

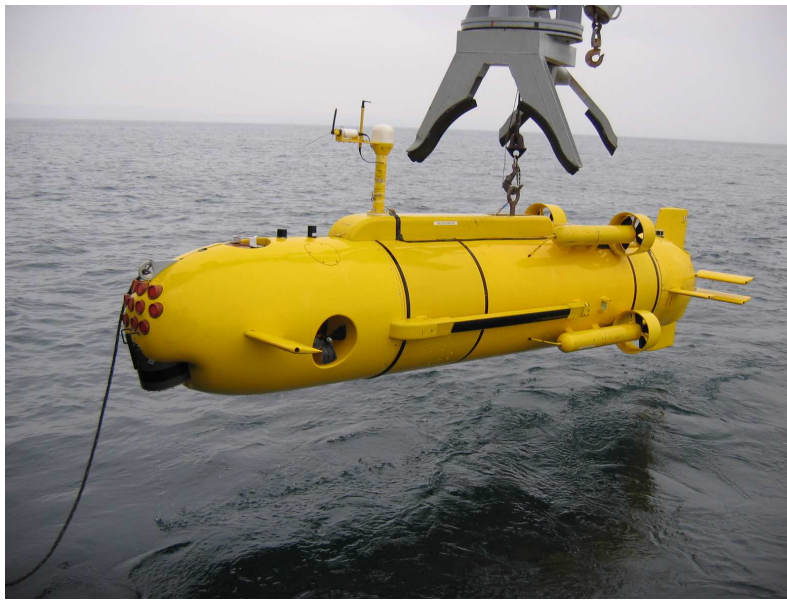
Calcul par intervalles pour la robotique mobile

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ITT, 28 octobre 2009

1 Redermor



The *Redermor*, GESMA



The *Redermor* at the surface

Show simulation

Why choosing an interval constraint approach for SLAM ?

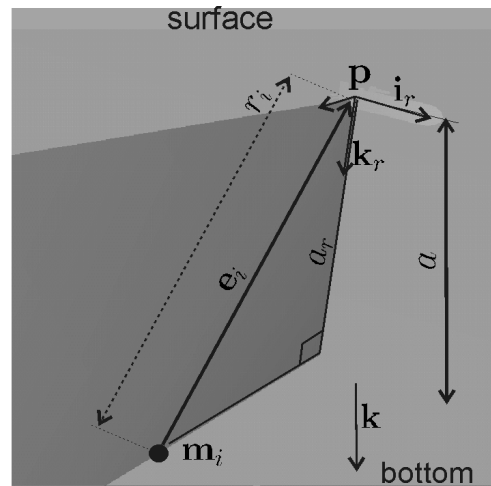
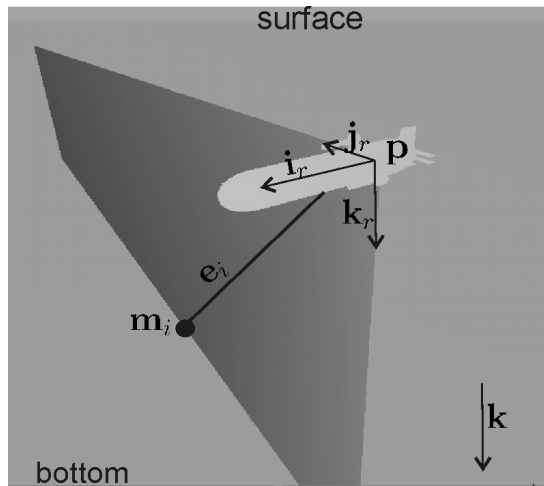
- 1) A reliable method is needed.
- 2) The model is nonlinear.
- 3) The pdf of the noises are unknown.
- 4) Reliable error bounds are provided by the sensors.
- 5) A huge number of redundant data are available.

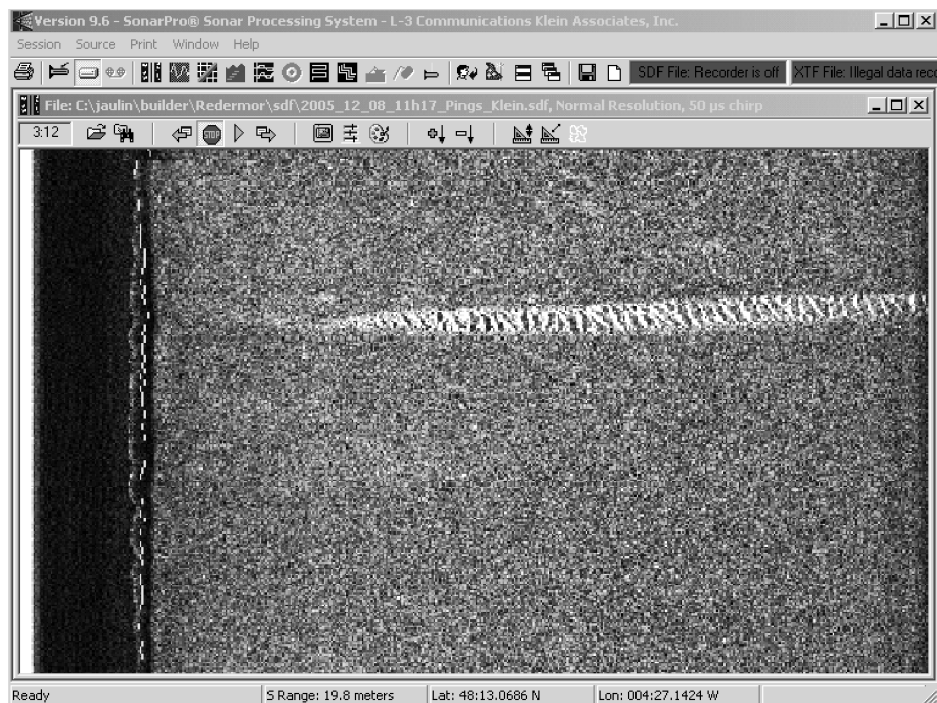
1.1 **Sensors**

A GPS (Global positioning system) at the surface only.

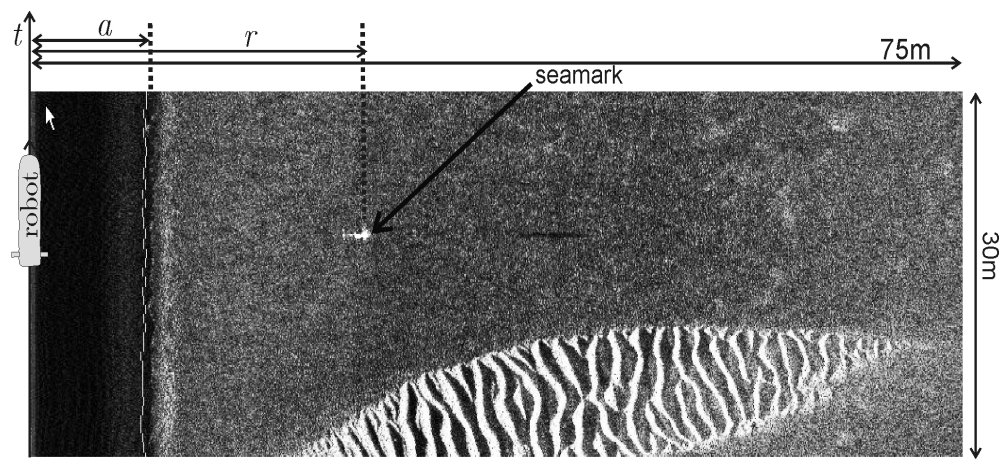
$$\begin{aligned} t_0 &= 6000 \text{ s}, & \ell^0 &= (-4.4582279^\circ, 48.2129206^\circ) \pm 2.5m \\ t_f &= 12000 \text{ s}, & \ell^f &= (-4.4546607^\circ, 48.2191297^\circ) \pm 2.5m \end{aligned}$$

A sonar (KLEIN 5400 side scan sonar). Gives the distance r between the robot to the detected object.





Screenshot of SonarPro



Detection of a mine using SonarPro

A Loch-Doppler. Returns the speed of the robot v_r and the altitude a of the robot $\pm 10\text{cm}$.

A Gyrocompass (Octans III from IXSEA). Returns the roll ϕ , the pitch θ and the head ψ .

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} \in \begin{pmatrix} \tilde{\phi} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} + \begin{pmatrix} 1.75 \times 10^{-4} \cdot [-1, 1] \\ 1.75 \times 10^{-4} \cdot [-1, 1] \\ 5.27 \times 10^{-3} \cdot [-1, 1] \end{pmatrix}.$$

1.2 Data

For each time $t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\}$, we get intervals for

$$\phi(t), \theta(t), \psi(t), v_r^x(t), v_r^y(t), v_r^z(t), a(t).$$

Six mines have been detected by the sonar:

i	0	1	2	3	4	5
$\tau(i)$	7054	7092	7374	7748	9038	9688
$\sigma(i)$	1	2	1	0	1	5
$\tilde{r}(i)$	52.42	12.47	54.40	52.68	27.73	26.98

6	7	8	9	10	11
10024	10817	11172	11232	11279	11688
4	3	3	4	5	1
37.90	36.71	37.37	31.03	33.51	15.05

1.3 Constraints satisfaction problem

$$t \in \{6000.0, 6000.1, 6000.2, \dots, 11999.4\},$$

$$i \in \{0, 1, \dots, 11\},$$

$$\begin{pmatrix} p_x(t) \\ p_y(t) \end{pmatrix} = 111120 \begin{pmatrix} 0 & 1 \\ \cos\left(\ell_y(t) * \frac{\pi}{180}\right) & 0 \end{pmatrix} \begin{pmatrix} \ell_x(t) - \ell_x^0 \\ \ell_y(t) - \ell_y^0 \end{pmatrix},$$

$$\mathbf{p}(t) = (p_x(t), p_y(t), p_z(t)),$$

$$\mathbf{R}_\psi(t) = \begin{pmatrix} \cos \psi(t) & -\sin \psi(t) & 0 \\ \sin \psi(t) & \cos \psi(t) & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mathbf{R}_\theta(t) = \begin{pmatrix} \cos \theta(t) & 0 & \sin \theta(t) \\ 0 & 1 & 0 \\ -\sin \theta(t) & 0 & \cos \theta(t) \end{pmatrix},$$

$$\mathbf{R}_\varphi(t)=\left(\begin{array}{ccc}1&0&0\\0&\cos\varphi(t)&-\sin\varphi(t)\\0&\sin\varphi(t)&\cos\varphi(t)\end{array}\right),$$

$$\mathbf{R}(t)=\mathbf{R}_\psi(t).\mathbf{R}_\theta(t).\mathbf{R}_\varphi(t),$$

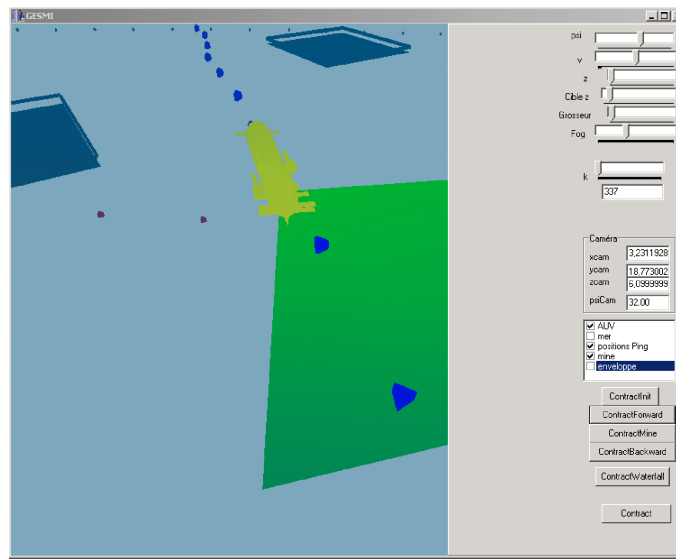
$$\dot{\mathbf{p}}(t)=\mathbf{R}(t).\mathbf{v}_r(t)$$

$$||\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))||\;=\;r(i),$$

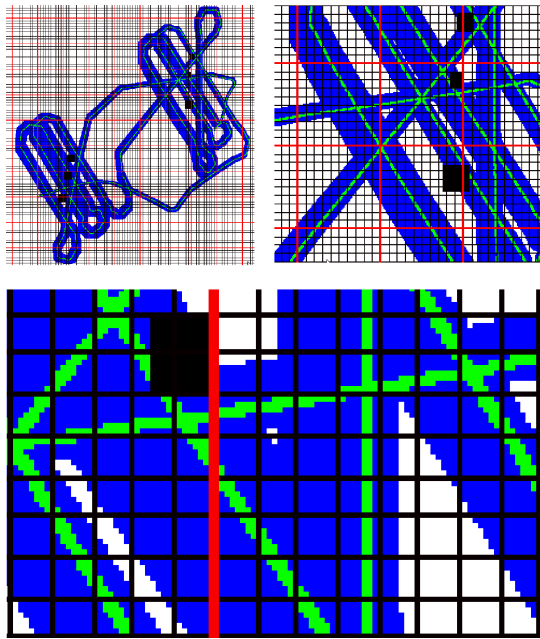
$$\mathbf{R}^\top(\tau(i))\left(\mathbf{m}(\sigma(i))-\mathbf{p}(\tau(i))\right)\in [0]\times [0,\infty]^{\times 2},$$

$$m_z(\sigma(i))-p_z(\tau(i))-a(\tau(i))\in [-0.5,0.5].$$

1.4 GESMI

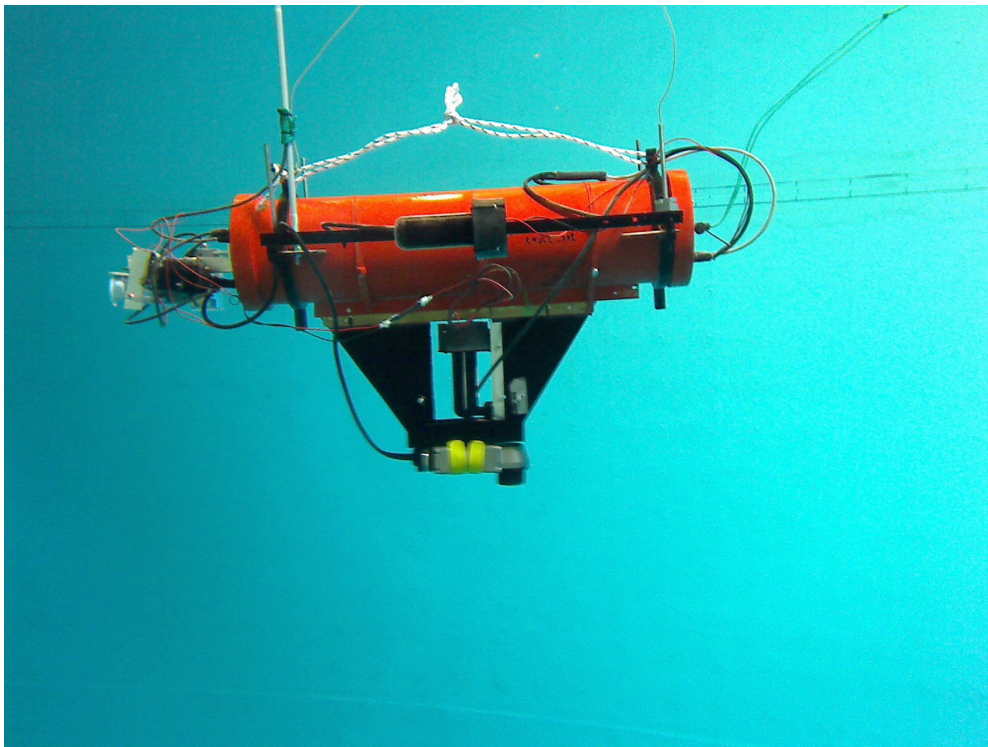


GESMI (Guaranteed Estimation of Sea Mines with Intervals)



Trajectory reconstructed by GESMI

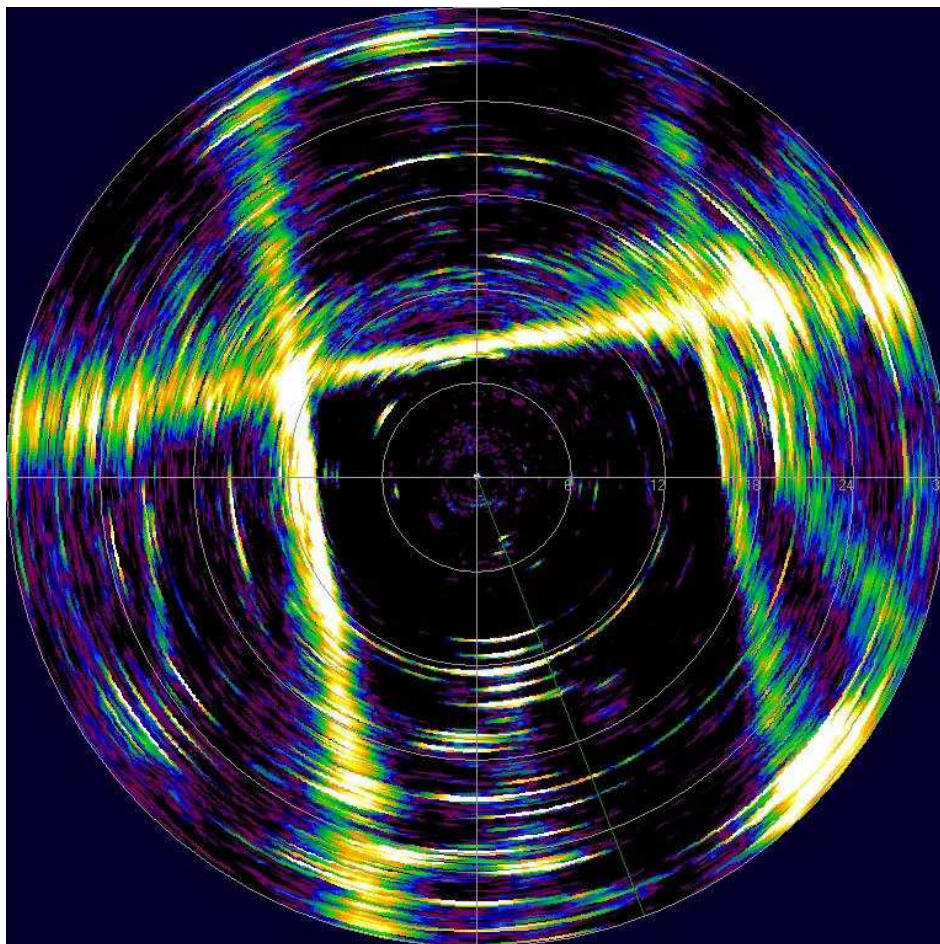
2 SAUC'ISSE



Robot SAUC'ISSE

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2.1 Localisation with sonar



2.2 Set-membership approach

$$\mathbf{y} = \psi(\mathbf{p}) + \mathbf{e},$$

where

- $\mathbf{e} \in \mathbb{E} \subset \mathbb{R}^m$ is the error vector,
- $\mathbf{y} \in \mathbb{R}^m$ is the data vector,
- $\mathbf{p} \in \mathbb{R}^n$ is the parameter vector.

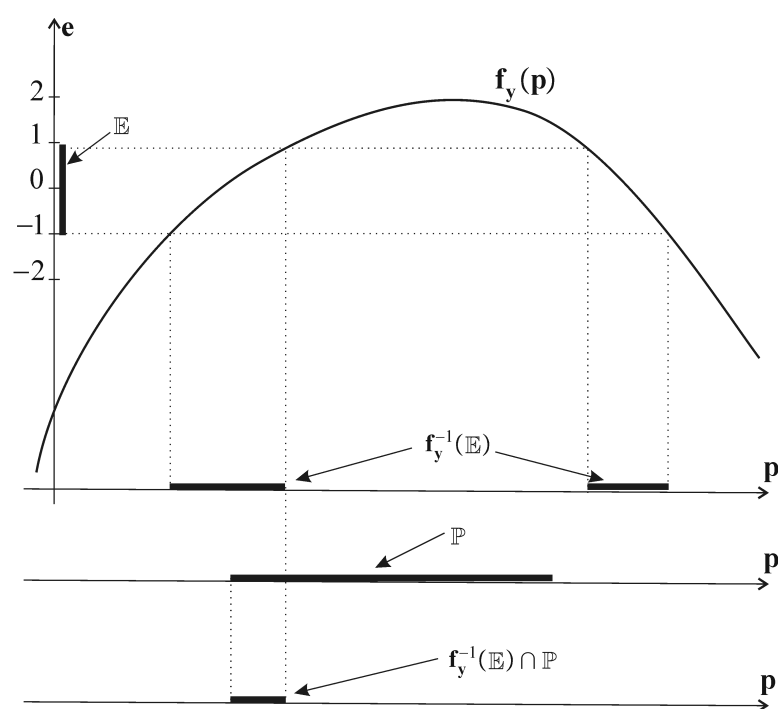
Show setdemo

The equation $\mathbf{y} = \psi(\mathbf{p}) + \mathbf{e}$, can be rewritten

$$\mathbf{e} = \underbrace{\mathbf{y} - \psi(\mathbf{p})}_{\mathbf{f}_y(\mathbf{p})}.$$

The *posterior feasible set* is

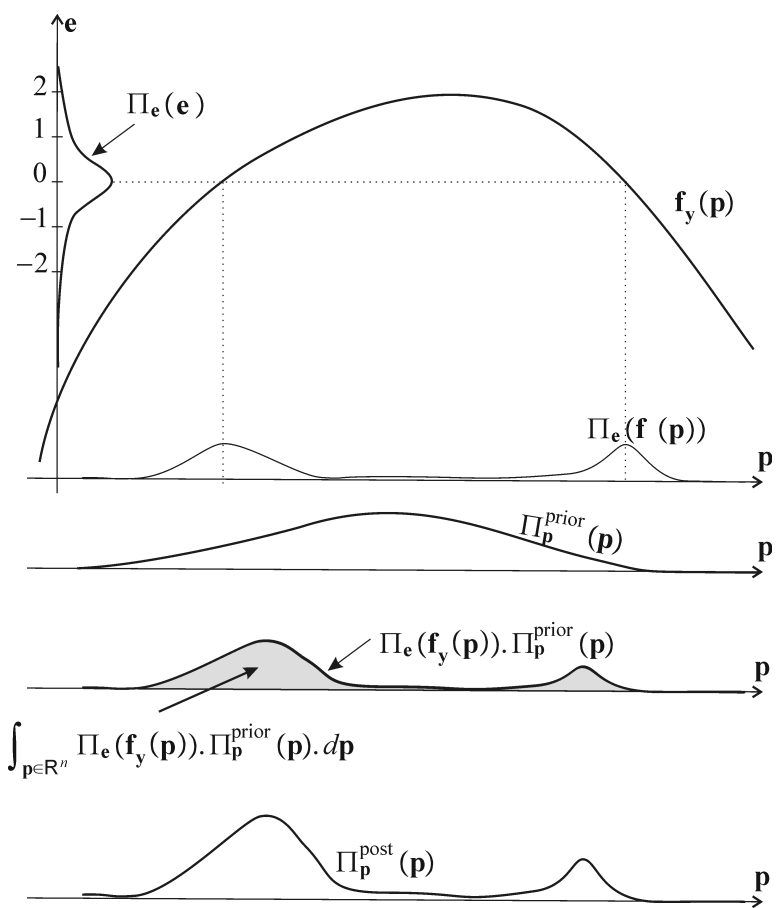
$$\hat{\mathbb{P}} = \mathbf{f}_{\mathbf{y}}^{-1}(\mathbb{E}) \cap \mathbb{P}.$$



In a Bayesian approach, prior pdf $\Pi_e, \Pi_p^{\text{prior}}$ are available for e, p .

The Bayes rule gives is the posterior pdf for p

$$\Pi_p^{\text{post}}(p) = \frac{\Pi_e(f_y(p)) \cdot \Pi_p^{\text{prior}}(p)}{\int_{p \in \mathbb{R}^n} \Pi_e(f_y(p)) \cdot \Pi_p^{\text{prior}}(p) \cdot dp}.$$



2.3 Computing with sets is easy !

$$[-1, 3] + [3, 5] = [2, 8]$$

$$[-1, 3] * [3, 5] = [-5, 15]$$

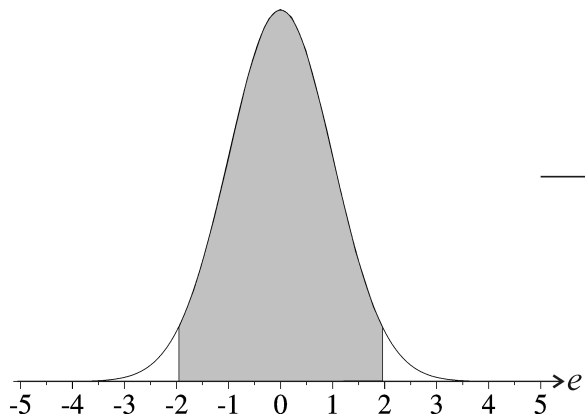
$$\sin([-1, 3]) = [\sin(-1), 1]$$

We can also solve nonlinear equations such as

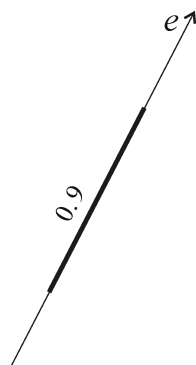
$$x.y.\sin(y\sqrt{x-y}+x)=1$$

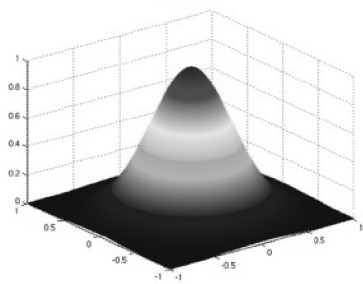
2.4 Error bounding

From Π_e , we can find some bounds for e ..

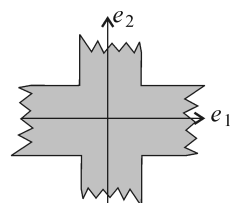
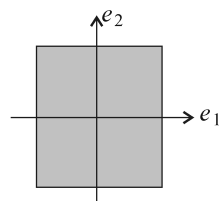
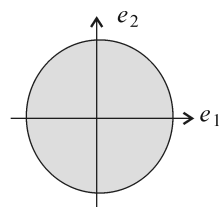


bornage



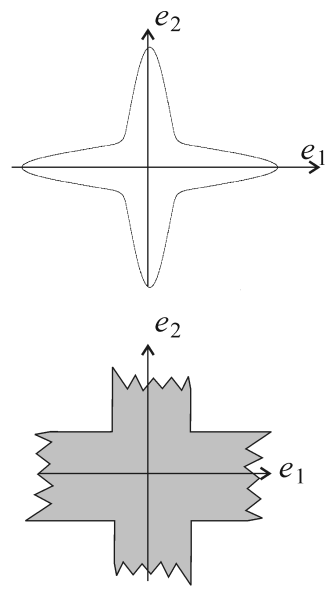


bornage →



$$\Pi(\mathbf{e}) \propto \left(\exp(-e_1^2) + \exp\left(-\frac{e_1^2}{10}\right) \right) \\ * \left(\exp(-e_2^2) + \exp\left(-\frac{e_2^2}{10}\right) \right)$$

bornage $\longrightarrow$



For $m = 100$ data, with $\pi = \Pr(e_k \in [e_k]) = 0.6$, the probability to have less than $q = 60$ outliers, is

$$\sum_{k=0}^{m-q-1} \frac{m!}{k! (m-k)!} \pi^k \cdot (1-\pi)^{m-k} = 0.99998.$$

2.5 State observer

$$\begin{cases} \mathbf{x}(k+1) &= \mathbf{f}_k(\mathbf{x}(k), \mathbf{n}(k)) \\ \mathbf{y}(k) &= \mathbf{g}_k(\mathbf{x}(k)), \end{cases}$$

where $\mathbf{n}(k) \in \mathbb{N}(k)$ and $\mathbf{y}(k) \in \mathbb{Y}(k)$.

If $\mathbb{X}(k)$ is the set of all $\mathbf{x}$ consistent with the following assumptions

(i) within all past time windows of length m , there is less than q outliers

(ii) $\mathbb{X}(0)$ contains $\mathbf{x}(0)$

Theorem.

$$\Pr(\mathbf{x}(k) \in \mathbb{X}(k)) \geq \alpha * \Pr(\mathbf{x}(k-1) \in \mathbb{X}(k-1))$$

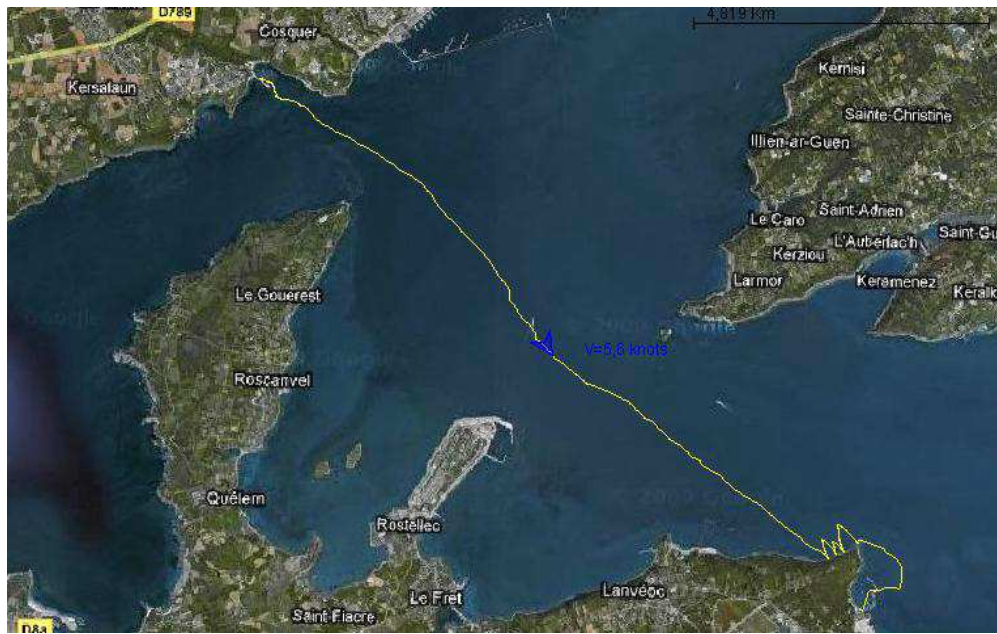
with

$$\alpha = \sqrt[m]{\sum_{i=m-q}^m \frac{m! \pi_y^i \cdot (1 - \pi_y)^{m-i}}{i! (m-i)!}}$$

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3 Breizh Spirit

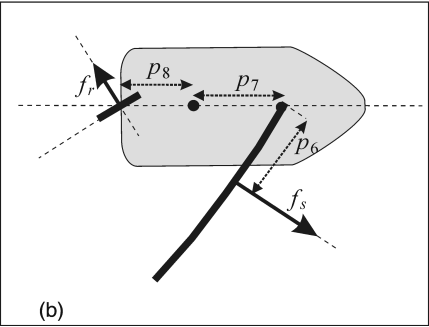
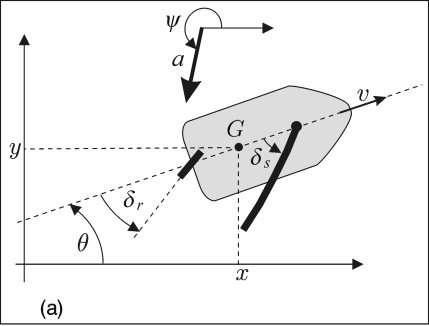




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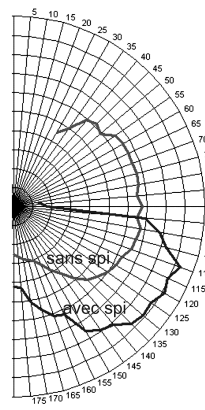
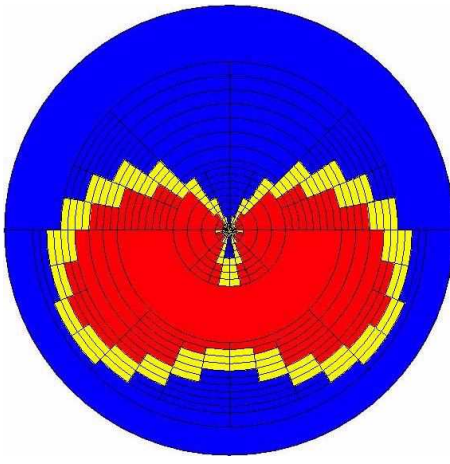
3.1 Normalized State equations

$$\left\{ \begin{array}{lcl} \dot{x} & = & v \cos \theta + a \cos \psi \\ \dot{y} & = & v \sin \theta + a \sin \psi \\ \dot{\theta} & = & \omega \\ \dot{v} & = & f_s \cdot \sin \delta_s - f_r \cdot \sin u_1 - v \\ \dot{\omega} & = & f_s \cdot (1 - \cos \delta_s) - f_r \cdot \cos u_1 - \omega \\ f_s & = & a \sin (\theta - \psi + \delta_s) \\ f_r & = & v \sin u_1 \\ \gamma & = & \cos (\theta - \psi) + \cos (u_2) \\ \delta_s & = & \left\{ \begin{array}{ll} \pi - \theta + \psi & \text{if } \gamma \leq 0 \\ \text{sign}(\sin (\theta - \psi)) \cdot u_2 & \text{otherwise.} \end{array} \right. \end{array} \right.$$

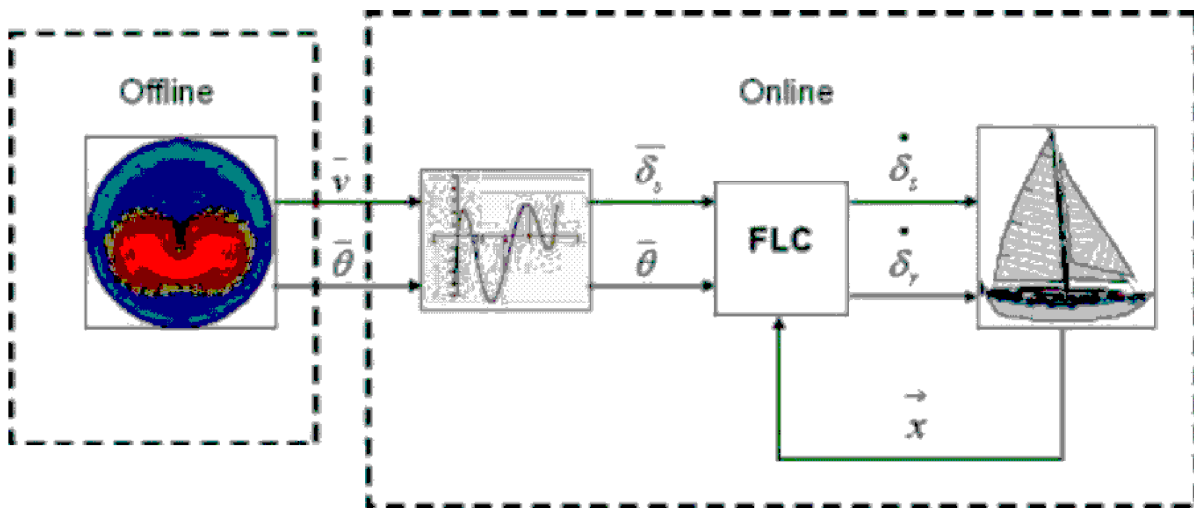


3.2 Polar speed diagram

$$\mathbb{W} = \{ (\theta, v) \mid \begin{array}{l} \exists(\omega, u_1, u_2, f_s, f_r, \delta_r, \delta_s) \\ \omega = 0, u_1 = 0, u_2 = 0 \\ f_s \sin \delta_s - f_r \sin \delta_r - v = 0 \\ (1 - \cos \delta_s) f_s - \cos \delta_r f_r = 0 \\ f_s = a \cos(\theta + \delta_s) - v \sin \delta_s \\ f_r = v \sin \delta_r \end{array} \}.$$



3.3 Control



From the state equations of the sailboat, it is easy to check that

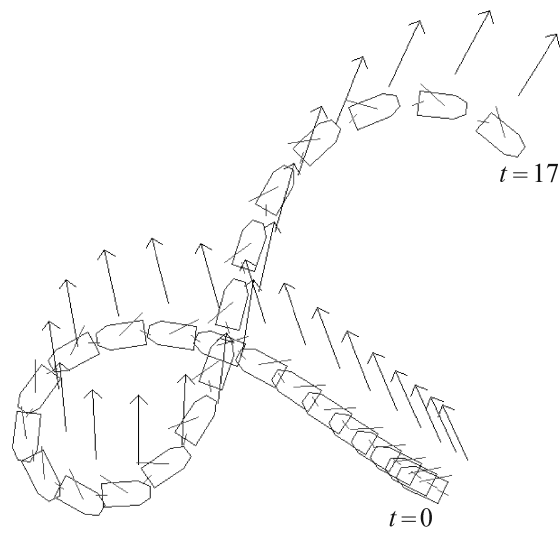
$$\underbrace{\begin{pmatrix} \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{pmatrix}}_{\mathbf{z}} = \Psi_t \underbrace{\begin{pmatrix} \theta \\ v \\ \omega \\ a \\ \psi \end{pmatrix}}_{\mathbf{x}}$$

with

$$\Psi_t(\mathbf{x}) = \begin{pmatrix} \theta \\ v \sin \theta + a \sin \psi \\ v \sin \theta + a \sin \psi \\ \omega \\ (f_s \sin \delta_s - f_r \sin u_1 - v) \cos \theta - \omega v \sin \theta \\ (f_s \sin \delta_s - f_r \sin u_1 - v) \sin \theta + \omega v \cos \theta \\ f_s (1 - \cos \delta_s) - f_r \cos u_1 - \omega \end{pmatrix}$$

and

$$\left\{ \begin{array}{lcl} f_s(\mathbf{x}) & = & a \sin(\theta - \psi + \delta_s) \\ f_r(\mathbf{x}, t) & = & v \sin u_1 \\ \delta_s(\mathbf{x}, t) & = & \begin{cases} \pi - \theta + \psi & \text{if } \gamma(\mathbf{x}, t) \leq 0 \\ \text{sign}(\sin(\theta - \psi)) \cdot u_2 & \text{otherwise} \end{cases} \\ \gamma(\mathbf{x}, t) & = & \cos(\theta - \psi) + \cos(u_2) . \end{array} \right.$$



Simulated experiment