Detection and capture of intruders using robots

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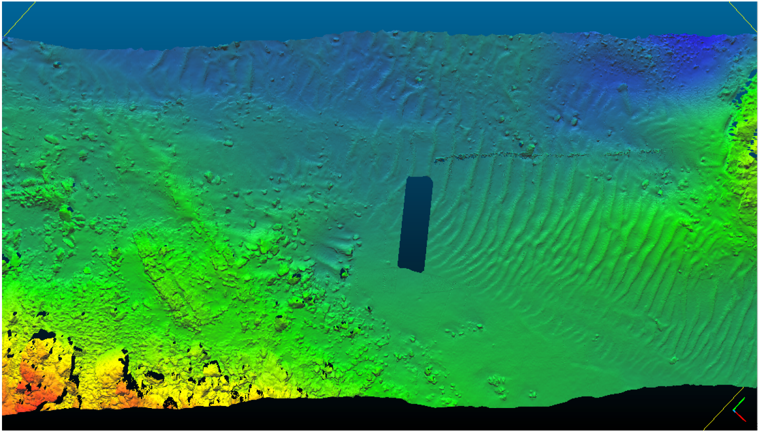
Journée Imagin, Jussieu

Abstract. The talk is motivated by the detection of submarine intruders inside the Bay of Biscay (golfe de Gascogne). In this project, we consider a group of underwater robots inside a zone with a known map (represented by an image). We assume that each robot is able to detect any intruder nearby. Moreover, the speed of the intruders is assumed to be bounded and is small with respect to that of our robots. The goal of this presentation is twofold: (1) to characterize an image representing the secure zone, i.e., the area for which we can guarantee that there is no intruder (2) to find a strategy for the group of robots in order to extend the secure zone as much as possible.

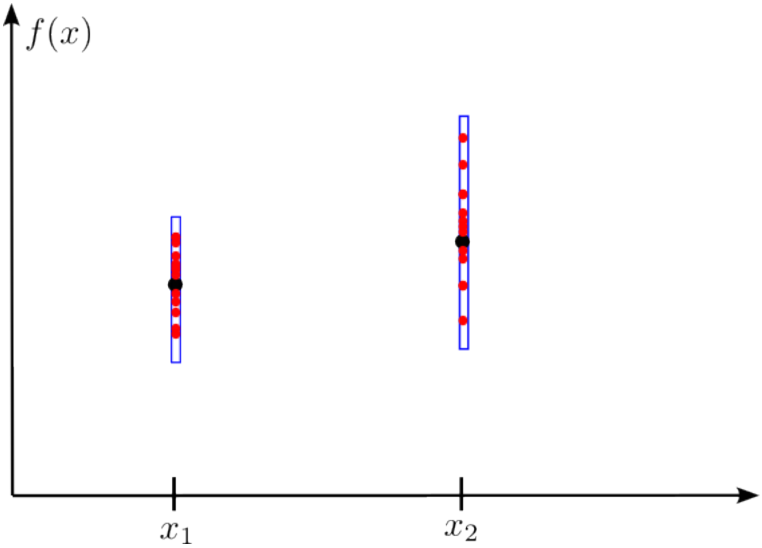
1. Image and maps



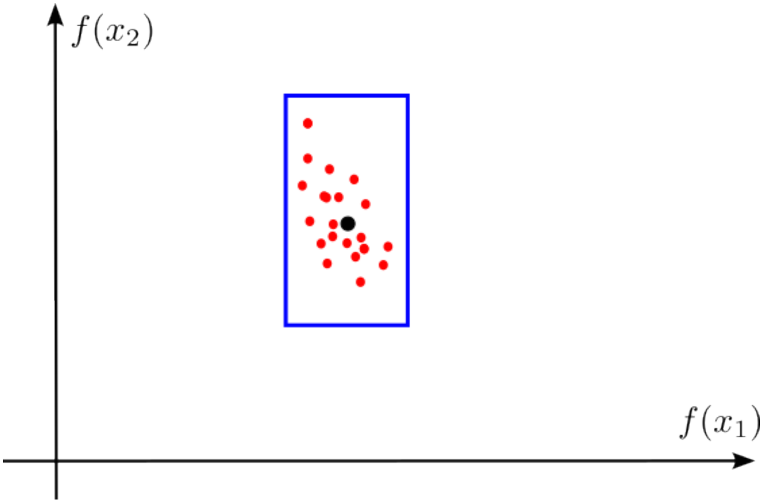
R2Sonic à 700kHz. MNT (3D)

Map of the currents :
https://www.ensta-bretagne.fr/jaulin/helios.html

What is an uncertain image?



An image with two pixels



An uncertain image

2. Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.

Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1, \dots, \mathscr{R}_n$ at positions $\mathbf{a}_1, \dots, \mathbf{a}_n$ are moving in the ocean.
- If the intruder is in the visibility zone of one robot, it is detected.[5]

Complementary approach

- We assume that a virtual intruder exists inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}(t)).$$

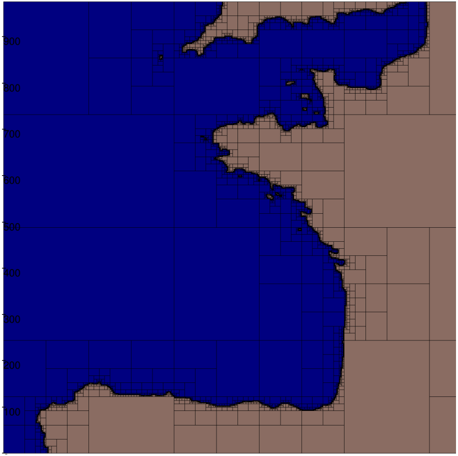
- Each robot $\mathscr{R}_i$ has the visibility zone $g_{\mathbf{a}_i}^{-1}([0,d_i])$ where d_i is the scope.

Theorem. An (undetected) intruder has a state vector $\mathbf{x}(t)$ inside the set

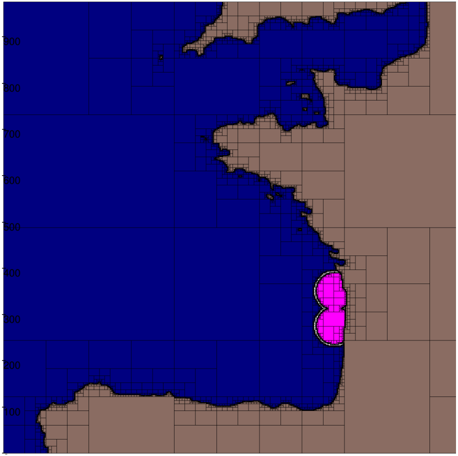
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

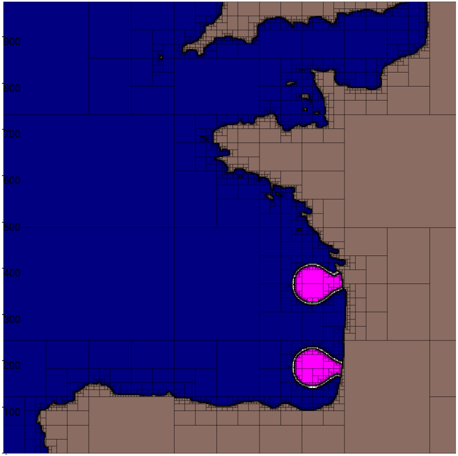
$$\mathbb{S}(t) = \overline{\mathbb{X}(t)}.$$



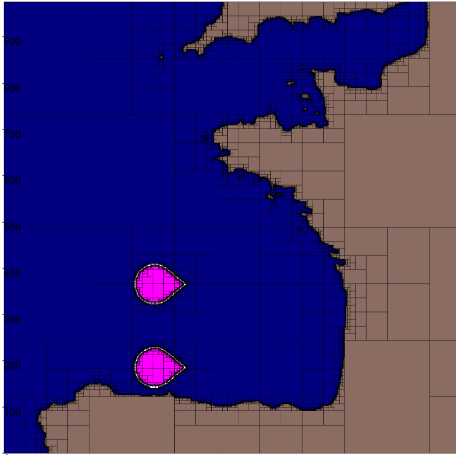
Set $\mathbb{G}$ in blue



Magenta: $\mathbb{G} \cap \bigcup_i g^{-1}_{\mathbf{a}_i(t)}([0, d_i(t)])$ Blue:$\mathbb{G} \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty])$



Blue: $\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty])$.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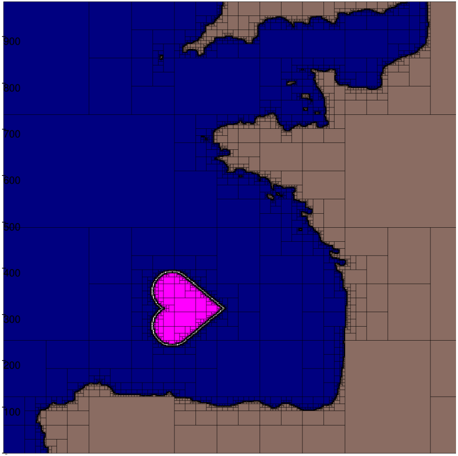
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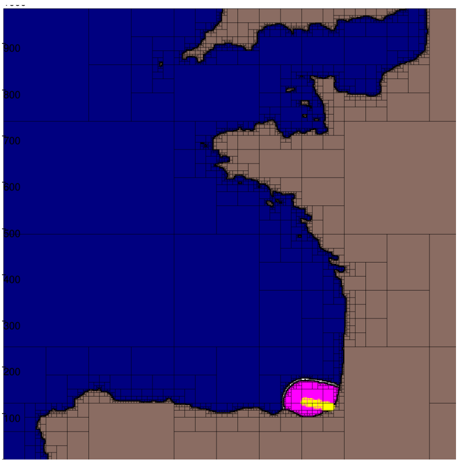
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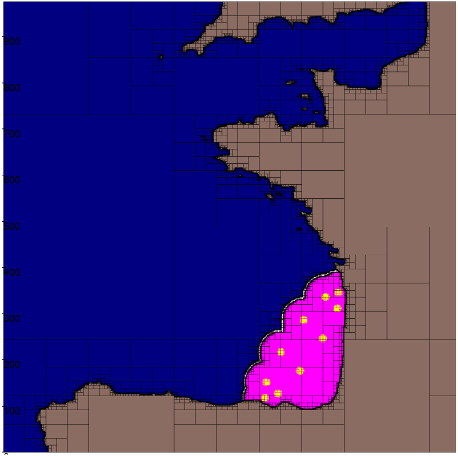
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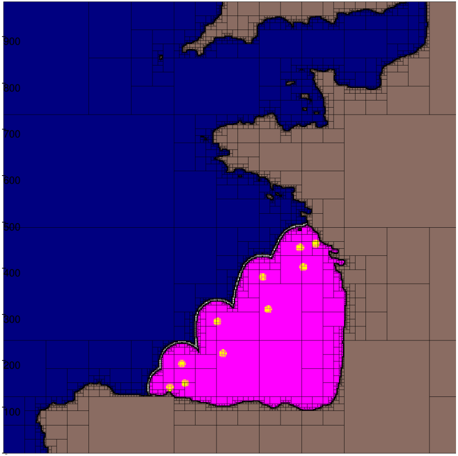
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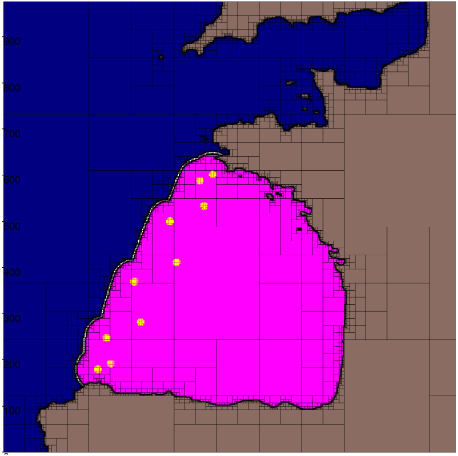
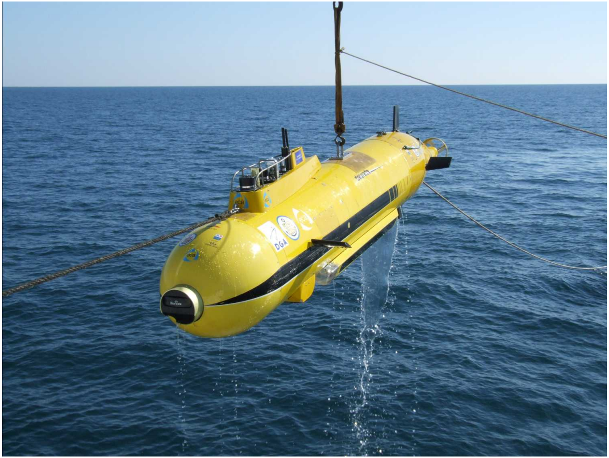
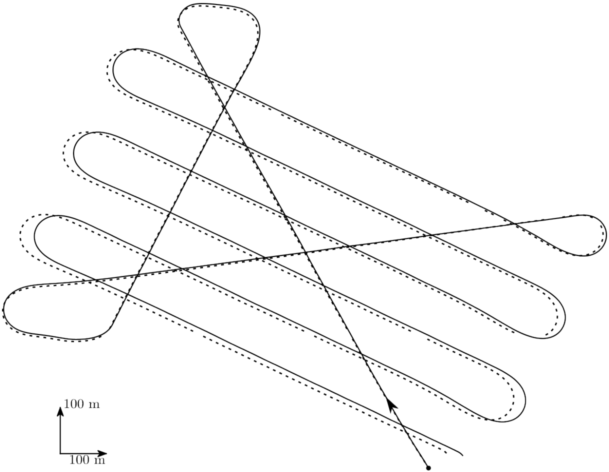


Video : https://youtu.be/rNcDW6npLfE

With uncertainties



Daurade


100 m
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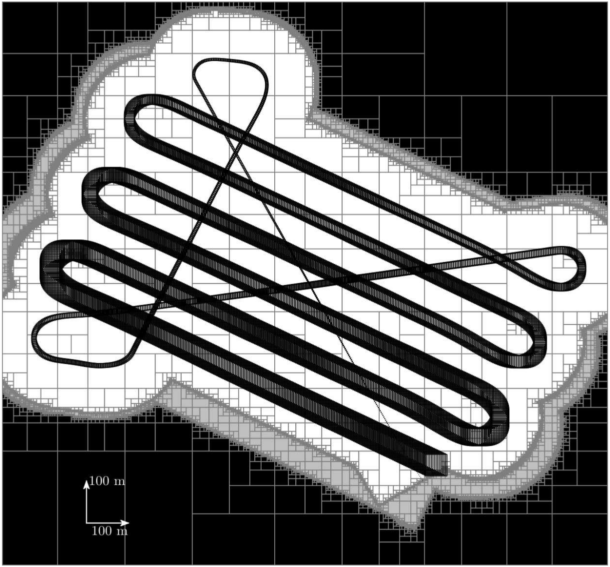
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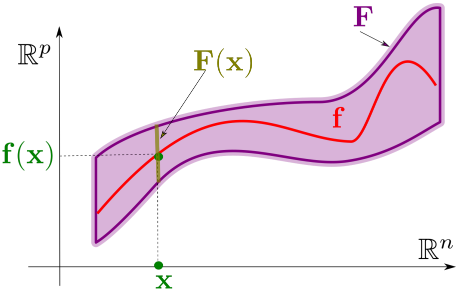
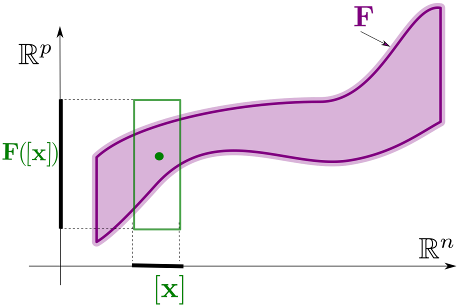
Uncertain image

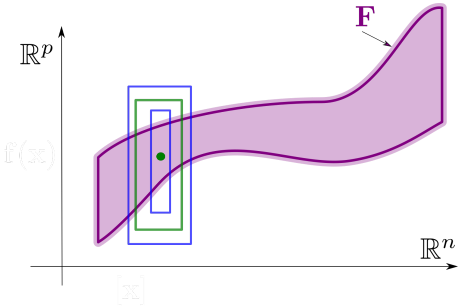
An image $\mathbf{f}:\mathbb{R}^2 \mapsto \mathbb{R}$
An interval image $\mathbf{F}$ is a thick function.



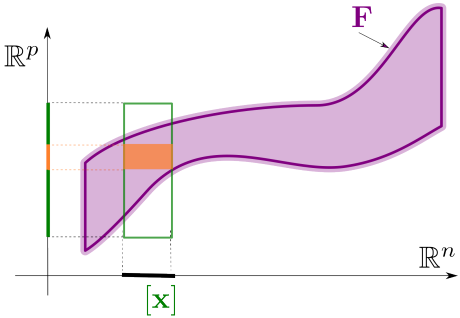
$$\mathbf{f} \in \mathbf{F}$$

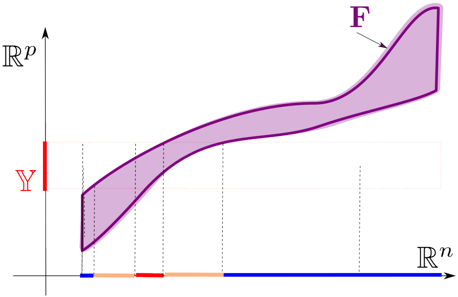


$$\mathbf{f}([\mathbf{x}]) \subset \mathbf{F}([\mathbf{x}])$$

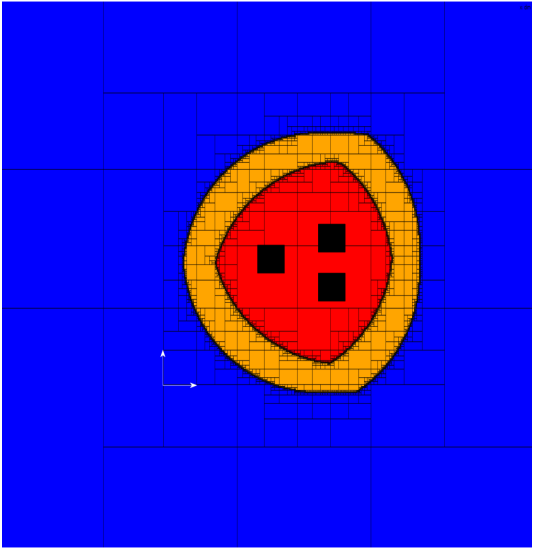


$\mathbf{F}$ is inclusion monotonic


F
$\mathbb{R}^p$
$\mathbb{R}^n$
$[\mathbf{x}]$



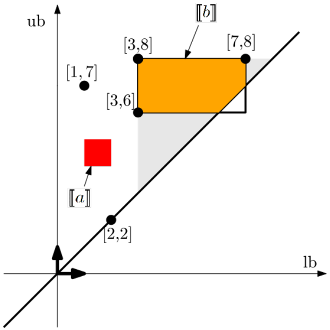
$\mathbf{F}^{-1}(\mathbb{Y})$ is a thick set

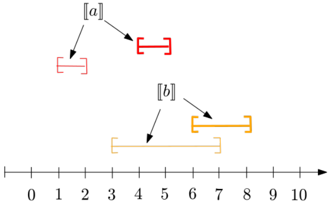


Twins

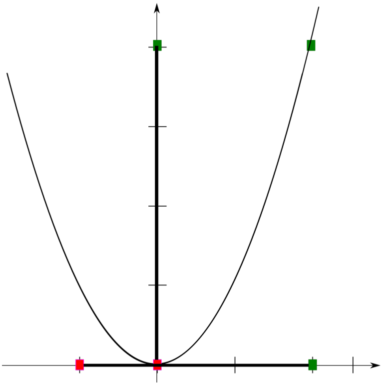
A *twin* (or *thick interval*) $[\![x]\!]$ ([2], Sainz et.al. [4], Chabert et.al. [1]) is a subset of $\mathbb{IR}$

$$\begin{aligned}[\![x]\!] &= [\![[x^-],[x^+]]\!] \\ &= \{[x^-,x^+] \in \mathbb{IR} \mid x^- \in [x^-] \text{ and } x^+ \in [x^+]\}.\end{aligned}$$

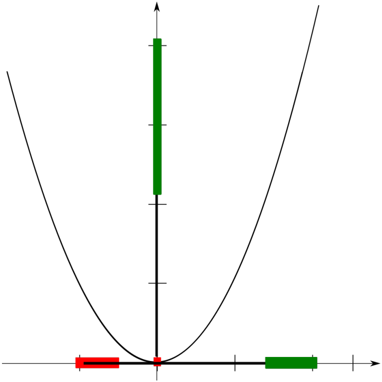

ub
[[b]]
[3,8]
[7,8]
[1,7]
[3,6]
[[a]]
[2,2]
lb


[[a]]
[[b]]
0 1 2 3 4 5 6 7 8 9 10

The square of a twin



$$x \in [x] \Rightarrow x^2 \in [x]^2$$



$$[x] \in [\![x]\!] \Rightarrow [x]^2 \in [\![x]\!]^2$$

Assume that

$$[x] \in [\![x]\!] = [\![\, [x^-]\, , [x^+] \,]\!] = [\![[-3,-1]\, , [-2,2]]\!]$$

What can we say about $[y] = [x]^2$?
What could represent $[\![x]\!]^2$?

Since

$$[\![x]\!] = [\![[-3,-1],[-2,2]]\!]$$

we have

$$\begin{array}{lcl} [-1,0] \in [\![x]\!] & \Rightarrow & [-1,0]^2 = [0,1] \in [\![x]\!]^2 \\ [-3,-2] \in [\![x]\!] & \Rightarrow & [-3,-2]^2 = [4,9] \in [\![x]\!]^2 \end{array}$$

Thus, we want $[\![[0,4],[1,9]]\!] \subset [\![[-3,-1],[-2,2]]\!]^2$.

The square of an interval [3]

$$
\begin{aligned}
[x]^2 &= [x^-, x^+]^2 \\
&= \{x^2 | x \in [x]\} \\
&= \begin{cases} [\min(x^{-2}, x^{+2}), \max(x^{-2}, x^{+2})] & \text{if } 0 \notin [x] \\ [0, \max(x^{-2}, x^{+2})] & \text{if } 0 \in [x] \end{cases}
\end{aligned}
$$

A closed form for interval square is

$$[x]^2 = [\max(0, \mathsf{sign}(x^- \cdot x^+)) \cdot \min(x^{-2}, x^{+2}), \max(x^{-2}, x^{+2})].$$

Assume that

$$[x] \in [\![[x^-], [x^+]]\!] = [\![[-3,-1], [-2,2]]\!]$$

what can we say about $[y] = [x]^2$?

Since $x^- \in [x^-] = [-3,-1]$ and $x^+ \in [x^+] = [-2,2]$, we have

$$\begin{array}{rcl} y^- & = & \max(0, \mathrm{sign}(x^- \cdot x^+)) \cdot \min(x^{-2}, x^{+2}) \\ & \in & \max(0, \mathrm{sign}([x^-] \cdot [x^+])) \cdot \min([x^-]^2, [x^+]^2) \\ & = & \max(0, \mathrm{sign}([-3,-1] \cdot [-2,2])) \cdot \min([-3,-1]^2, [-2,2]^2) \\ & = & [0,1] \cdot [0,4] = [0,4] \\ y^+ & = & \max(x^{-2}, x^{+2}) \\ & \in & \max([x^-]^2, [x^+]^2) \\ & = & \max([1,9],[0,4]) = [1,9] \end{array}$$

Therefore $[x]^2 \in [\![[0,4], [1,9]]\!]$.

G. Chabert and L. Jaulin.
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