Computing Capture Sets

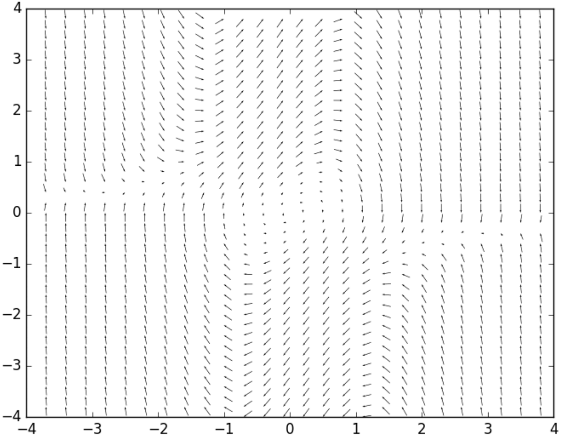
T. Le Mézo, L. Jaulin, B. Zerr

Capture set

We consider a state equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$.

Example: The Van der Pol system

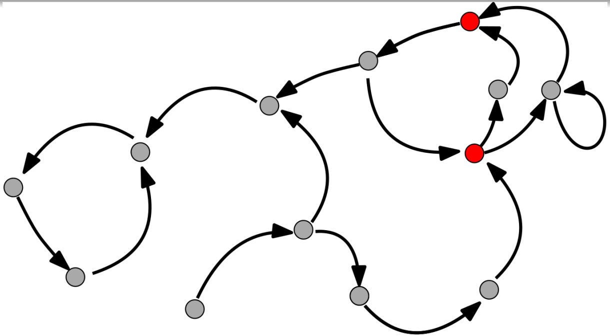
$$\begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & \left(1 - x_1^2\right) \cdot x_2 - x_1 \end{cases}$$


4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4

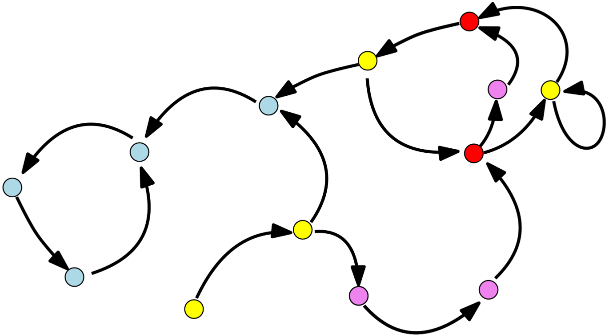
Let φ be the flow map.
The *capture* set of the *target* $\mathbb{T} \subset \mathbb{R}^n$ is:

$$\overleftarrow{\mathbb{T}} = \{\mathbf{x}_0 \mid \exists t \geq 0, \varphi(t, \mathbf{x}_0) \in \mathbb{T}\}.$$

Graph analogy

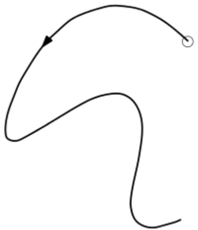


Using a backward method, we can enclose $\overleftarrow{\mathbb{T}}$.

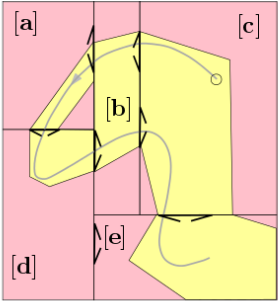


Maze

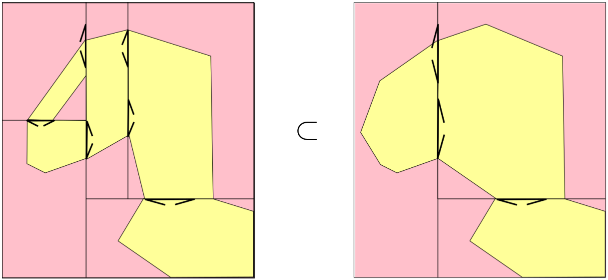
A maze



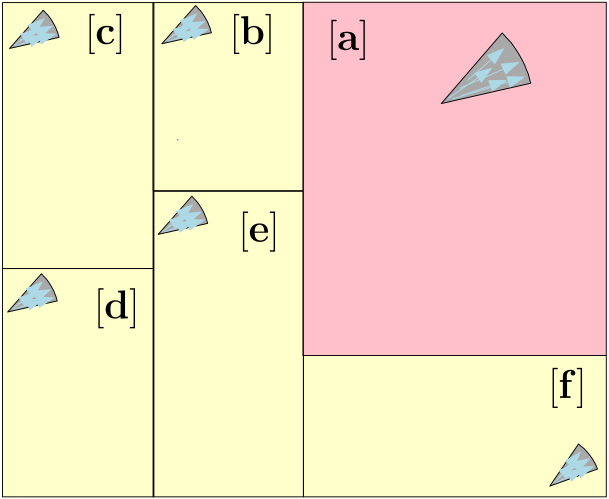
$\in$

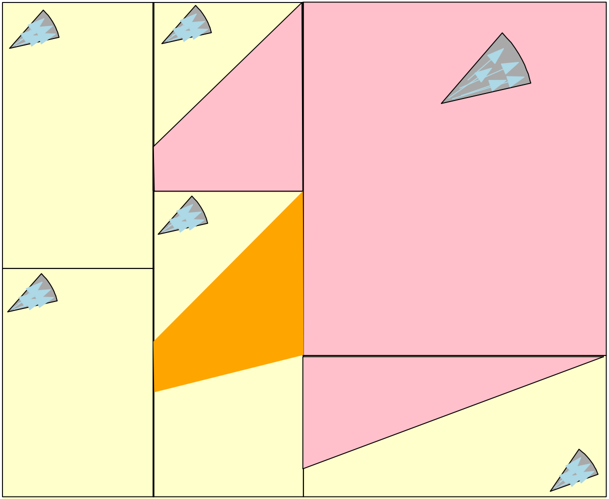


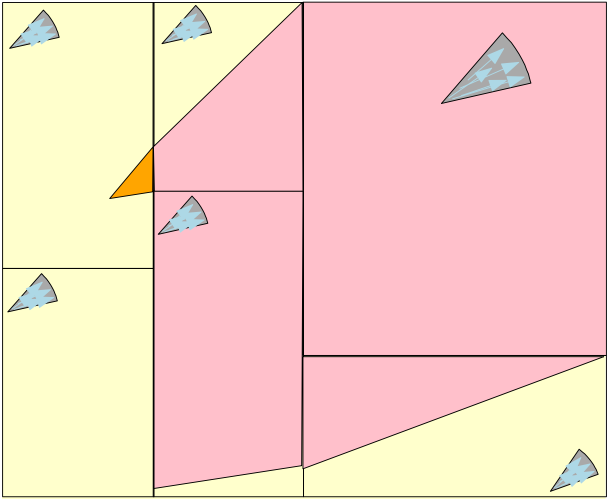
The left maze contains less paths than the right maze.

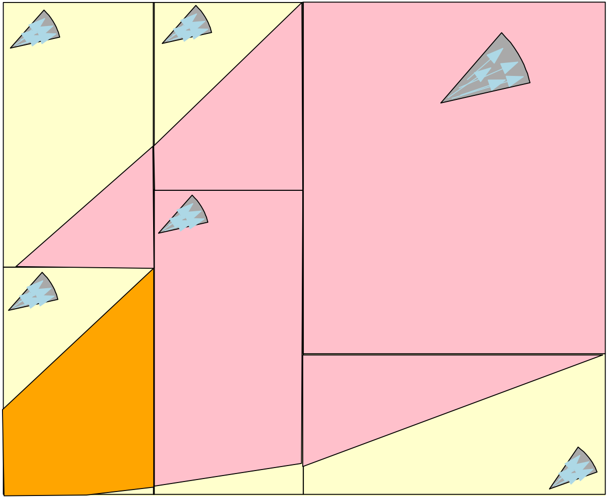


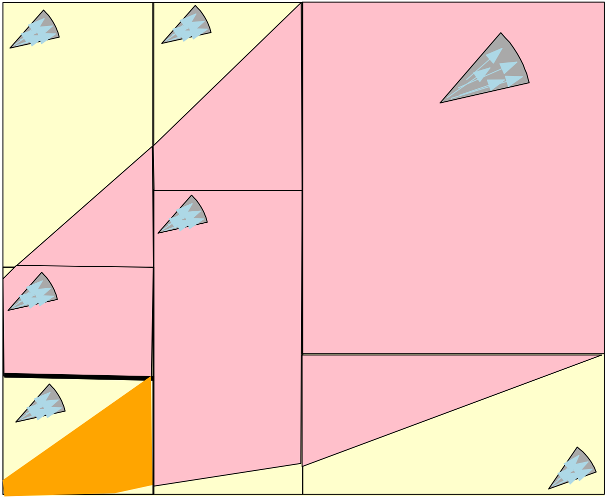
Inner approximation of $\overleftarrow{\mathbb{T}}$

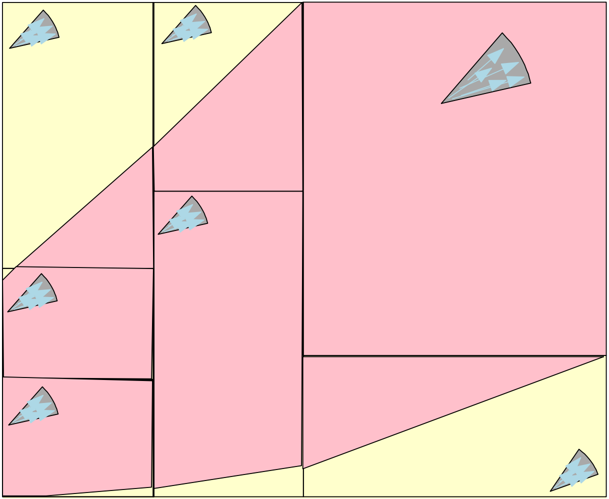

[c]
[b]
[a]
[e]
[d]
[f]



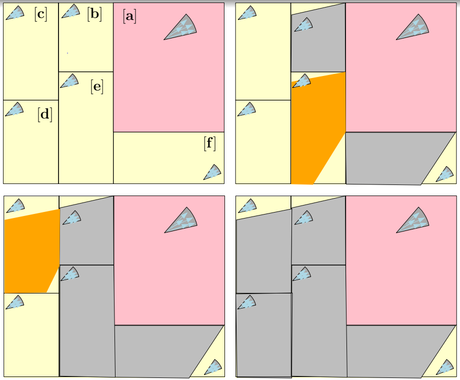








Outer propagation

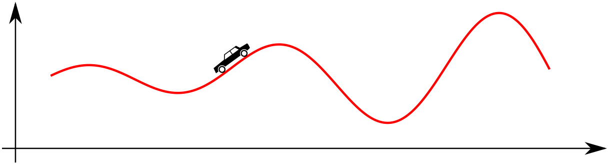


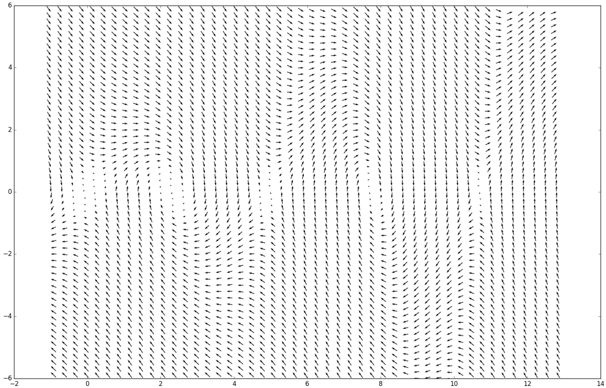
An interpretation can be given only when the fixed point is reached.

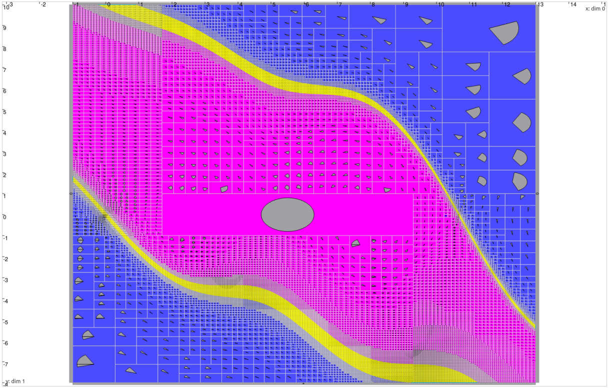
Applications

Car on the hill

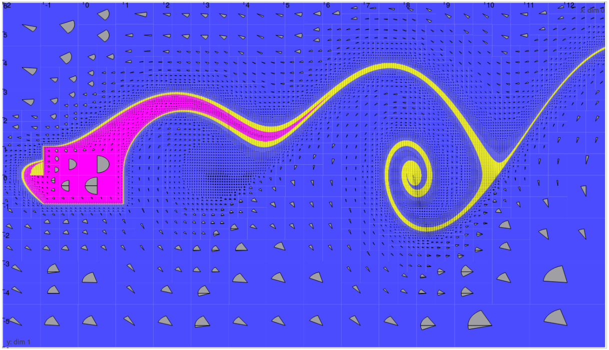
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 9.81 \sin\left(\frac{11}{24} \cdot \sin x_1 + 0.6 \cdot \sin(1.1 \cdot x_1)\right) - 0.7 \cdot x_2 \end{cases}$$




6
4
2
0
−2
−4
−6
−2
0
2
4
6
8
10
12
14


x: dim 0
y: dim 1

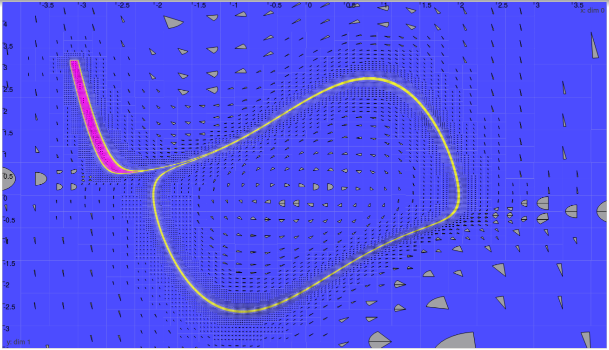
Combined with an outer propagation



Van der Pol system

Consider the system

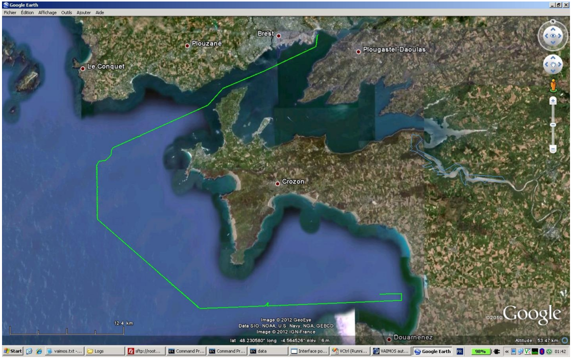
$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \left(1 - x_1^2\right) \cdot x_2 - x_1 \end{cases}$$


x: dim 0
y: dim 1

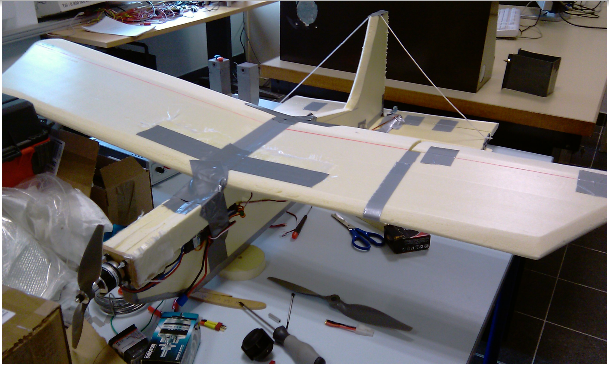
Vaimos (ENSTA-Ifremer)

Brest-Douarnenez. January 17, 2012


miniji
F78
F78

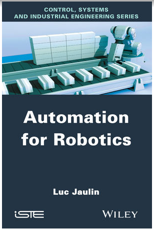

Google Earth
Fichier Édition Affichage Outils Ajouter Aide
Brest
Plouzané
Le Conquet
Plougastel-Daoulas
Crozon
Douarnenez
12.4 km
Image © 2012 GeoEye
Data SIO, NOAA, U.S. Navy, NGA, GEBCO
Image © 2012 IGN-France
©2010 Google

Avions

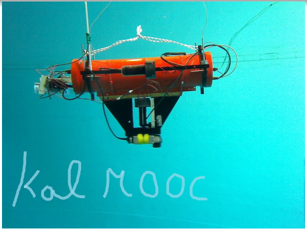


Enseignements


Mobile
Robotics
Luc Jaulin
iSTE


CONTROL, SYSTEMS
AND INDUSTRIAL ENGINEERING SERIES
Automation
for Robotics
Luc Jaulin
iSTE
WILEY

MOOC


Kal Mooc


Rob Mooc

Club robotique