Caractérisation de la zone explorée par un robot sous-marin

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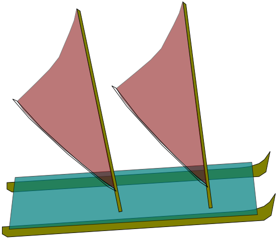
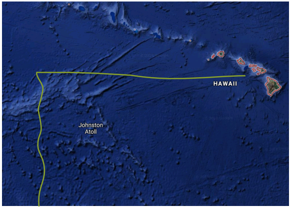
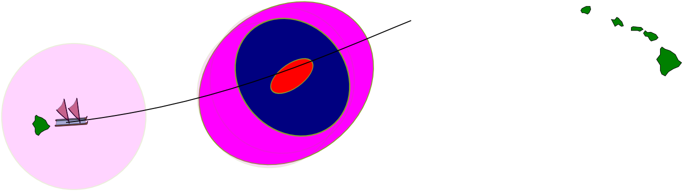
Polynesian navigation

- Given a set of point islands $\mathbf{m}_1, \mathbf{m}_2, \dots$
- Boats have a visibility area.
- Boats have to move in this environment without being lost, find new islands, etc.

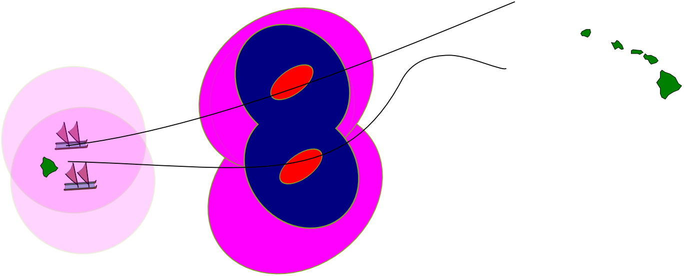

Arctic Ocean
Russia
Canada
Sea of Okhotsk
Bering Sea
Mongolia
Sea of Japan
China
South Korea
Japan
East China Sea
North Pacific Ocean
Myanmar (Burma)
Philippine Sea
Thailand
Vietnam
Malaysia
Indonesia
Banda Sea
Papua New Guinea
Arafura Sea
Coral Sea
Australia
Great Australian Bight
Tasman Sea
New Zealand
South Pacific Ocean
Google



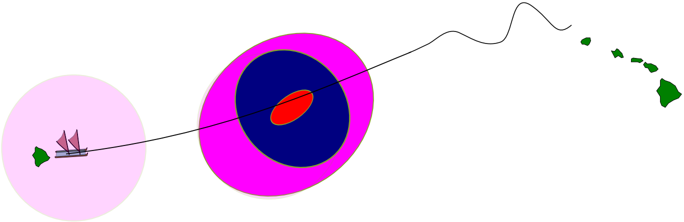

HAWAII
Johnston Atoll



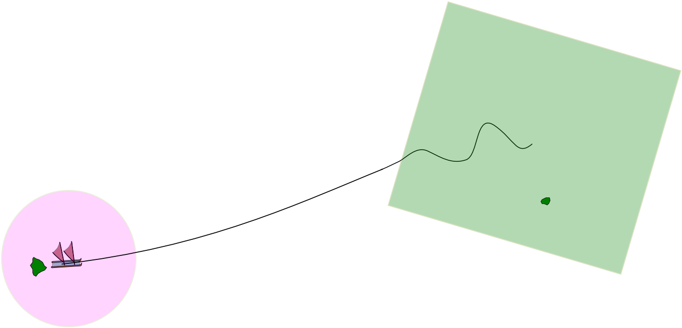
Prove that geo-localized islands will be reached by one boat



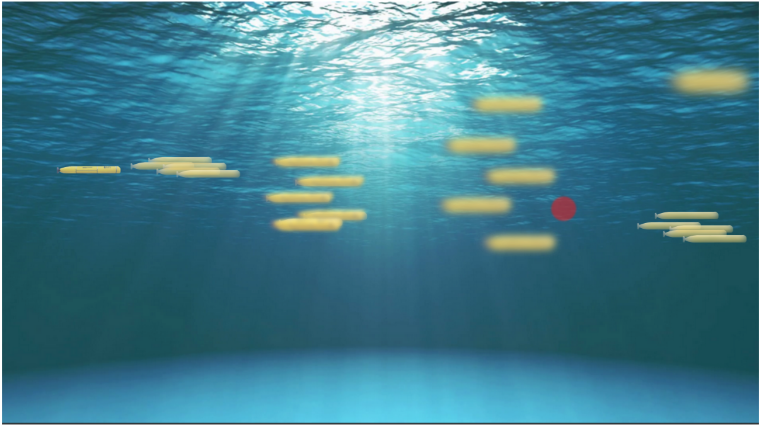
Prove that geo-localized islands will be reached by the n boats



Find a control to reach the geo-localized islands



Explore a given area entirely to find new islands



Visible area

The robot has a state $\mathbf{x}$. The visible area is $\mathbb{V}(\mathbf{x})$

Example. The robot is able to see all up to 3 meters

$$\mathbb{V}(\mathbf{x}) = \left\{ (z_1, z_2) \,|\, (z_1 - x_1)^2 + (z_2 - x_2)^2 \leq 9 \right\}.$$

Blind observer

Given

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}), & \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbf{y} = \mathbf{g}(\mathbf{x}), & \mathbf{y}(t) \in [\mathbf{y}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$

An observer is *blind* if $\dim \mathbf{y} = 0$.

Predict is a blind estimation problem

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & \\ \mathbf{y}(t) & = & \mathbf{g}(\mathbf{x}(t))+\mathbf{e}_y(t) & \mathbf{e}_y(t)\in[\mathbf{e}_y] \\ \mathbf{u}(t) & = & \mathbf{r}(\mathbf{y}(t))+\mathbf{e}_u(t) & \mathbf{e}_u(t)\in[\mathbf{e}_u] \end{cases}$$

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t), \mathbf{r}(\mathbf{g}(\mathbf{x}(t)) + \mathbf{e}_y(t)) + \mathbf{e}_u(t)\right) \\ \mathbf{e}_y(t) \in [\mathbf{e}_y], \mathbf{e}_u(t) \in [\mathbf{e}_u] \end{cases}$$

$$\begin{array}{rcl} \dot{\mathbf{x}}(t) & = & \mathbf{h}(\mathbf{x}(t),\mathbf{v}(t)) \\ & & \mathbf{v}(t)\in[\mathbf{v}] \end{array}$$

with

$$\begin{array}{l} \mathbf{v}=(\mathbf{e}_y,\mathbf{e}_u) \\ \mathbf{h}(\mathbf{x},\mathbf{v})=\mathbf{f}\left(\mathbf{x},\mathbf{r}(\mathbf{g}(\mathbf{x})+\mathbf{e}_y)+\mathbf{e}_u\right) \\ [\mathbf{v}]=[\mathbf{e}_y]\times[\mathbf{e}_u] \end{array}$$

Blind exploration

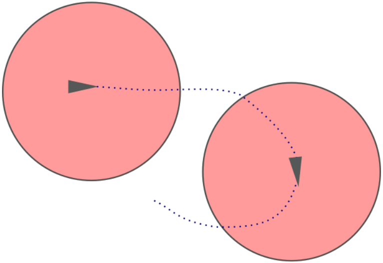
The blind explored zone $\mathbb{Z}$ is defined by [2]

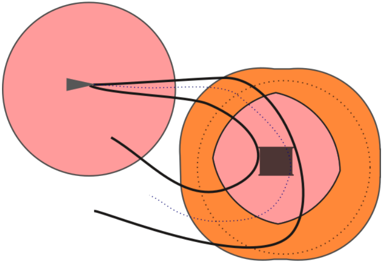
$$\left\{ \begin{array}{c} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}),\ \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbb{Z} = \bigcup_{t\geq 0}\ \mathbb{V}(\mathbf{x}(t)) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{array} \right.$$

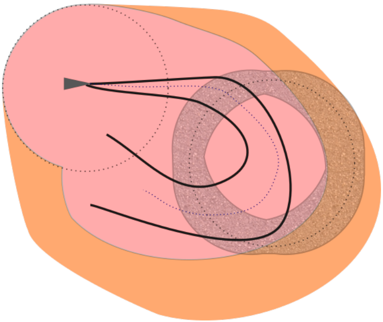
We have

$$\underbrace{\bigcap_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^-} \subset \mathbb{Z} \subset \underbrace{\bigcup_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^+}.$$

$\mathbb{Z}^-$ is the *certainly explored zone.*
$\mathbb{Z}^+$ is the *maybe explored zone*.
$\mathbb{Z}^+\backslash\mathbb{Z}^-$ is the *penumbra.*



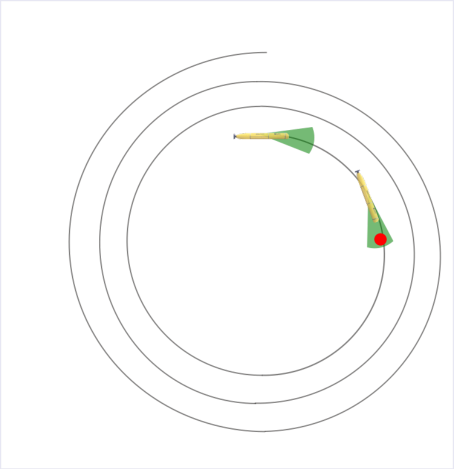


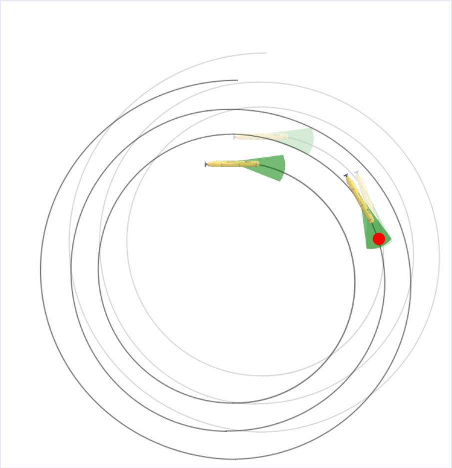


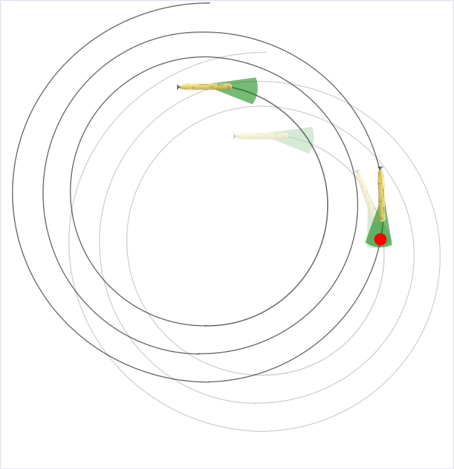
Spiral scan

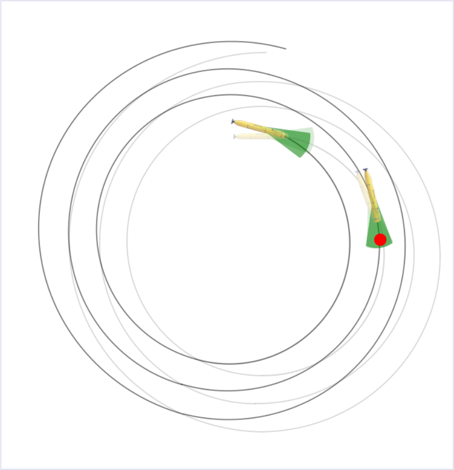
We have [4]

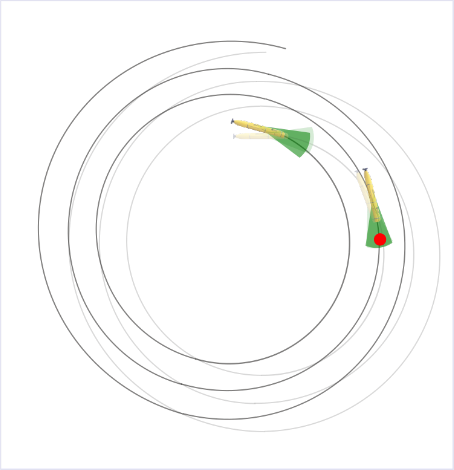
$$\underbrace{\bigcup_{t\geq 0}\ \bigcap_{\mathbf{x}\in\mathscr{X}(t)} \mathbb{V}(\mathbf{x})}_{\{\mathbf{z}\ \mid\ \exists t\ \forall \mathbf{x}\in\mathscr{X}(t),\ \mathbf{z}\in\mathbb{V}(\mathbf{x})\}} \subset \mathbb{Z}^{-} = \underbrace{\bigcap_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)}\ \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\{\mathbf{z}\ \mid\ \forall \mathbf{x}(\cdot)\in\mathscr{X}(\cdot), \exists t,\ \mathbf{z}\in\mathbb{V}(\mathbf{x})\}}$$

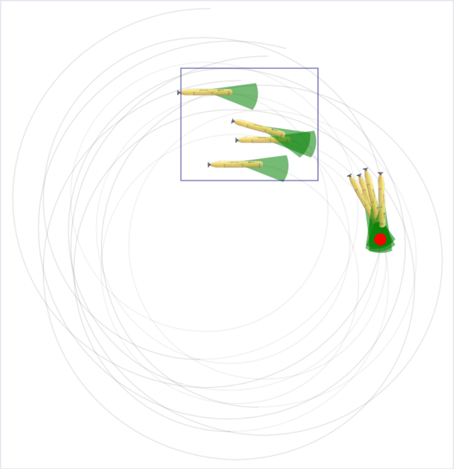






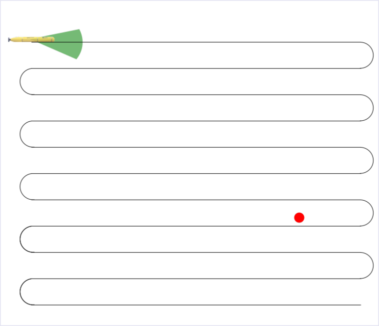


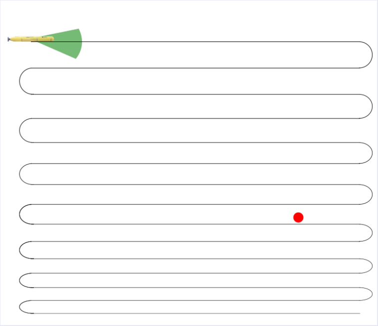




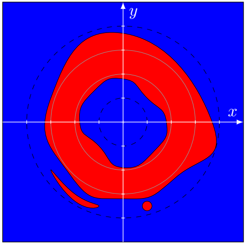
Boustrophedon

Which pattern is the best for exploration?





Thick sets



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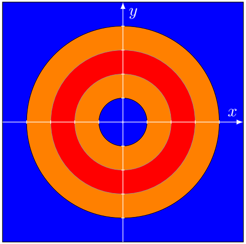


Interval analysis

Problem. Given $f : \mathbb{R}^n \to \mathbb{R}$ and a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Interval arithmetic can solve efficiently this problem.

Example. Is the function

$$f(\mathbf{x}) = x_1 x_2 - (x_1 + x_2)\cos x_2 + \sin x_1 \cdot \sin x_2 + 2$$

always positive for $x_1, x_2 \in [-1, 1]$?

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= ?, \\ [-1,3]\cdot[2,5] &= ?, \\ \text{abs}([-7,1]) &= ? \end{aligned}$$

Interval arithmetic

$$\begin{aligned} [-1,3]+[2,5] &= [1,8], \\ [-1,3]\cdot[2,5] &= [-5,15], \\ \mathrm{abs}([-7,1]) &= [0,7] \end{aligned}$$

The interval extension of

$$f(x_1, x_2) = x_1 \cdot x_2 - (x_1 + x_2) \cdot \cos x_2 + \sin x_1 \cdot \sin x_2 + 2$$

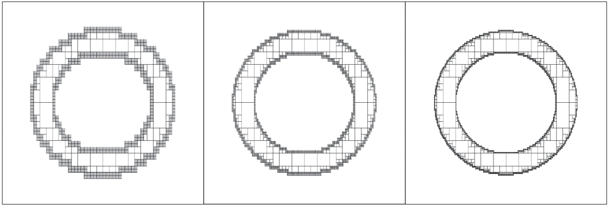
is

$$\begin{aligned} [f]([x_1], [x_2]) &= [x_1] \cdot [x_2] - ([x_1] + [x_2]) \cdot \cos [x_2] \\ &\quad + \sin [x_1] \cdot \sin [x_2] + 2. \end{aligned}$$

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^+ \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Set Inversion

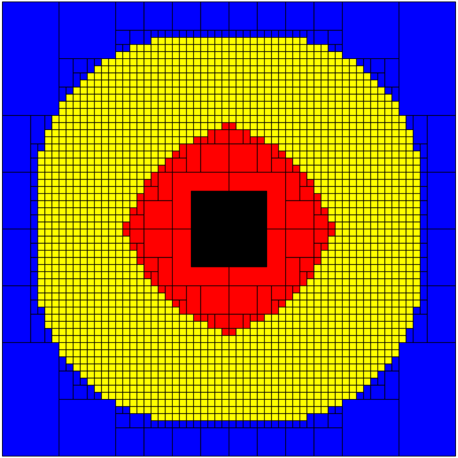


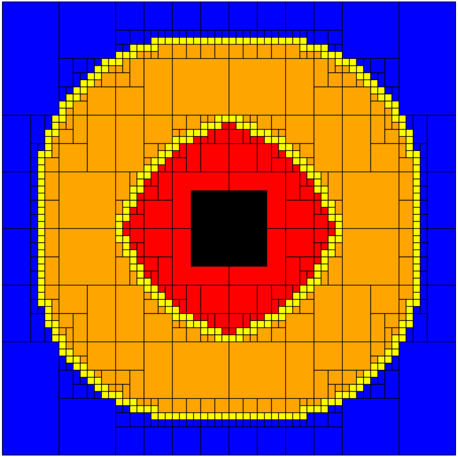
$$\mathbb{X} = \{(x_1, x_2) \mid x_1^2 + x_2^2 \in [1, 2]\}$$

Thick sets

The boat at $\mathbf{a} \in [\mathbf{a}]$ sees all at distance in $[y^{\subset}] = [0, 10]$ and nothing outside $[y^{\supset}] = [0, 20]$.
What is the visible zone $\mathbb{Z}$?
We apply interval arithmetic to characterize $\mathbb{Z}$.

$$\begin{array}{lccc} (i) & ([z_1]-[a_1])^2+([z_2]-[a_2])^2 \subset [0,10] & \Rightarrow & [\mathbf{z}] \subset \mathbb{Z} \\ (ii) & ([z_1]-[a_1])^2+([z_2]-[a_2])^2 \cap [0,20] = \emptyset & \Rightarrow & [\mathbf{z}] \cap \mathbb{Z} = \emptyset. \end{array}$$





Thick set inversion problem [3] can be written as

$$\mathbb{X} = \mathbf{f}^{-1}(\mathbb{Y}), \mathbf{f} \in [\mathbf{f}] \text{ and } \mathbb{Y} \in [\![\mathbb{Y}]\!].$$

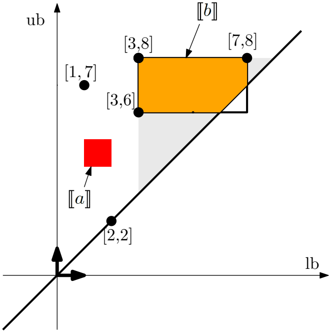
The set $\mathbb{X}$ is a *solution* if

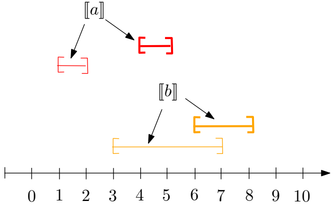
$$\exists \mathbf{f} \in [\mathbf{f}], \exists \mathbb{Y} \in [\![\mathbb{Y}]\!], \mathbb{X} = \mathbf{f}^{-1}(\mathbb{Y}).$$

Thick intervals

A *thick interval* $[\![x]\!]$ is a set of intervals [1]

$$\begin{aligned} [\![x]\!] &= [\![[x^-],[x^+]]\!] \\ &= \left\{[x^-,x^+]\in \mathbb{IR} \mid x^- \in [x^-] \text{ and } x^+ \in [x^+]\right\}. \end{aligned}$$





In the endpoints diagram, an interval is a point

Operations

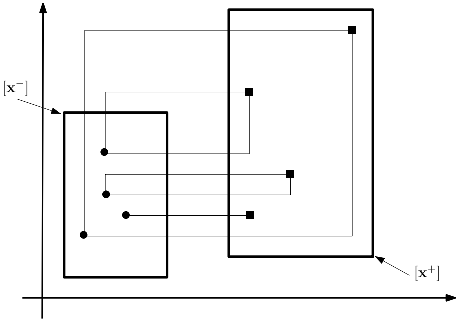
Multiplication [5]. Since

$$[a,b]\cdot[c,d]=[\min(ac,ad,bc,bd),\max(ac,ad,bc,bd)]$$

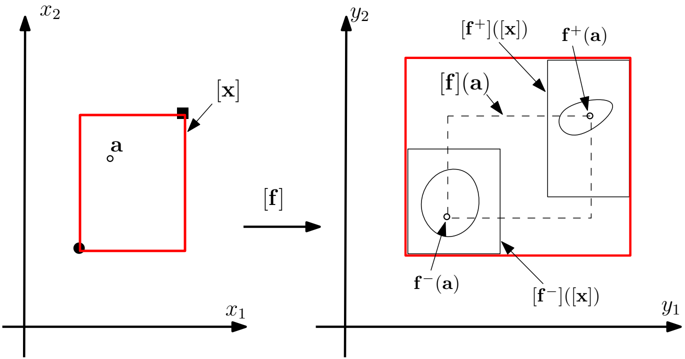
we have

$$\begin{aligned} & [\![[a],[b]]\!]\cdot[\![[c],[d]]\!] \\ = \; & [\![\min([a]\cdot[c],[a]\cdot[d],[b]\cdot[c],[b]\cdot[d]), \\ & \max([a]\cdot[c],[a]\cdot[d],[b]\cdot[c],[b]\cdot[d])]\!]\end{aligned}$$

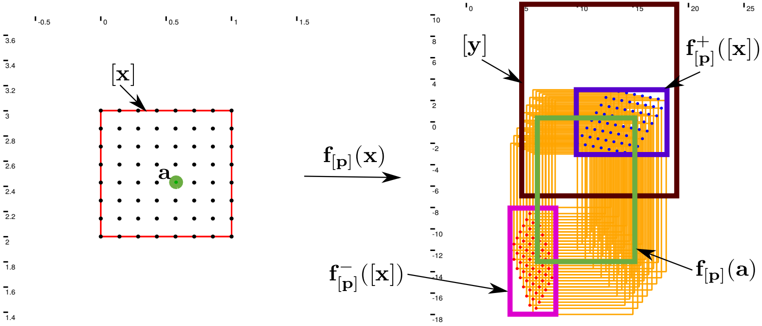
Thick boxes



The four boxes belong to the thick box $[\![\mathbf{x}]\!] = [\![[\mathbf{x}^-],[\mathbf{x}^+]]\!]$

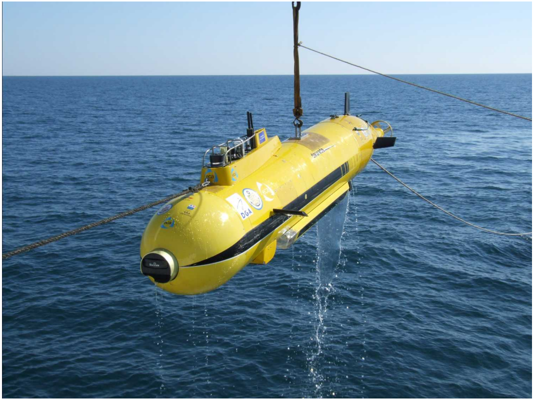


$$[\mathsf{f}](\mathsf{a}) \in [\![[\mathsf{f}^-]([\mathsf{x}]), [\mathsf{f}^+]([\mathsf{x}])]\!]$$

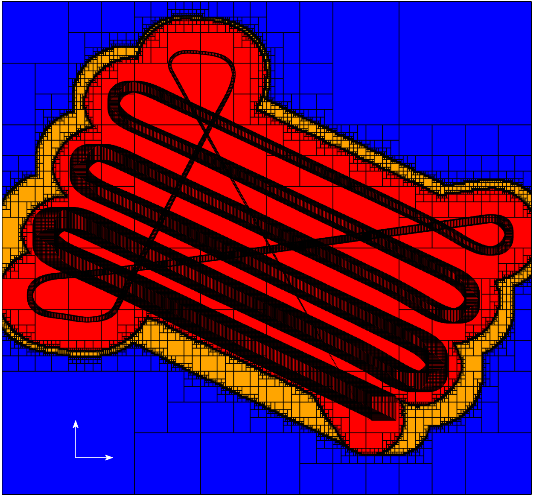


We conclude that [x] is inside the penumbra

Applications



Daurade DGA-TN



During its boustrophedon Daurade explored $\mathbb{Z} \in [\![\mathbb{Z}]\!]$

A la recherche de la Cordelière

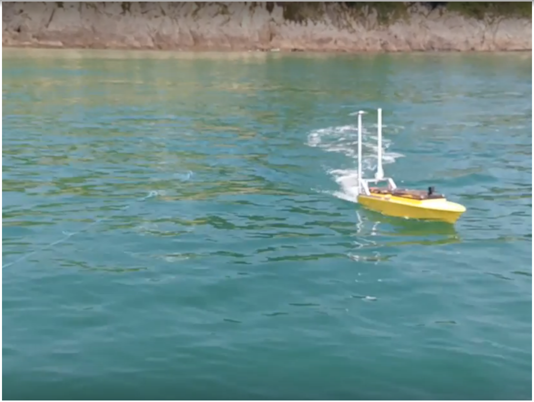
Reconstitution de la bataille

youtu.be/yP4cM1UGrqY

Expériences à Guerlédan

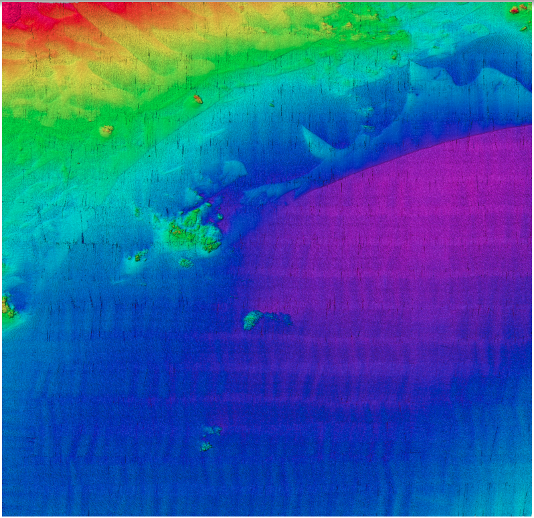
youtu.be/KXHyYeWYGOI

Submeeting 2018



youtu.be/VqXG9zO_q1A

submeeting2018.html



Boatbot

Objectif: remorquer le SeaSpy du Shom avec un zodiac autonome.

Boatbot tracte un magnétomètre

youtu.be/cxVs1fDdm1s

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