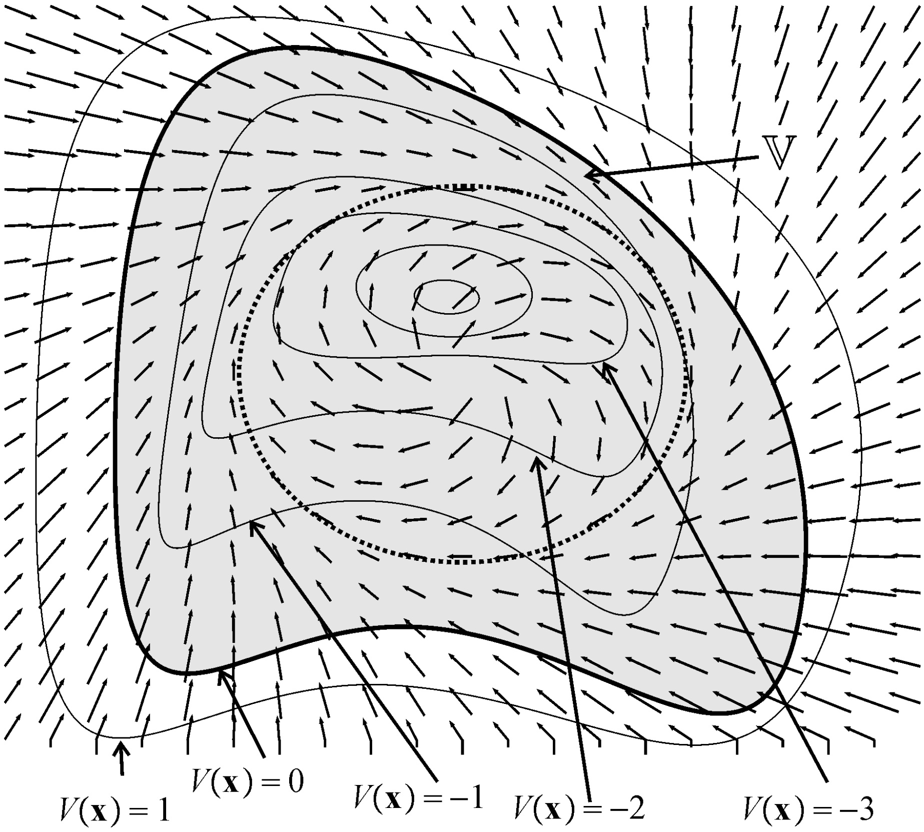
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

is Lyapunov-stable (1892) is there exists $V(\mathbf{x}) \geq 0$ such that

$$\begin{aligned} \dot{V}(\mathbf{x}) &< 0 \text{ if } \mathbf{x} \neq \mathbf{0}, \\ V(\mathbf{x}) &= 0 \text{ iff } \mathbf{x} = \mathbf{0}. \end{aligned}$$

Definition. Consider a differentiable function $V(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$. The system is V-stable if

$$\Big(V(\mathbf{x}) \geq 0 \;\Rightarrow\; \dot{V}(\mathbf{x}) < 0\Big).$$



$\mathbb{V}$
$V(\mathbf{x}) = 1$
$V(\mathbf{x}) = 0$
$V(\mathbf{x}) = -1$
$V(\mathbf{x}) = -2$
$V(\mathbf{x}) = -3$

Theorem. If the system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ is V-stable then

(i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
(ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0,\ V(\mathbf{x}(t + \tau)) < 0$.

Now,

$$
\begin{aligned}
& \left(V\left(\mathbf{x}\right) \geq 0 \;\Rightarrow\; \dot{V}\left(\mathbf{x}\right) < 0\right) \\
\Leftrightarrow\; & \left(V\left(\mathbf{x}\right) \geq 0 \Rightarrow \frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{x}\right).\mathbf{f}\left(\mathbf{x}\right) < 0\right) \\
\Leftrightarrow\; & \forall \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{x}\right).\mathbf{f}\left(\mathbf{x}\right) < 0 \text{ or } V(\mathbf{x}) < 0 \\
\Leftrightarrow\; & \neg\left(\exists \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{x}\right).\mathbf{f}\left(\mathbf{x}\right) \geq 0 \text{ and } V(\mathbf{x}) \geq 0\right)
\end{aligned}
$$

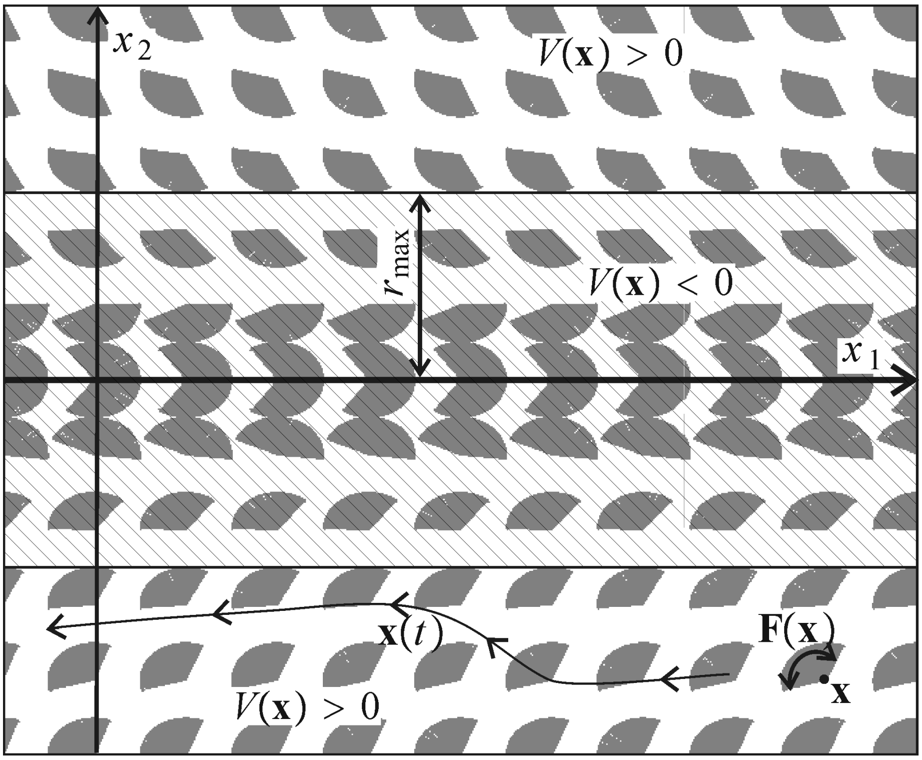
Theorem. We have

$$\left\{ \begin{array}{c} \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) \, .\mathbf{f}(\mathbf{x}) \geq 0 \\ V(\mathbf{x}) \geq 0 \end{array} \right. \text{ inconsistent } \Leftrightarrow \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \text{ is } V\text{-stable.}$$

Interval method could easily prove the V-stability.

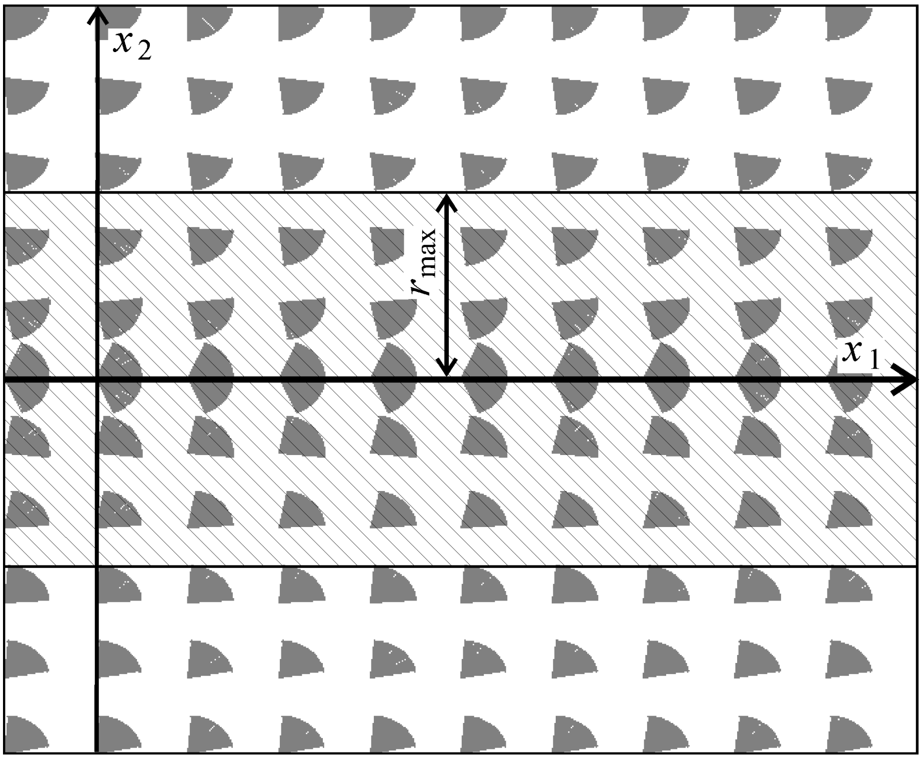
Theorem. We have

$$\left\{ \begin{array}{l} \frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{x}\right).\mathbf{a} \geq 0 \\ \quad \mathbf{a} \in \mathbf{F}\left(\mathbf{x}\right) \\ \quad V(\mathbf{x}) \geq 0 \end{array} \right. \quad \text{inconsistent} \quad \Leftrightarrow \quad \dot{\mathbf{x}} \in \mathbf{F}\left(\mathbf{x}\right) \text{ is } V\text{-stable}$$



Differential inclusion $\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$ for the sailboat.

$$V(x) = x_2^2 - r_{\text{max}}^2.$$



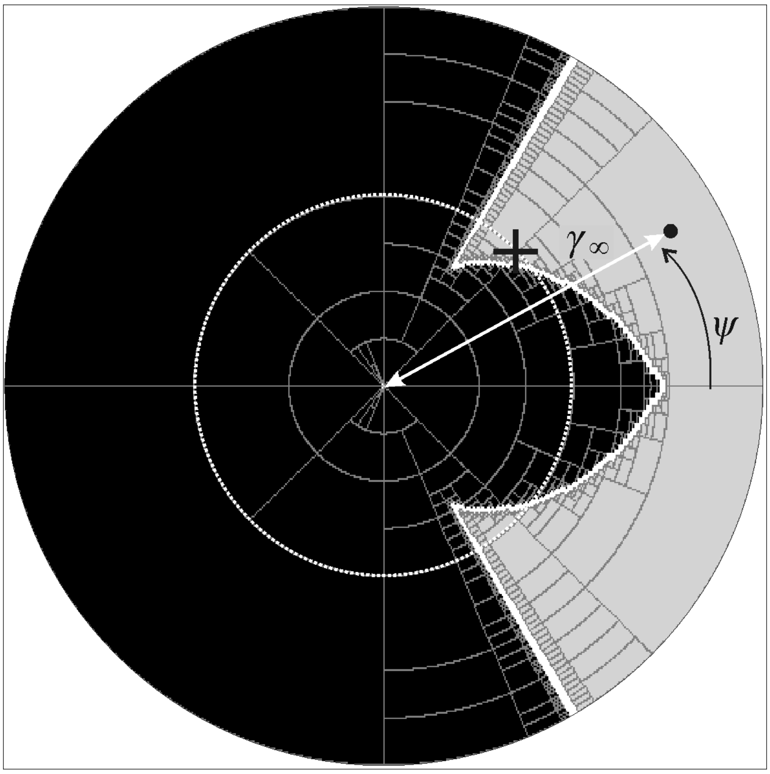
x_2
$r_{\max}$
x_1

3.7 Parametric case

Consider the differential inclusion

$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x}, \mathbf{p}).$$

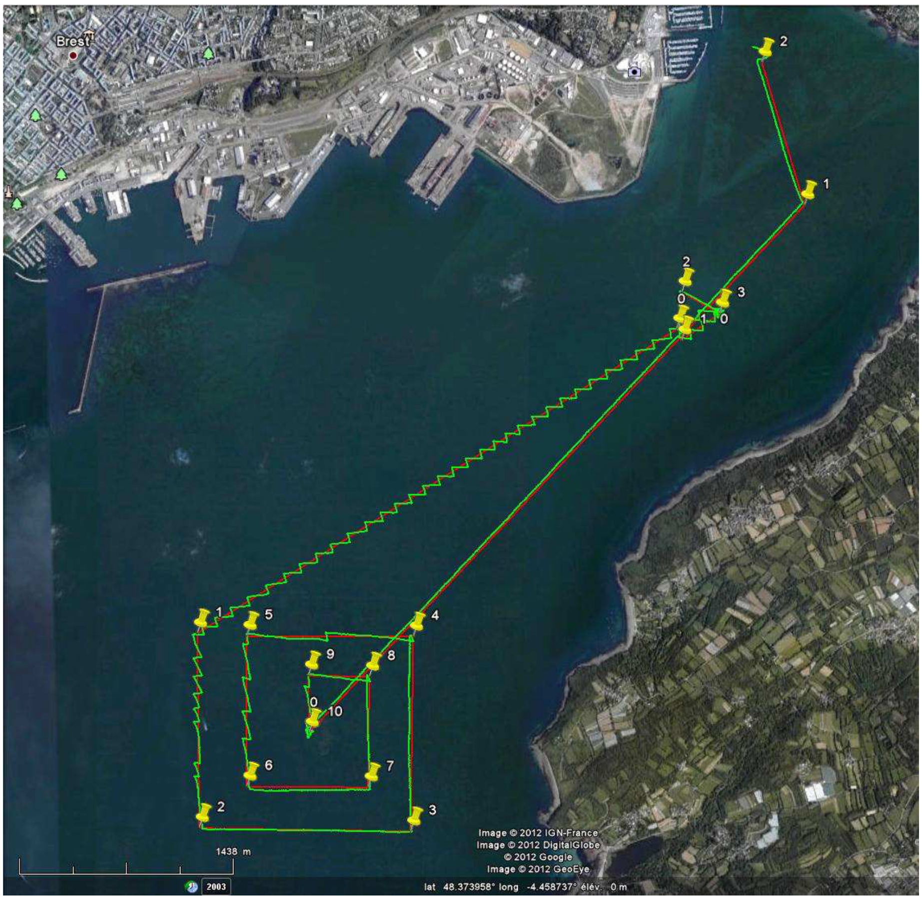
We characterize the set $\mathbb{P}$ of all $\mathbf{p}$ such that the system is V-stable.



+
γ_∞
ψ

3.8 Experimental validation

Brest


Brest
2
1
2
0
3
1
0
1
5
4
9
8
0
10
6
7
2
3
Image © 2012 IGN-France
Image © 2012 DigitalGlobe
© 2012 Google
Image © 2012 GeoEye
1438 m
2003
lat 48.373958° long -4.458737° élév. 0 m

Show Dashboard

Brest-Douarnenez. January 17, 2012, 8am


F7
8


Brest
PRO
BR D98 505
YAMAHA


miniji
F78


Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Le Conquet
Plougastel-Daoulas
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
lat 48.230580° long -4.564526° élév 6 m
Altitude 53.47 km
©2010 Google
Start
Google Earth
01:42

Montrer la mise à l'eau

Middle of Atlantic ocean

350 km made by Vaimos in 53h, September 6-9, 2012.

Consequence.

It is possible for a sailboat robot to navigate inside a corridor.

Essential, to create circulation rules when robot swarms are considered.

Essential to determine who has to pay in case of accident.

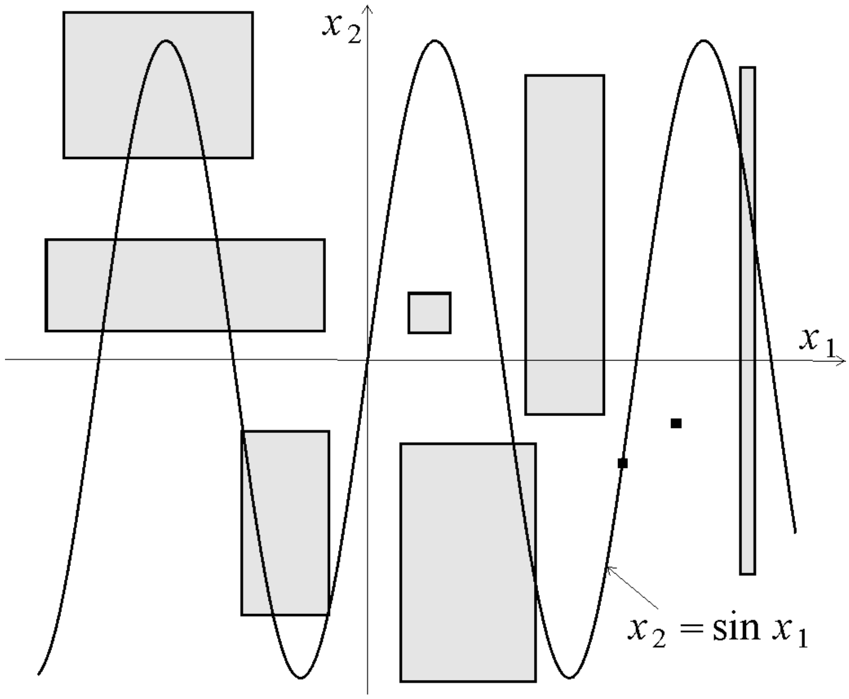
4 Contractors

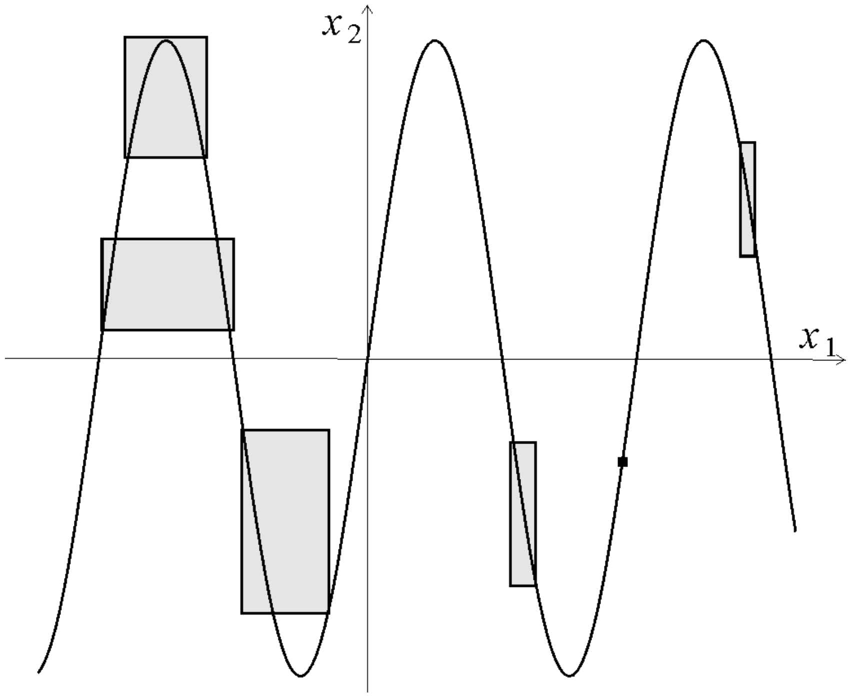
The operator $\mathcal{C} : \mathbb{IR}^n \rightarrow \mathbb{IR}^n$ is a *contractor* for the equation $f(\mathbf{x}) = 0$, if

$$\left\{ \begin{array}{ll} \mathcal{C}([\mathbf{x}]) \subset [\mathbf{x}] & \text{(contractance)} \\ \mathbf{x} \in [\mathbf{x}] \text{ and } f(\mathbf{x}) = 0 \Rightarrow \mathbf{x} \in \mathcal{C}([\mathbf{x}]) & \text{(consistence)} \end{array} \right.$$

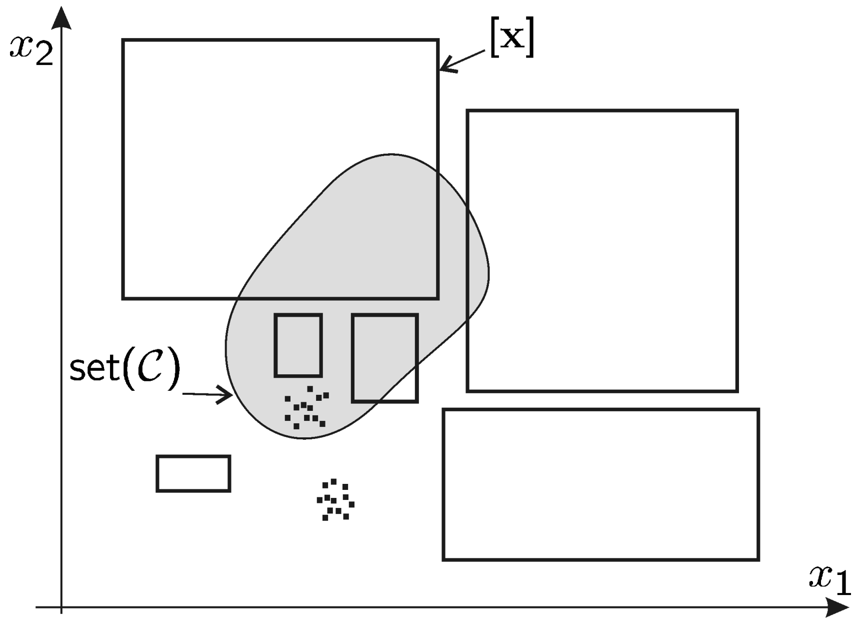
Example. Consider the primitive equation:

$$x_2 = \sin x_1.$$

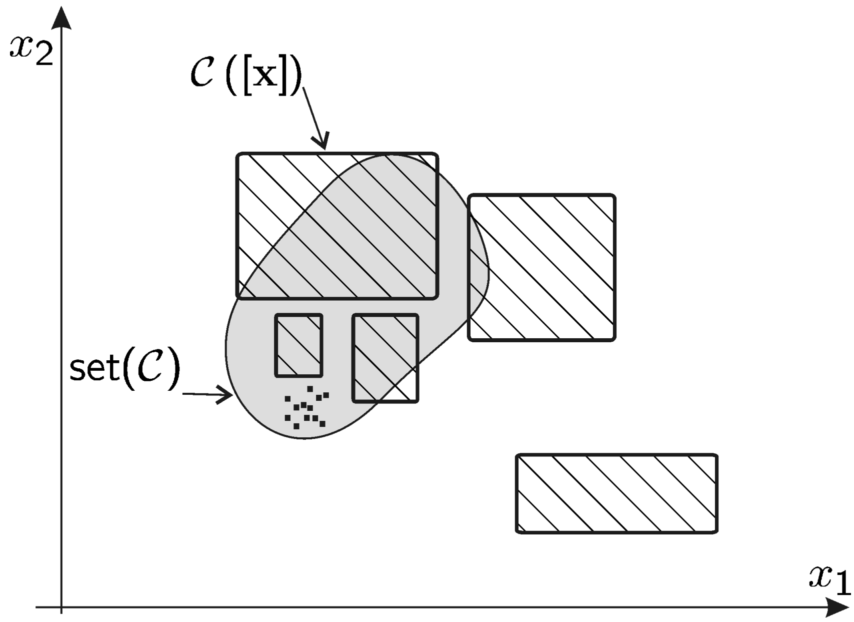

x_2
x_1
$x_2 = \sin x_1$



x_2
x_1



x_2
$[\mathbf{x}]$
set($\mathcal{C}$)
x_1



x_2
$\mathcal{C}([\mathbf{x}])$
$\mathsf{set}(\mathcal{C})$
x_1

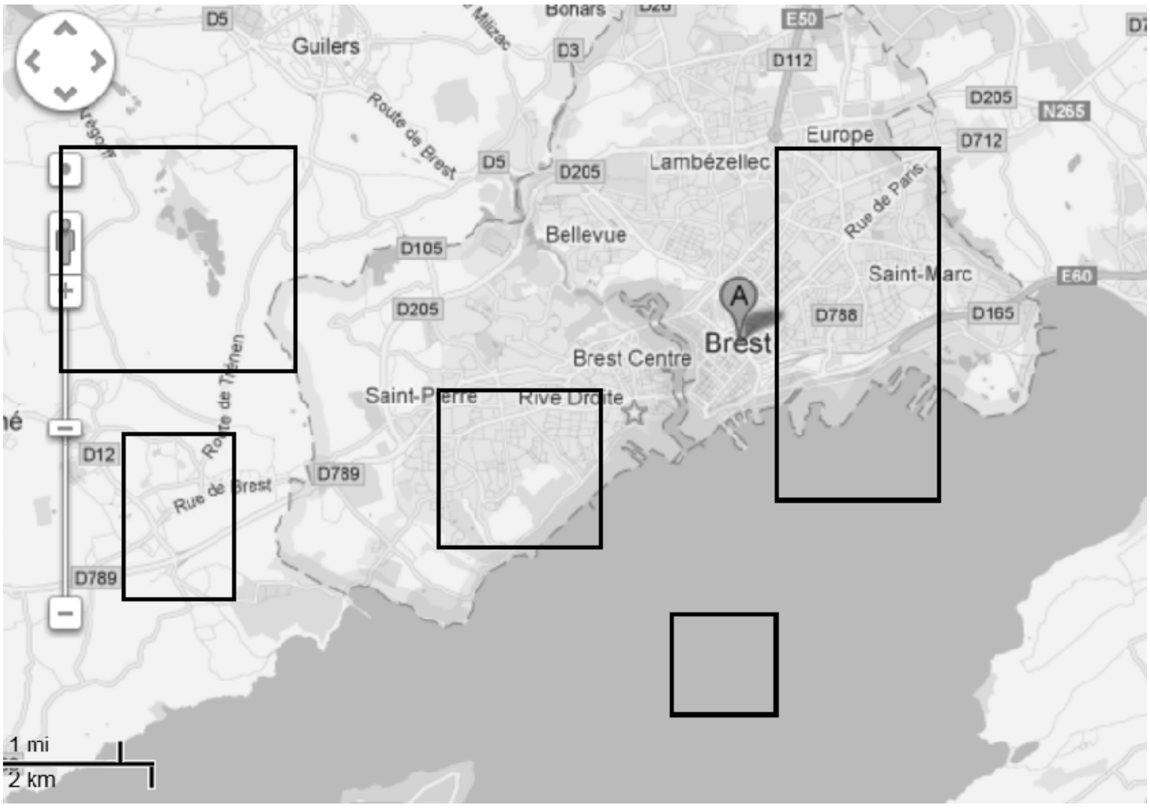
$\mathcal{C}$ is *monotonic* if	$[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow \mathcal{C}([\mathbf{x}]) \subset \mathcal{C}([\mathbf{y}])$
$\mathcal{C}$ is *minimal* if	$\mathcal{C}([\mathbf{x}]) = [[\mathbf{x}] \cap \mathsf{set}\,(\mathcal{C})]$
$\mathcal{C}$ is *idempotent* if	$\mathcal{C}\,(\mathcal{C}([\mathbf{x}])) = \mathcal{C}([\mathbf{x}])$
$\mathcal{C}$ is *continuous* if	$\mathcal{C}\,(\mathcal{C}^{\infty}([\mathbf{x}])) = \mathcal{C}^{\infty}([\mathbf{x}])$.

Contractor algebra

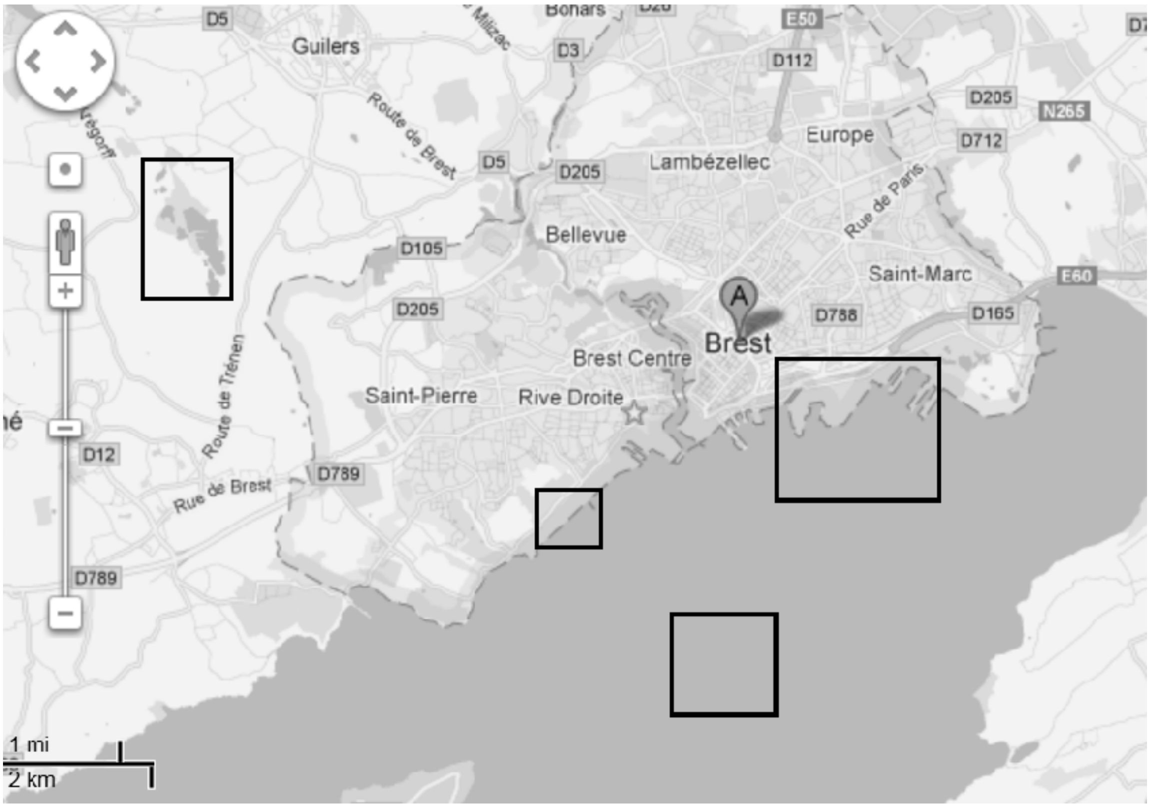
intersection	$(\mathcal{C}_1 \cap \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1([\mathbf{x}]) \cap \mathcal{C}_2([\mathbf{x}])$
union	$(\mathcal{C}_1 \cup \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} [\mathcal{C}_1([\mathbf{x}]) \cup \mathcal{C}_2([\mathbf{x}])]$
composition	$(\mathcal{C}_1 \circ \mathcal{C}_2)([\mathbf{x}]) \overset{\text{def}}{=} \mathcal{C}_1(\mathcal{C}_2([\mathbf{x}]))$
reiteration	$\mathcal{C}^\infty \overset{\text{def}}{=} \mathcal{C} \circ \mathcal{C} \circ \mathcal{C} \circ \dots$

Contractor on images

The robot with coordinates (x_1, x_2) is in the water.



Guilers
Bohars
Lambézellec
Europe
Bellevue
Saint-Marc
Brest
Brest Centre
Saint-Pierre
Rive Droite
Route de Brest
Rue de Paris
Route de Trénen
Rue de Brest
D5
D3
D26
E50
D112
D205
N265
D712
D105
E60
D788
D165
D12
D789
1 mi
2 km



D5
Guilers
Bohars
D3
E50
D112
Route de Brest
D205
Europe
D712
N265
D5
D205
Lambézellec
Rue de Paris
Bellevue
D105
Saint-Marc
E60
D205
A
Brest
D788
D165
Brest Centre
Saint-Pierre
Rive Droite
Route de Trénen
D12
Rue de Brest
D789
D789
1 mi
2 km

Building contractors for equations

Consider the primitive equation

$$x_1 + x_2 = x_3$$

with $x_1 \in [x_1]$, $x_2 \in [x_2]$, $x_3 \in [x_3]$.

We have

$$\begin{array}{llll} x_3 = x_1 + x_2 \Rightarrow & x_3 \in & [x_3] \cap ([x_1] + [x_2]) & \text{// forward} \\ x_1 = x_3 - x_2 \Rightarrow & x_1 \in & [x_1] \cap ([x_3] - [x_2]) & \text{// backward} \\ x_2 = x_3 - x_1 \Rightarrow & x_2 \in & [x_2] \cap ([x_3] - [x_1]) & \text{// backward} \end{array}$$

The contractor associated with $x_1 + x_2 = x_3$ is thus

$$\mathcal{C}\begin{pmatrix} [x_1] \\ [x_2] \\ [x_3] \end{pmatrix} = \begin{pmatrix} [x_1] \cap ([x_3] - [x_2]) \\ [x_2] \cap ([x_3] - [x_1]) \\ [x_3] \cap ([x_1] + [x_2]) \end{pmatrix}$$

4.1 Solver

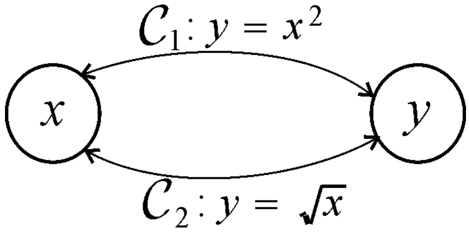
Example 1. Solve the system

$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

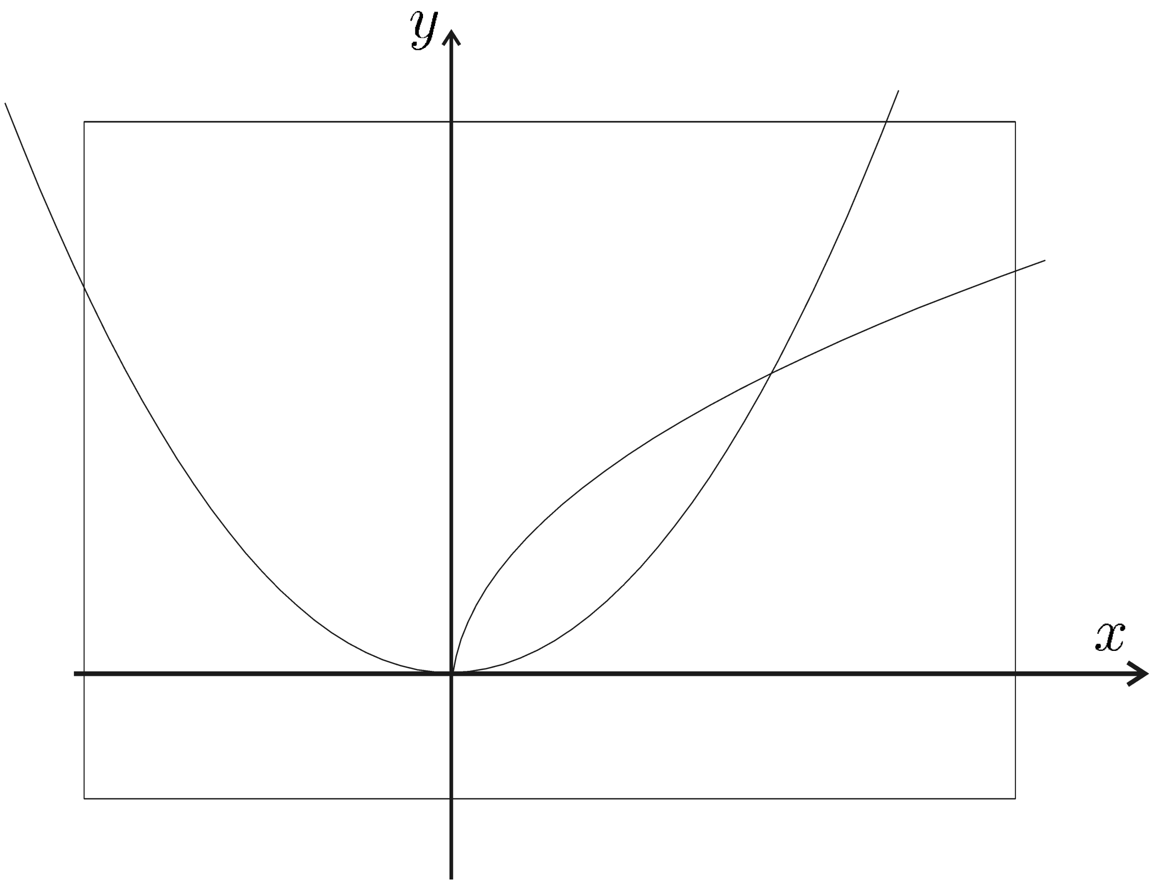
We build two contractors

$$\mathcal{C}_1 : \begin{cases} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{cases} \quad \text{associated to } y = x^2$$

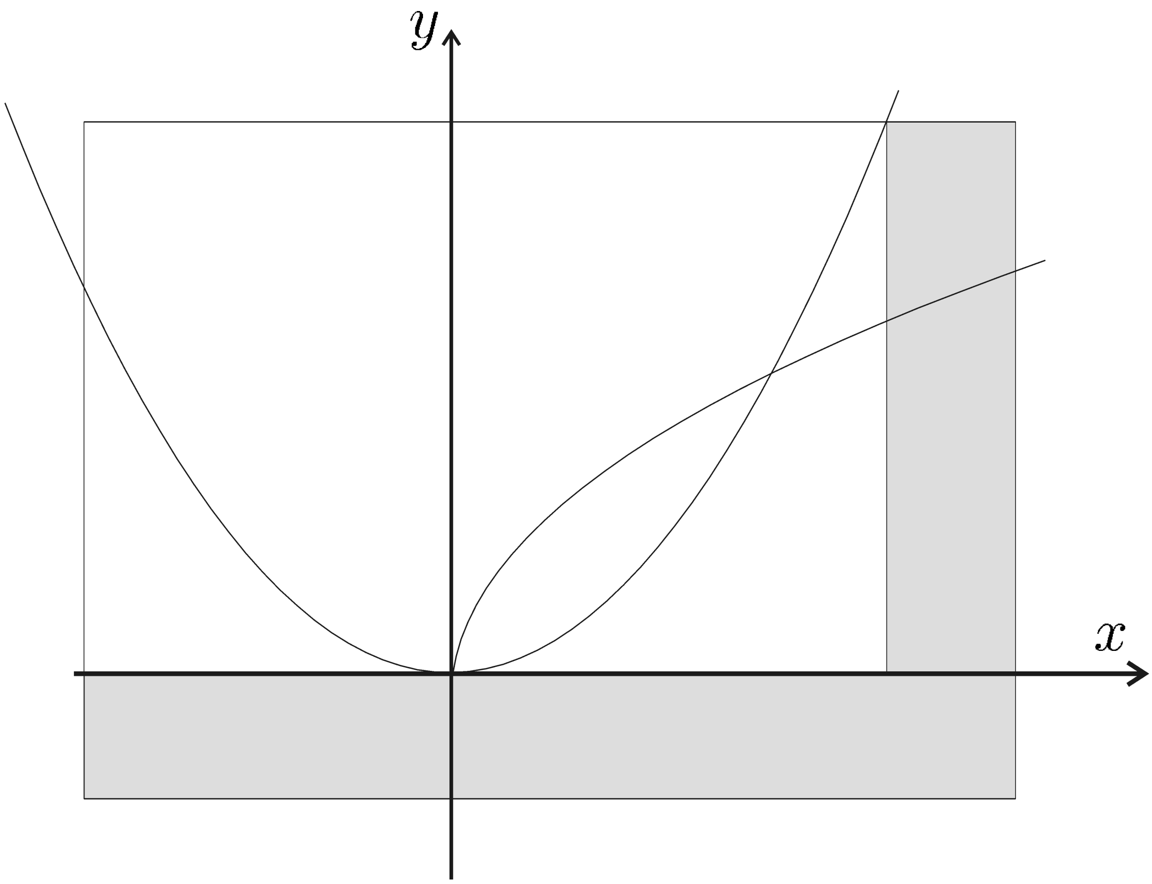
$$\mathcal{C}_2 : \begin{cases} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{cases} \quad \text{associated to } y = \sqrt{x}$$

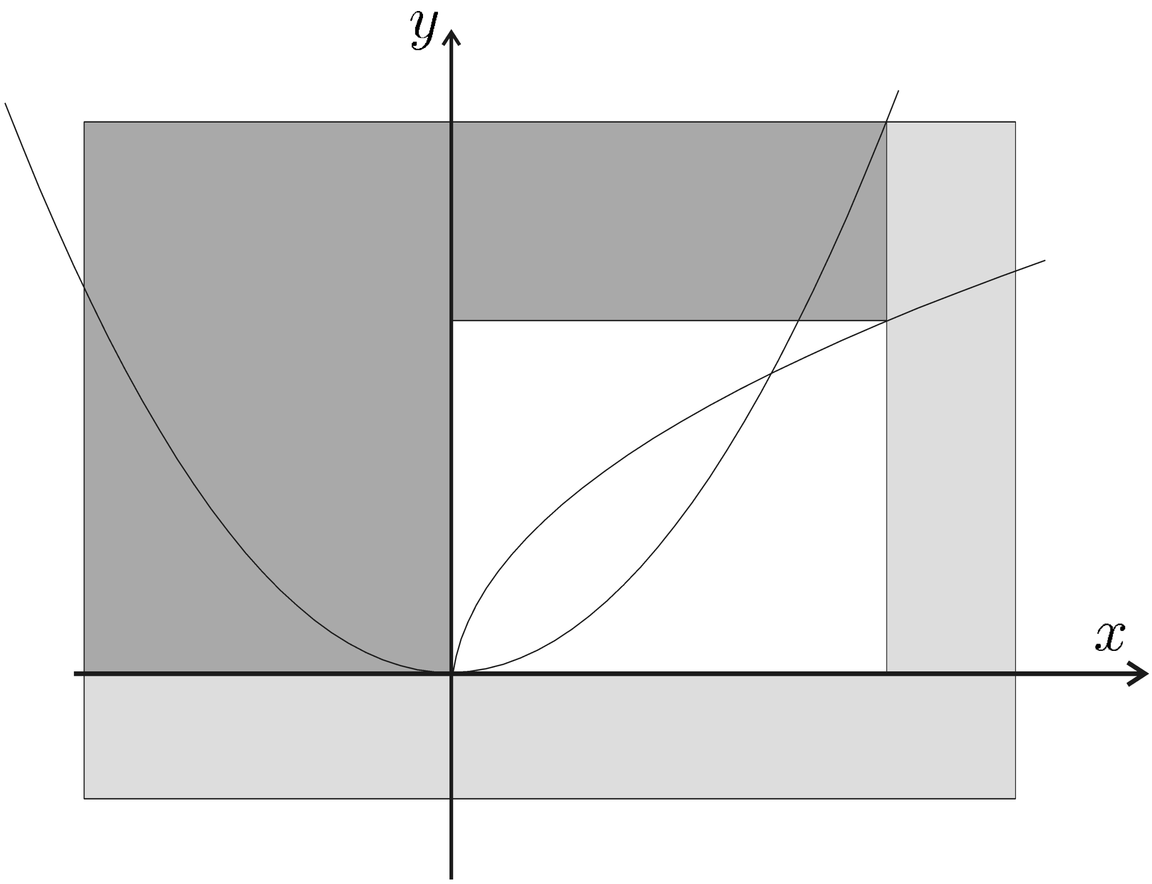


Contractor graph

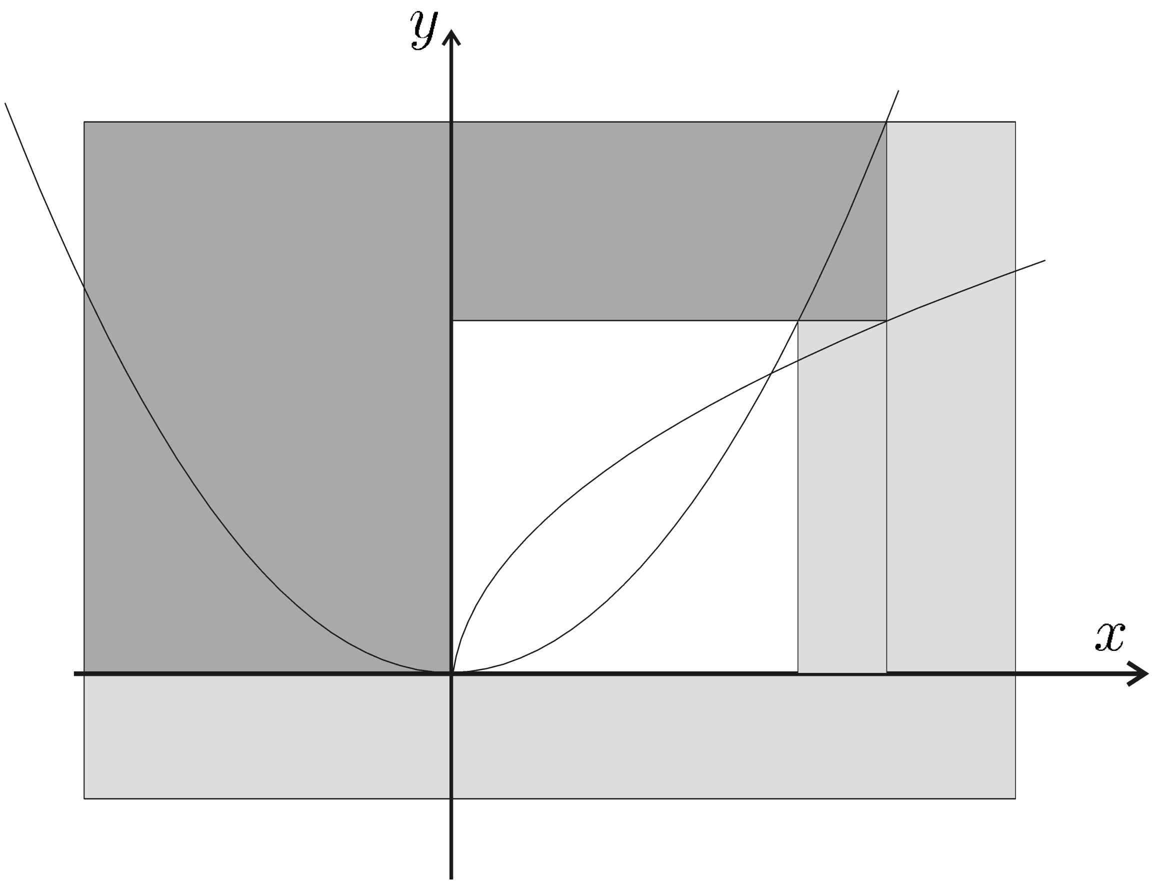


y
x

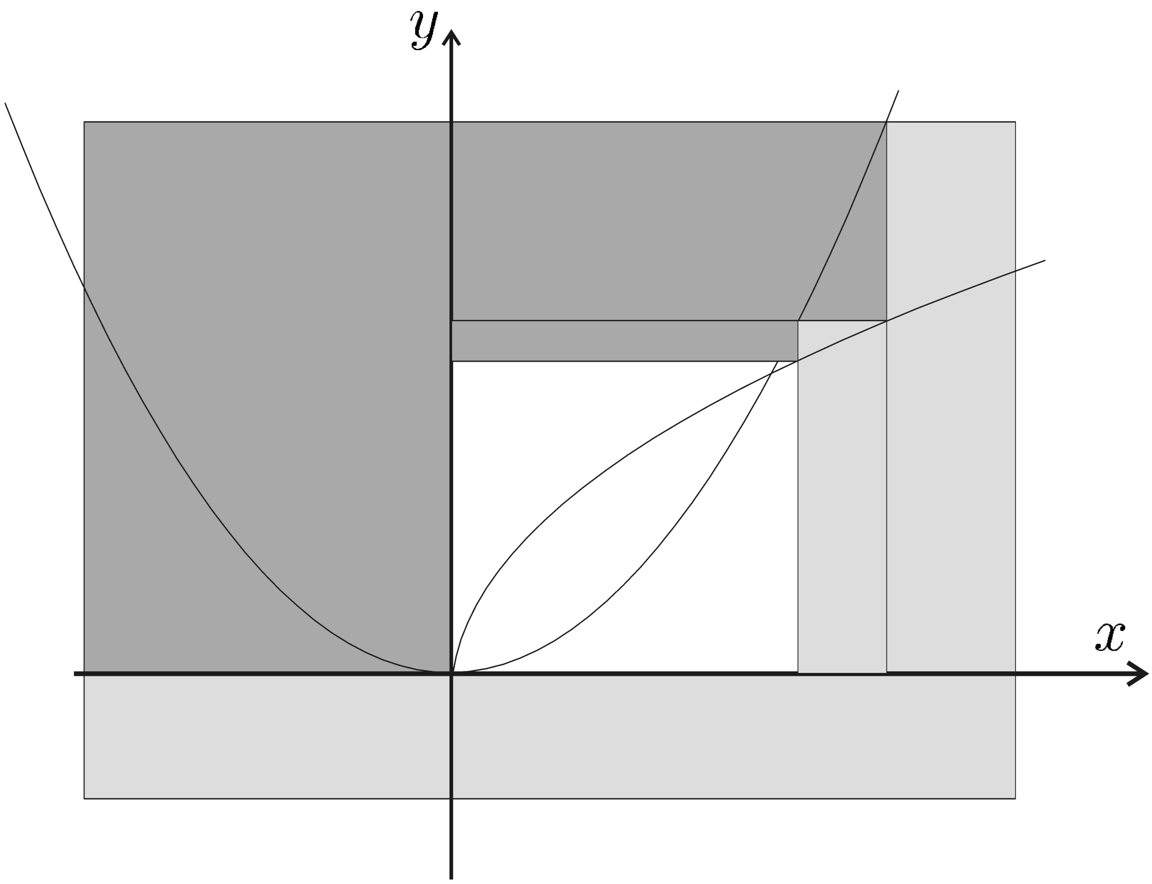

y
x



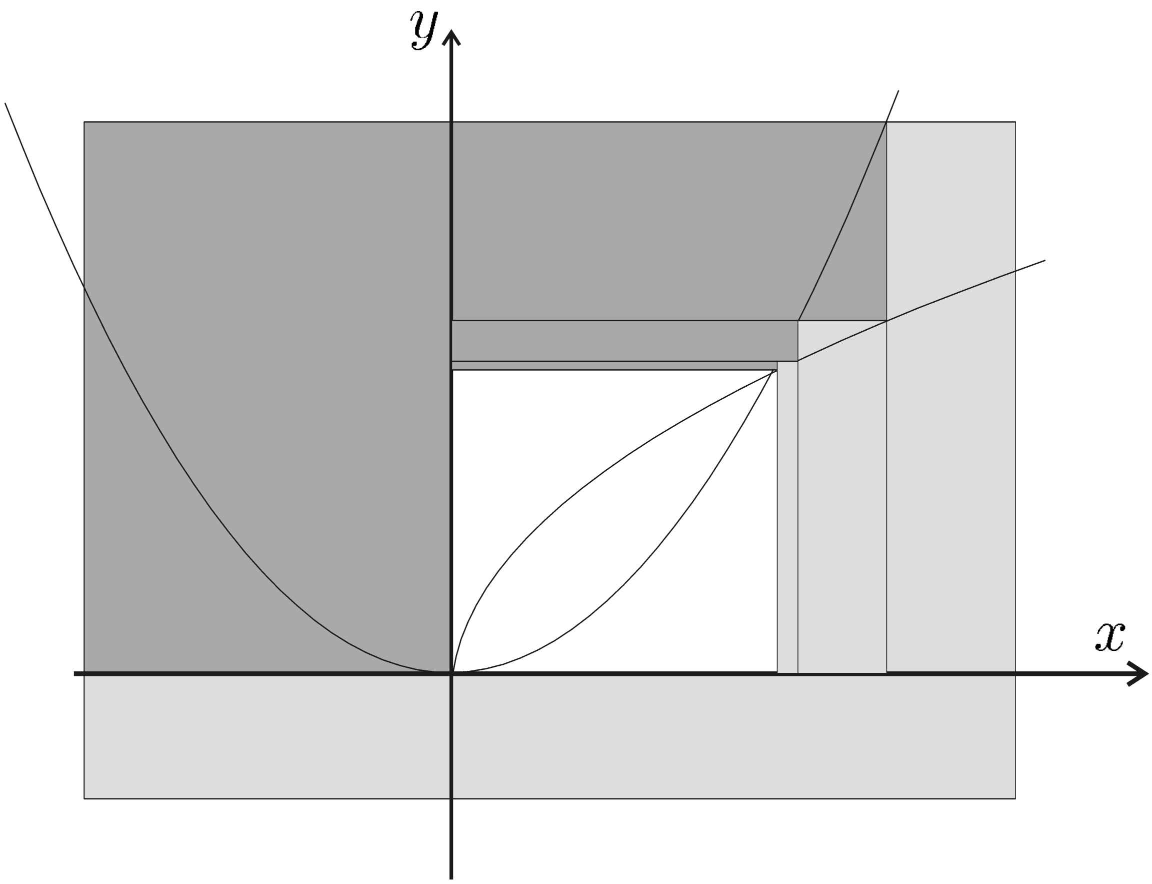
y
x

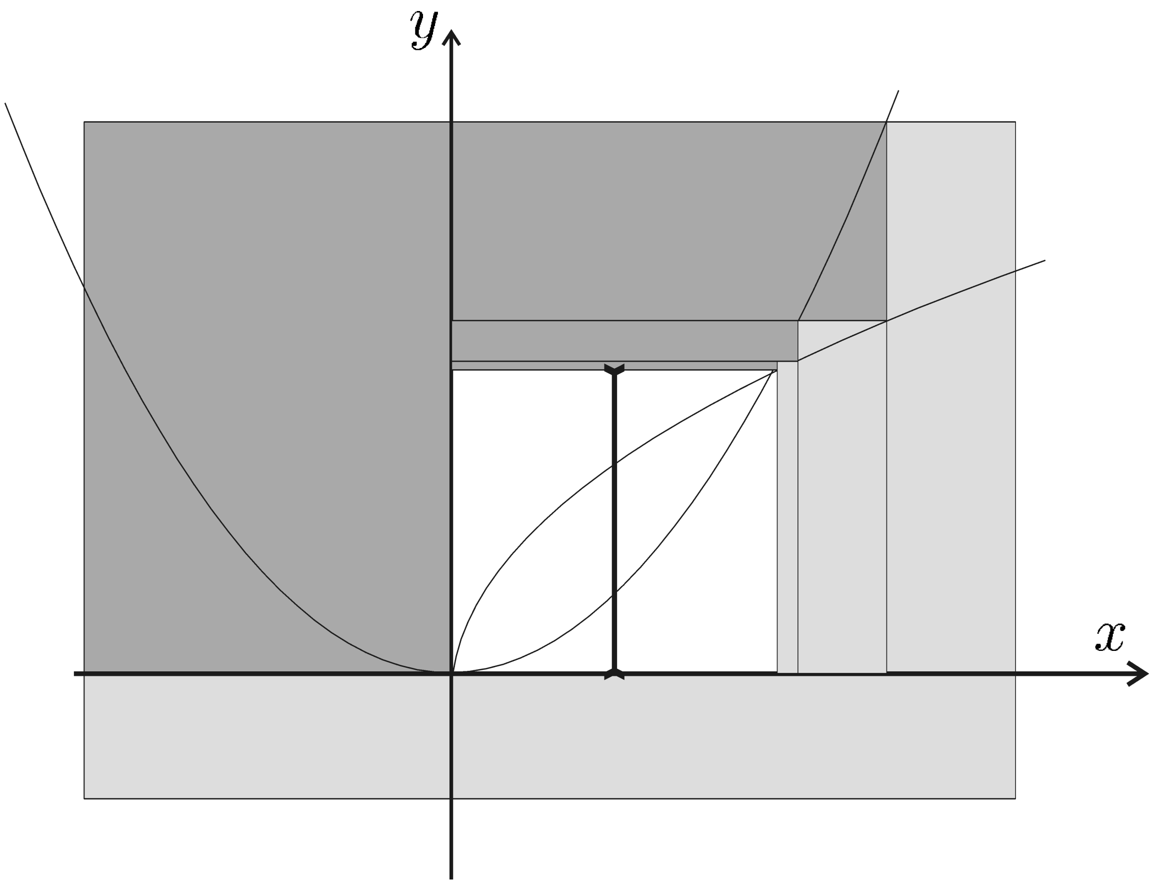


y
x

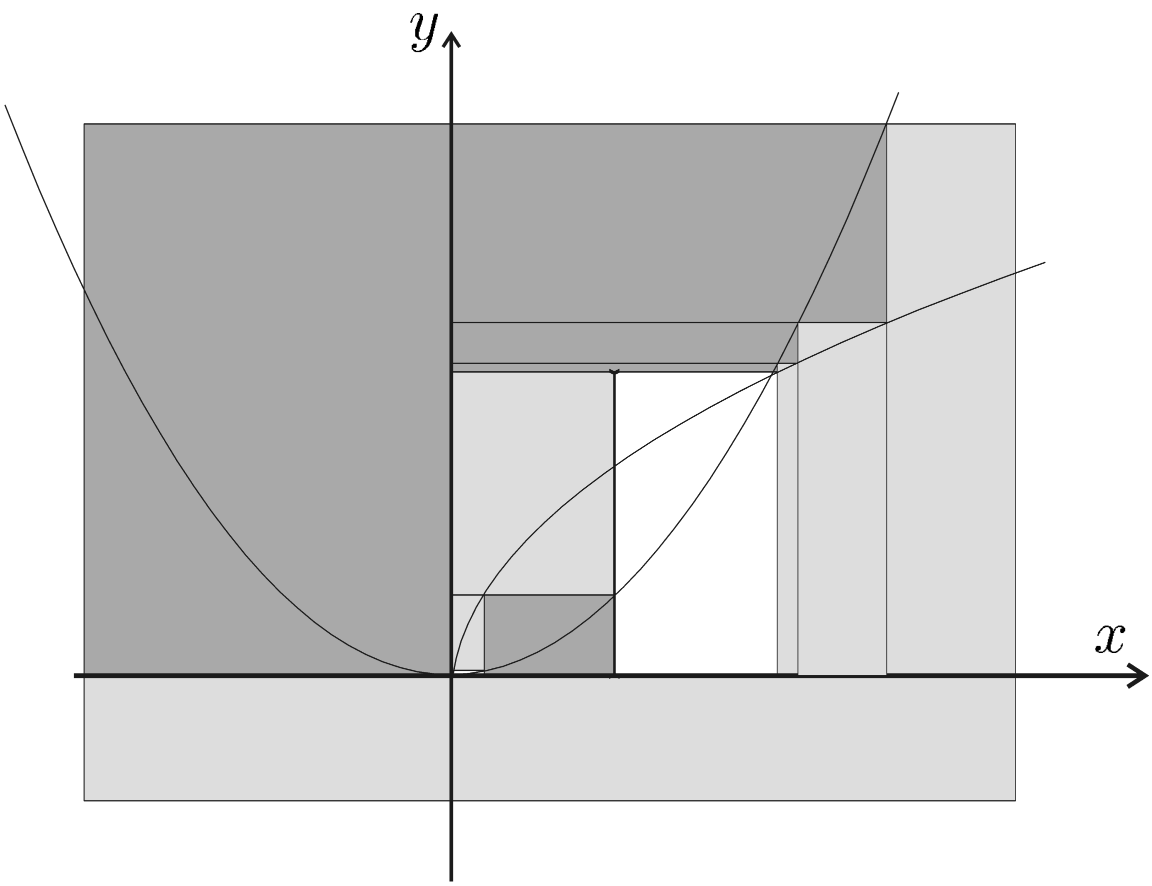


y
x

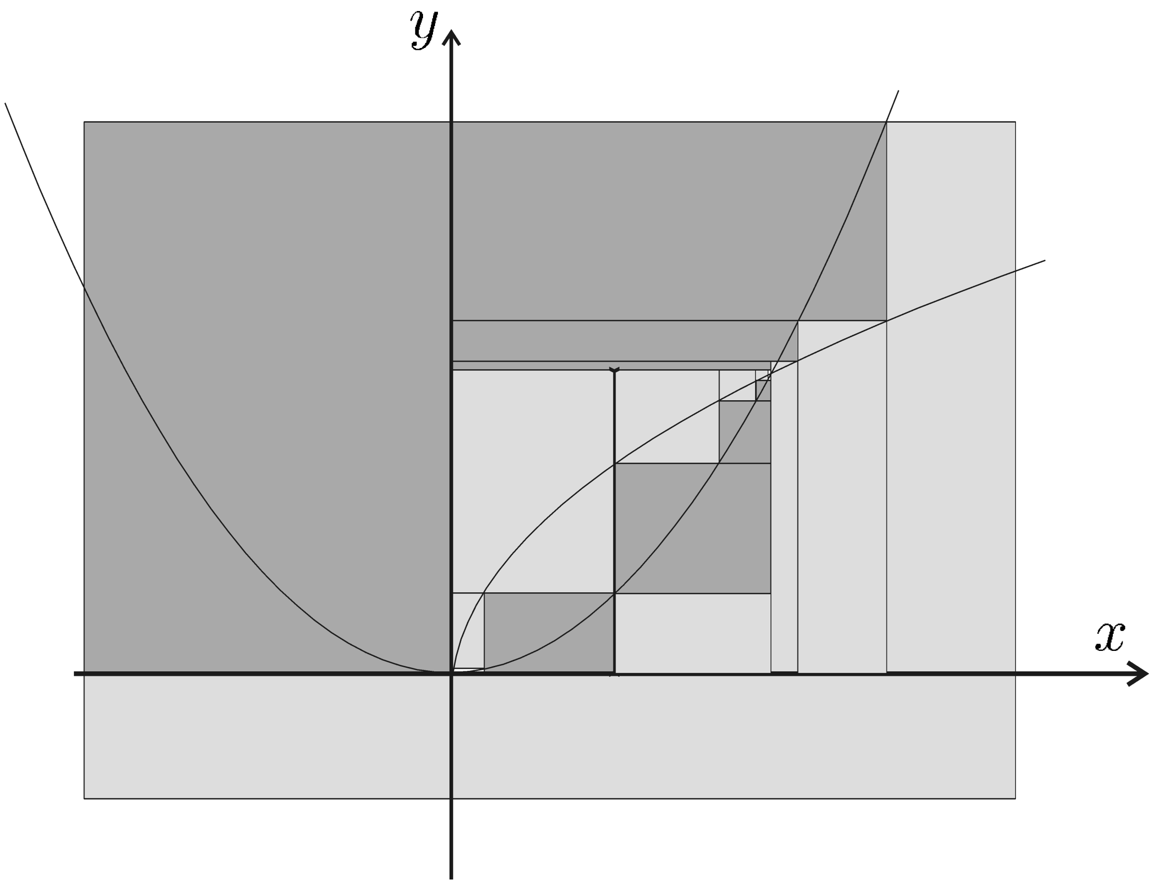

y
x



y
x

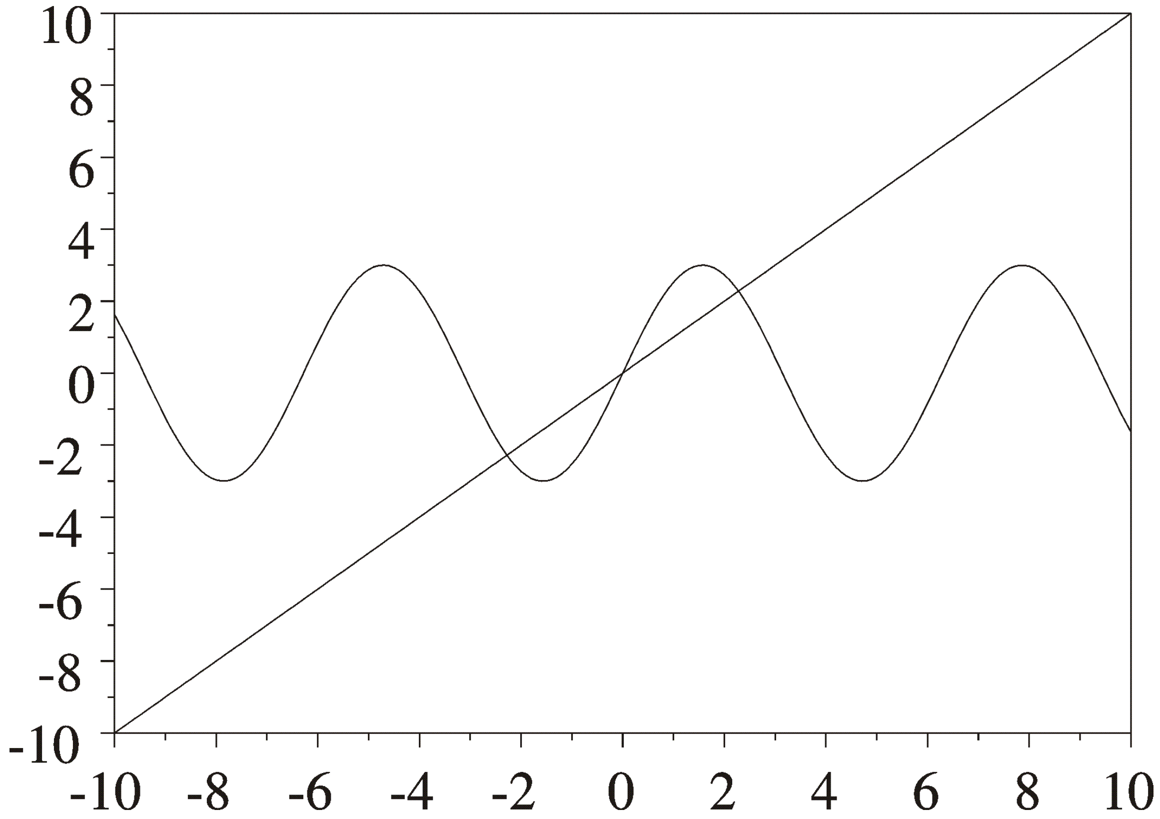


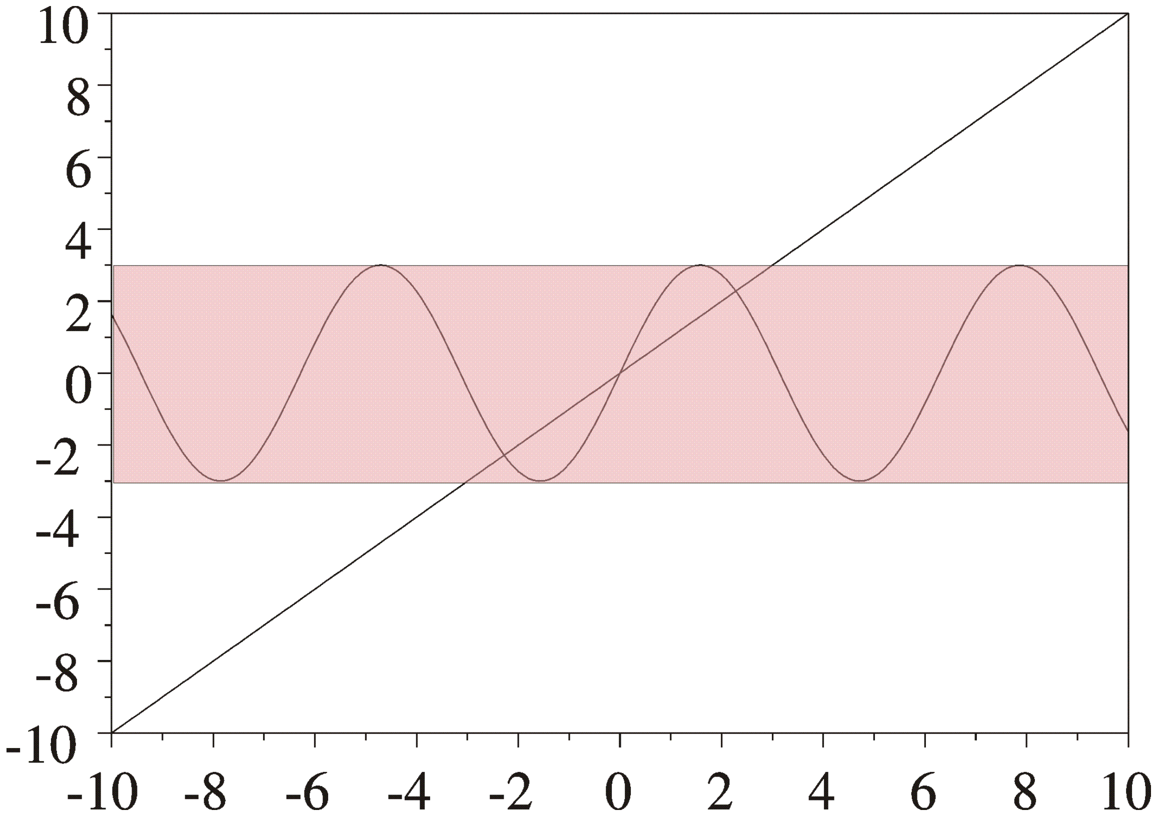
y
x

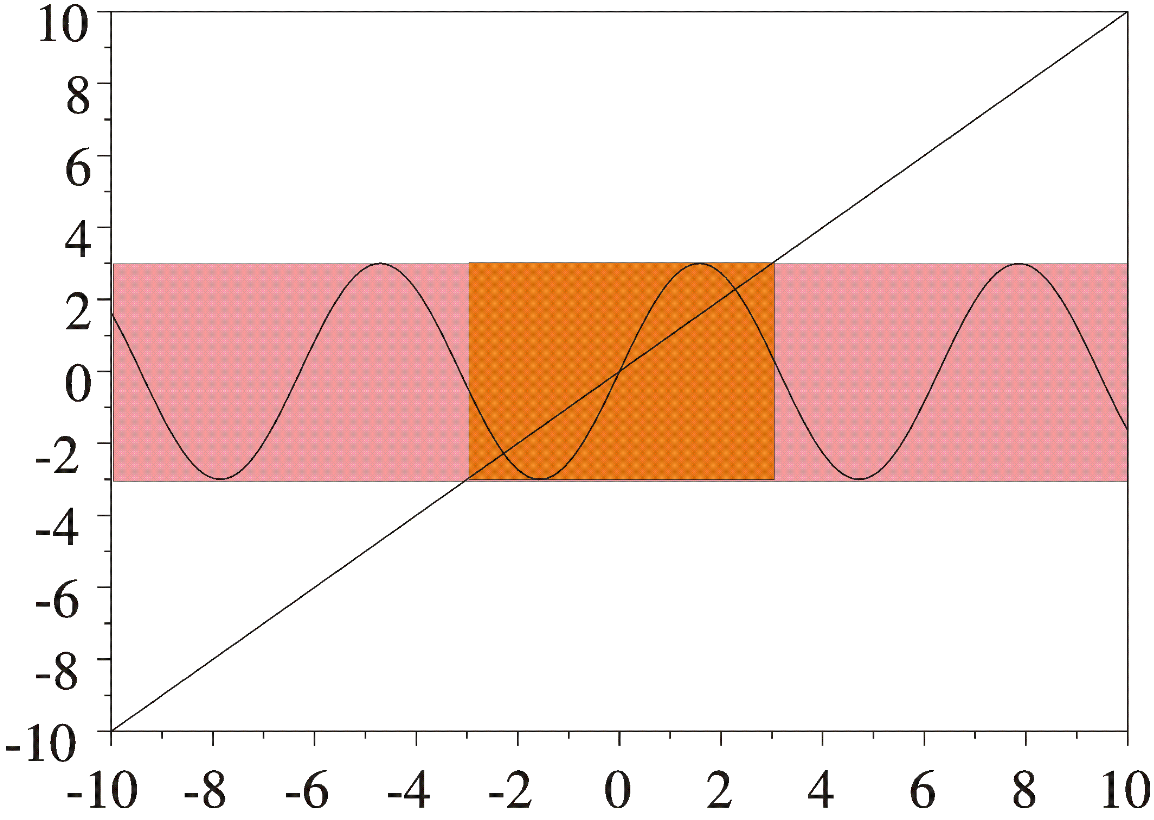

y
x

Example 2. Consider the system

$$\begin{cases} y &= 3\sin(x) \\ y &= x \end{cases} \qquad x \in \mathbb{R},\ y \in \mathbb{R}.$$


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 0 2 4 6 8 10


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

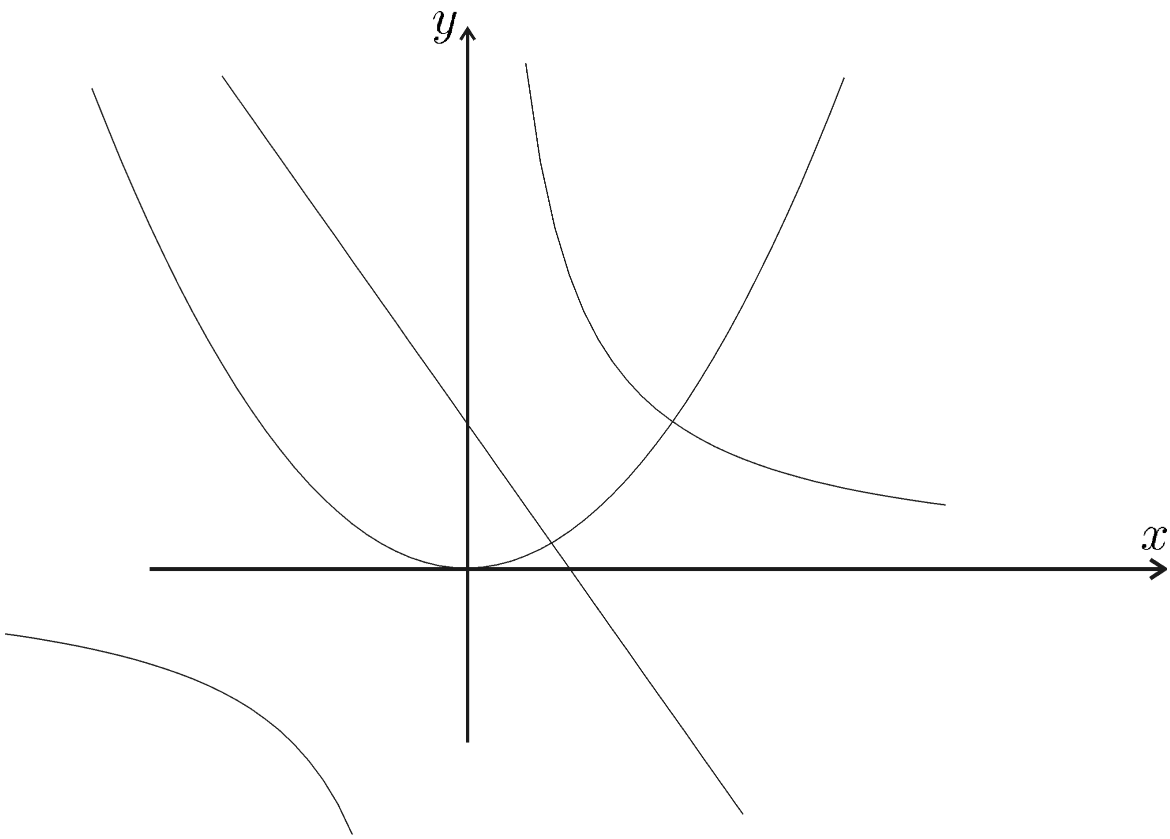


We converge the largest box $[\mathbf{x}]$ such that

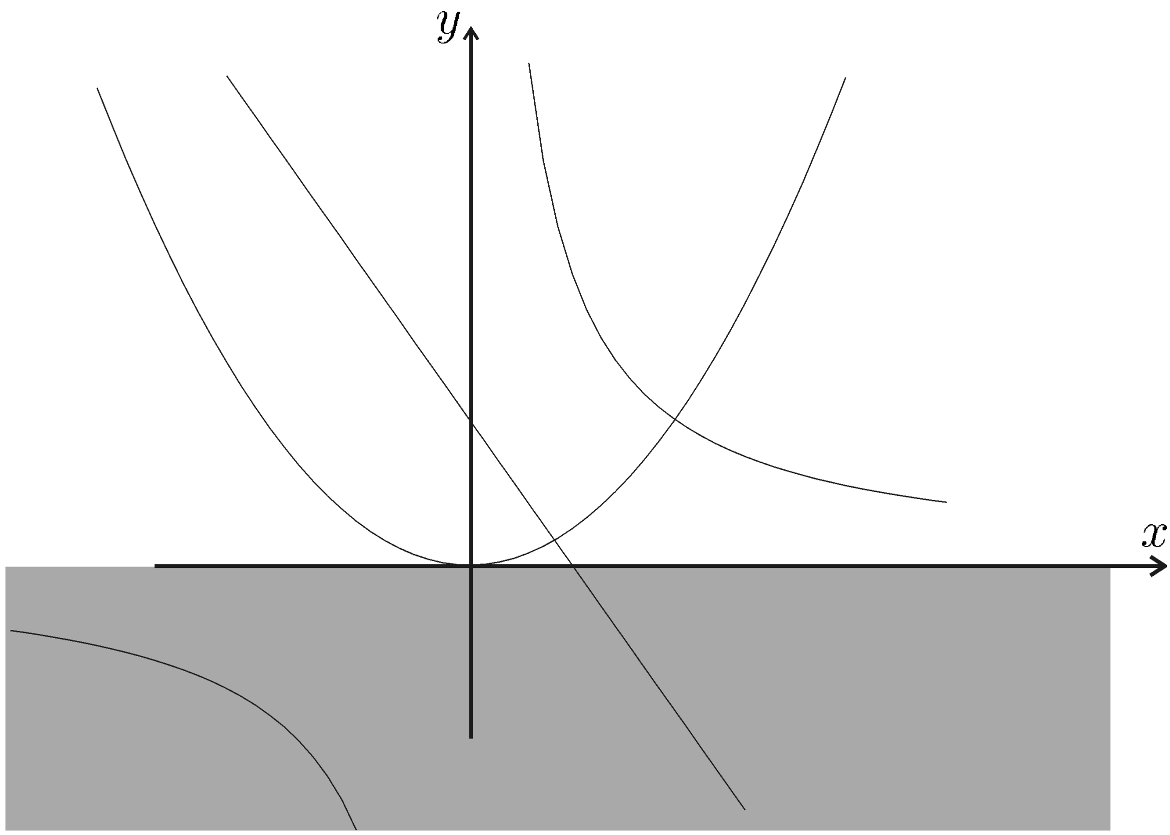
$$\mathcal{C}_1([\mathbf{x}]) = \mathcal{C}_2([\mathbf{x}]) = [\mathbf{x}].$$

Example 3. Consider the following problem

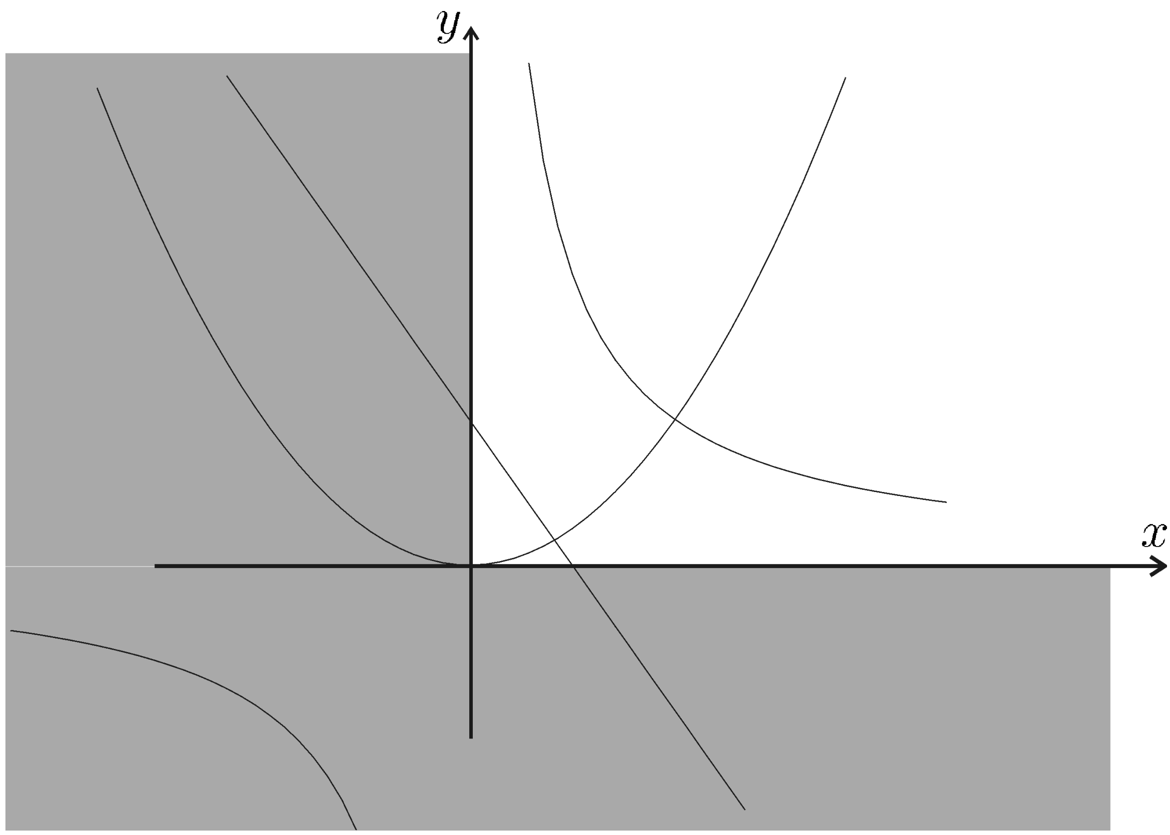
$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$



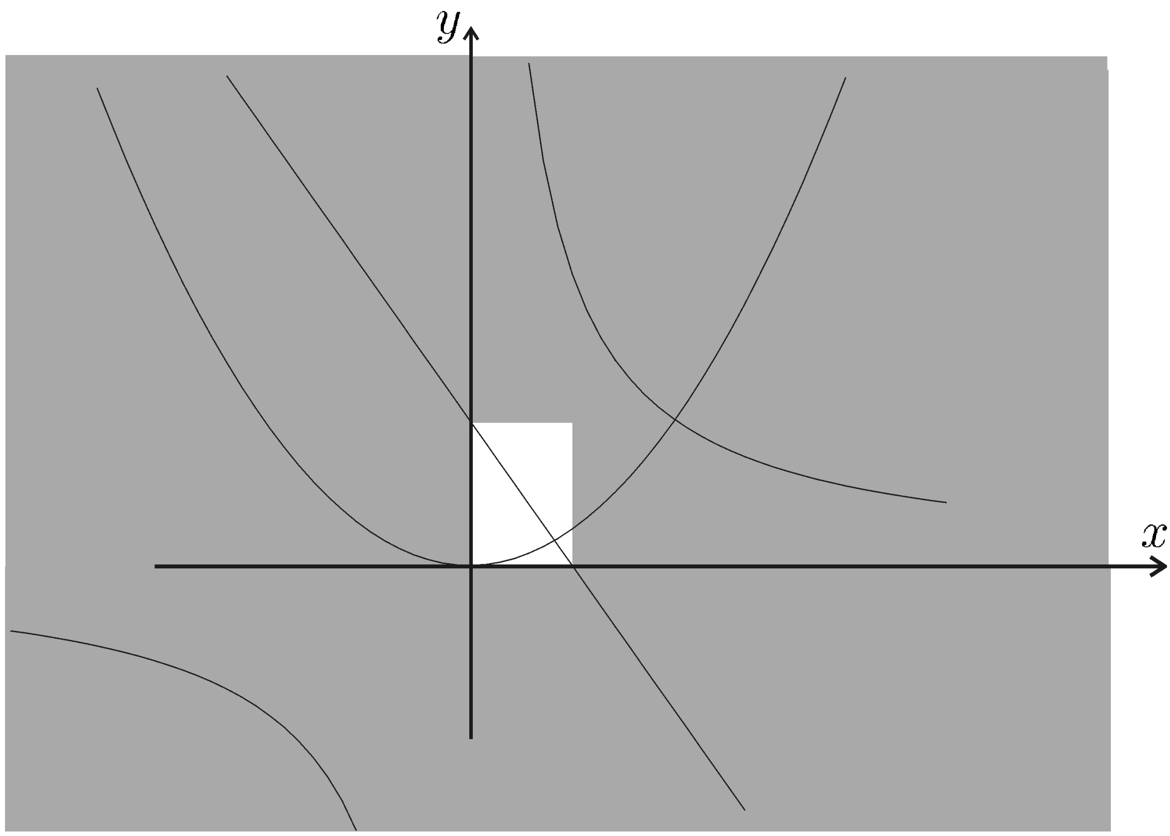
y
x

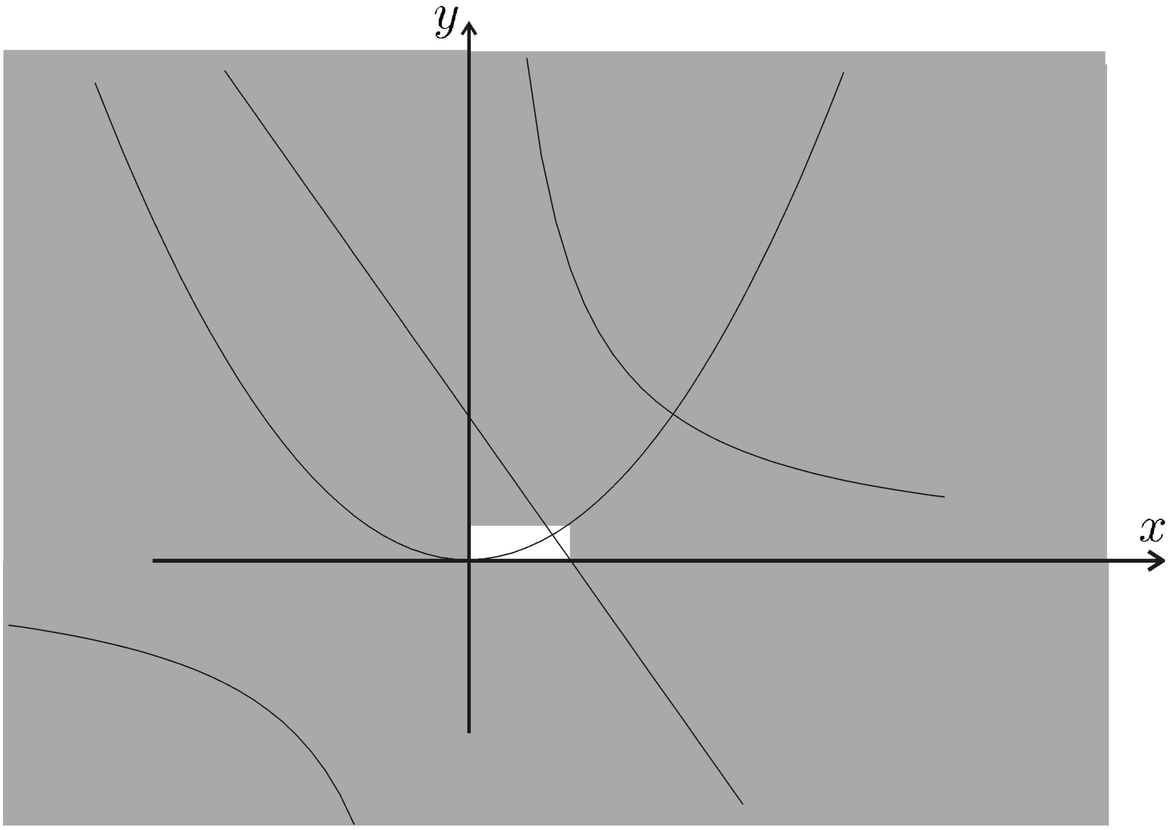


y
x

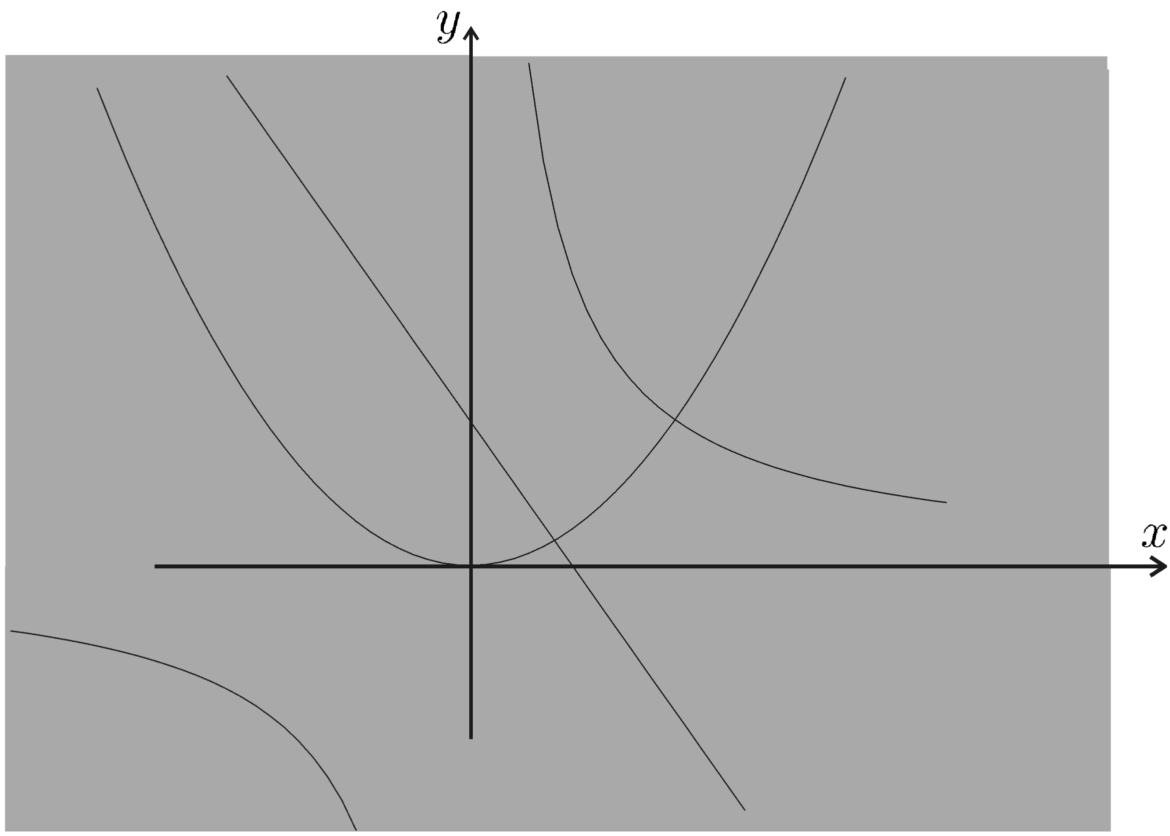


y
x


y
x



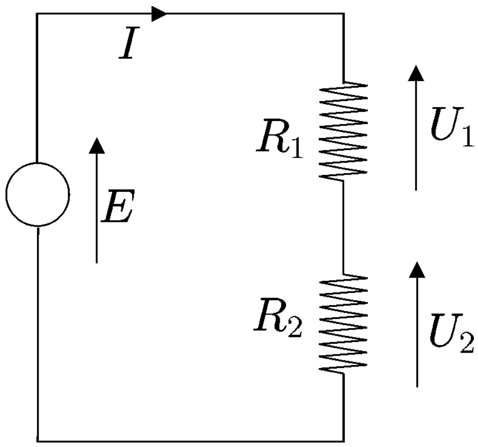
y
x


y
x

$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty,\infty]^2 \ = \ [0,\infty] \\
(C_2) \Rightarrow & x \in & 1/[0,\infty] \ = [0,\infty] \\
(C_3) \Rightarrow & y \in & [0,\infty] \ \cap \ ((-2)\,.[0,\infty] + 1) \\
 & & = [0,\infty] \ \cap \ ([-\infty,1]) = [0,1] \\
 & x \in & [0,\infty] \ \cap \ (-[0,1]/2 + 1/2) = [0,\frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0,1] \cap [0,1/2]^2 = [0,1/4] \\
(C_2) \Rightarrow & x \in & [0,1/2] \ \cap \ 1/[0,1/4] = \emptyset \\
 & y \in & [0,1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

4.2 Circuits

Example 1



Domains

$$
\begin{aligned}
E &\in [23V, 26V]; I \in [4A, 8A]; \\
U_1 &\in [10V, 11V]; U_2 \in [14V, 17V]; \\
P &\in [124W, 130W]; R_1 \in [0, \infty[\text{ and } R_2 \in [0, \infty[.
\end{aligned}
$$

Constraints

(i) $P = EI$, (ii) $E = (R_1 + R_2)\, I$, (iii) $U_1 = R_1 I$,
(iv) $U_2 = R_2 I$, (v) $E = U_1 + U_2$.

Solution set

$$\mathbb{S} = \left\{ \begin{pmatrix} E \\ R_1 \\ R_2 \\ I \\ U_1 \\ U_2 \\ P \end{pmatrix} \in \begin{pmatrix} [23, 26] \\ [0, \infty[\\ [0, \infty[\\ [4, 8] \\ [10, 11] \\ [14, 17] \\ [124, 130]; \end{pmatrix}, \left\{ \begin{array}{l} P = EI \\ E = (R_1 + R_2)\, I \\ U_1 = R_1 I \\ U_2 = R_2 I \\ E = U_1 + U_2 \end{array} \right. \right\}$$

```
variables
 E in [23 ,26];
 I in [4,8];
 U1 in [10,11];
 U2 in [14 ,17];
 P in [124,130];
 R1 in [0 ,1e08 ];
 R2 in [0 ,1e08 ];
contractor_list L
 P=E*I;
 E=(R1+R2)*I;
 U1=R1*I;
 U2=R2*I;
 E=U1+U2;
end
```

```
contractor C
   compose(L);
end
contractor epsilon
   precision(1);
end
```

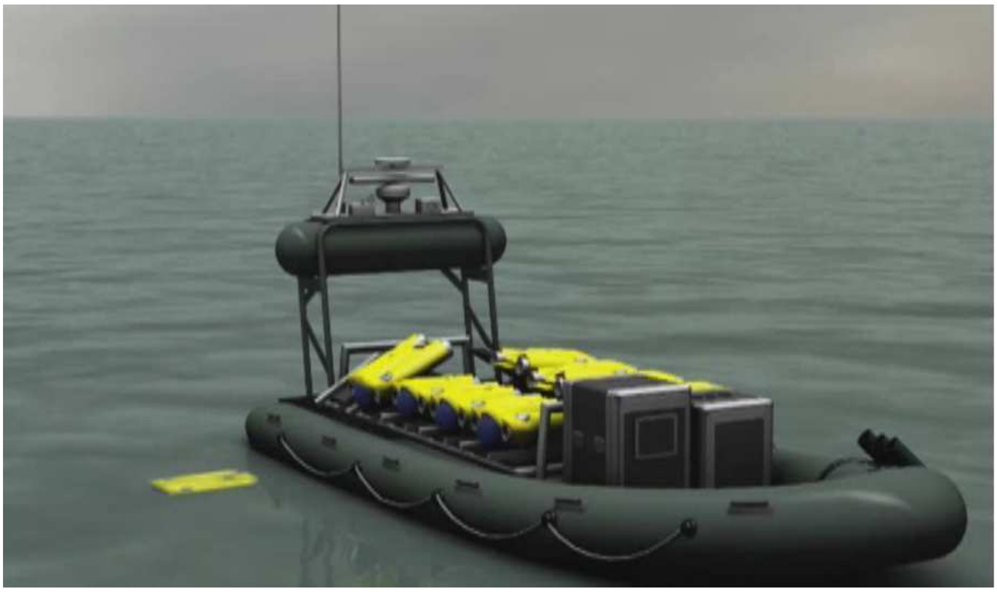
Quimper returns

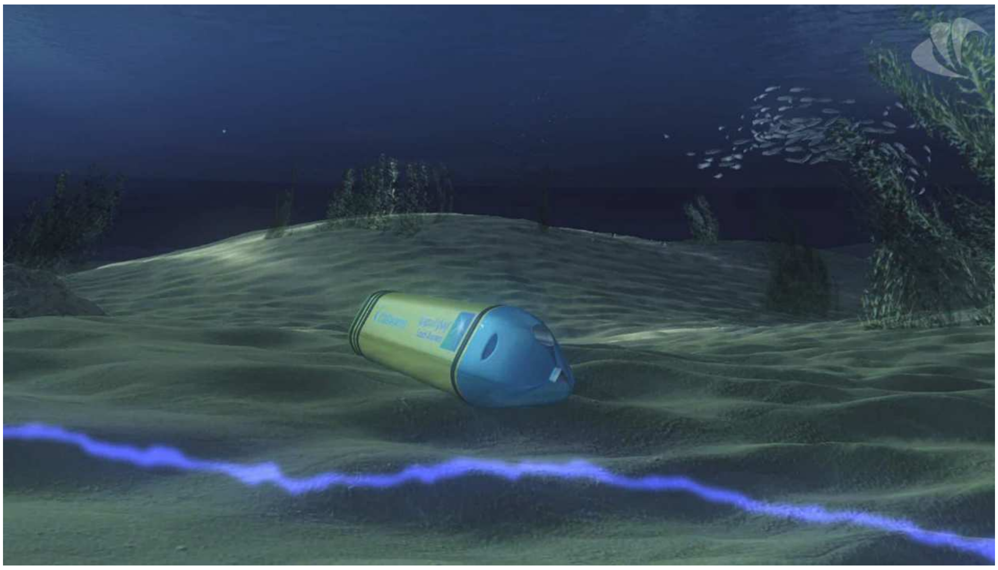
$$[24; 26] \times [1.846; 2.307] \times [2.584; 3.355]$$
$$\times [4.769; 5.417] \times [10; 11] \times [14; 16] \times [124; 130]\,,$$

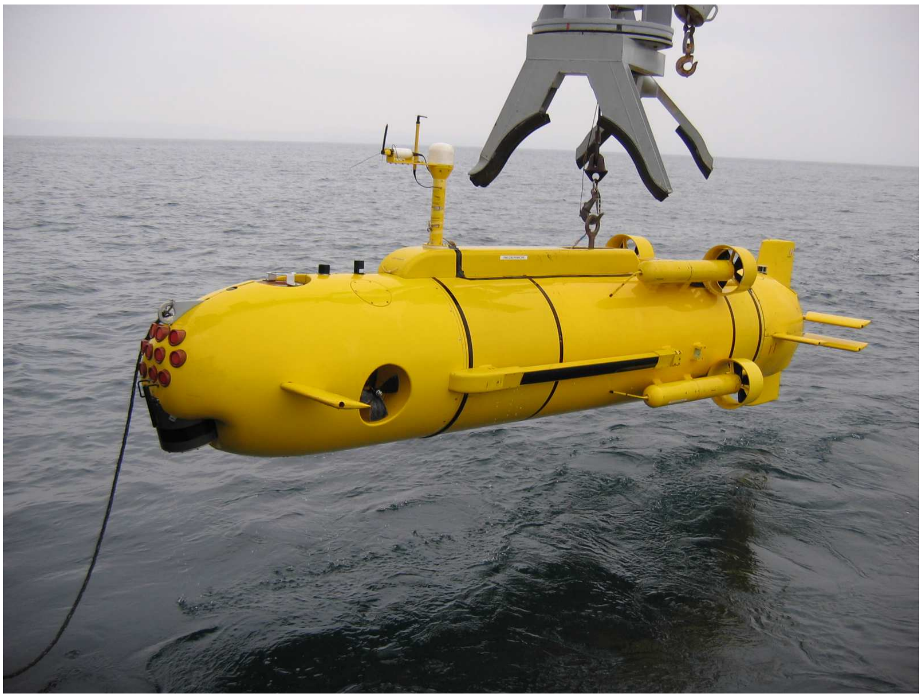
i.e.,

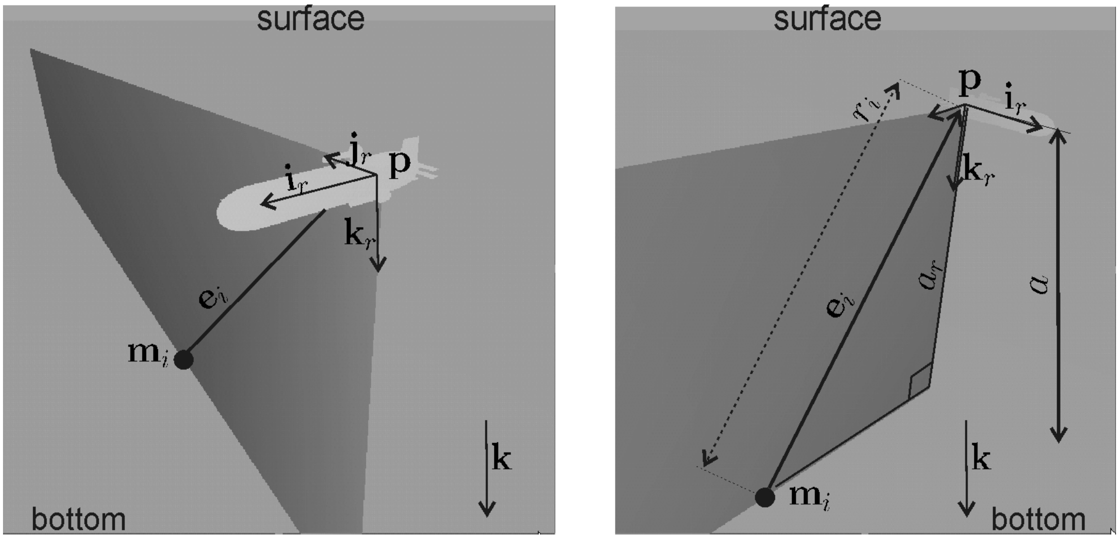
$$\begin{array}{ll} E \in [24; 26]\,, & R_1 \in [1.846; 2.307]\,, \\ R_2 \in [2.584; 3.355], & I \in [4.769; 5.417]\,, \\ U_1 \in [10; 11]\,, & U_2 \in [14; 16]\,, \\ P \in [124; 130]\,. & \end{array}$$

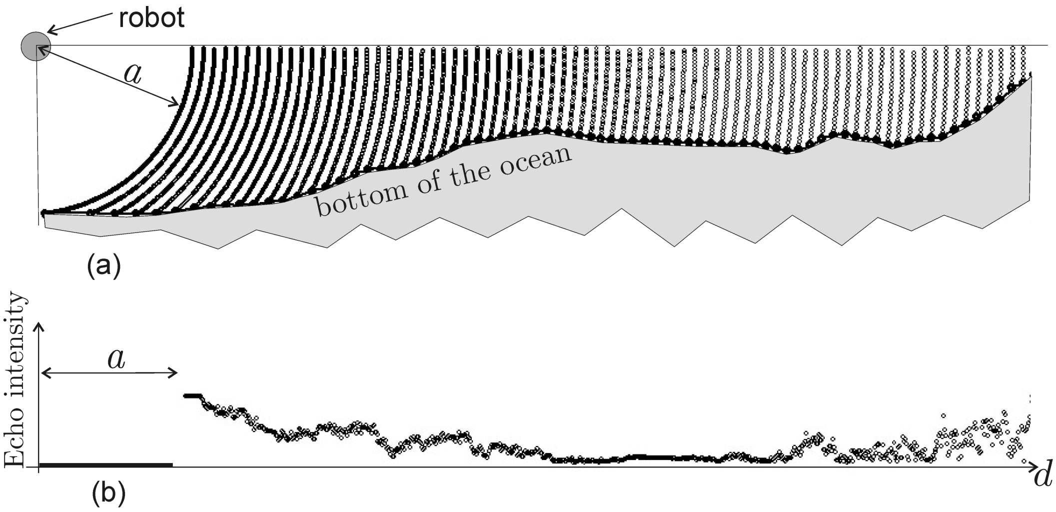
5 SLAM



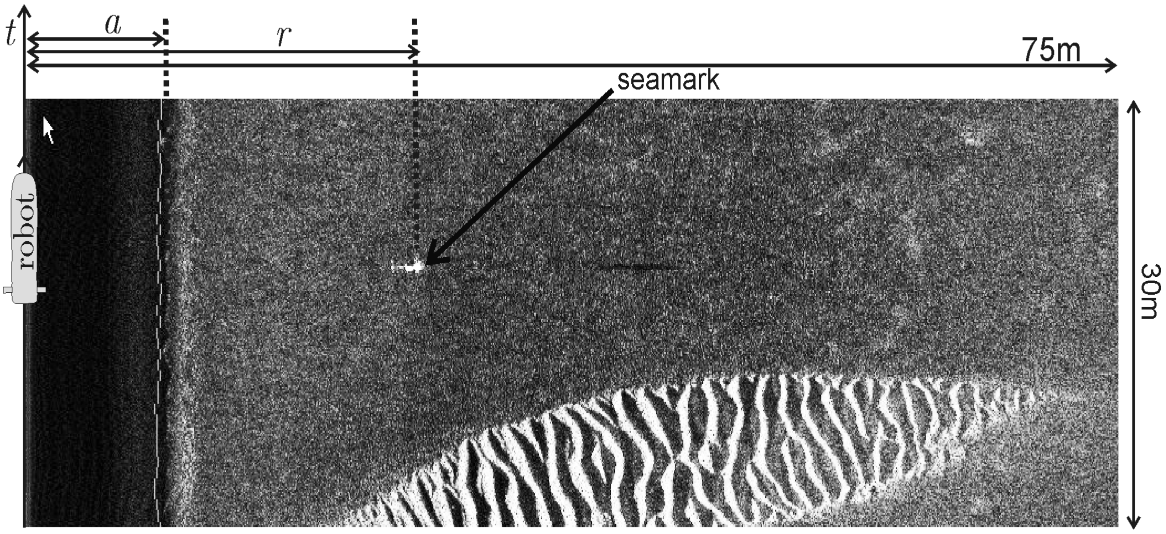





surface
$\mathbf{j}_r$
$\mathbf{p}$
$\mathbf{i}_r$
$\mathbf{k}_r$
$\mathbf{e}_i$
$\mathbf{m}_i$
$\mathbf{k}$
bottom
surface
$\mathbf{p}$
r_i
$\mathbf{i}_r$
$\mathbf{k}_r$
a_r
$\mathbf{e}_i$
a
$\mathbf{k}$
$\mathbf{m}_i$
bottom



robot
a
bottom of the ocean
(a)
Echo intensity
a
d
(b)



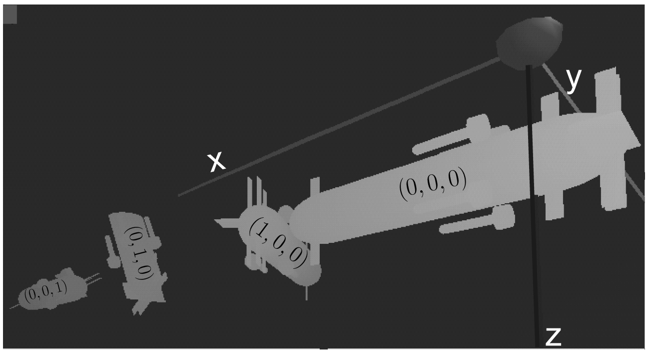
Mine detection with SonarPro

Loch-Doppler returns the speed robot $\mathbf{v}_r$.

$$\mathbf{v}_r \in \tilde{\mathbf{v}}_r + 0.004 * [-1,1] \,.\tilde{\mathbf{v}}_r + 0.004 * [-1,1]$$

Inertial central (Octans III from IXSEA).

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4}.\,[-1,1] \\ 1.75 \times 10^{-4}.\,[-1,1] \\ 5.27 \times 10^{-3}.\,[-1,1] \end{pmatrix}.$$



Six mines have been detected.

i	0	1	2	3	4	5	6	7	8	9	10	11
$\tau(i)$	7054	7092	7374	7748	9038	9688	10024	10817	11172	11232	11279	11688
$\sigma(i)$	1	2	1	0	1	5	4	3	3	4	5	1
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98	37.90	36.71	37.37	31.03	33.51	15.05

5.1 Constraints

$$t \in \{6000.0, 6000.1, 6000.2, \ldots, 11999.4\},$$

$$i \in \{0, 1, \ldots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \cdot \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) \cdot \frac{\pi}{180}\right) & 0 \end{pmatrix} \cdot \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos\psi(t) & -\sin\psi(t) & 0 \\ \sin\psi(t) & \cos\psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos\theta(t) & 0 & \sin\theta(t) \\ 0 & 1 & 0 \\ -\sin\theta(t) & 0 & \cos\theta(t) \end{pmatrix},$$

$$\mathbf{R}_{\varphi}(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi(t) & -\sin\varphi(t) \\ 0 & \sin\varphi(t) & \cos\varphi(t) \end{pmatrix},$$

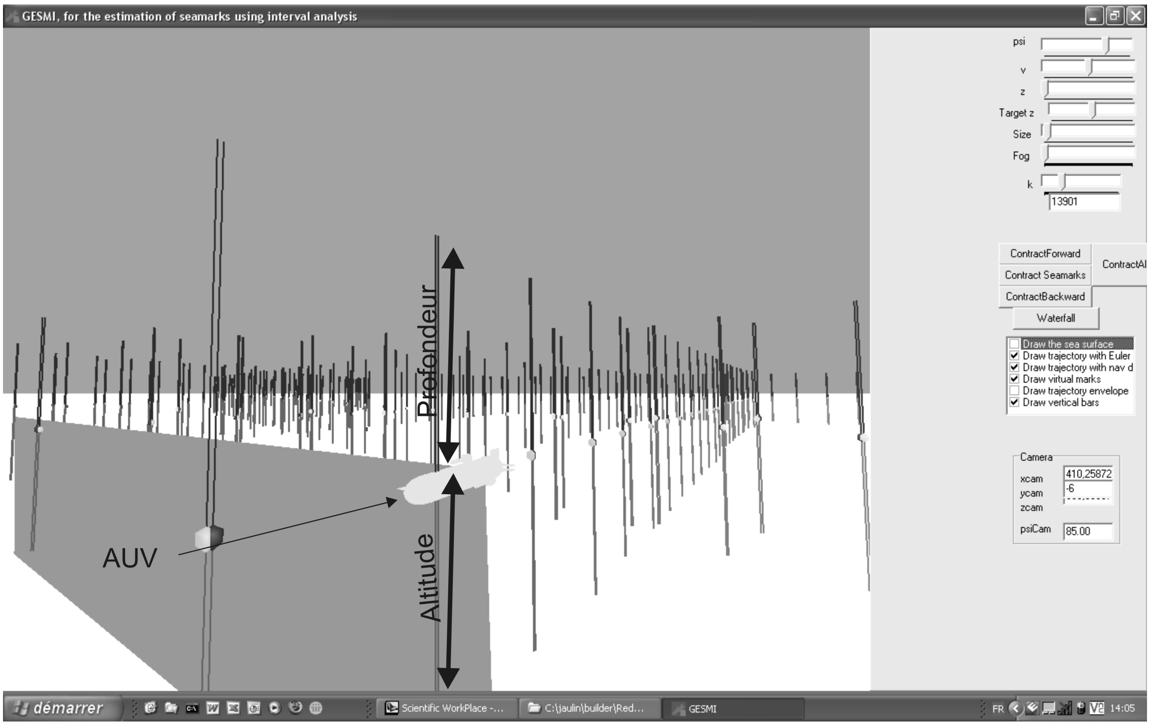
$$\mathbf{R}(t) = \mathbf{R}_{\psi}(t) \cdot \mathbf{R}_{\theta}(t) \cdot \mathbf{R}_{\varphi}(t),$$

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t) \cdot \mathbf{v}_r(t),$$

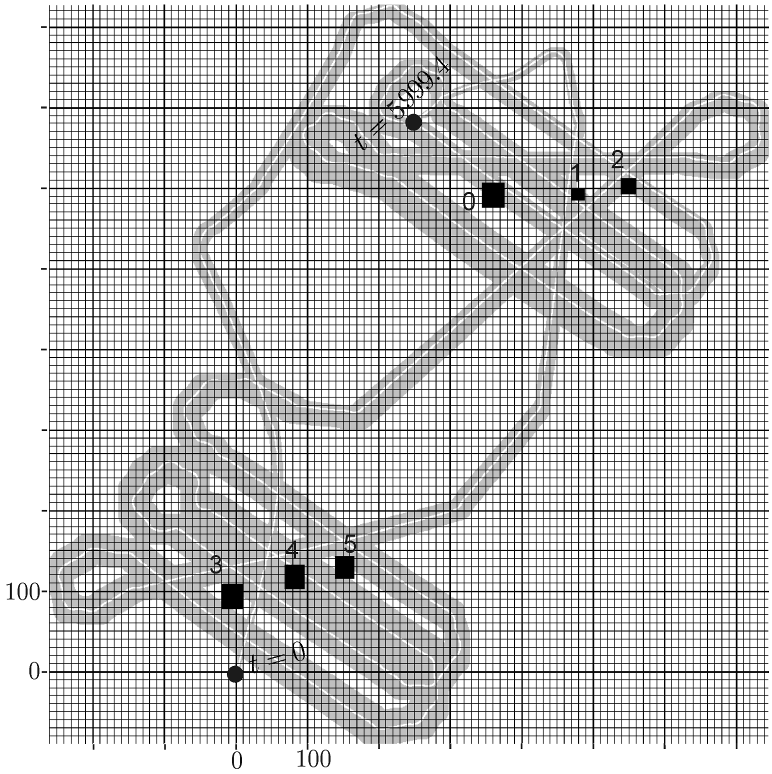
$$||\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))|| \ = r(i),$$

$$\mathbf{R}^{\mathsf{T}}(\tau(i)) \cdot (\mathbf{m}(\sigma(i)) - \mathbf{p}(\tau(i))) \in [0] \times [0, \infty]^{\times 2}.$$

5.2 GESMI



GESMI, for the estimation of seamarks using interval analysis
psi
v
z
Target z
Size
Fog
k
13901
ContractForward
Contract Seamarks
ContractBackward
Waterfall
Draw the sea surface
Draw trajectory with Euler
Draw virtual marks
Draw trajectory envelope
Draw vertical bars
Camera
xcam
410,25872
ycam
-6
zcam
psiCam
85.00
Profondeur
Altitude
AUV
démarrer
GESMI
FR
14:05



t = 5990.4
0
1
2
3
4
5
t = 0
100
0
0
100

References.

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