Exploration based on the electric sense

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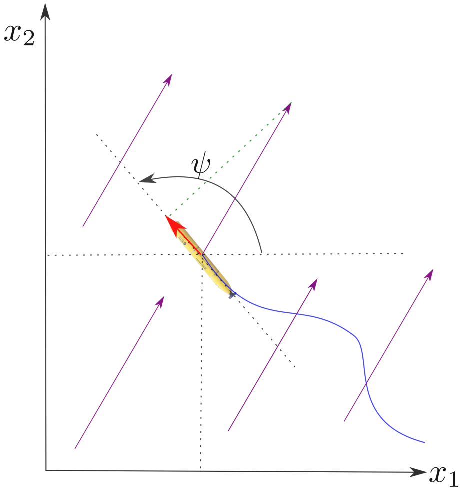
Réunion ANR Karst

1. Problem


x_2
ψ
x_1

$$\begin{cases} \dot{x}_1 & = & \cos\psi \\ \dot{x}_2 & = & \sin\psi \\ \dot{\psi} & = & u \end{cases}$$

Or more generally

$$\begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \end{pmatrix} = \mathbf{f}(\mathbf{p}, \mathbf{q}, \mathbf{u})$$

where $\mathbf{p} \in \mathbb{R}^2$ is the position vector.

We have a map

$$\varphi = \mathbf{m}(\mathbf{p})$$

Example 1

$$\varphi = \int_{\Omega} \eta(\mathbf{z} - \mathbf{p}) \cdot d\mathbf{z}$$

Example 2

$$\varphi = \begin{pmatrix} \cos \hat{\psi} \\ \sin \hat{\psi} \end{pmatrix}$$

where

$$\hat{\psi} = \underset{\psi}{\operatorname{argmax}} \; \eta(\mathbf{p}, \psi)$$

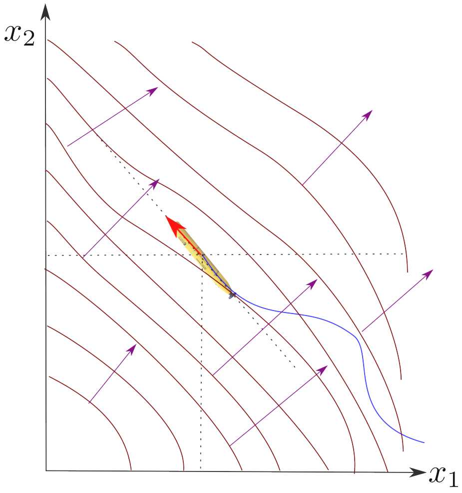
Observation equation

$$\begin{aligned}\mathbf{y} &= \mathbf{g}(\mathbf{p}, \mathbf{q}) \\ &= \mathbf{g}(\mathbf{m}(\mathbf{p}), \mathbf{q})\end{aligned}$$

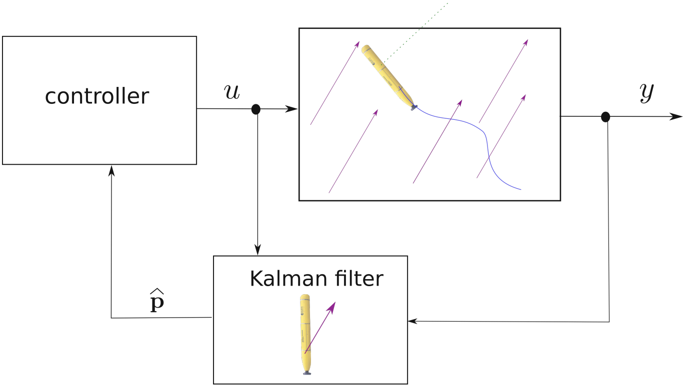
Example. Tensor-based observation

$$y = (\cos\psi \; \sin\psi) \cdot \mathbf{Q} \cdot \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

where $\mathbf{Q}$ is known.
With oscillations, φ can be estimated at the current position.


x_2
x_1

Use for navigation


controller
u
y
Kalman filter
$\widehat{\mathbf{p}}$

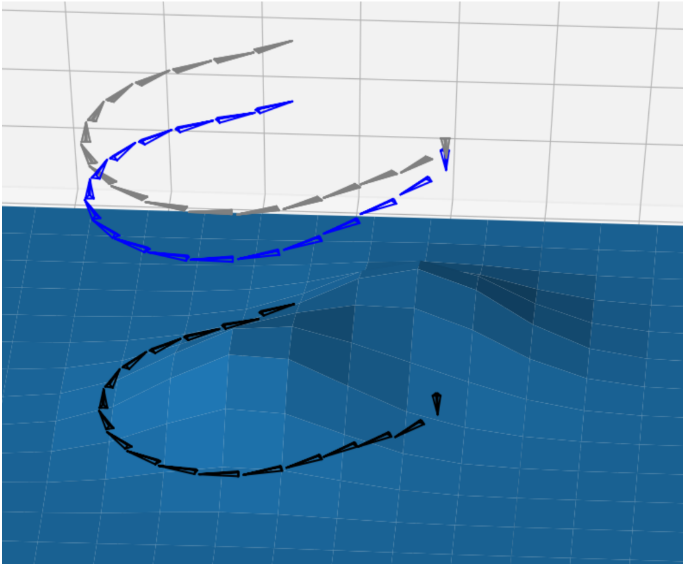
No loop constraint

$$\mathbf{m}(\mathbf{p}(t_1)) \neq \mathbf{m}(\mathbf{p}(t_2)) \Rightarrow \mathbf{p}(t_1) \neq \mathbf{p}(t_2)$$

Stable cycles

- Poincaré Bendixon (no chaos for φ in 2D)
- We can probably find stable cycles

Question: Do we have a potential to derive φ?



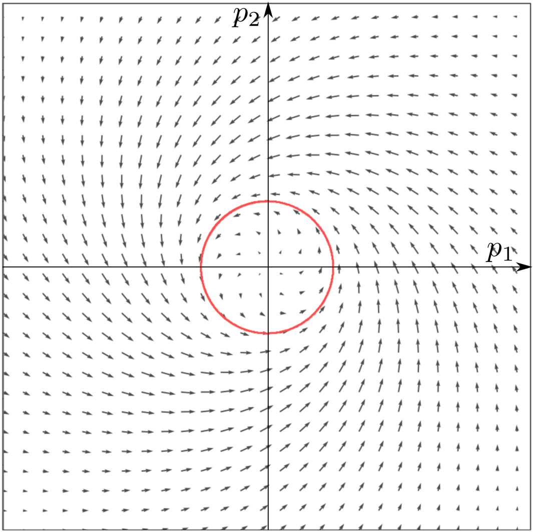
If yes, we can follow an isopotential forward and backward.
Otherwise, difficult to come back

2. Generating a map

We need to create electric like world

Do we have a realistic electric world simulator ?

$$\varphi_0(\mathbf{p}) = e^{-\sqrt{p_1^2+p_2^2}} \begin{pmatrix} p_2 & p_1 \\ -p_1 & p_2 \end{pmatrix} \begin{pmatrix} -\sqrt{p_1^2+p_2^2} \\ 1-\sqrt{p_1^2+p_2^2} \end{pmatrix}$$



Seed vector field

We take

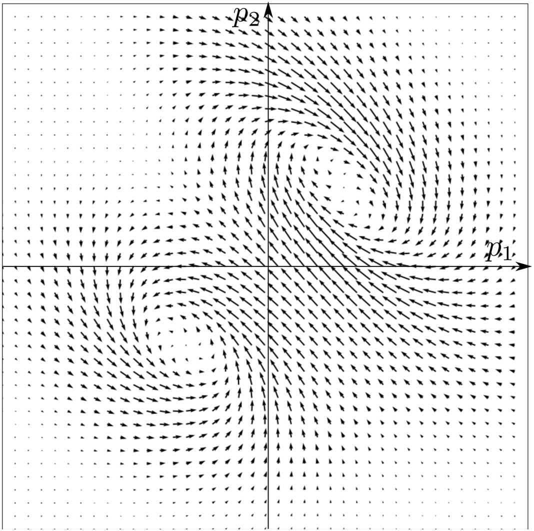
$$\varphi(\mathbf{p}) = \mathbf{A}_1 \cdot \varphi_0(\mathbf{A}_1^{-1} \cdot (\mathbf{p} - \mathbf{c}_1)) + \mathbf{A}_2 \cdot \varphi_0(\mathbf{A}_2^{-1} \cdot (\mathbf{p} - \mathbf{c}_2))$$

where

$$\mathbf{A}_i = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix} \cdot \begin{pmatrix} s_{i1} & 0 \\ 0 & s_{i2} \end{pmatrix}$$

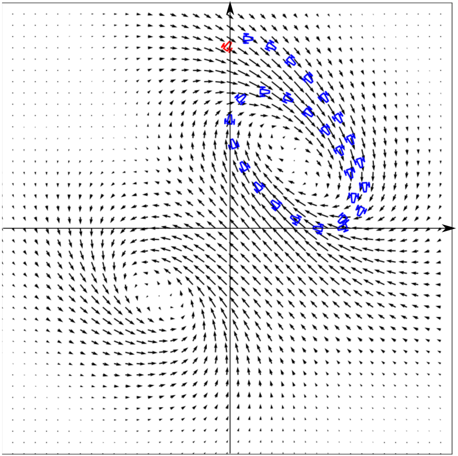
and

$$\begin{array}{lll} \mathbf{s}_1 = (2, -1)^\mathsf{T}; & \theta_1 = -\frac{\pi}{4} & ; \mathbf{c}_1 = (3,3)^\mathsf{T} \\ \mathbf{s}_2 = (1, 1.5)^\mathsf{T}; & \theta_2 = \frac{\pi}{4} & ; \mathbf{c}_2 = (-3,-3)^\mathsf{T} \end{array}$$

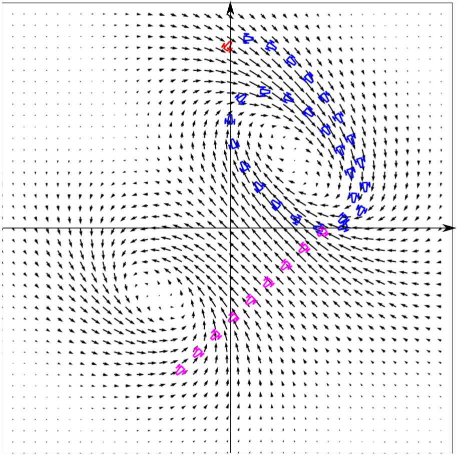


Vector field of the environment

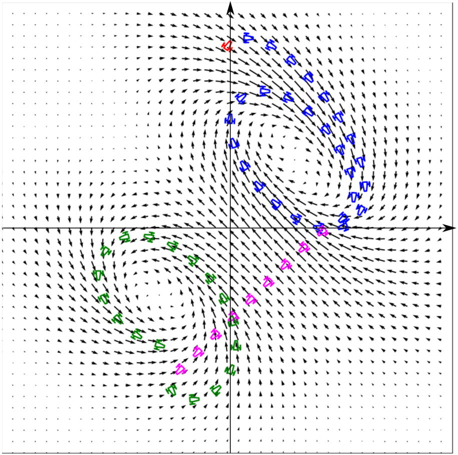
3. Exploration



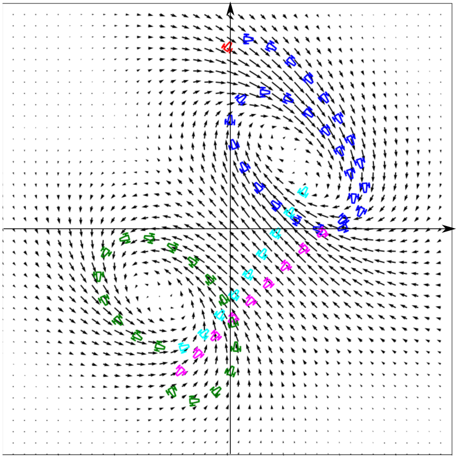
The robot follows the vector field and converges toward an attractor



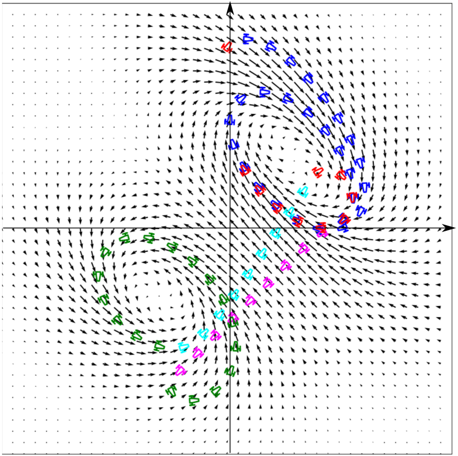
For exploration, the car goes randomly to a given heading



It searches and finds another stable cycle $\mathbb{C}_2$.
It follows the field and find a new cycle.



The robot comes back home



It follows its initial cycle.

The robot was never lost.