Interval robotics

Laas, Toulouse le 25 avril 2012.

L. Jaulin, Labsticc.

http://www.ensta-bretagne.fr/jaulin/

1 Interval analysis

Problem. Given $f : \mathbb{R}^n \to \mathbb{R}$, a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}]\,, f(\mathbf{x}) \geq 0.$$

Interval arithmetic can solve efficiently this problem.

Interval arithmetic

$$\begin{aligned}
[-1,3]+[2,5] &= [1,8],\\
[-1,3]*[2,5] &= [-5,15],\\
\mathrm{abs}\,([-7,1]) &= [0,7]
\end{aligned}$$

If f is given

Algorithm f(in: $\mathbf{x} = (x_1, x_2, x_3)$, out: y)

$z := x_1$;
for $k := 0$ to 100
    $z := (\cos x_2)\,(\sin(z) + kx_3)$;
next;
$y := \sin(zx_1)$;

Its interval extension is

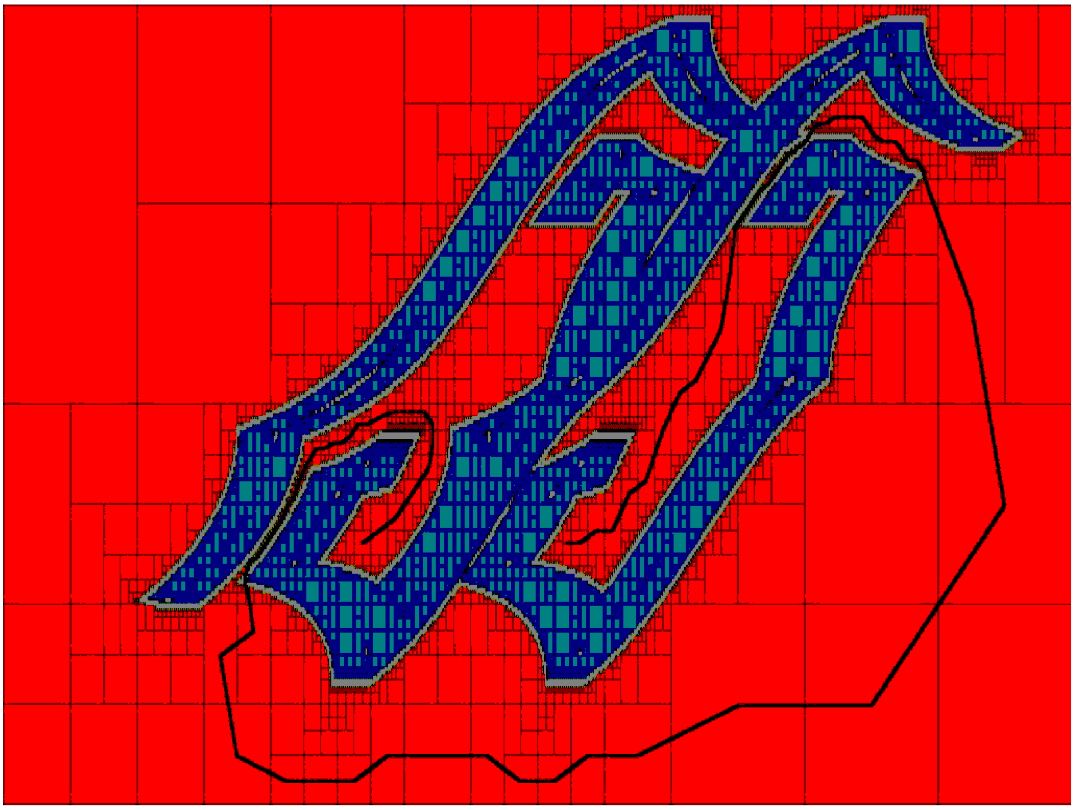
Algorithm $[f]$(in: $[\mathbf{x}] = ([x_1], [x_2], [x_3])$, out: $[y]$)

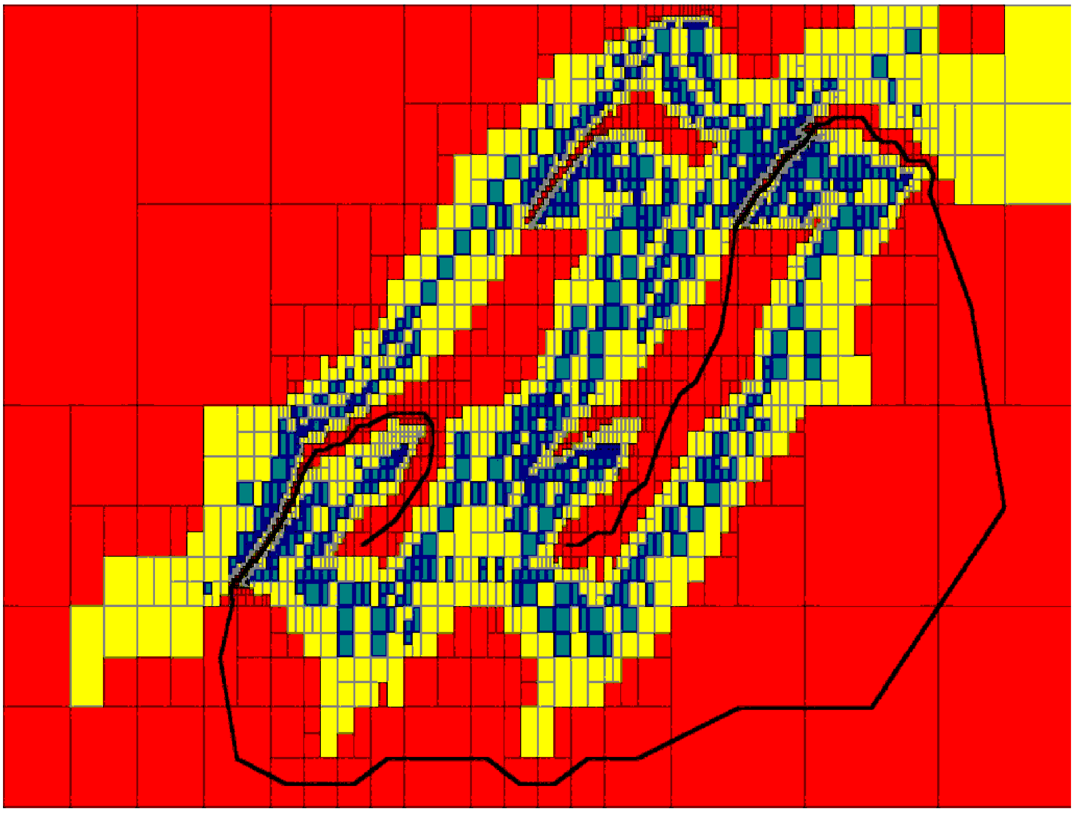
$[z] := [x_1]$;
for $k := 0$ to 100
    $[z] := (\cos[x_2]) * (\sin([z]) + k * [x_3])$;
next;
$[y] := \sin([z] * [x_1])$;

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^{+} \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq \mathbf{0}.$$

2 Path planning





Associated video

`www.ensta-bretagne.fr/jaulin/cameleon.avi`

3 Contractors

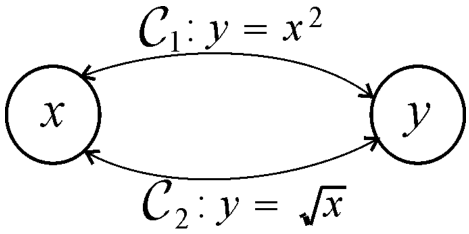
Example. Solve the system

$$\begin{aligned} y &= x^2 \\ y &= \sqrt{x}. \end{aligned}$$

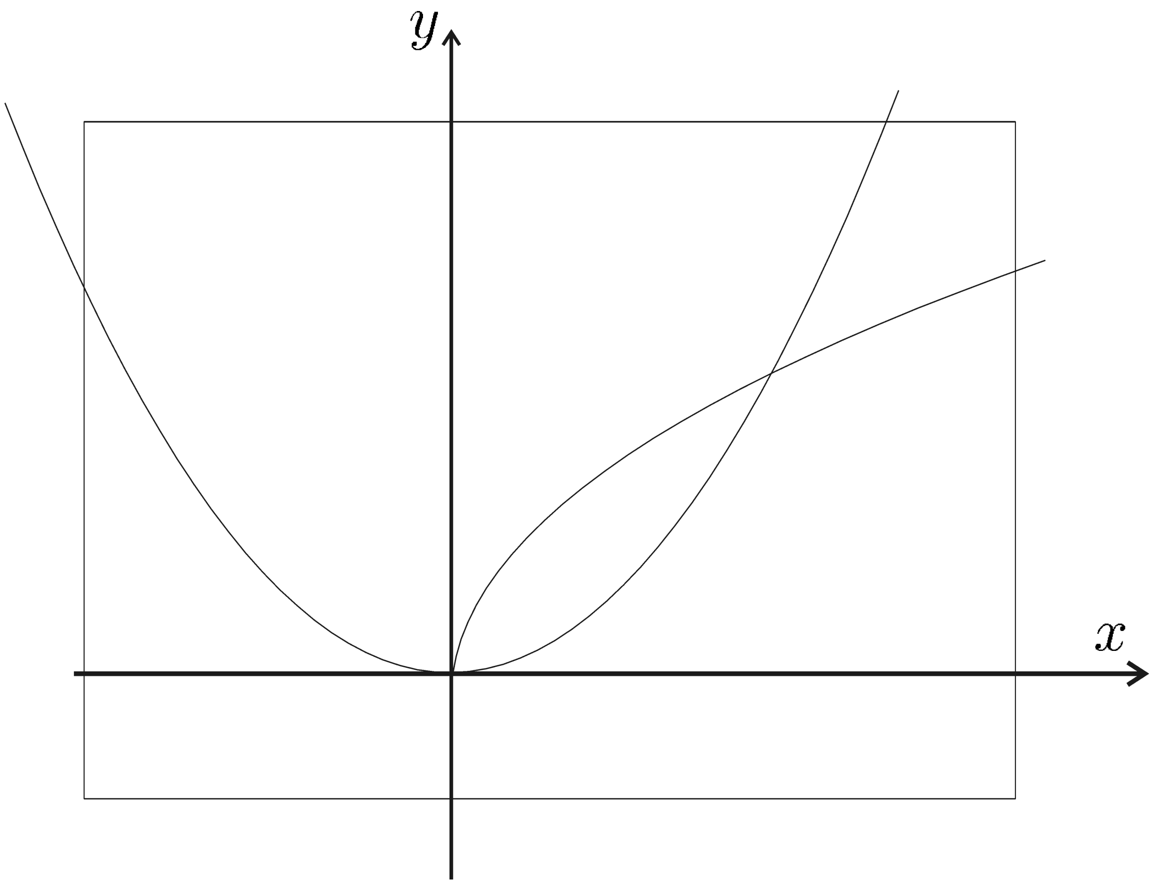
We build two contractors

$$\mathcal{C}_1 : \left\{ \begin{array}{l} [y] = [y] \cap [x]^2 \\ [x] = [x] \cap \sqrt{[y]} \end{array} \right. \quad \text{associated to } y = x^2$$

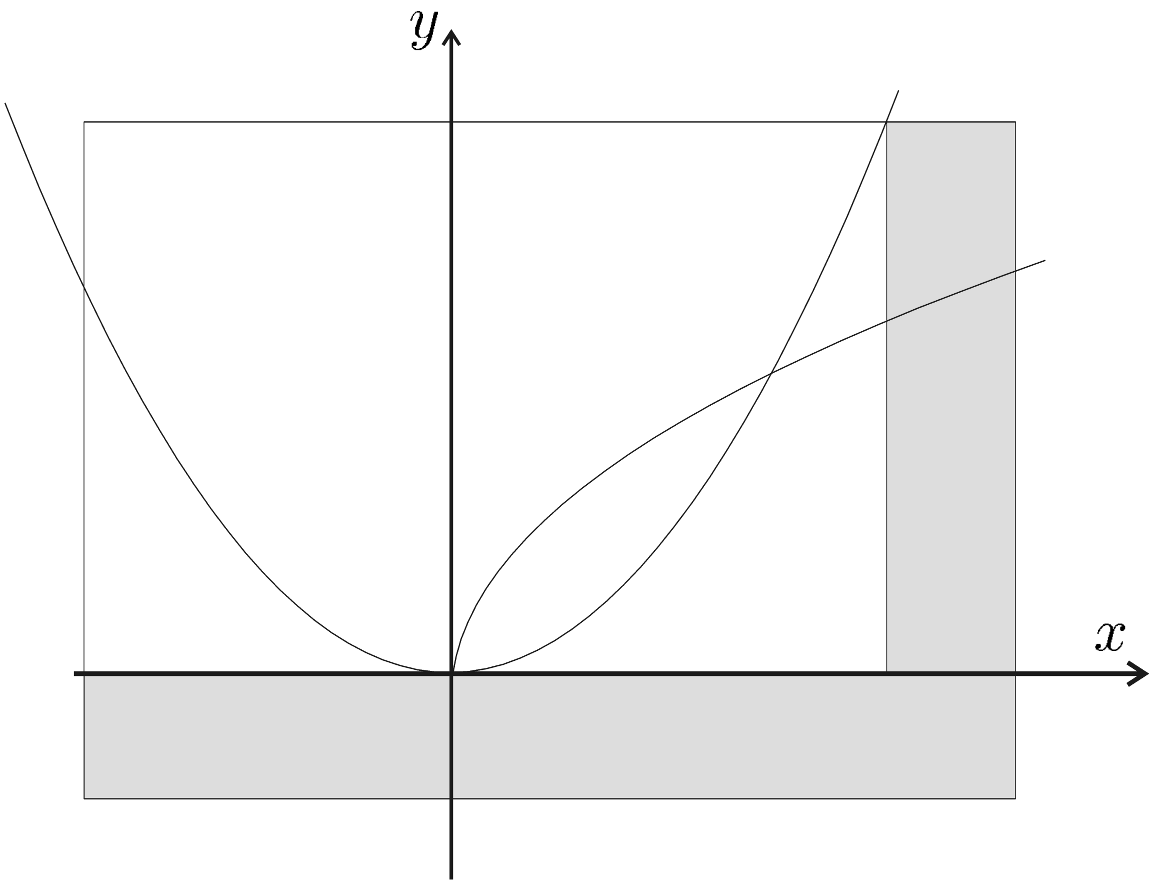
$$\mathcal{C}_2 : \left\{ \begin{array}{l} [y] = [y] \cap \sqrt{[x]} \\ [x] = [x] \cap [y]^2 \end{array} \right. \quad \text{associated to } y = \sqrt{x}$$

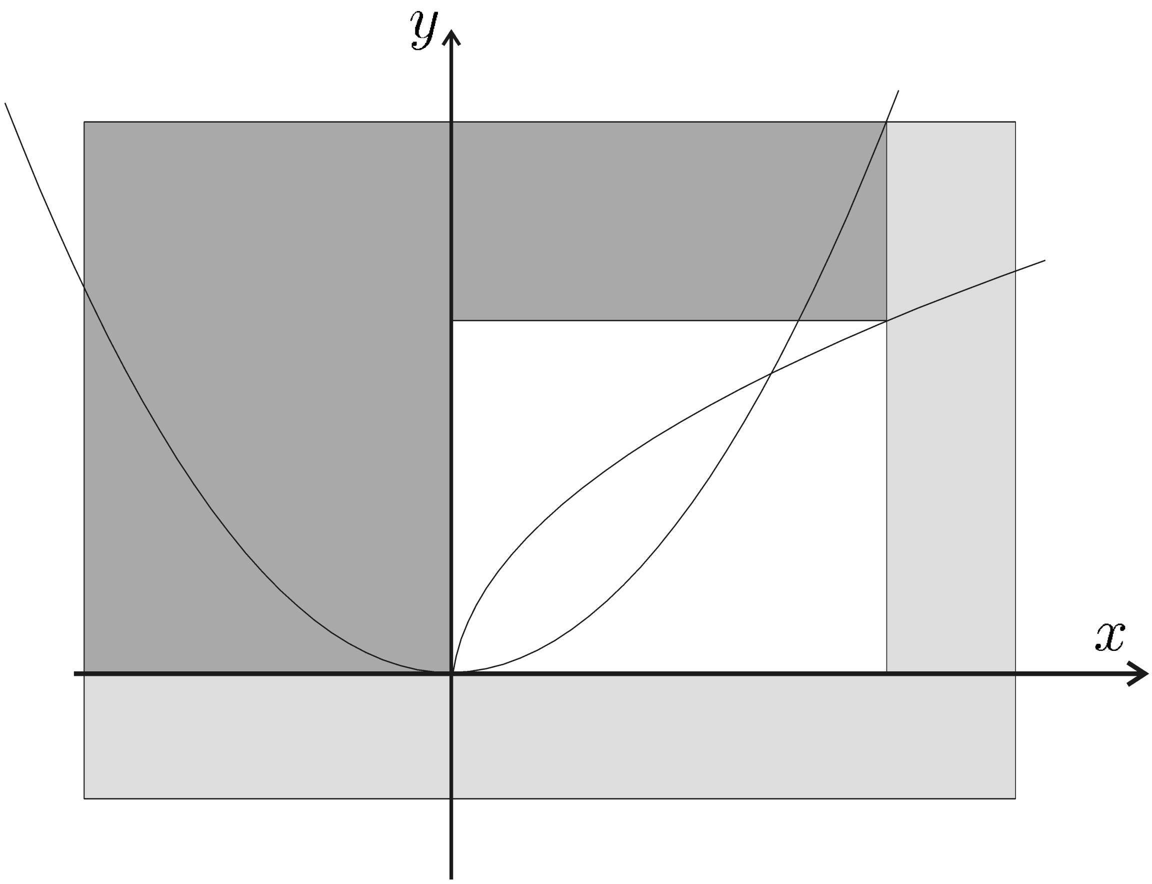


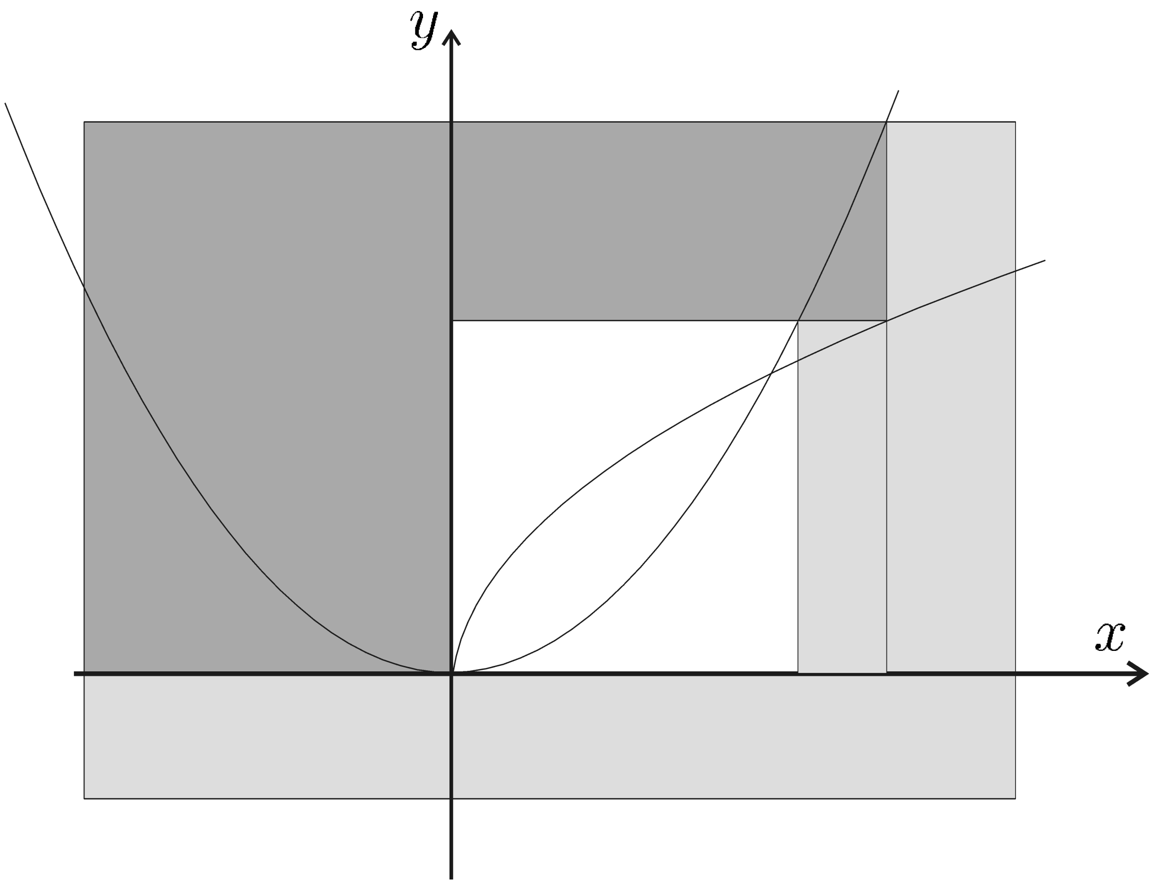
Contractor graph



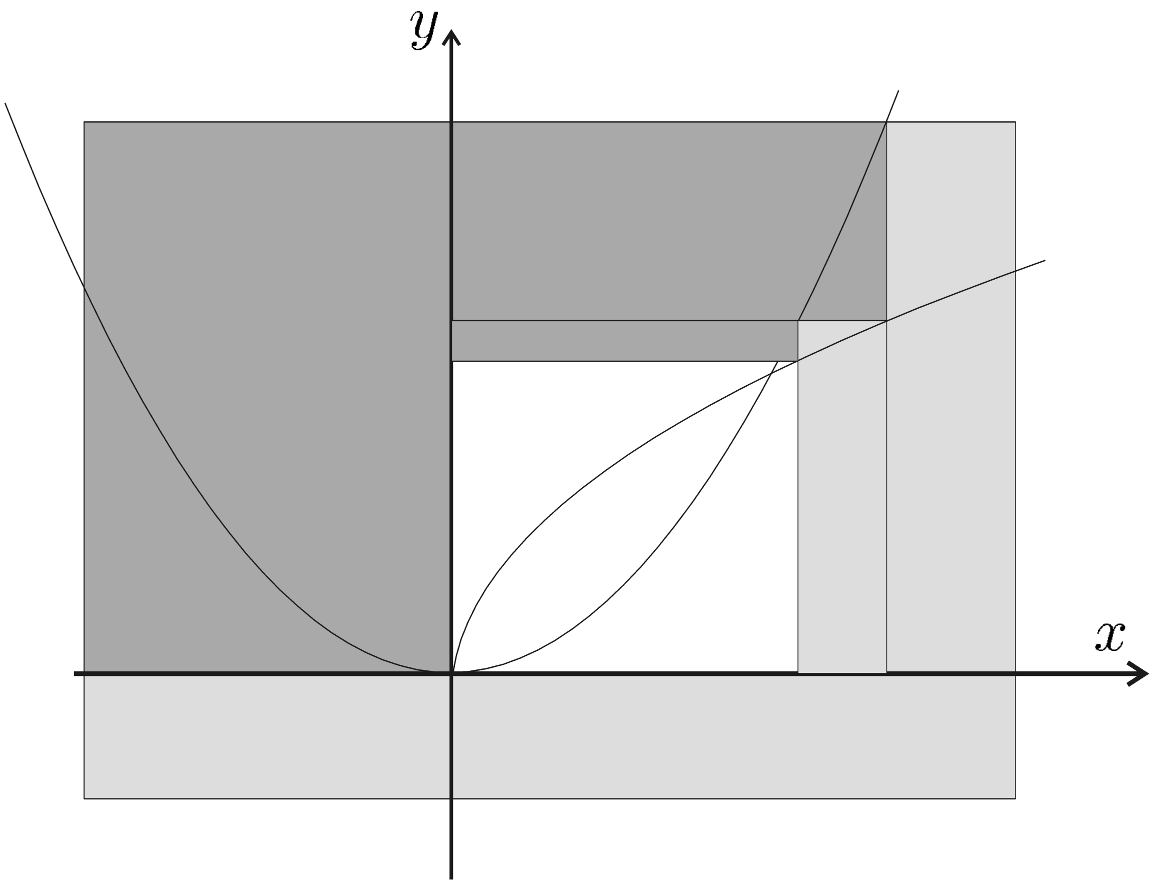
y
x

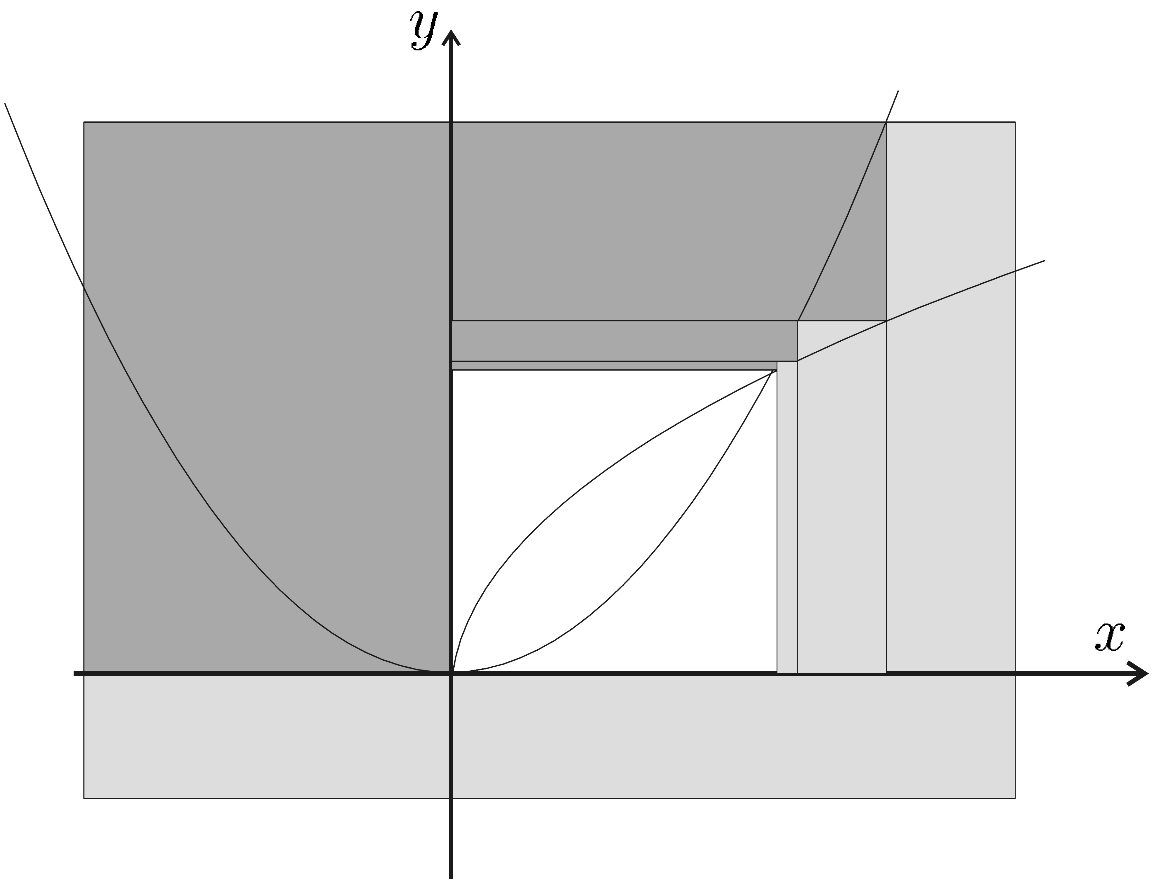

y
x

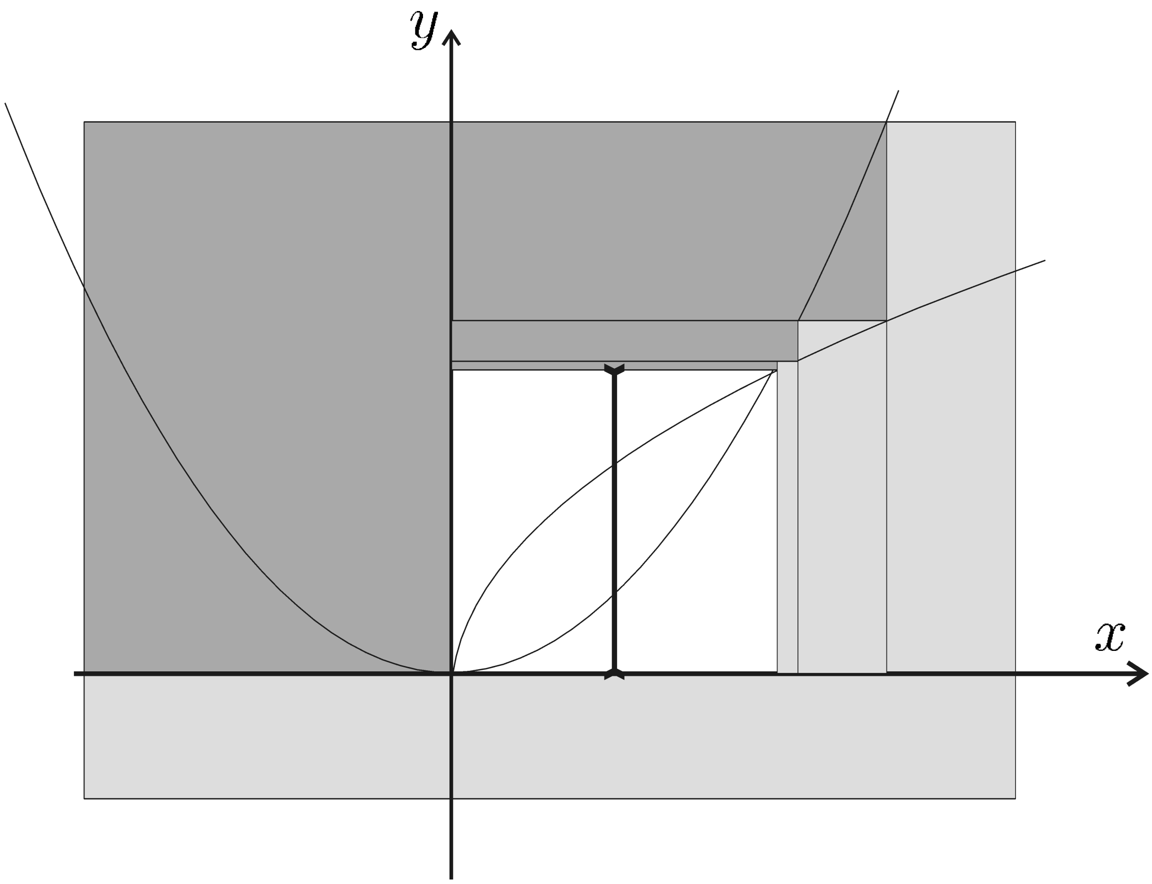

y
x

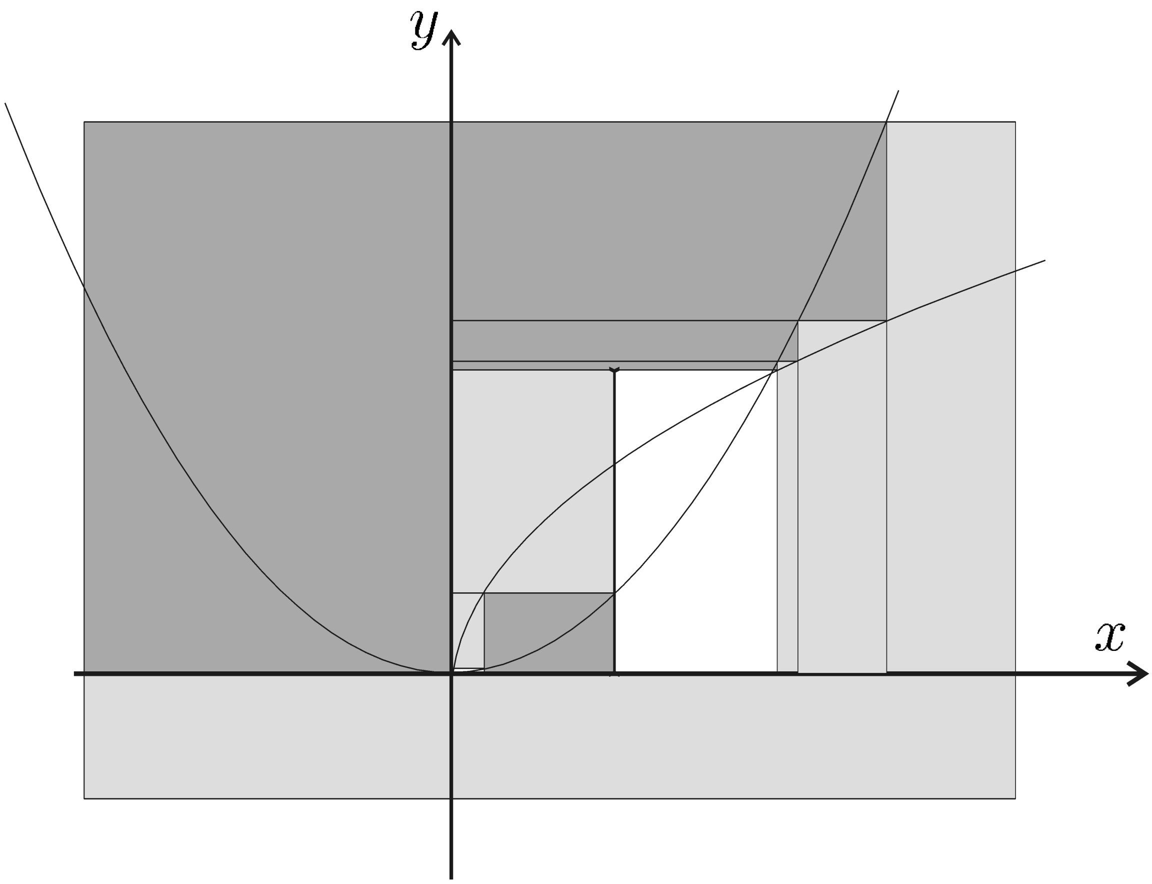


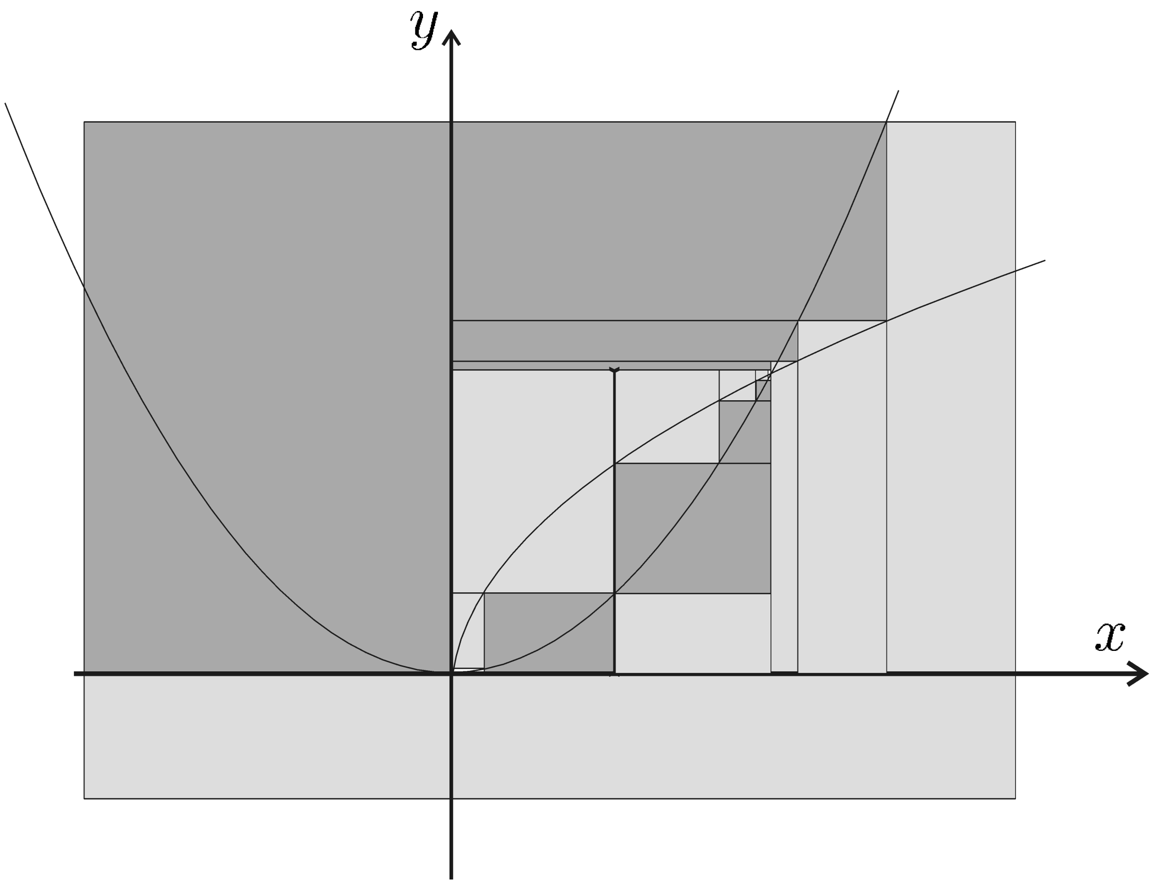
y
x


y
x


y
x

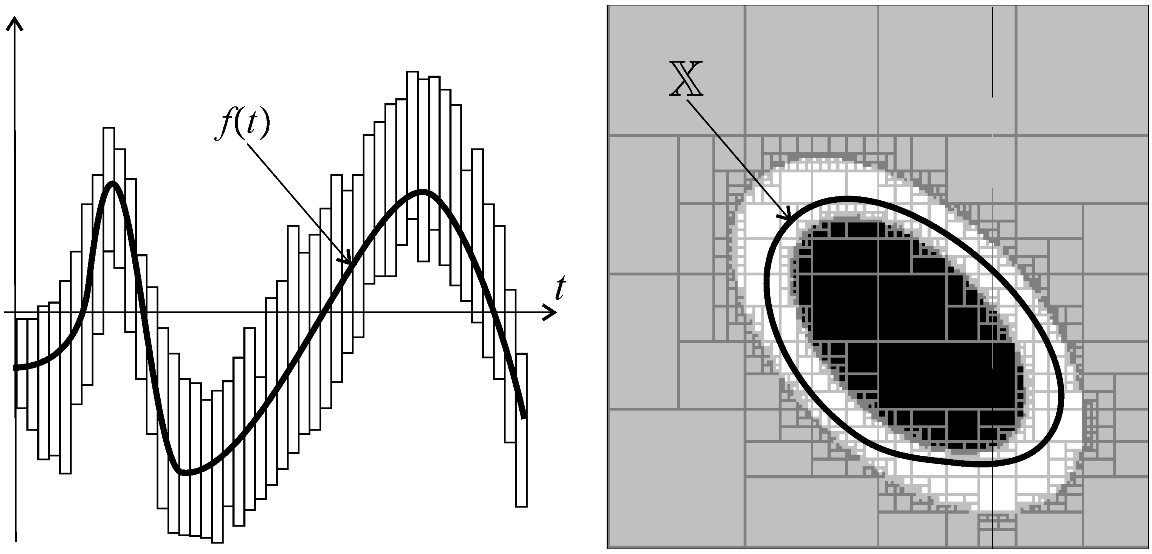

y
x


y
x

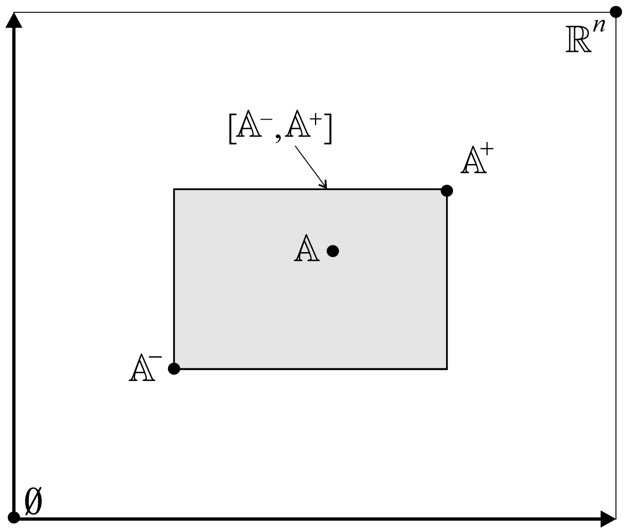


y
x

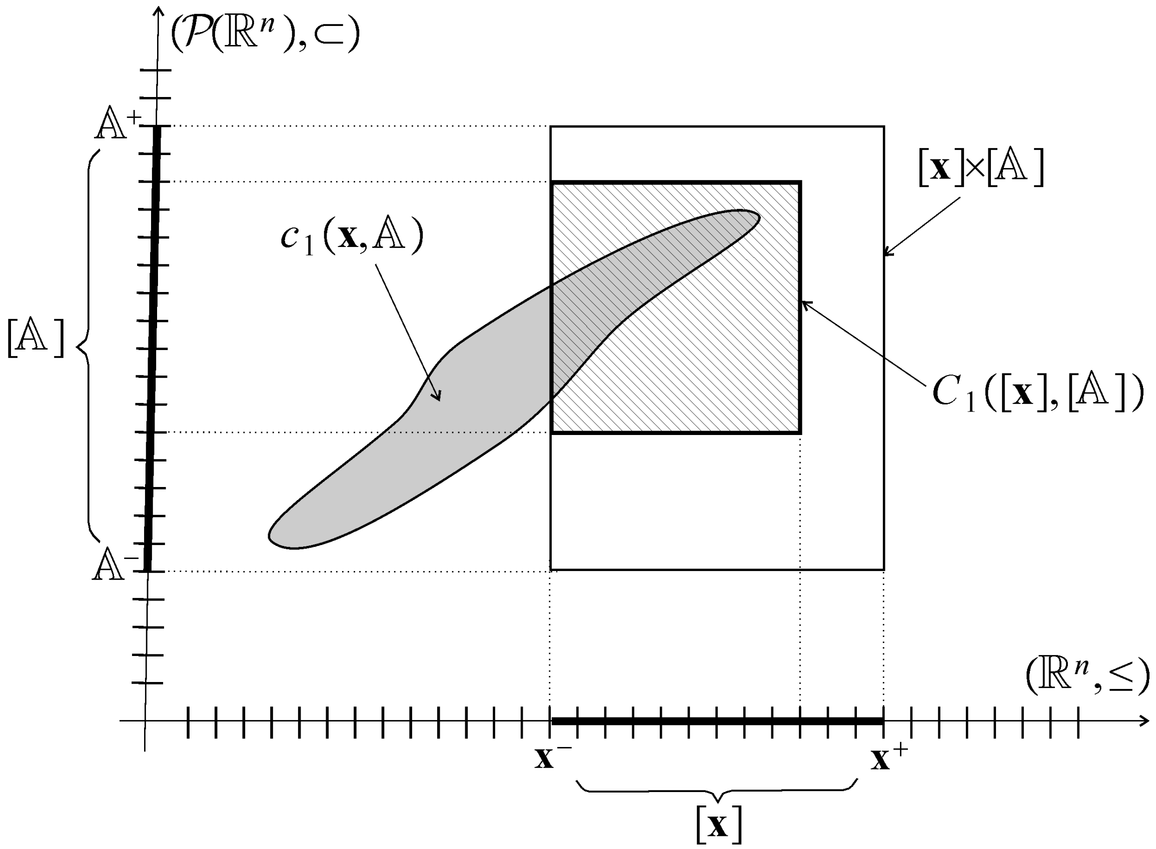
4 Generalized intervals



An interval function (or tube) and a set interval



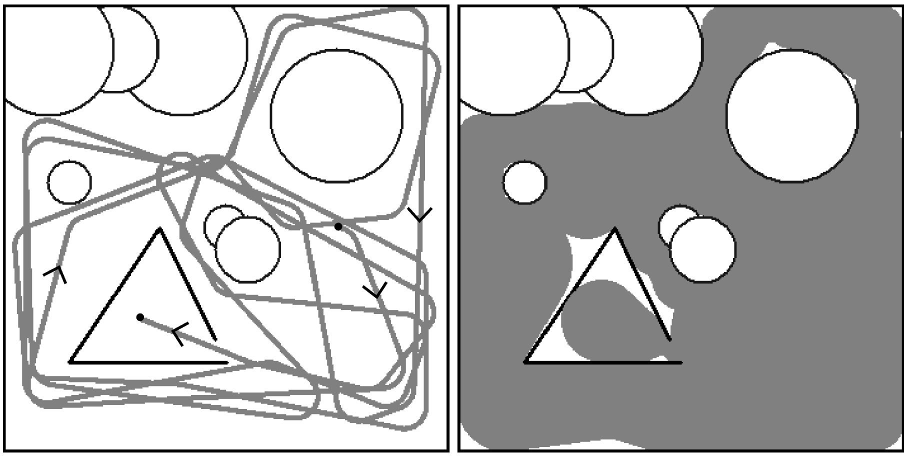
$\mathbb{R}^n$
$[\mathbb{A}^-, \mathbb{A}^+]$
$\mathbb{A}^+$
$\mathbb{A}$
$\mathbb{A}^-$
Ø



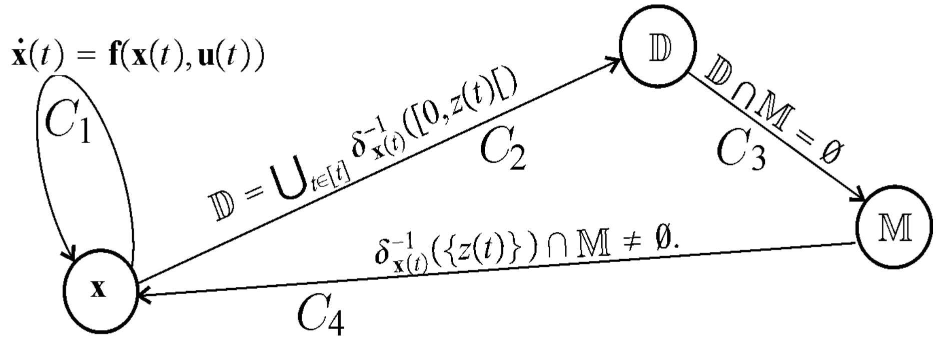
(𝒫(ℝ^n),⊂)
𝔸+
[𝔸]
𝔸−
c1(x,𝔸)
[x]×[𝔸]
C1([x],[𝔸])
(ℝ^n,≤)
x−
x+
[x]

5 Range-only SLAM

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}\left(\mathbf{x}\left(t\right), \mathbf{u}\left(t\right)\right) \\ z\left(t\right) & = & d\left(\mathbf{x}\left(t\right), \mathbb{M}\right). \end{cases}$$



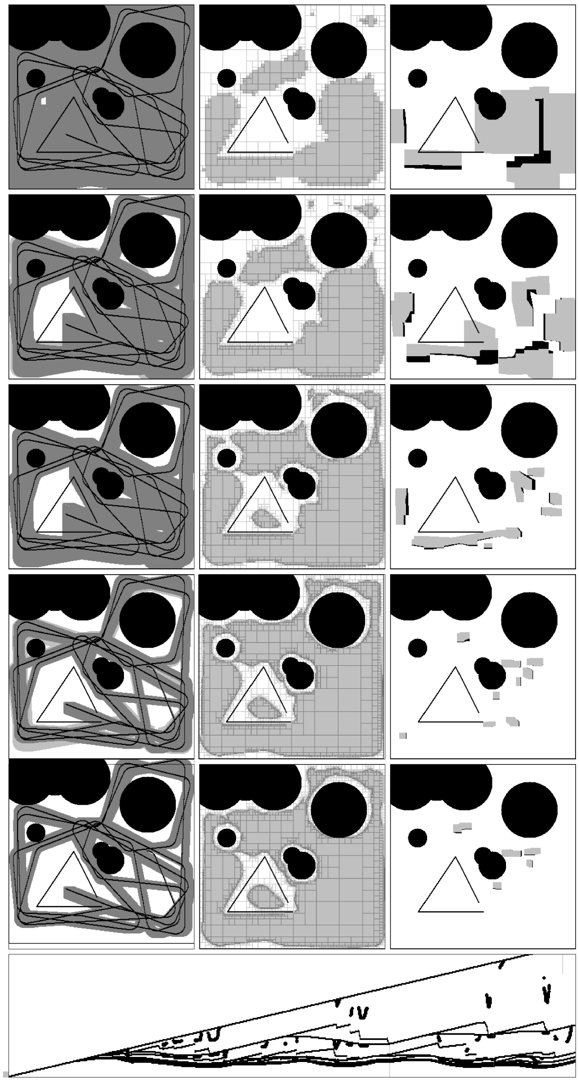
True trajectory and dug space



Constraint diagram of the SLAM problem

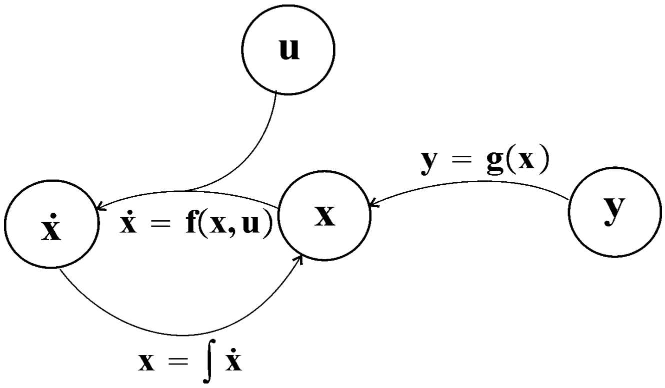
Associated video

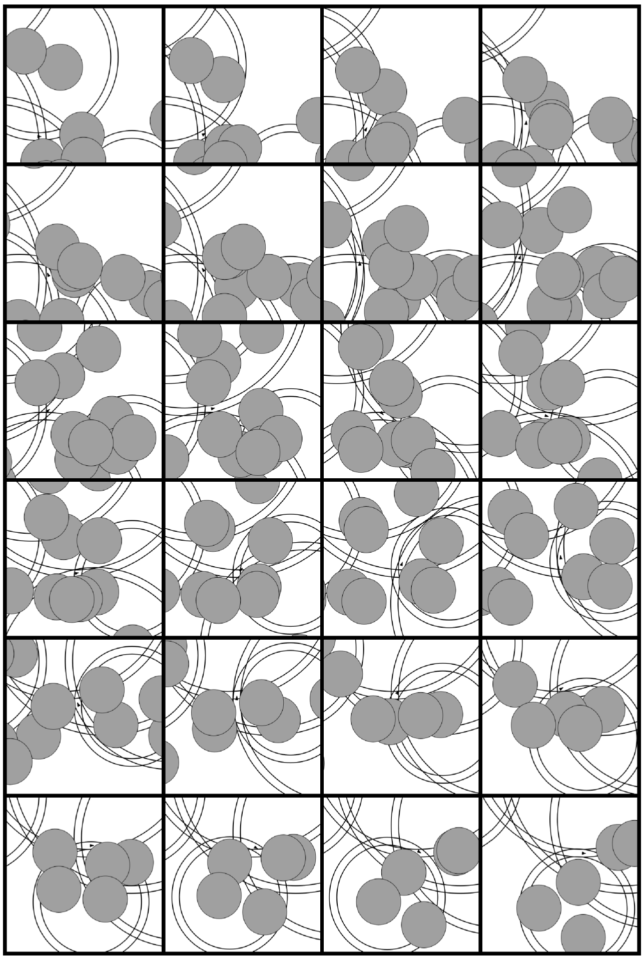
`www.ensta-bretagne.fr/jaulin/dig2.avi`



6 Motion Planning

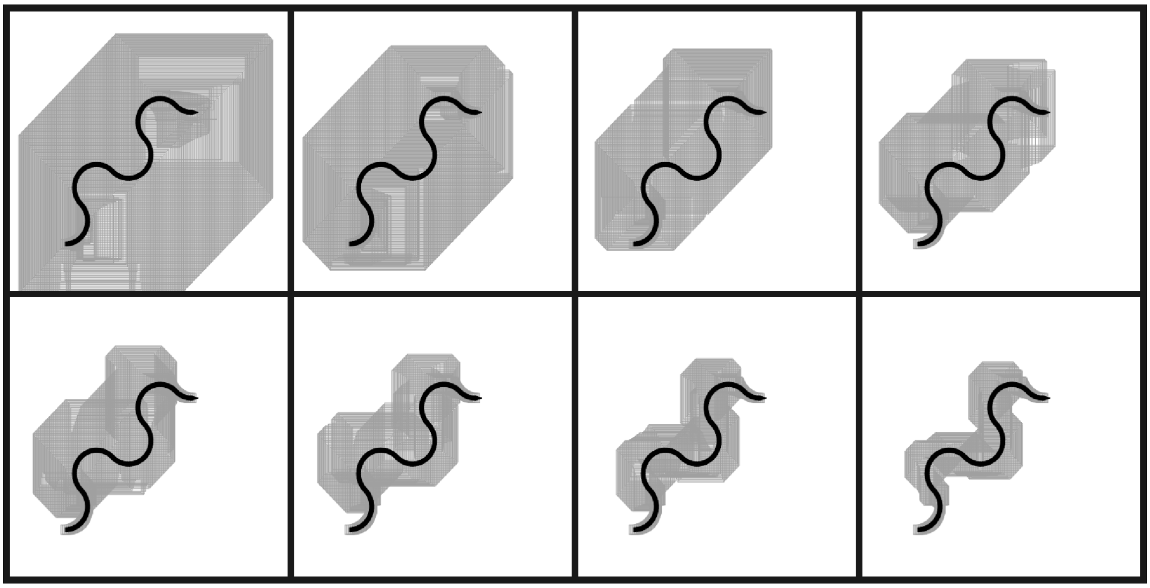
$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), & \mathbf{u}(t) \in \mathbb{U} \\ \mathbf{y} = \mathbf{g}(\mathbf{x}), & \mathbf{y}(t) \in \mathbb{Y}(t) \end{cases}$$


u
ẋ
ẋ = f(x, u)
x
y = g(x)
y
x = ∫ ẋ



Associated video

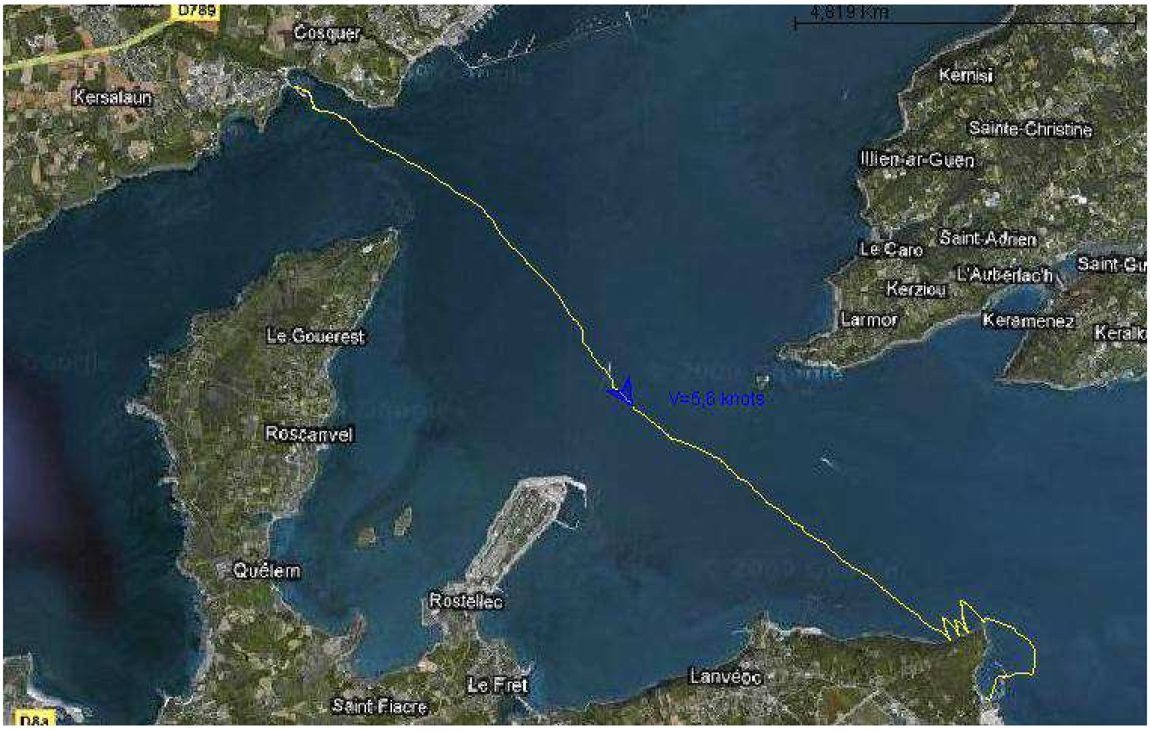
www.ensta-bretagne.fr/jaulin/docking.avi



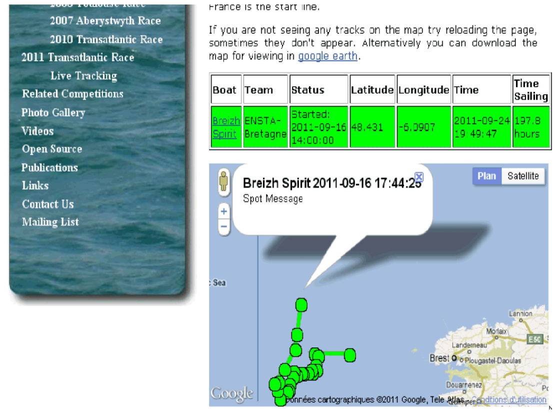
7 Sailboat robotics

L. Jaulin, F. Lebars, J. Sliwka, O. Ménage, P. Rousseau, S. Prigent, T. Terre, Y. Gallou, B. Clément, O. Reynet, B. Zerr, . . .


BREIZH
ENSIETA
SPIRIT



D789
Cosquer
Kersalaun
4,819 Km
Kernisi
Sainte-Christine
Illien-ar-Guen
Saint-Adrien
Le Caro
L'Auberlac'h
Kerziou
Larmor
Keramenez
Le Gouerest
V=5,6 knots
Roscanvel
Quélern
Rostellec
Le Fret
Lanvéoc
Saint Fiacre
D8a



2007 Aberystwyth Race
2010 Transatlantic Race
2011 Transatlantic Race
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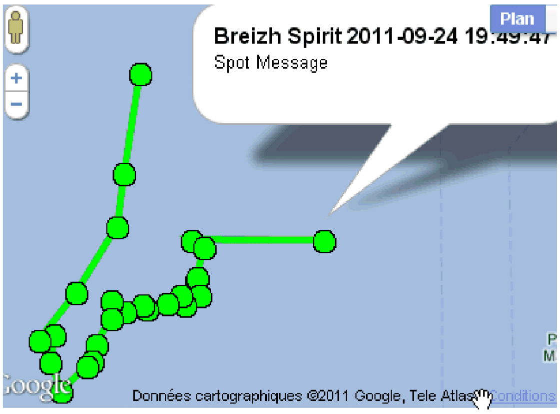
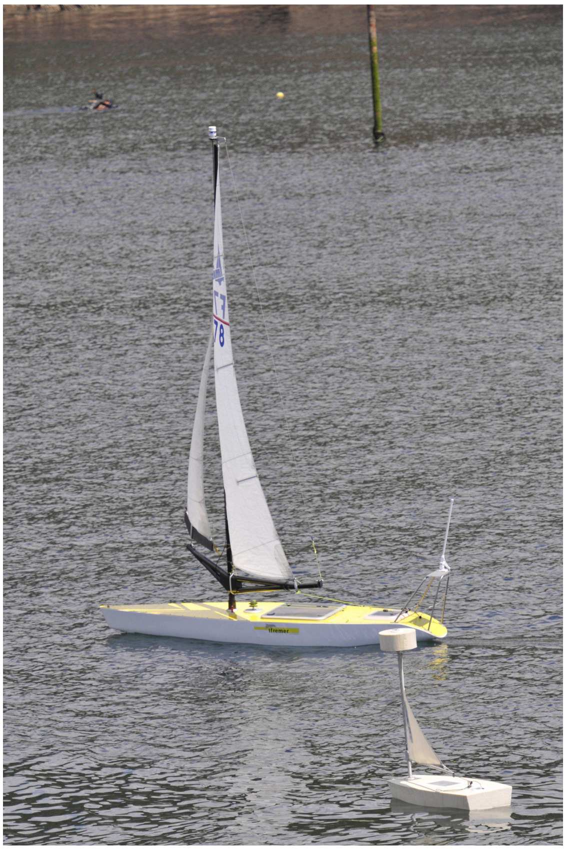
France is the start line.

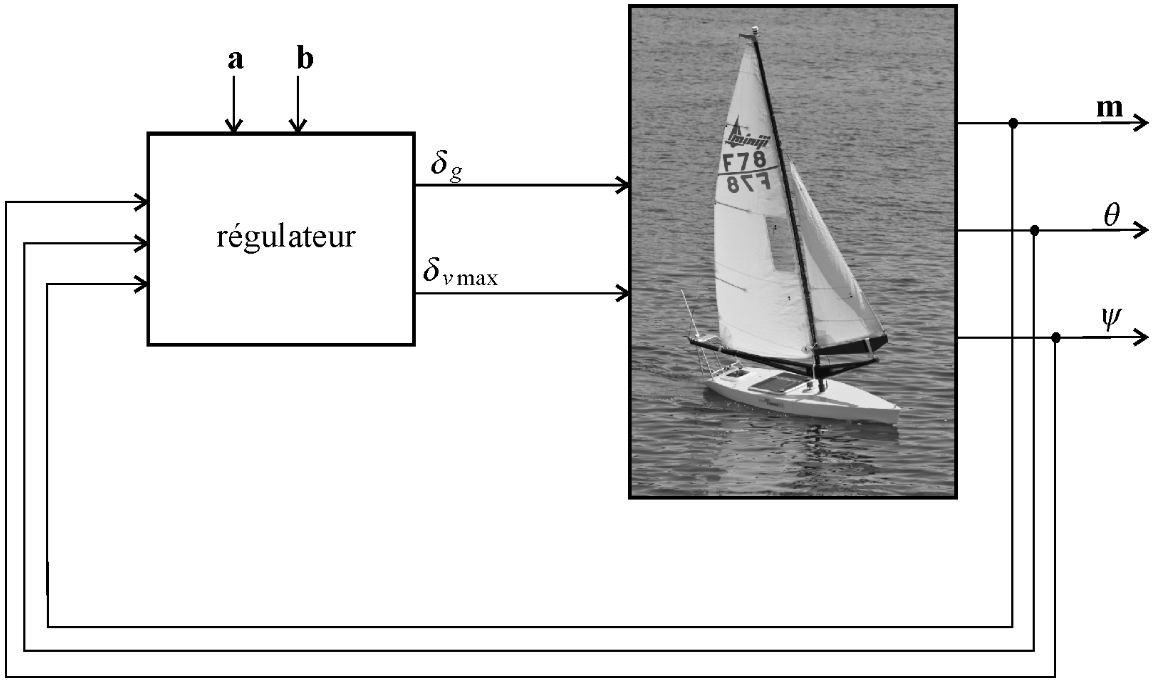
If you are not seeing any tracks on the map try reloading the page, sometimes they don't appear. Alternatively you can download the map for viewing in google earth.

Boat	Team	Status	Latitude	Longitude	Time	Time Sailing
Breizh Spirit	ENSTA-Bretagne	Started: 2011-09-16 14:00:00	48.431	-6.0907	2011-09-24 19:49:47	197.8 hours




F78




a
b
régulateur
δ_g
$\delta_{v\,max}$
m
θ
ψ

8 Brest-Douarnenez

Départ le 17 janvier 2012 à 8h.


F7
8


Challenger


Brest
PRO
YAMAHA


miniji
F78

Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Le Conquet
Plougastel-Daoulas
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
lat 48.230580° long -4.564526° élév 6 m
Altitude 53.47 km
©2010 Google

Associated video

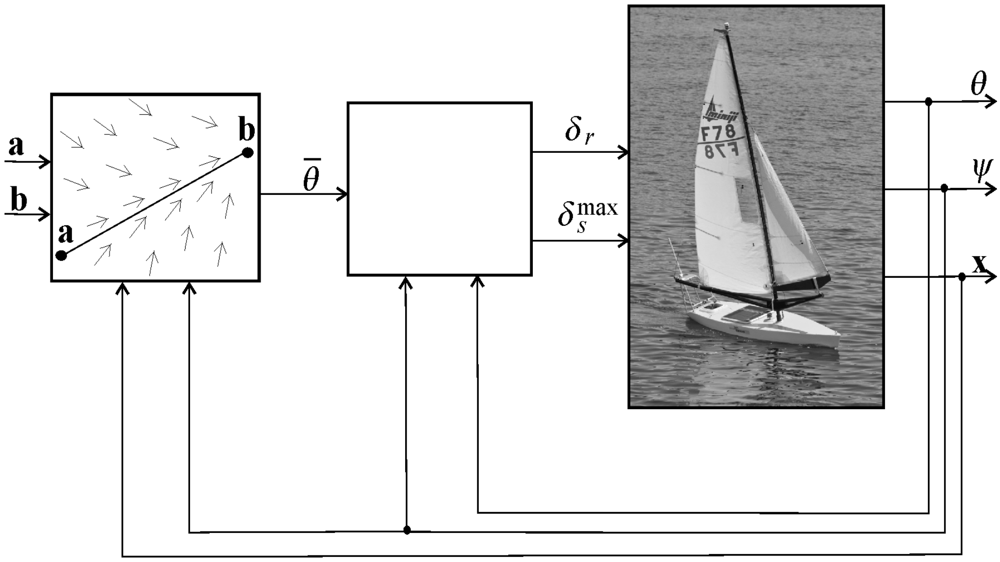
www.ensta-bretagne.fr/jaulin/vaimosdash.avi

Il est donc possible pour un robot voilier de rester dans sa bande.

Indispensable pour créer des règles de circulation lorsqu'on travaille avec des meutes.

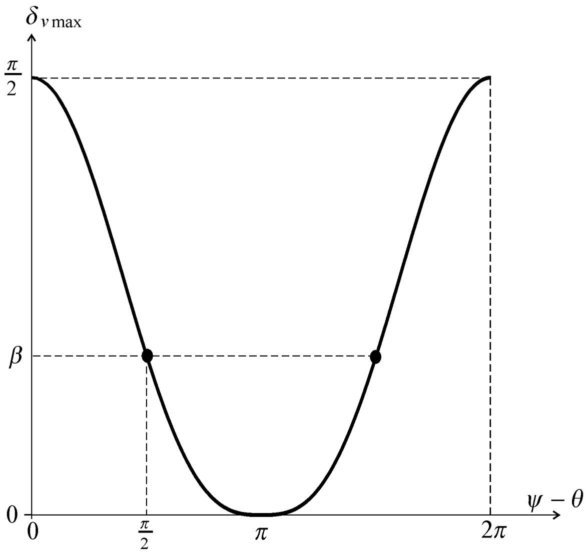
Indispensable pour déterminer les responsabilités en cas d'accident.

9 Line following

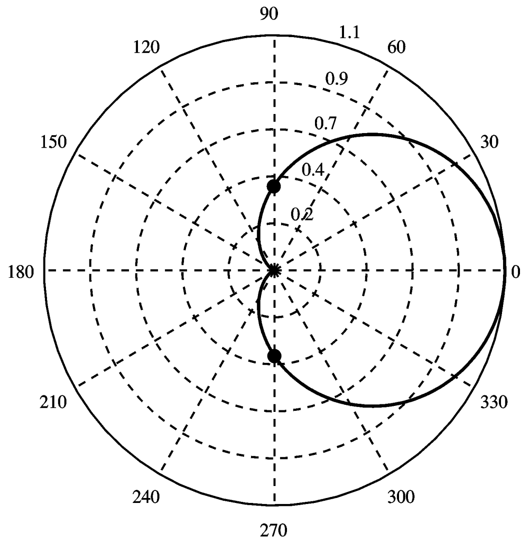


a
b
a
b
$\bar{\theta}$
δ_r
δ_s^{max}
θ
ψ
x

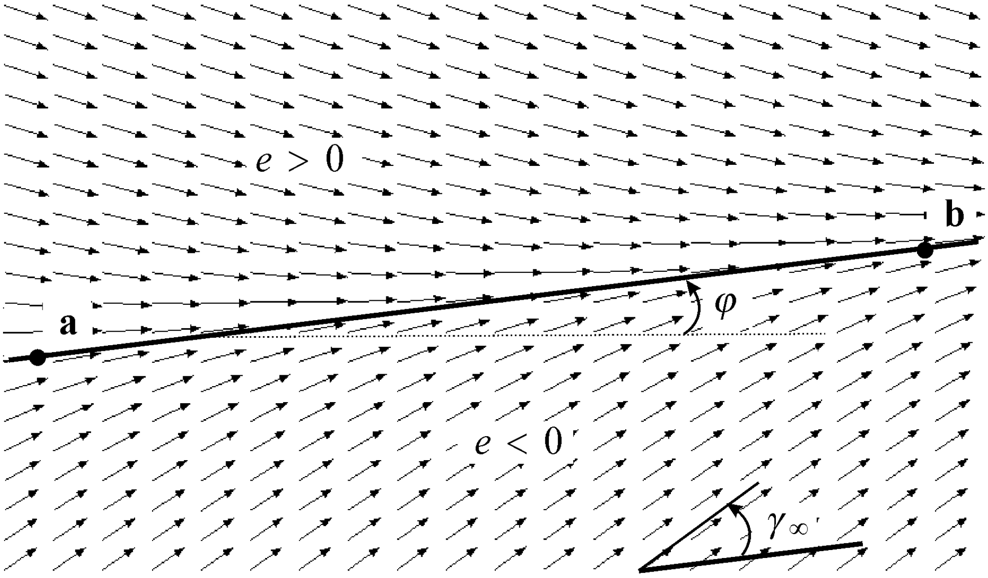
$$\left\{\begin{array}{lll} \delta_g & = & \left\{\begin{array}{ll} \delta_g^{\mathsf{max}}.\,\mathsf{sin}\left(\theta-\bar{\theta}\right) & \text{if } \mathsf{cos}\left(\theta-\bar{\theta}\right)\geq 0 \\ \delta_g^{\mathsf{max}}.\mathsf{sign}\left(\mathsf{sin}\left(\theta-\bar{\theta}\right)\right) & \text{otherwize} \end{array}\right. \\ \delta_s^{\mathsf{max}} & = & \frac{\pi}{2}.\left(\frac{\mathsf{cos}(\psi-\bar{\theta})+1}{2}\right) \end{array}\right.$$



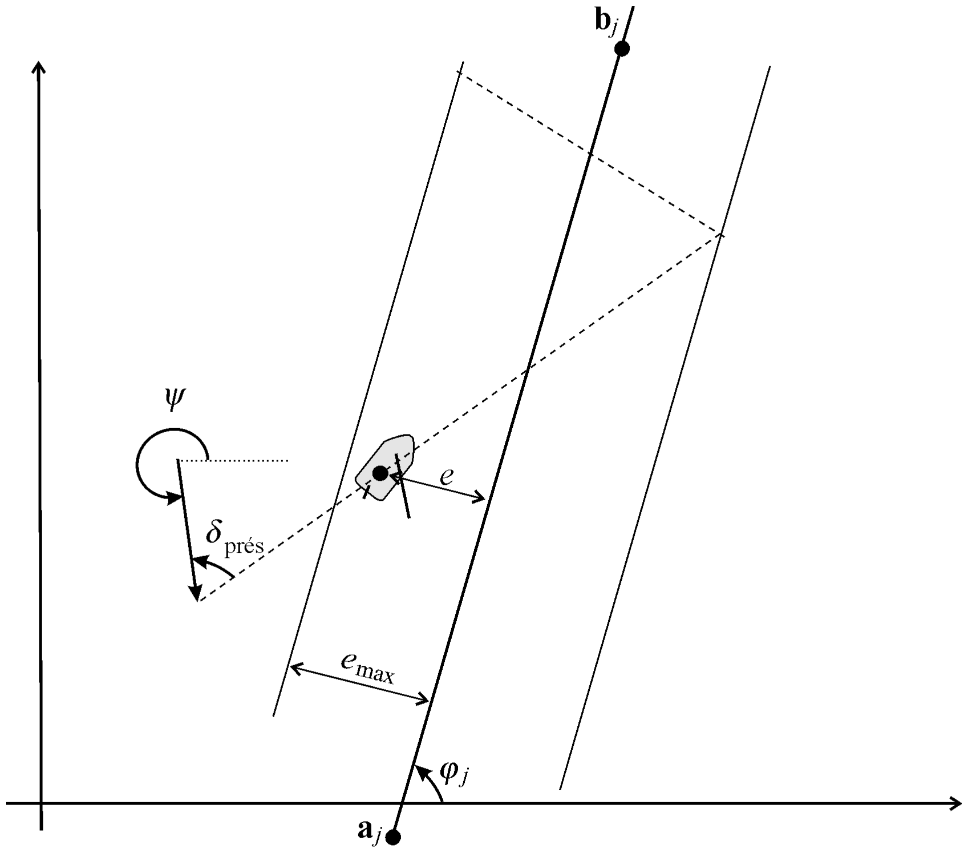
$\delta_{v\,\max}$
$\frac{\pi}{2}$
β
0
0
$\frac{\pi}{2}$
π
2π
$\psi - \theta$



90
1.1
60
120
0.9
0.7
30
150
0.4
0.2
180
0
210
330
240
300
270



$$\bar{\theta} = \varphi - \frac{2.\gamma_\infty}{\pi}.\mathrm{atan}\left(\frac{e}{r}\right)$$



$\mathbf{b}_j$
ψ
e
$\delta_{\text{prés}}$
$e_{\max}$
φ_j
$\mathbf{a}_j$

A course $\bar{\theta}$ is feasible if

$$\cos\left(\psi - \bar{\theta}\right) + \cos\zeta \geq 0.$$

Function $\bar{\theta}\left(\mathbf{x}, \mathbf{a}, \mathbf{b}, \psi, \gamma_{\infty}, r, \zeta\right)$

$e = \det\left(\frac{\mathbf{b}-\mathbf{a}}{\|\mathbf{b}-\mathbf{a}\|}, \mathbf{x}-\mathbf{a}\right)$

$\varphi = \text{atan2}(\mathbf{b}-\mathbf{a})$

$\theta^{*} = \varphi - \frac{2.\gamma_{\infty}}{\pi}.\text{atan}\left(\frac{e}{r}\right)$

if $\cos\left(\psi - \theta^{*}\right) + \cos\zeta < 0$

    or $(|e| < r$ and $(\cos(\psi - \varphi) + \cos(\zeta) < 0))$

      then $\bar{\theta} = \pi + \psi - \zeta.\text{sign}(e)$;

      else $\bar{\theta} = \theta^{*}$;

end

Choose a frame $\mathcal{R}(\mathbf{a}, \mathbf{i}, \mathbf{j})$ based on the line.

Function $\bar{\theta}(\mathbf{x}, \psi, \gamma_\infty, r, \zeta)$

$\theta^* = -\frac{2.\gamma_\infty}{\pi}.\text{atan}\left(\frac{x_2}{r}\right)$

if $\cos(\psi - \theta^*) + \cos\zeta < 0$

or $(|e| < r$ and $(\cos(\psi - \varphi) + \cos(\zeta) < 0))$

then $\bar{\theta} = \pi + \psi - \zeta.\text{sign}(x_2)$;

else $\bar{\theta} = \theta^*$;

end

$$\dot{\mathbf{x}} = \begin{pmatrix} \cos\bar{\theta}\left(\mathbf{x}, \psi\right) \\ \sin\bar{\theta}\left(\mathbf{x}, \psi\right) \end{pmatrix}$$

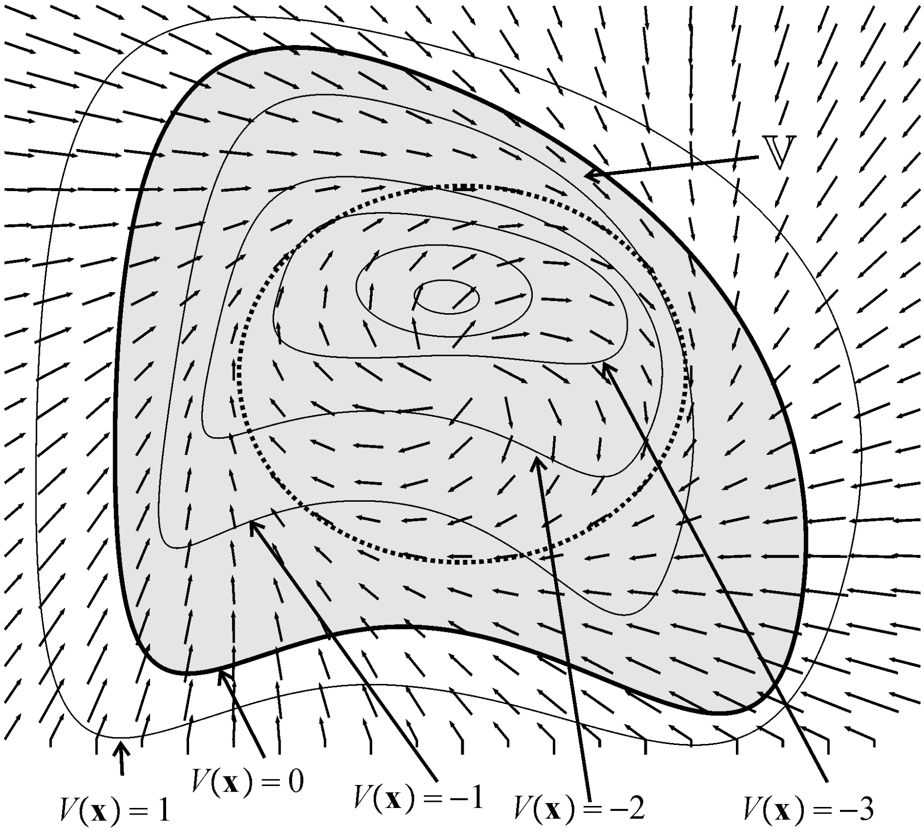
10 V-stability

Controlled robot:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}).$$

Definition. Consider a differentiable function $V(\mathbf{x})$. The system is V-stable if $\exists \varepsilon > 0$ such that

$$\left(V(\mathbf{x}) \geq 0 \ \Rightarrow \ \dot{V}(\mathbf{x}) \leq -\varepsilon \right).$$



𝕍
V(x) = 1
V(x) = 0
V(x) = −1
V(x) = −2
V(x) = −3

Theorem. If the system is V-stable then

(i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
(ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0$, $V(\mathbf{x}(t+\tau)) < 0$.

Now,

$$
\begin{array}{ll}
& \left(V(\mathbf{x}) \geq 0 \ \Rightarrow \ \dot{V}(\mathbf{x}) < -\varepsilon\right) \\
\Leftrightarrow & \left(V(\mathbf{x}) \geq 0 \Rightarrow \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) + \varepsilon < 0\right) \\
\Leftrightarrow & \forall \mathbf{x}, \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) + \varepsilon < 0 \text{ or } V(\mathbf{x}) < 0 \\
\Leftrightarrow & \forall \mathbf{x}, \min\left(\frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) . \mathbf{f}(\mathbf{x}) + \varepsilon, V(\mathbf{x})\right) < 0 \\
\Leftrightarrow & \forall \mathbf{x}, g_{\varepsilon}(\mathbf{x}) < 0 \\
\Leftrightarrow & g_{\varepsilon}^{-1}([0, \infty[) = \emptyset.
\end{array}
$$

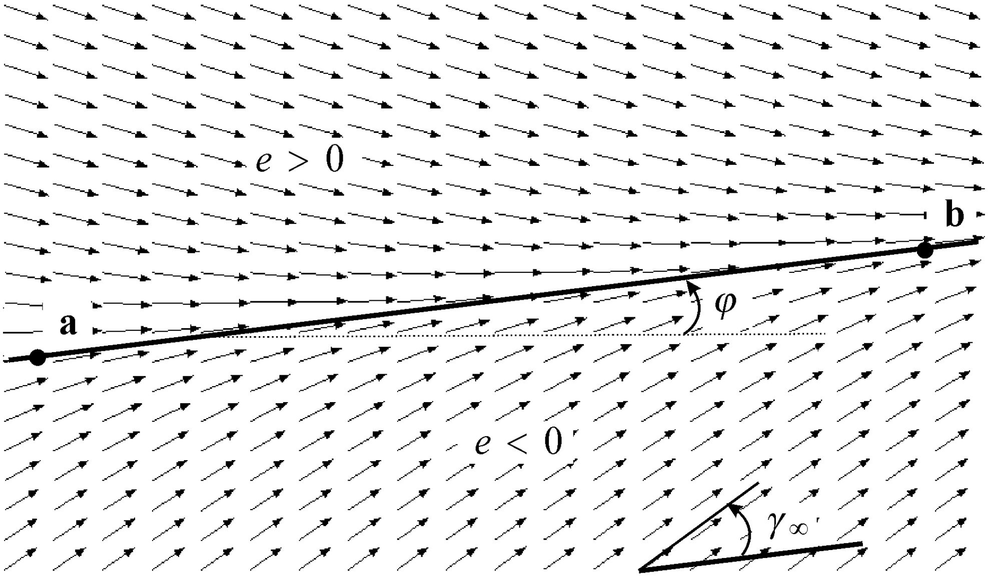
11 Robust case

The system also depends on a perturbation $\mathbf{w} \in \mathbb{W}$

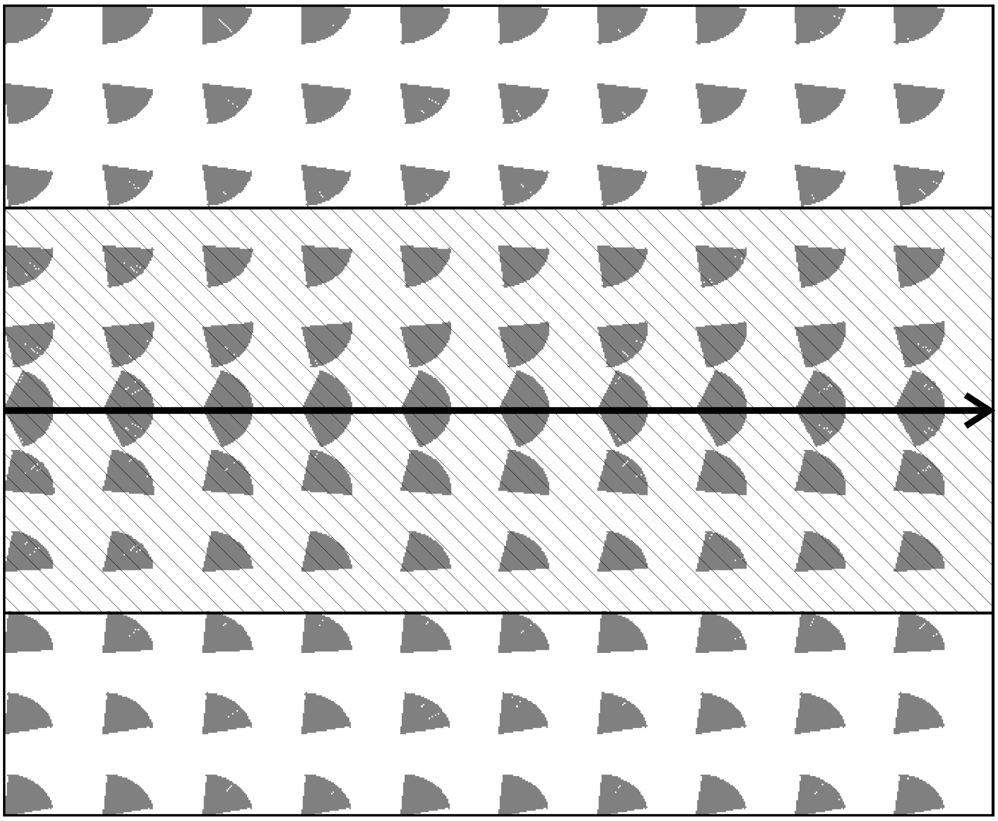
$$\mathcal{S}: \quad \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{w}).$$

Define

$$\mathbb{P} = \{\mathbf{p}, \forall \mathbf{w} \in \mathbb{W}, \mathcal{S} \text{ is } V\text{-stable}\}.$$



$e > 0$
b
a
φ
$e < 0$
γ_∞



Differential inclusion associated to *Vaimos*

12 Test-case

Assumption. The heading controller generates an actual heading of $\theta + w$ with $w \in \mathbb{W}$.

The system is

$$\dot{\mathbf{x}} = \begin{pmatrix} \cos\left(\bar{\theta} + w\right) \\ \sin\left(\bar{\theta} + w\right) \end{pmatrix}, \text{ with } \bar{\theta} = \bar{\theta}\left(\mathbf{x}, \psi, \gamma_{\infty}, r, \zeta\right).$$

For $\mathbb{W} = \pm 5^{\circ}$, $\zeta = \frac{\pi}{6}$rad, $\gamma_{\infty} = \frac{\pi}{8}$ we have

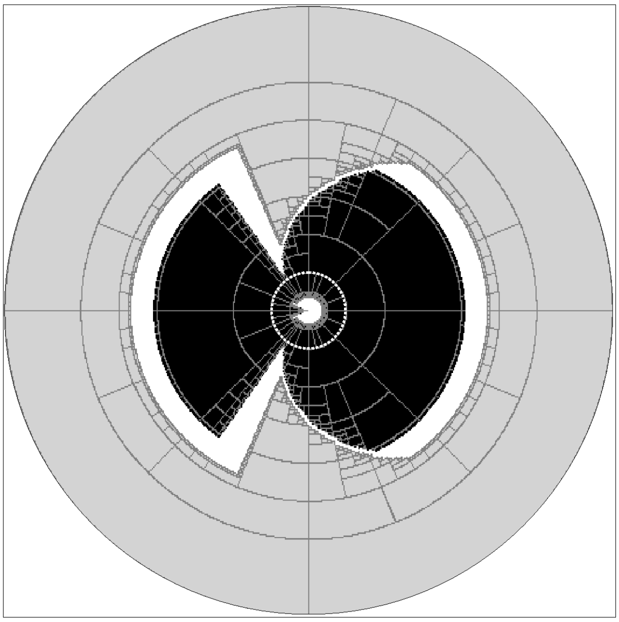
Property 1. If $|e(\mathbf{x})| < r_{\max}$ then, it will be the case for ever.

Property 2. If $|e(\mathbf{x})| > r_{\max}$ then $|e(\mathbf{x})|$ will decrease until $|e(\mathbf{x})| < r_{\max}$.

Property 3. The course should be feasible, i.e., $\cos\left(\psi - \bar{\theta}\right) + \cos\zeta \geq 0$.

Property 4. The robot always moves toward the right direction, i.e. $\dot{x}_1 > 0$.

Take $V(\mathbf{x}) = x_2^2 - r_{\text{max}}^2$, $\mathbb{W} = \pm 5^\circ = \pm 0.085$rad.



The parameter vector is $\mathbf{p} = (\gamma_\infty, \psi)$.

5th edition of the Small Workshop on Interval Methods
SWIM 2012
will be held on 4-6 June 2012 in Oldenburg, Germany
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