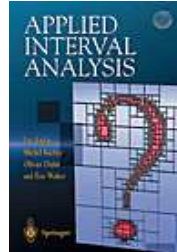


Calcul par intervalles pour la résolution garantie de problèmes non-linéaires



Luc Jaulin, ENSIETA
Séminaire au Lab-STICC, Brest
Mardi 26 mai 2008, 10h-10h45.

1 Motivations

Modèle : $y_m(\mathbf{p}, t) = p_1 e^{-p_2 t}$.

Paramètres : p_1, p_2 .

Temps de mesure : $t_1, t_2, \dots, t_m$

Barres de mesure : $[y_1^-, y_1^+], [y_2^-, y_2^+], \dots, [y_m^-, y_m^+]$

Ensemble de vraisemblance :

$$\begin{aligned} \mathbb{S} &= \left\{ \mathbf{p} \in \mathbb{R}^2, \forall i \in \{1, \dots, m\}, y_m(\mathbf{p}, t_i) \in [y_i^-, y_i^+] \right\} . \\ &= \mathbf{y}_m^{-1}([\mathbf{y}]). \end{aligned}$$

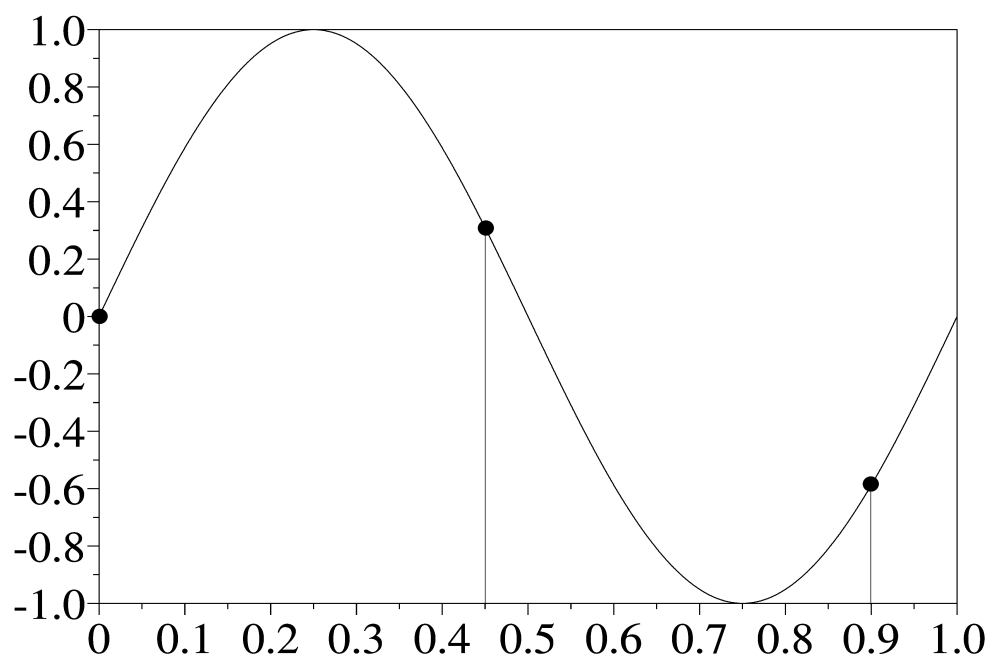
Logiciel SetDemo

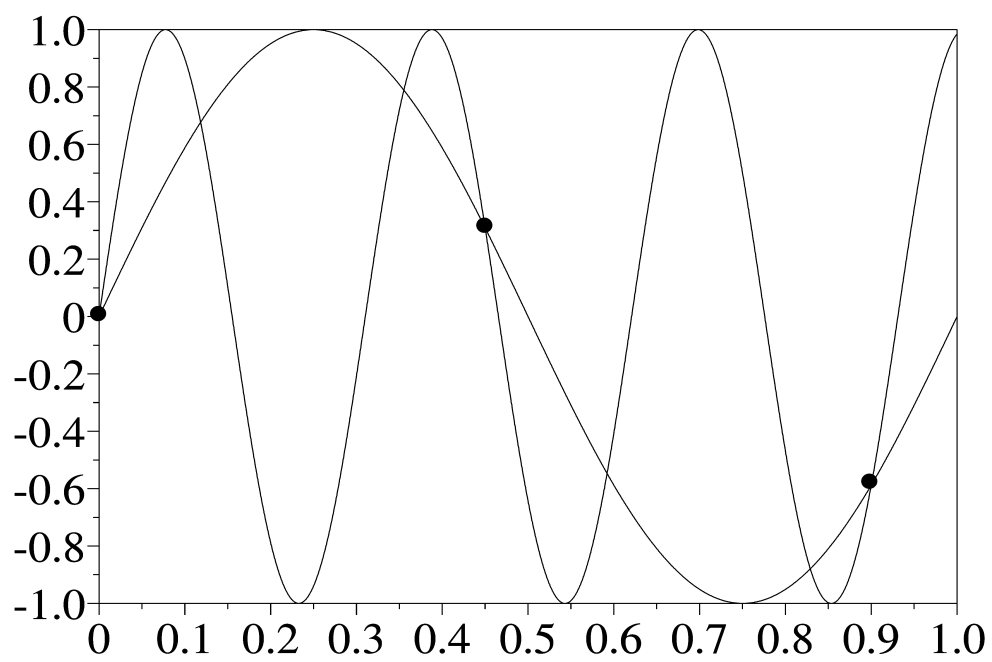
Modèle : $y_m(\mathbf{p}, t) = p_1 \sin(2\pi p_2 t)$.

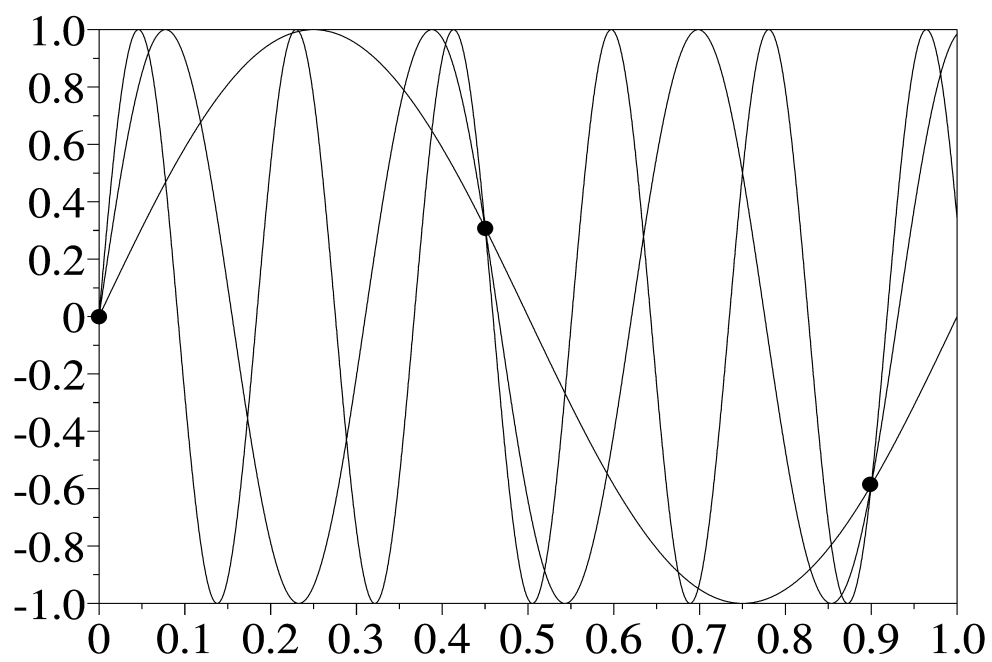
Paramètres : p_1, p_2 .

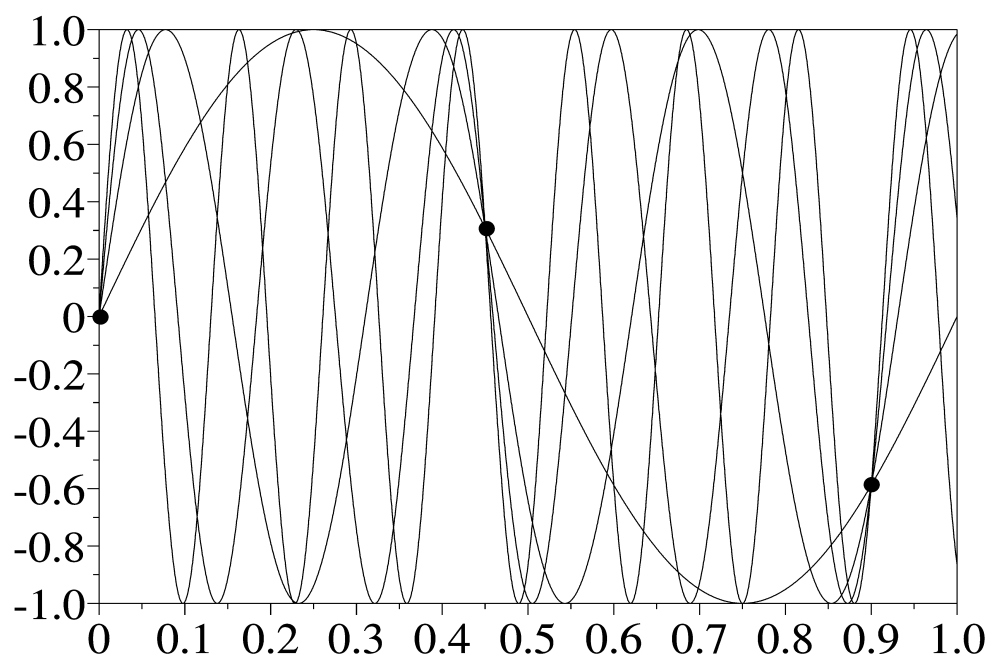
Temps de mesure : $t_1 = 0, t_2 = T_e, t_3 = 2T_e, \dots$

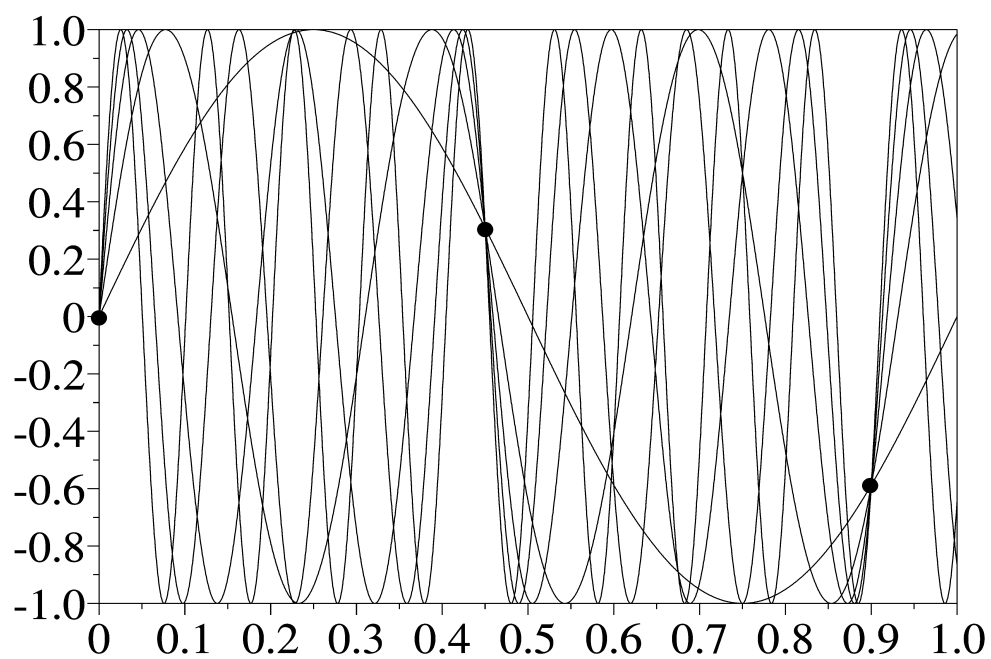
Barres de mesure : $[y_1^-, y_1^+], [y_2^-, y_2^+], [y_3^-, y_3^+], \dots$











(Montrer l'ensemble de vraisemblance avec SetDemo).

2 Méthode

2.1 Calcul par intervalles

Si $\diamond \in \{+, -, \cdot, /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [\cdot, \cdot], \\ [-1, 3] \cdot [2, 5] &= [\cdot, \cdot], \\ [-1, 3] / [2, 5] &= [\cdot, \cdot], \\ [-1, 3] \vee [2, 5] &= [\cdot, \cdot]. \end{aligned}$$

Si $\diamond \in \{+, -, \cdot, /, \max, \min\}$

$$[x] \diamond [y] = [\{x \diamond y \mid x \in [x], y \in [y]\}].$$

Par exemple,

$$\begin{aligned} [-1, 3] + [2, 5] &= [1, 8], \\ [-1, 3] \cdot [2, 5] &= [-5, 15], \\ [-1, 3] / [2, 5] &= [-\frac{1}{2}, \frac{3}{2}], \\ [-1, 3] \vee [2, 5] &= [2, 5]. \end{aligned}$$

$$\begin{array}{rcl}
[x^-, x^+] + [y^-, y^+] & = & [?, ?], \\
[x^-, x^+].[y^-, y^+] & = & [?, ?], \\
[-1, 3] \vee [2, 5] & = & [?, ?].
\end{array}$$

$$\begin{aligned}
[x^-, x^+] + [y^-, y^+] &= [x^- + y^-, x^+ + y^+], \\
[x^-, x^+].[y^-, y^+] &= [x^-y^- \wedge x^+y^- \wedge x^-y^+ \wedge x^+y^+, \\
&\quad x^-y^- \vee x^+y^- \vee x^-y^+ \vee x^+y^+], \\
[x^-, x^+] \vee [y^-, y^+] &= [\vee(x^-, y^-), \vee(x^+, y^+)].
\end{aligned}$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\sin([0, \pi]) = [?, ?],$$

$$\text{sqr}([-1, 3]) = [?, ?],$$

$$\text{abs}([-7, 1]) = [?, ?],$$

$$\text{sqrt}([-10, 4]) = [?, ?],$$

$$\log([-2, -1]) = [?, ?].$$

Si $f \in \{\cos, \sin, \text{sqr}, \text{sqrt}, \log, \exp, \dots\}$

$$f([x]) = [\{f(x) \mid x \in [x]\}].$$

Par exemple,

$$\sin([0, \pi]) = [0, 1],$$

$$\text{sqr}([-1, 3]) = [-1, 3]^2 = [0, 9],$$

$$\text{abs}([-7, 1]) = [0, 7],$$

$$\text{sqrt}([-10, 4]) = \sqrt{[-10, 4]} = [0, 2],$$

$$\log([-2, -1]) = \emptyset.$$

2.2 Projection de contraintes

Soient x, y, z trois variables telles que

$$x \in [-\infty, 5],$$

$$y \in [-\infty, 4],$$

$$z \in [6, \infty],$$

$$z = x + y.$$

Les valeurs < 2 pour x , < 1 pour y et > 9 pour z sont inconsistantes.

2.3 Méthode numérique de projection

En effet, puisque $x \in [-\infty, 5]$, $y \in [-\infty, 4]$, $z \in [6, \infty]$ et $z = x + y$, nous avons

$$z = x + y \Rightarrow z \in [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ = [6, \infty] \cap [-\infty, 9] = [6, 9].$$

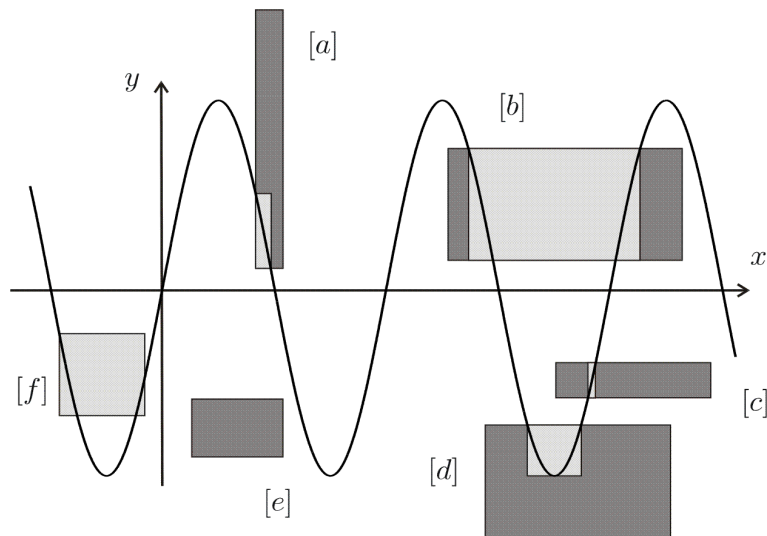
$$x = z - y \Rightarrow x \in [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ = [-\infty, 5] \cap [2, \infty] = [2, 5].$$

$$y = z - x \Rightarrow y \in [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ = [-\infty, 4] \cap [1, \infty] = [1, 4].$$

Pour la contrainte

$$y = \sin x, \quad x \in [x], y \in [y]$$

le problème est un peu plus difficile.



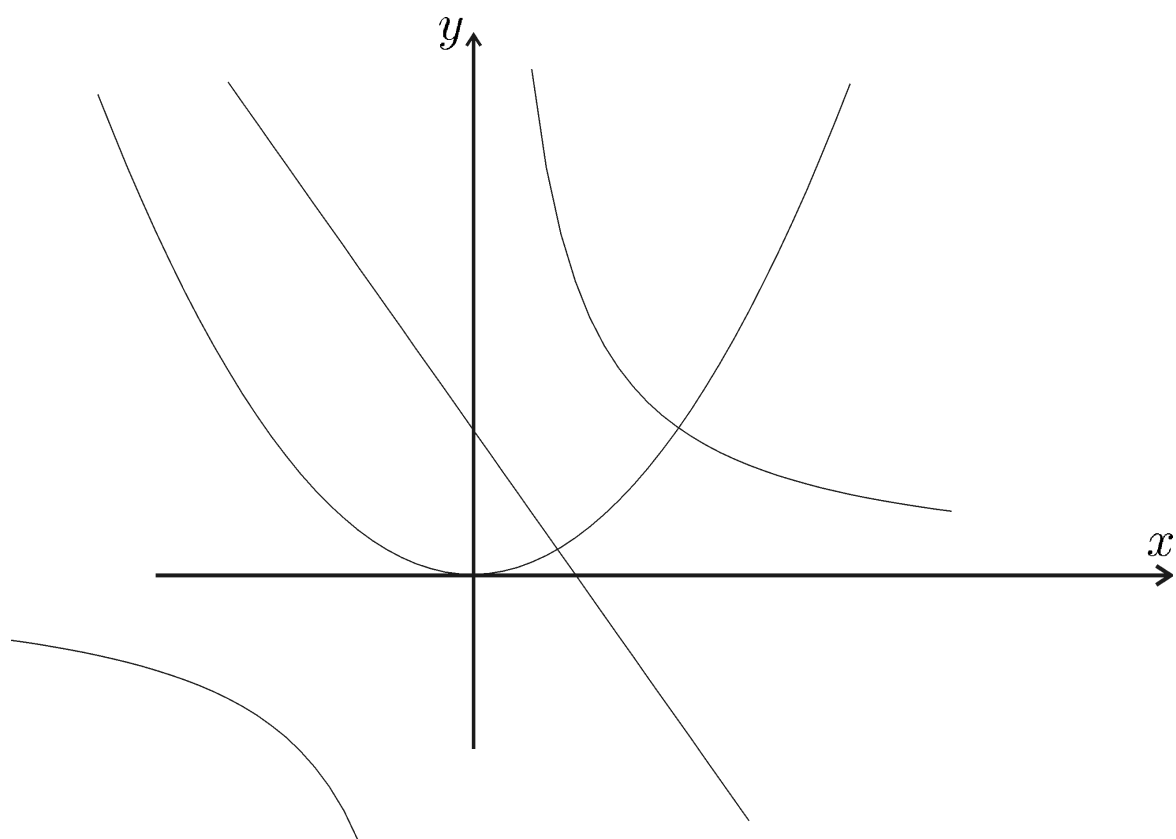
2.4 Propagation de contraintes

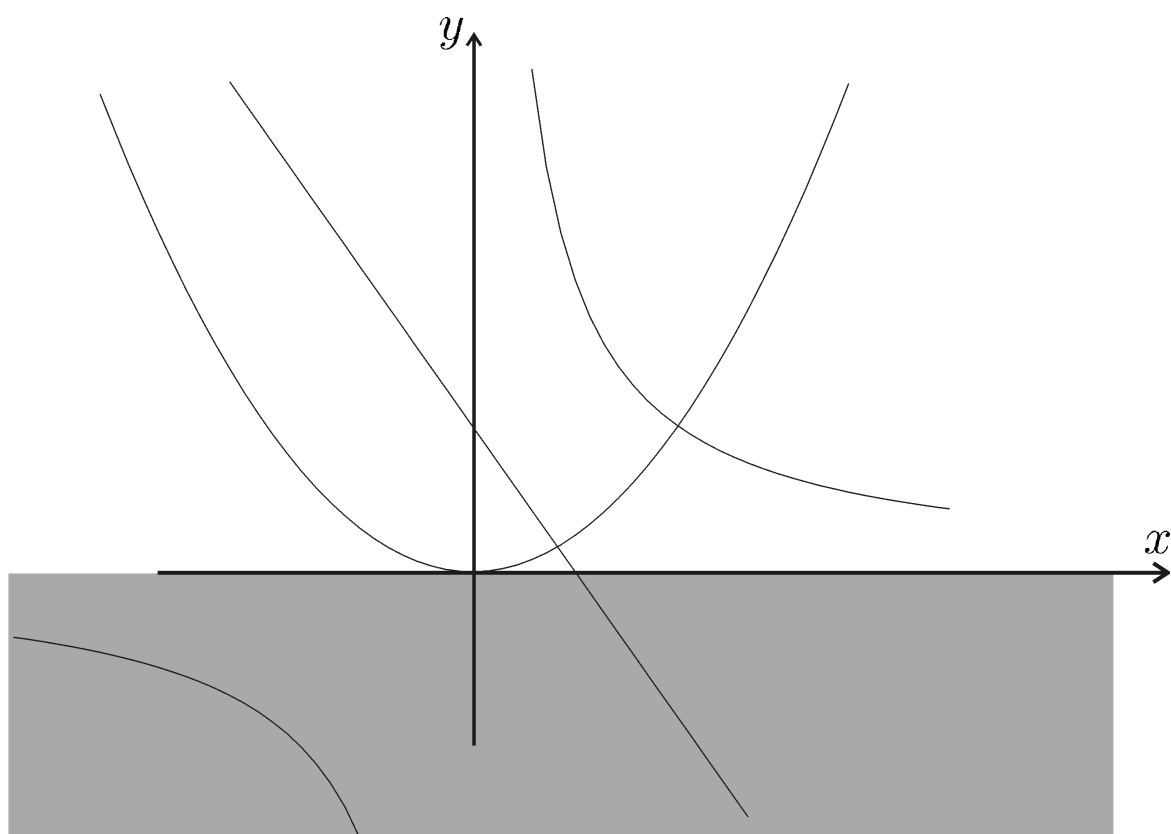
Considérons le problème à trois contraintes

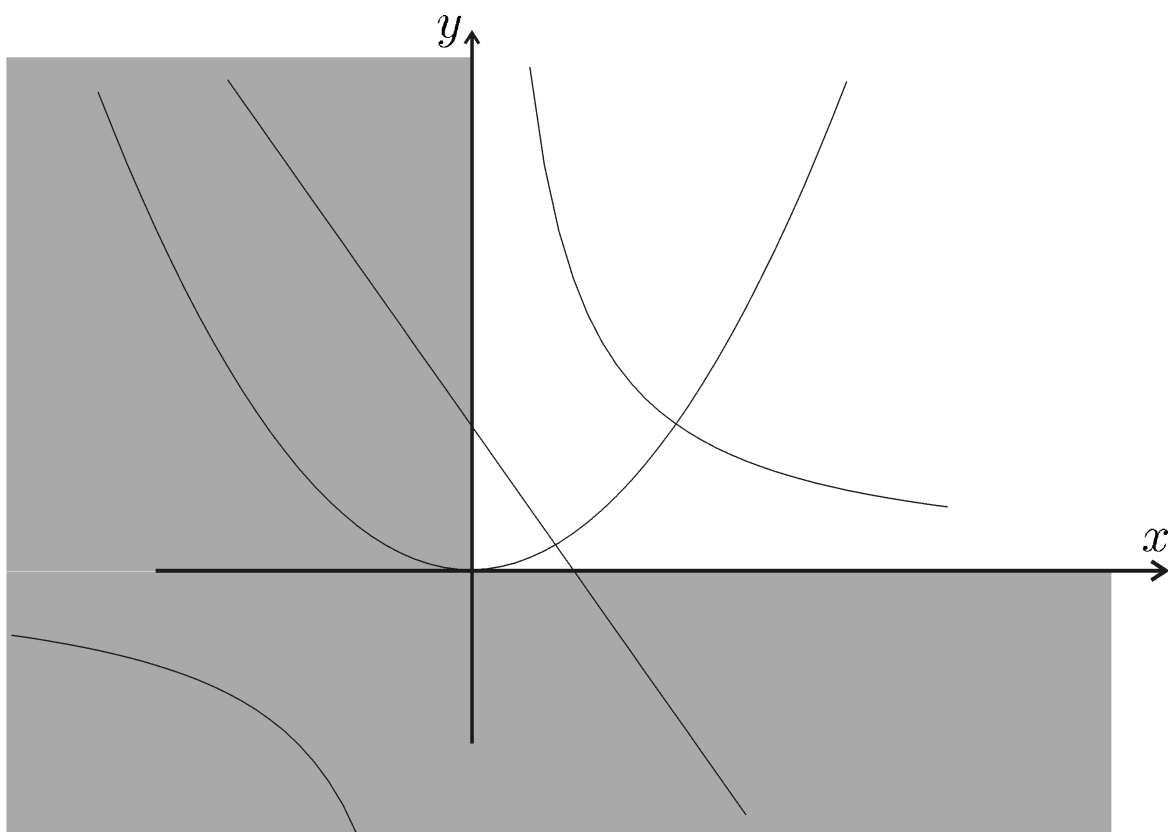
$$\begin{cases} (C_1) : & y = x^2 \\ (C_2) : & xy = 1 \\ (C_3) : & y = -2x + 1 \end{cases}$$

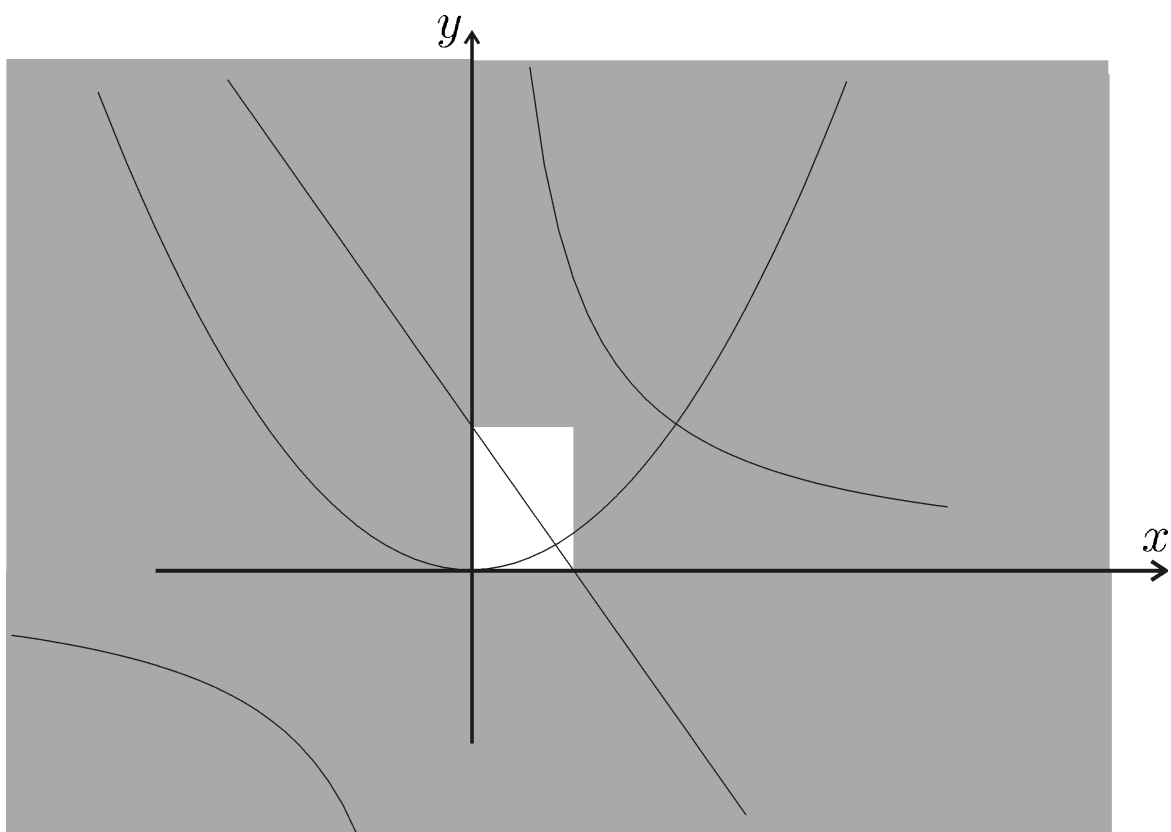
Pour chacune de ces variables, nous affectons le domaine $[-\infty, \infty]$.

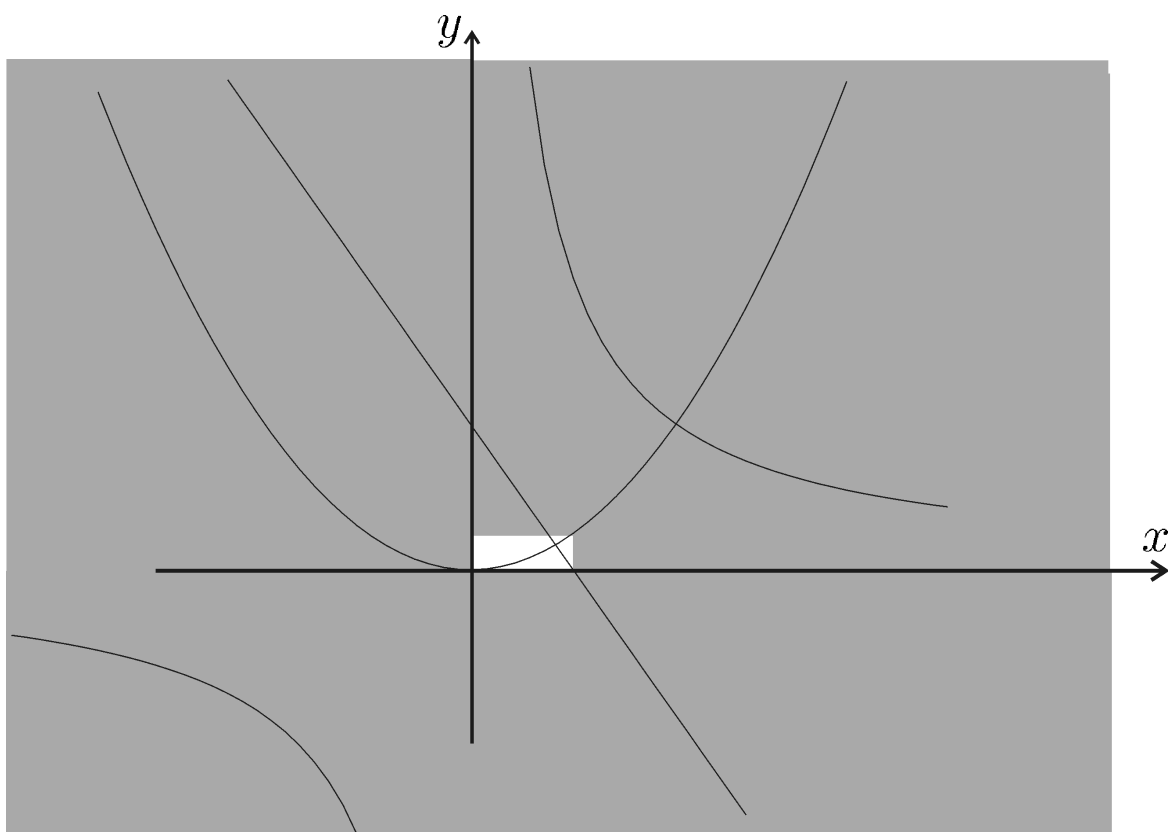
La *propagation* consiste à projeter ces contraintes jusqu'à l'équilibre.

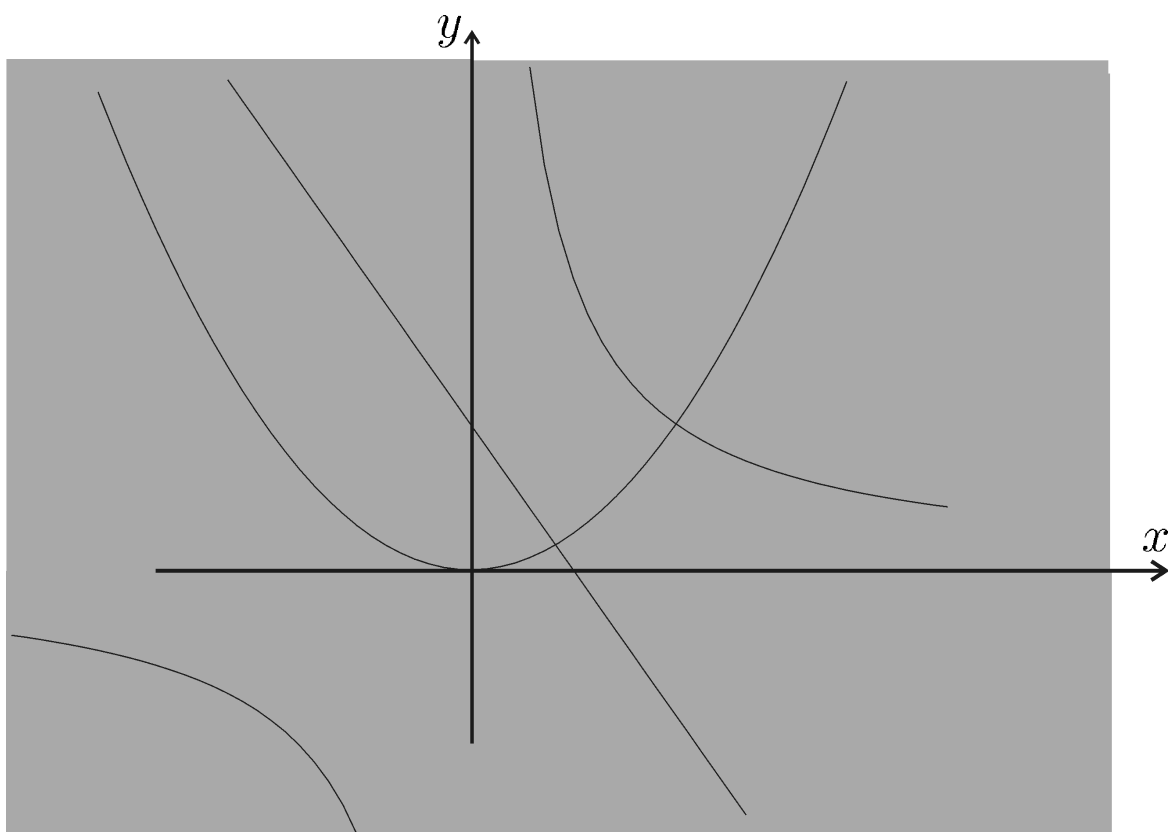


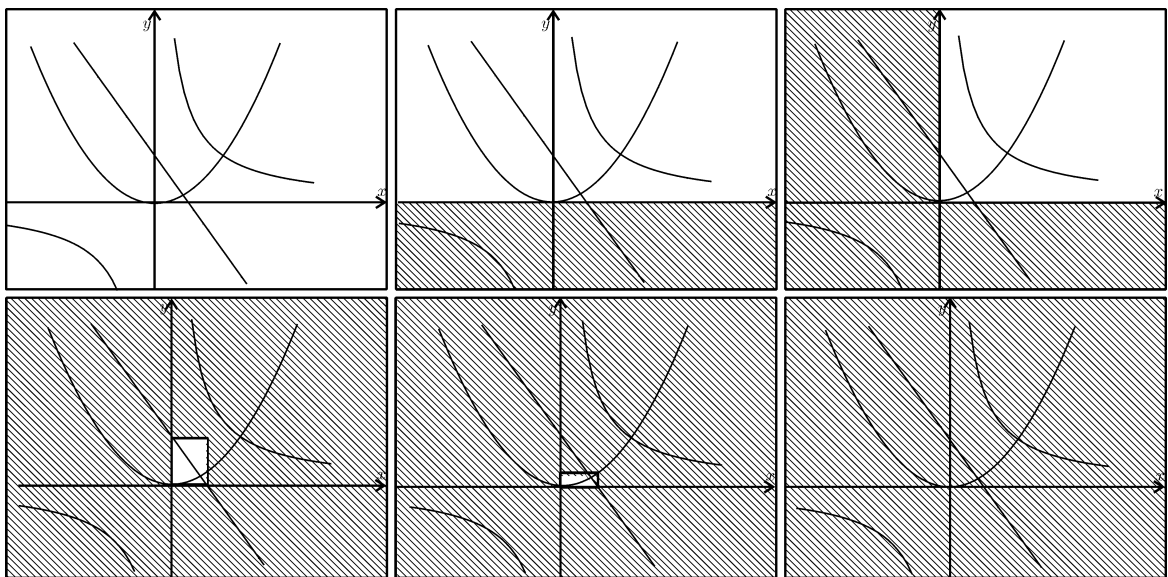












$$(C_1) \Rightarrow y \in [-\infty, \infty]^2 = [0, \infty]$$

$$(C_2) \Rightarrow x \in 1/[0, \infty] = [0, \infty]$$

$$(C_3) \Rightarrow y \in [0, \infty] \cap ((-2) \cdot [0, \infty] + 1) \\ = [0, \infty] \cap ([-\infty, 1]) = [0, 1]$$

$$x \in [0, \infty] \cap (-[0, 1]/2 + 1/2) = [0, \frac{1}{2}]$$

$$(C_1) \Rightarrow y \in [0, 1] \cap [0, 1/2]^2 = [0, 1/4]$$

$$(C_2) \Rightarrow x \in [0, 1/2] \cap 1/[0, 1/4] = \emptyset$$

$$y \in [0, 1/4] \cap 1/\emptyset = \emptyset$$

2.5 Décomposition

Pour les contraintes plus complexes, il nous faut effectuer une décomposition

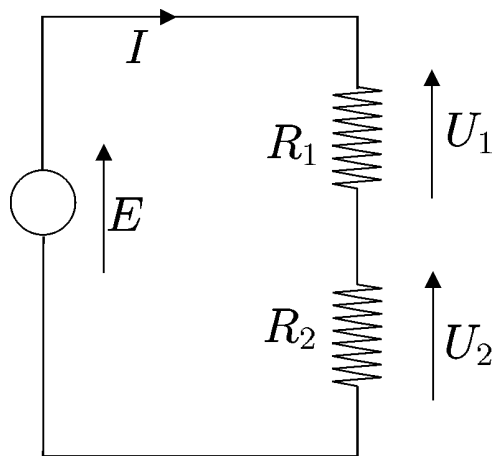
$$\begin{aligned} x + \sin(y) - xz &\leq 0, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{aligned}$$

se décompose en

$$\left\{ \begin{array}{ll} a = \sin(y) & x \in [-1, 1] \quad a \in]-\infty, \infty[\\ b = x + a & y \in [-1, 1] \quad b \in]-\infty, \infty[\\ c = xz & z \in [-1, 1] \quad c \in]-\infty, \infty[\\ b - c = d & d \in]-\infty, 0] \end{array} \right.,$$

3 Applications

3.1 Problème d'estimation



Contraintes

$$P = EI; E = (R_1 + R_2) I;$$

$$U_1 = R_1 I; U_2 = R_2 I; E = U_1 + U_2.$$

Domaines initiaux

$$\begin{aligned} R_1 &\in [0, \infty] \Omega, & R_2 &\in [0, \infty] \Omega, \\ E &\in [23, 26] \text{V}, & I &\in [4, 8] \text{A}, \\ U_1 &\in [10, 11] \text{V}, & U_2 &\in [14, 17] \text{V}, \\ P &\in [124, 130] \text{W}, \end{aligned}$$

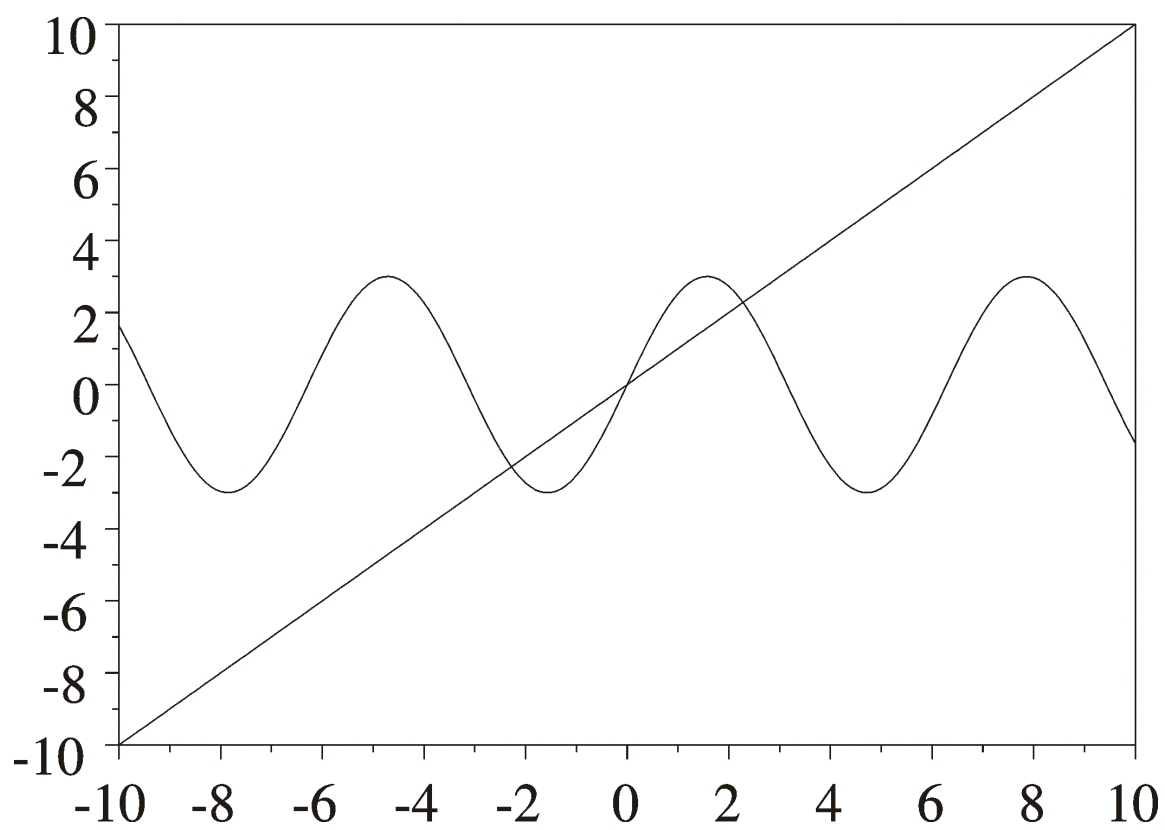
Les domaines contractés sont

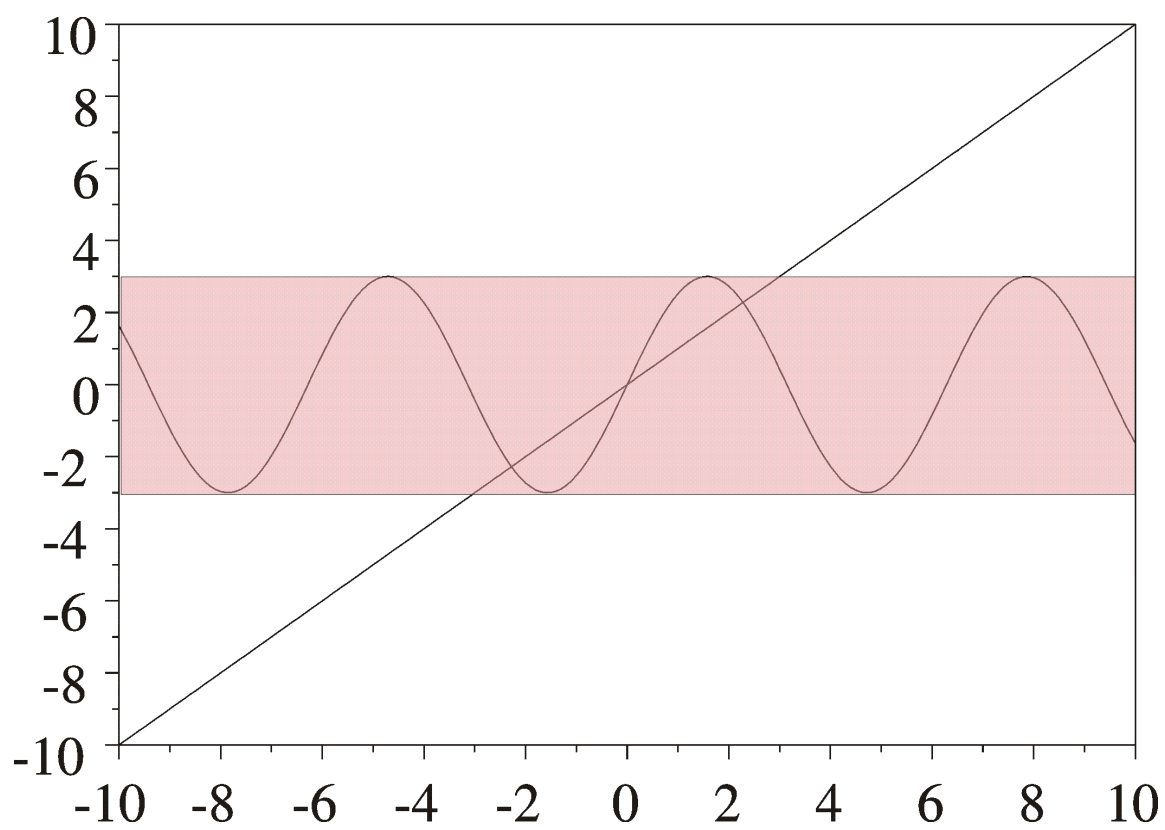
$$\begin{aligned} R_1 &\in [1.84, 2.31] \Omega, & R_2 &\in [2.58, 3.35] \Omega, \\ E &\in [24, 26] \text{V}, & I &\in [4.769, 5.417] \text{A}, \\ U_1 &\in [10, 11] \text{V}, & U_2 &\in [14, 16] \text{V}, \\ P &\in [124, 130] \text{W}, \end{aligned}$$

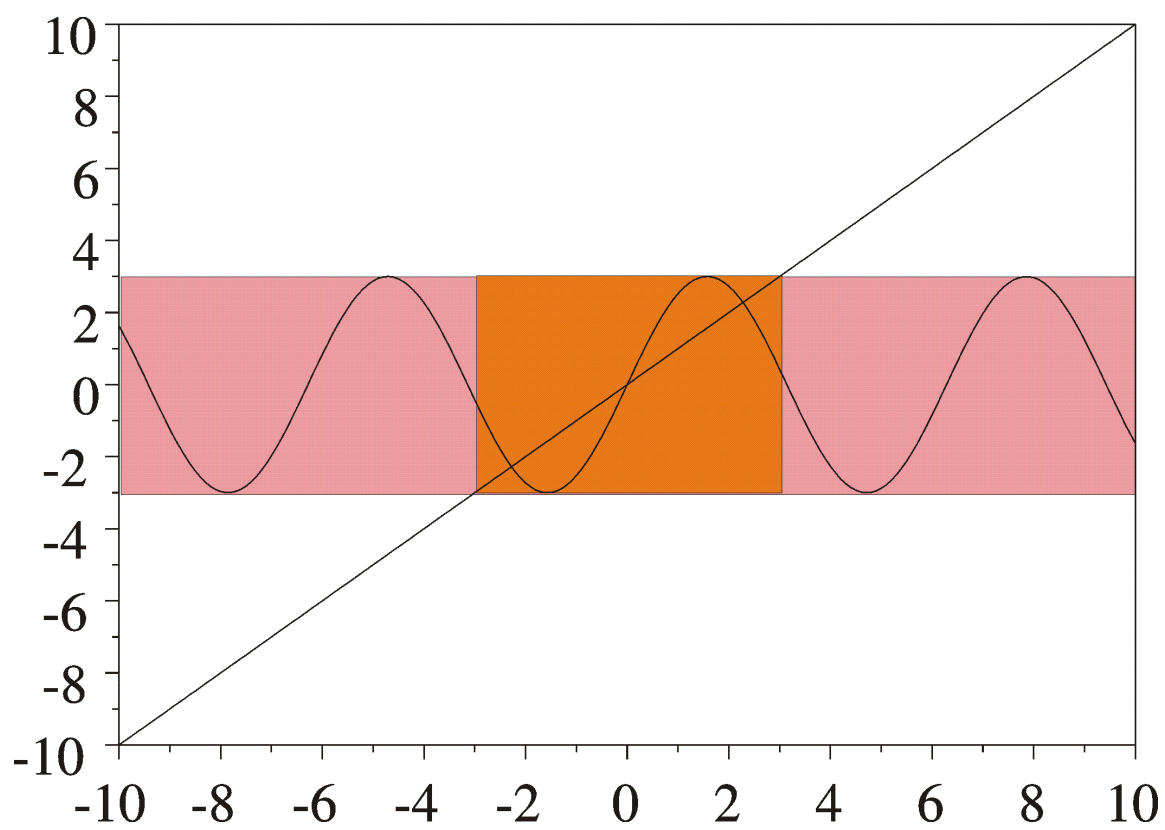
3.2 Résolution d'équations non-linéaires

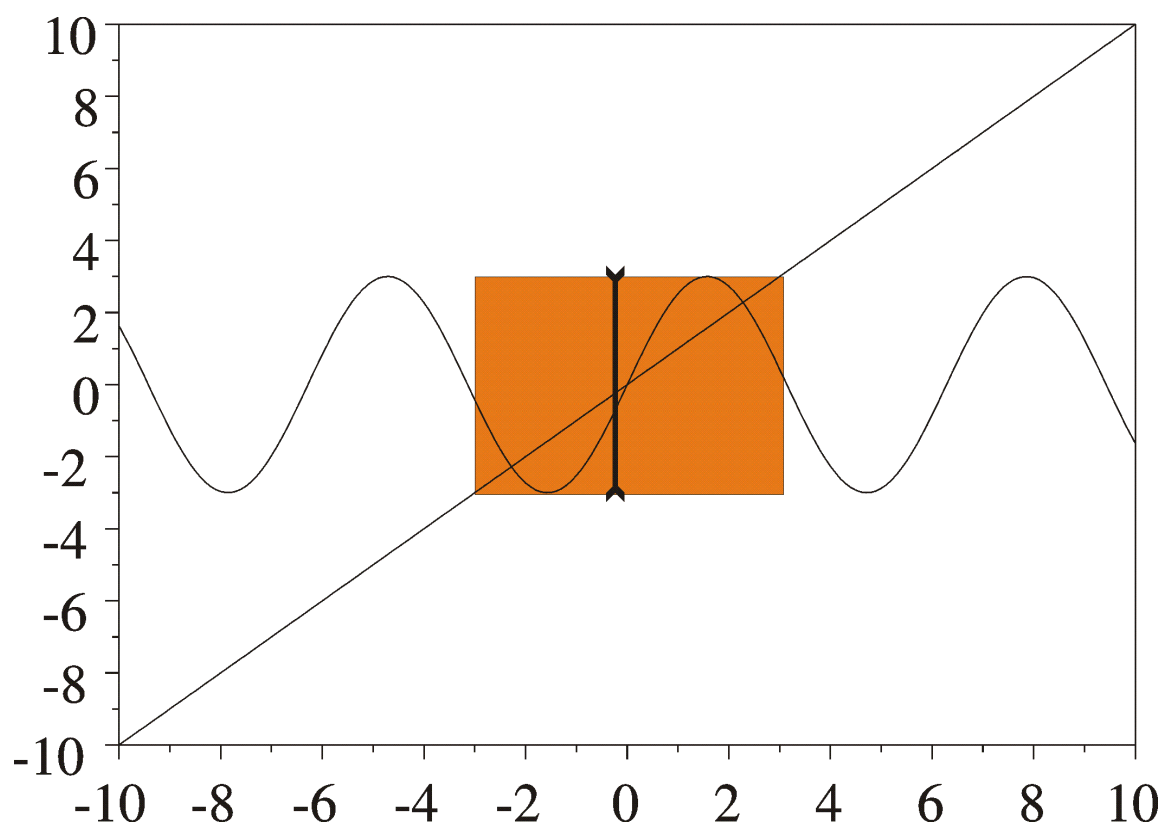
Considérons le système

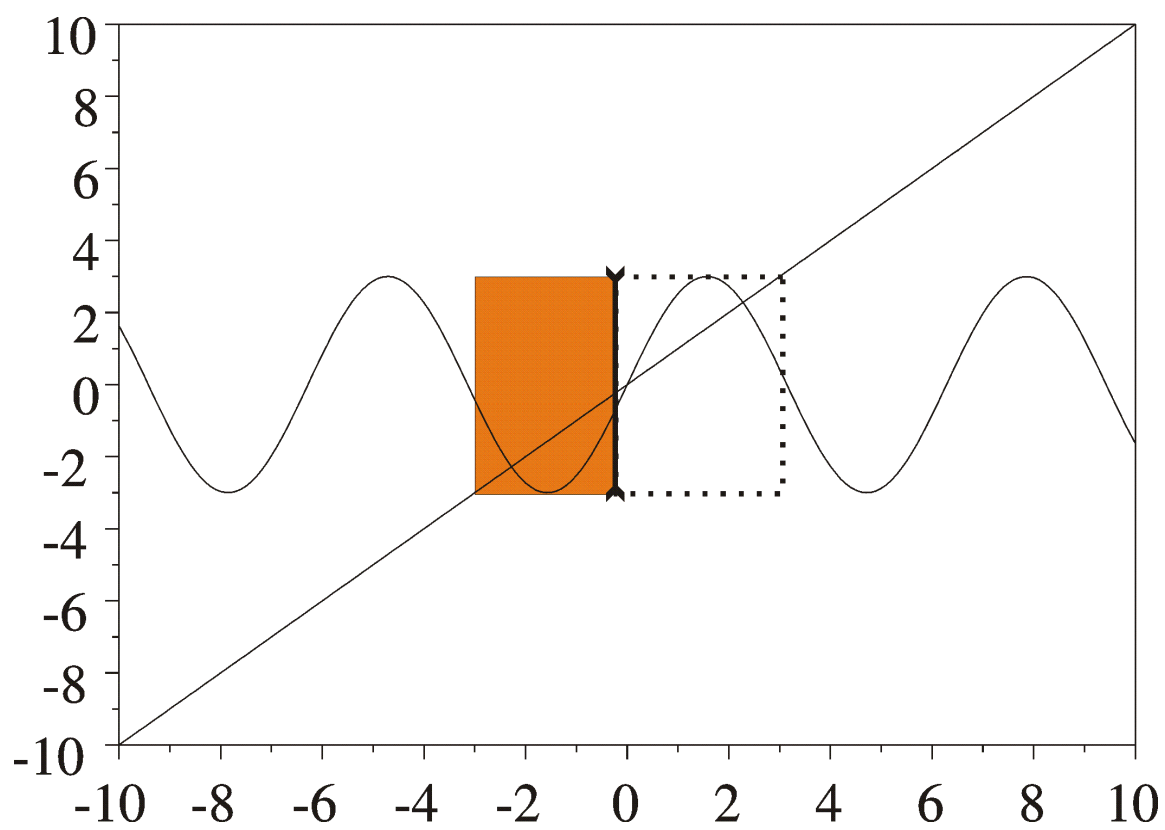
$$\begin{cases} y = 3 \sin(x) \\ y = x \end{cases} \quad x \in \mathbb{R}, y \in \mathbb{R}.$$

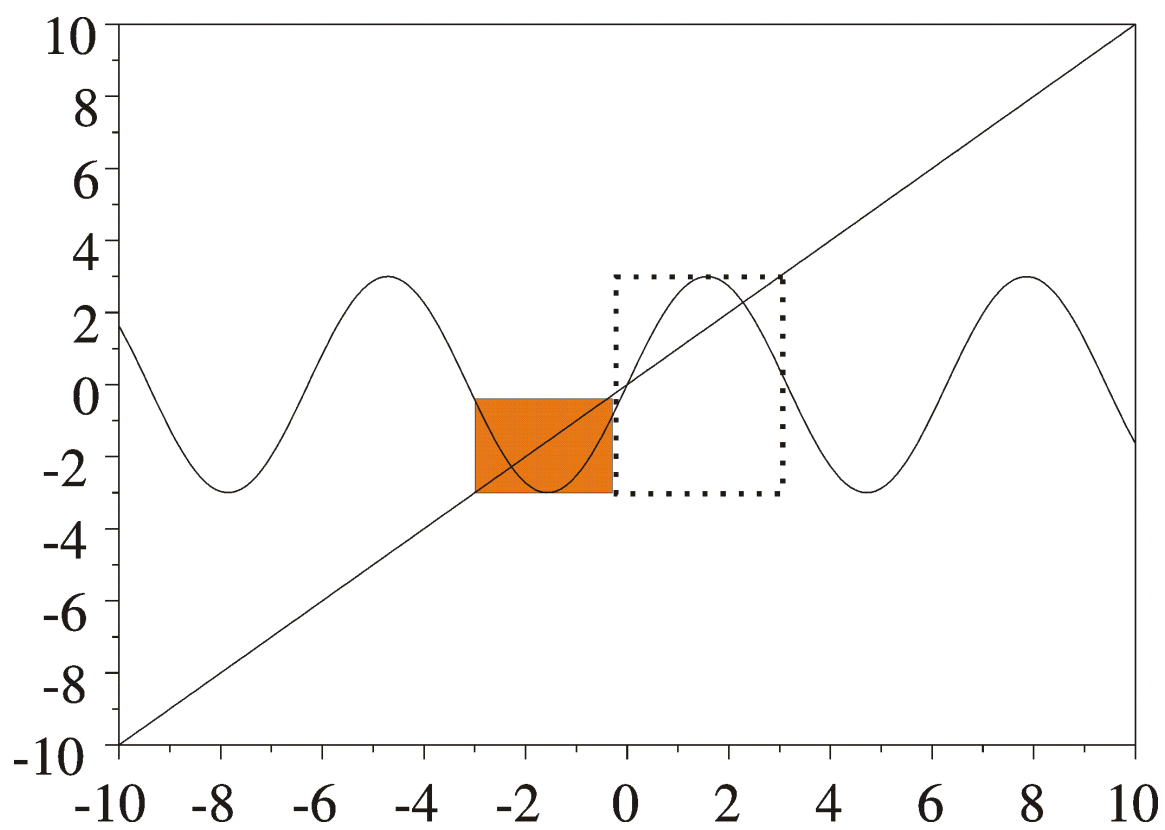


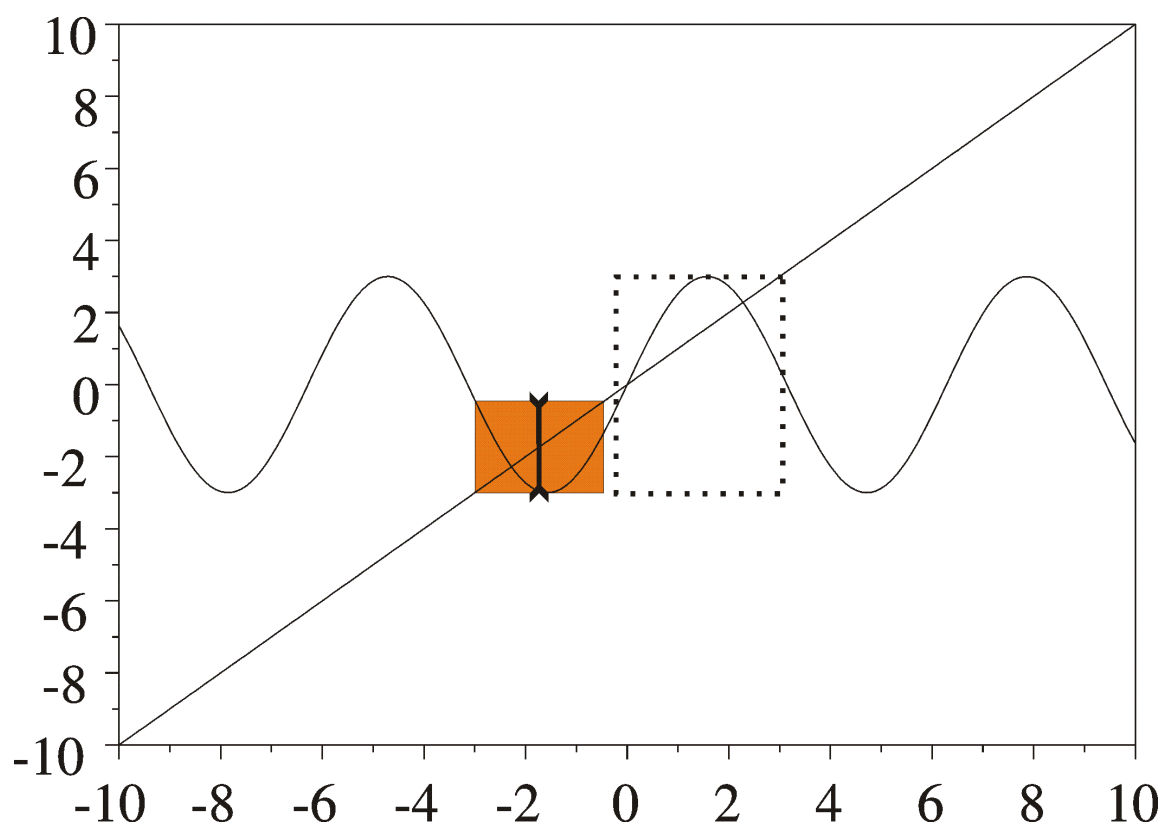


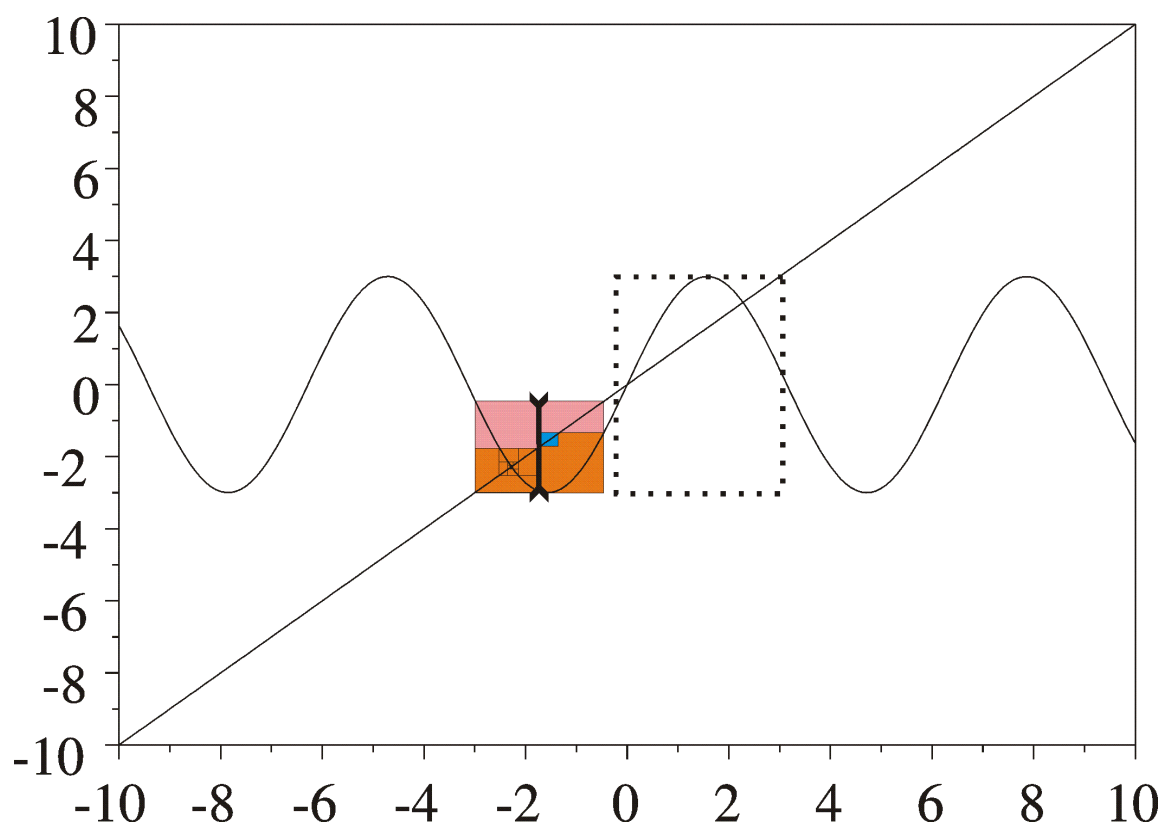


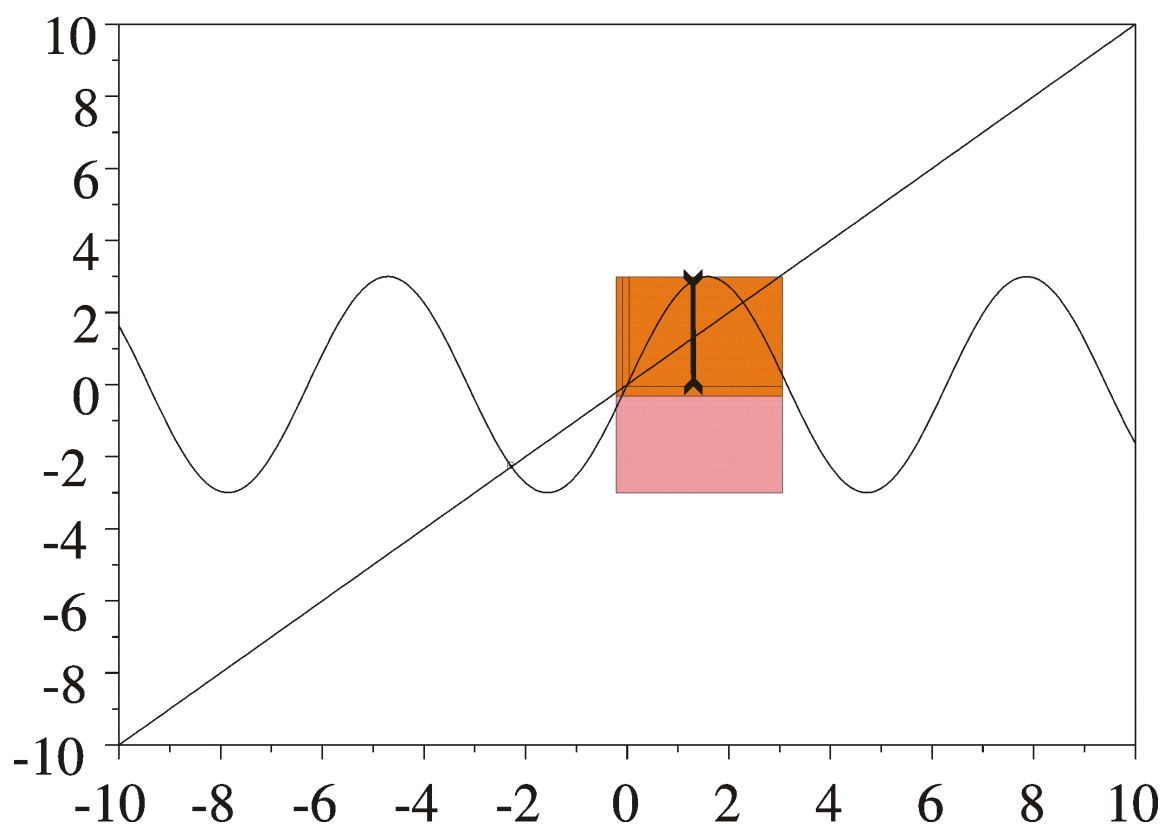


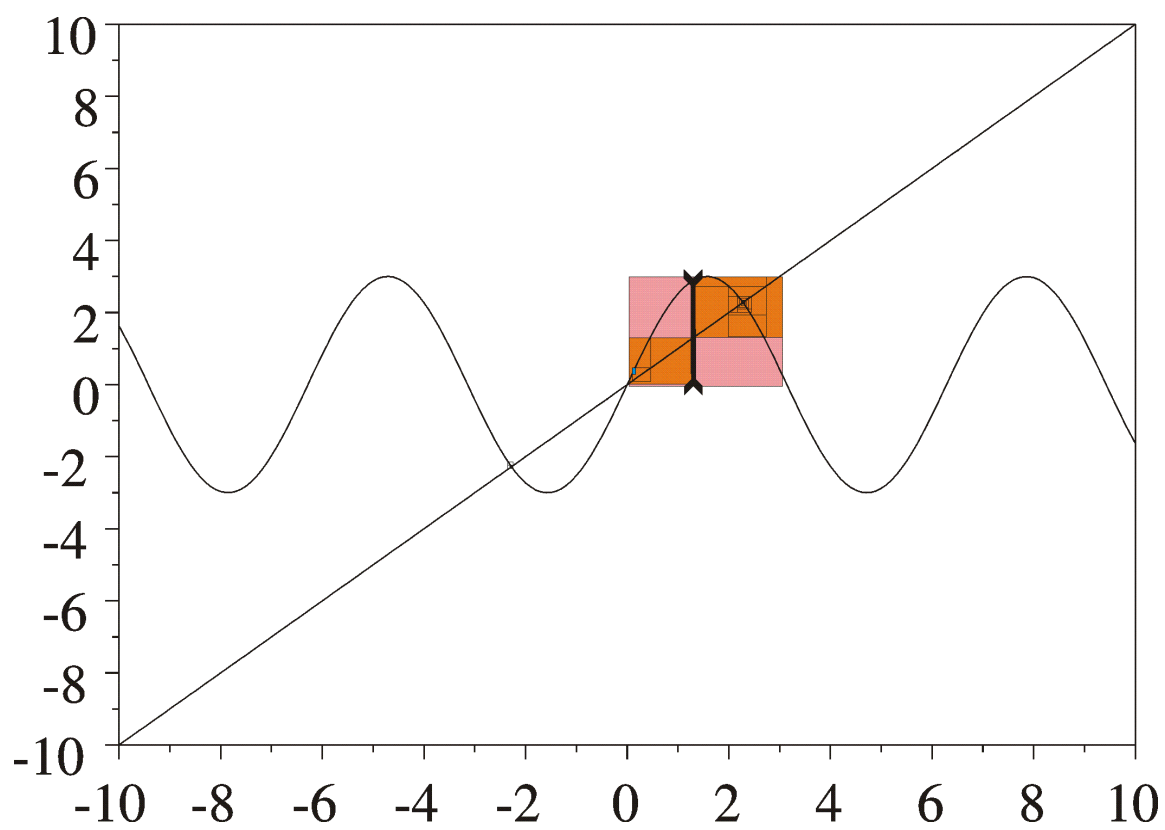






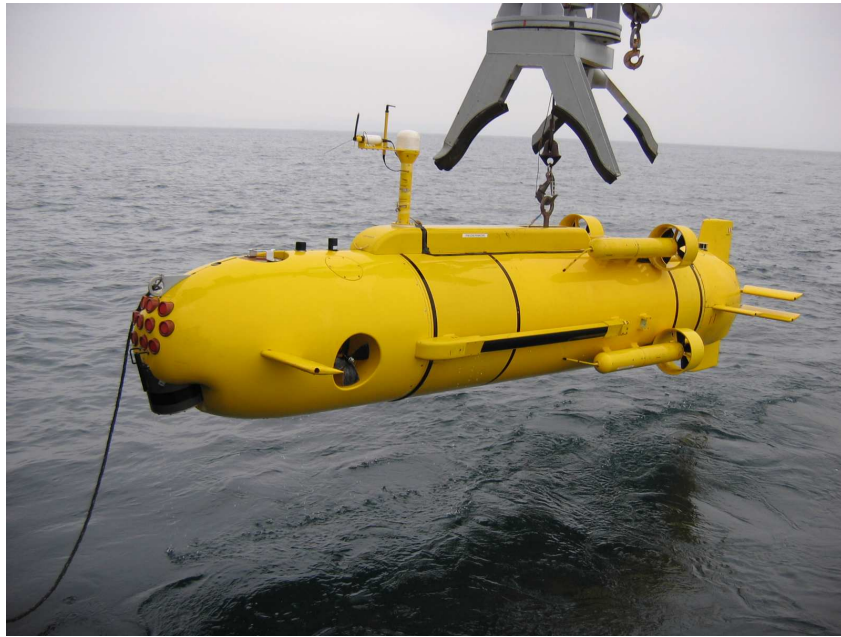






(Illustrer la résolution à l'aide du solveur Proj2D)

4 Localisation de mines



Le *Redermor*, fabriqué par le GESMA
(Groupe d'Etude Sous-Marine de l'Atlantique)



Le *Redermor* à la surface

Montrer la simulation

Pourquoi une approche par intervalles ?

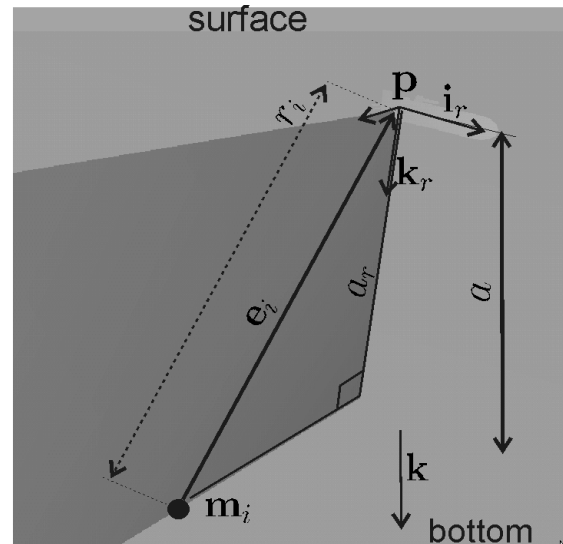
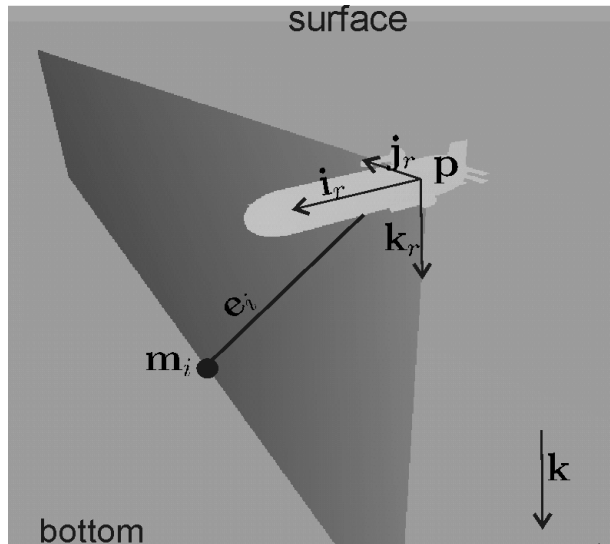
- 1) Besoin d'une approche fiable.
- 2) Les équations du robot sont non linéaires.
- 3) Les bruits de mesure sont non gaussiens.
- 4) Des bornes sur les erreurs sont fournies par les constructeurs des capteurs.
- 5) Beaucoup de mesures redondantes sont disponibles.

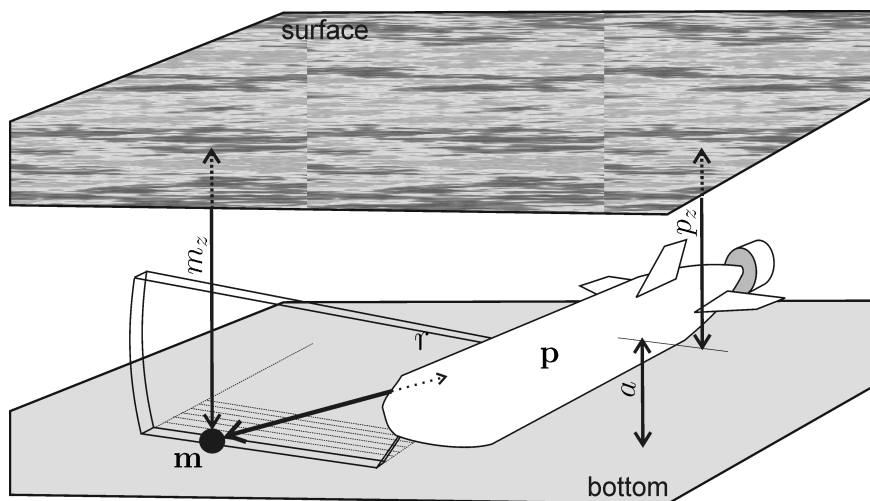
4.1 Capteurs

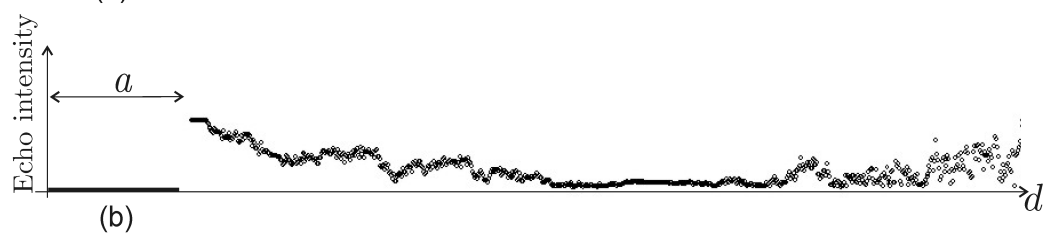
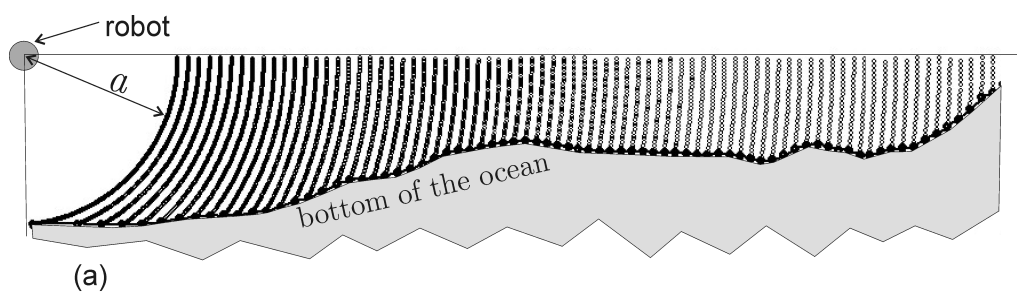
Un GPS (Global positioning system), disponible à la surface.

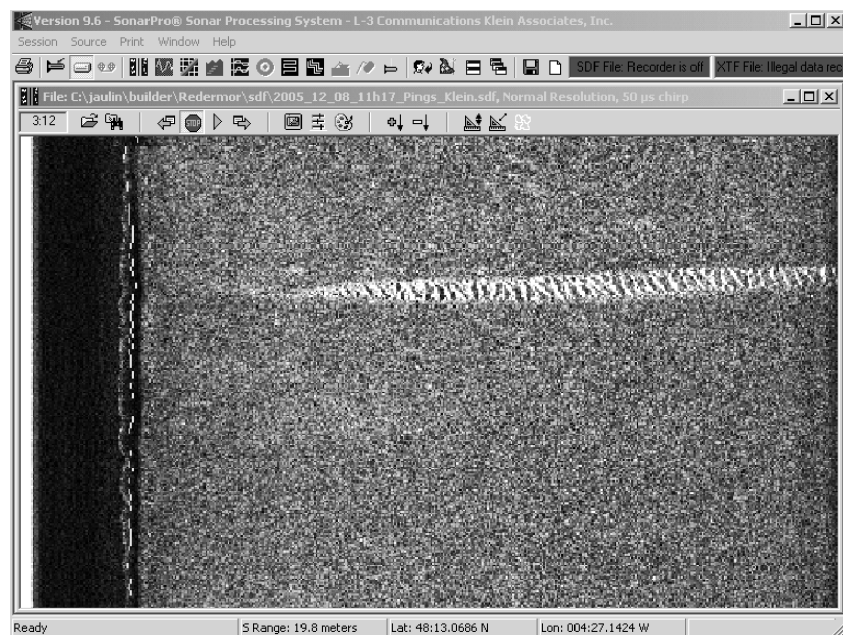
$$\begin{aligned} t_0 &= 6000 \text{ s}, & \ell^0 &= (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\ t_f &= 12000 \text{ s}, & \ell^f &= (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m \end{aligned}$$

Un sonar (KLEIN 5400 side scan sonar). Donne la distance r entre le robot et la mine

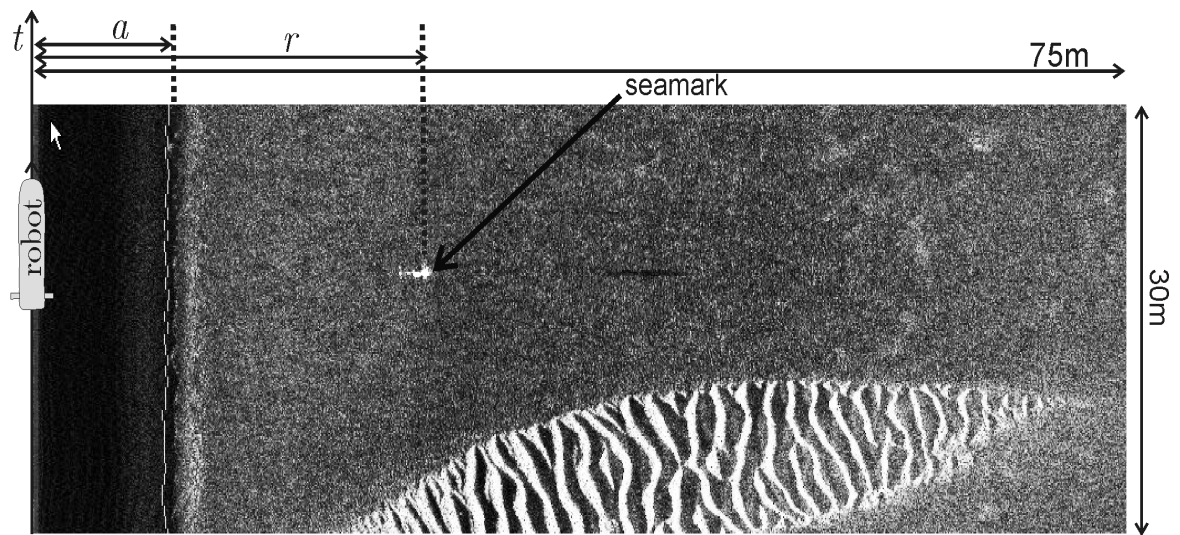








Screenshot du logiciel SonarPro

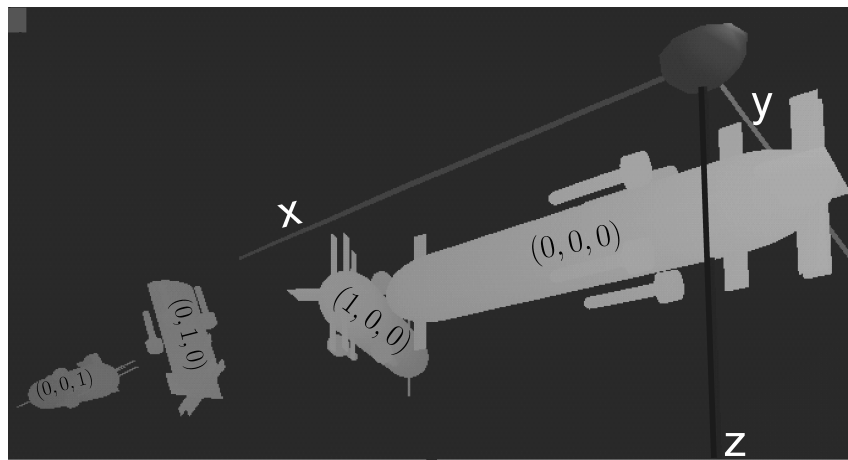


Détection d'une mine à l'aide de SonarPro

Le Loch-Doppler renvoie la vitesse du robot v_r et son altitude a ..

Une centrale inertielle (Octans III from IXSEA) renvoie le roulis ϕ , le tangage θ et le cap ψ du robot.

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$



4.2 Données

Pour chaque $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$, nous avons des intervalles pour

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines ont été détectées par un opérateur humain, à l'aide de SonarPro.

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

4.3 Contraintes

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix}$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi(t) & -\sin \varphi(t) \\ 0 & \sin \varphi(t) & \cos \varphi(t) \end{pmatrix},$$

$$\mathbf{R}(t) = \mathbf{R}_\psi(t)\mathbf{R}_\theta(t)\mathbf{R}_\varphi(t),$$

$$\dot{\mathbf{p}}(t) = \mathbf{R}(t).\mathbf{v}_r(t),$$

$$||\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))||\;=\;r(i),$$

$$\mathbf{R}^\top(\tau(i))\left(\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))\right)\in [0]\times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i))-p_z(\tau(i))-a(\tau(i))\in [-0.5,0.5]$$

5 SAUC'E

In Portsmouth, July 12-15, 2007.





