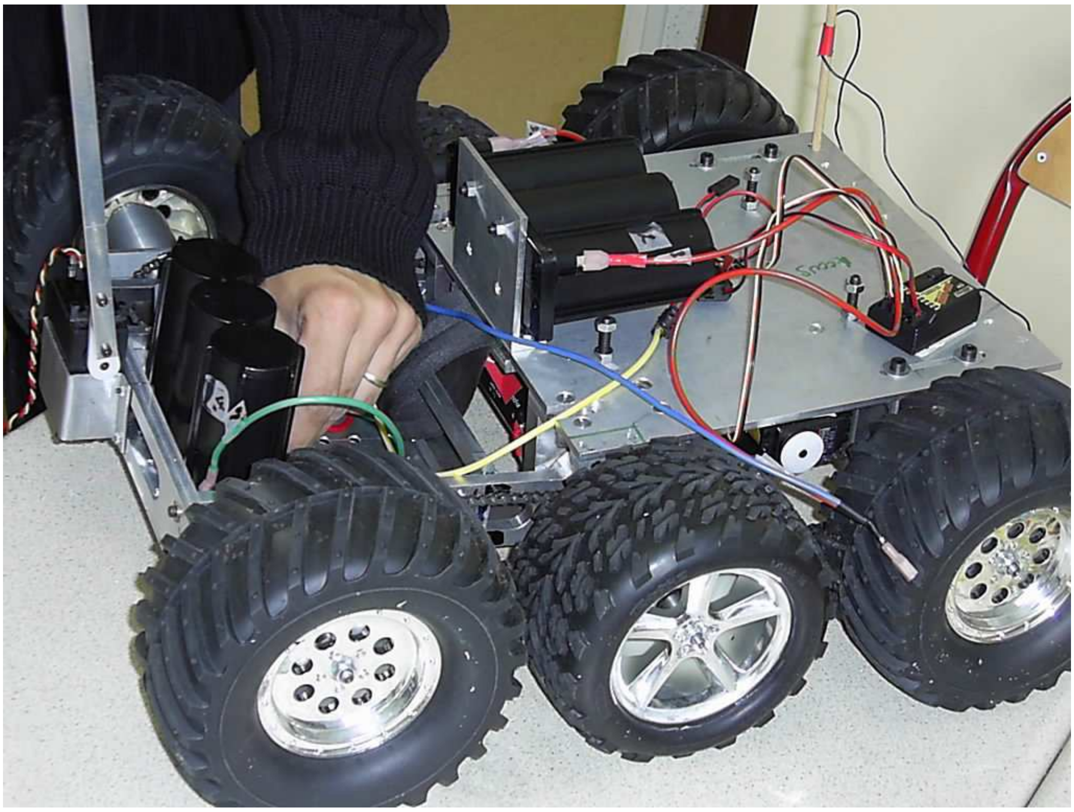
Control of a wheeled stair-climbing robot using linear programming

L. Jaulin,

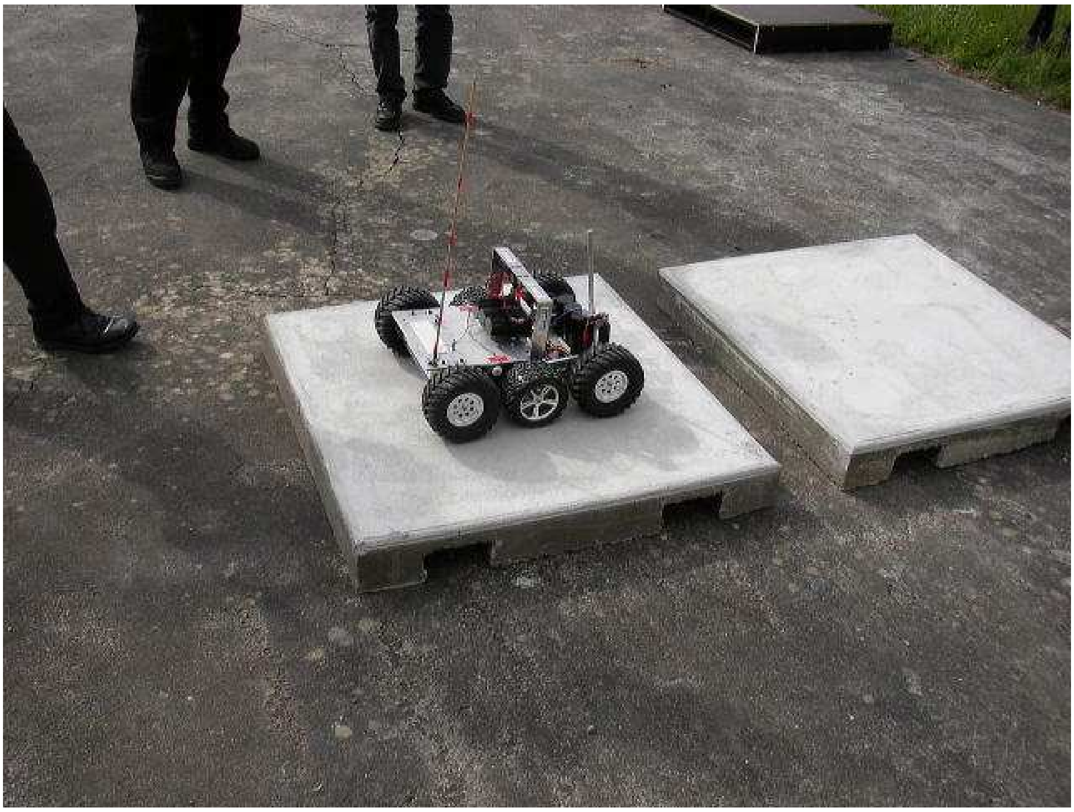
ENSIETA, E3I2, Brest

April 12, 2007.

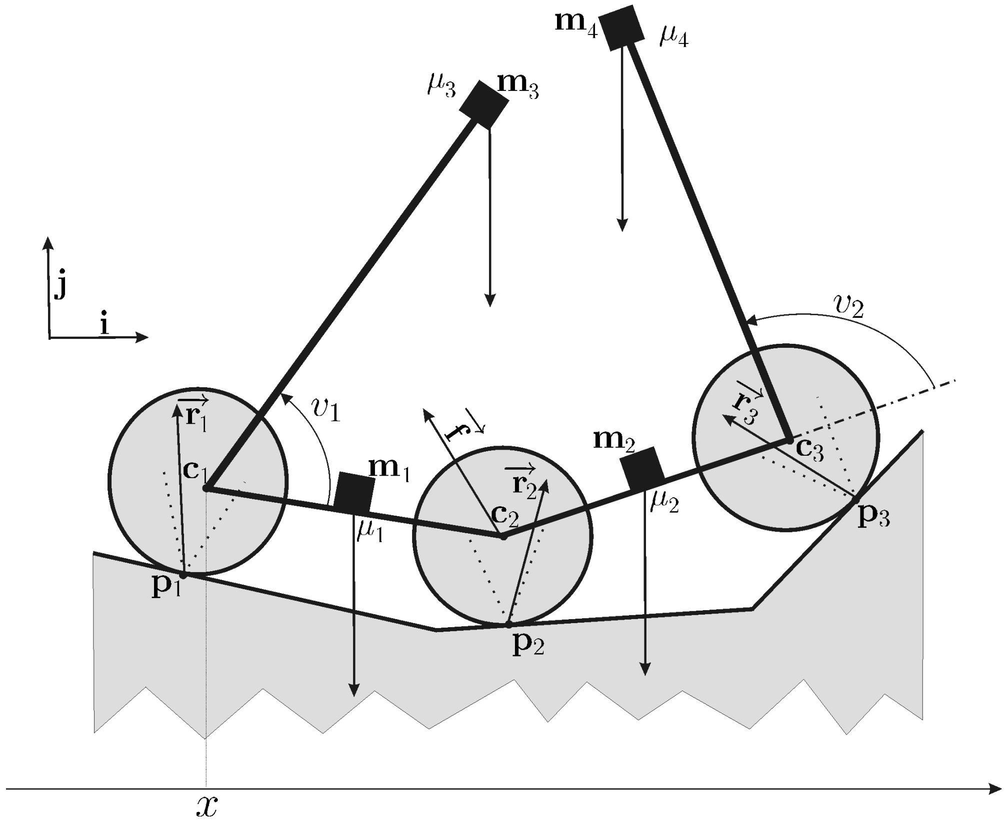
1 ETAS competition



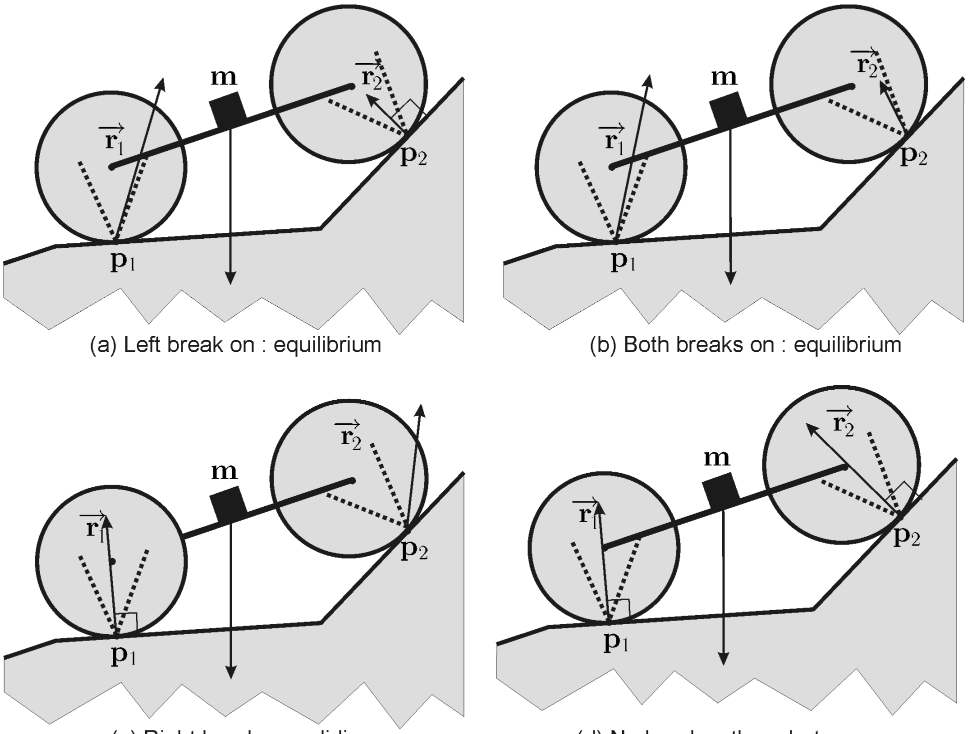

NAT-SIL



2 Idea



Mass transfer system to avoid any sliding



(a) Left break on : equilibrium

(b) Both breaks on : equilibrium

(c) Right break on : sliding

(d) No breaks : the robot moves

For (a), (b), (c) the fundamental principle of static is satisfied

3 Formalization

Consider the class of constrained dynamic robots

$$\begin{gathered}\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{g}(\mathbf{x}(t), \mathbf{v}(t)) \leq \mathbf{0}.\end{gathered}$$

$\mathbf{u}(t)$ is the *evolution input vector*,
$\mathbf{x}(t)$ is the *state vector*,
$\mathbf{v}(t)$ is the *viable input vector*.

- If $\mathbf{g}(\mathbf{x}, \mathbf{v}) = \mathbf{A}(\mathbf{x}).\mathbf{v} + \mathbf{b}(\mathbf{x}) \leq \mathbf{0}$ then a simplex method can find a feasible $\mathbf{v}$.

- Otherwise, interval methods can be used to find a feasible $\mathbf{v}$.

4 Interval constraint propagation

4.1 Constraint contraction

Let x, y, z be 3 variables such that

$$x \in [-\infty, 5],\ y \in [-\infty, 4],\ z \in [6, \infty], \qquad z = x + y.$$

We have

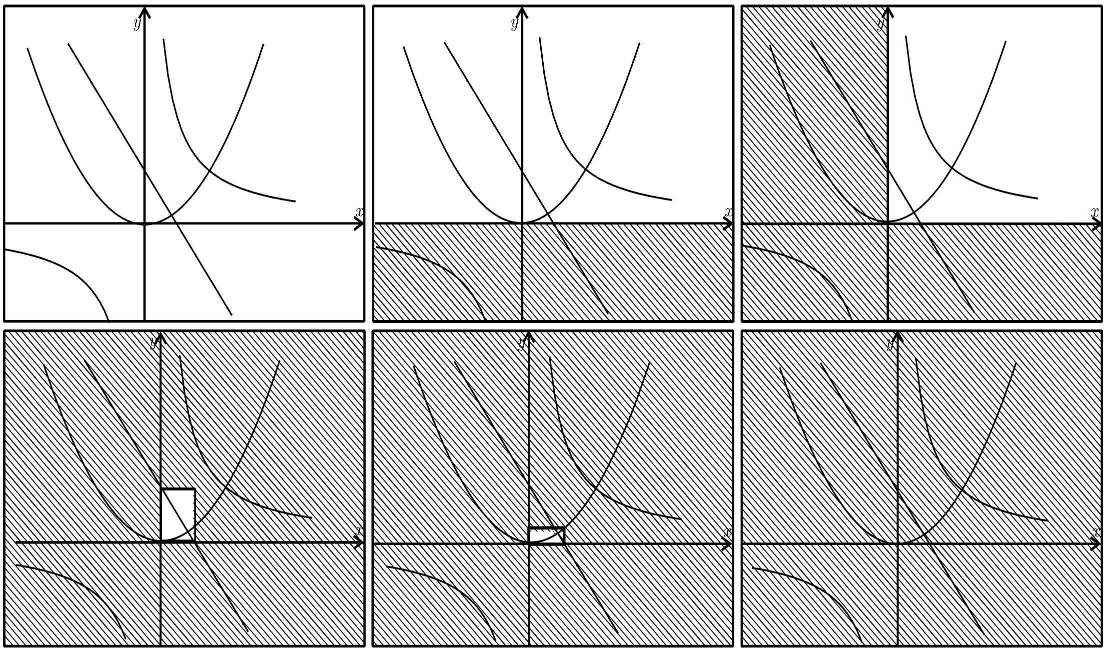
$$\begin{array}{rrl} z = x + y \Rightarrow & z \in & [6, \infty] \cap ([-\infty, 5] + [-\infty, 4]) \\ & & = [6, \infty] \cap [-\infty, 9] = [6, 9]. \\ x = z - y \Rightarrow & x \in & [-\infty, 5] \cap ([6, \infty] - [-\infty, 4]) \\ & & = [-\infty, 5] \cap [2, \infty] = [2, 5]. \\ y = z - x \Rightarrow & y \in & [-\infty, 4] \cap ([6, \infty] - [-\infty, 5]) \\ & & = [-\infty, 4] \cap [1, \infty] = [1, 4]. \end{array}$$

4.2 Constraint propagation

Consider the three constraints

$$\begin{cases} (C_1): & y = x^2 \\ (C_2): & xy = 1 \\ (C_3): & y = -2x + 1 \end{cases}$$

To each variable we assign the domain $[-\infty, \infty]$. Then, we contract all constraints until equilibrium.



y
x
y
x
y
x
y
x
y
x
y
x

$$
\begin{array}{lll}
(C_1) \Rightarrow & y \in & [-\infty,\infty]^2 \ = \ [0,\infty] \\
(C_2) \Rightarrow & x \in & 1/[0,\infty] \ = [0,\infty] \\
(C_3) \Rightarrow & y \in & [0,\infty] \ \cap \ ((-2)\,.[0,\infty]+1) \\
 & & = [0,\infty] \ \cap \ ([-\infty,1]) = [0,1] \\
 & x \in & [0,\infty] \ \cap \ (-[0,1]/2+1/2) = [0,\frac{1}{2}] \\
(C_1) \Rightarrow & y \in & [0,1] \cap [0,1/2]^2 = [0,1/4] \\
(C_2) \Rightarrow & x \in & [0,1/2] \ \cap \ 1/[0,1/4] = \emptyset \\
 & y \in & [0,1/4] \cap 1/\emptyset = \emptyset
\end{array}
$$

4.3 Decomposition

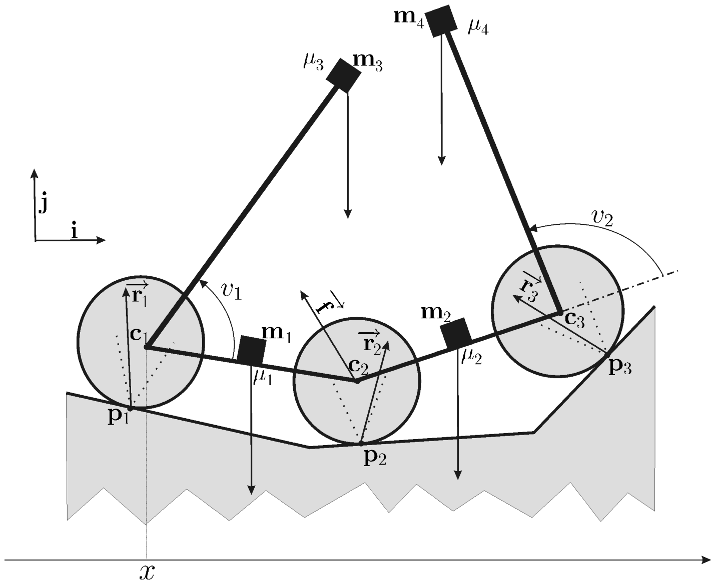
For more complex constraints, we have to perform a decomposition. For instance

$$\begin{array}{l} x + \mathsf{sin}(y) - xz \leq \mathsf{0}, \\ x \in [-1, 1], y \in [-1, 1], z \in [-1, 1] \end{array}$$

can be decomposed into

$$\left\{ \begin{array}{l} a = \mathsf{sin}(y) \\ b = x + a \\ c = xz \\ b - c = d \end{array} \right. , \qquad \begin{array}{ll} x \in [-1, 1] & a \in [-\infty, \infty] \\ y \in [-1, 1] & b \in [-\infty, \infty] \\ z \in [-1, 1] & c \in [-\infty, \infty] \\ & d \in [-\infty, \mathsf{0}] \end{array}$$

5 Resolution of the mass transfer problem



$$\dot{x} = u,$$
$$\mathbf{g}(x, v_1, v_2) \leq \mathbf{0}.$$

5.1 Fundamental principle of static

When the robot does not move,

$$\left\{\begin{array}{rcl}
-\overrightarrow{\mathbf{p}_1\mathbf{m}_1} \wedge \mu_1\mathbf{j} + \overrightarrow{\mathbf{p}_1\mathbf{c}_2} \wedge \overrightarrow{\mathbf{f}} - \overrightarrow{\mathbf{p}_1\mathbf{m}_3} \wedge \mu_3\mathbf{j} & = & 0 \\
\overrightarrow{\mathbf{p}_2\mathbf{m}_2} \wedge \mu_2\mathbf{j} + \overrightarrow{\mathbf{p}_2\mathbf{c}_2} \wedge \overrightarrow{\mathbf{f}} - \overrightarrow{\mathbf{p}_2\mathbf{p}_3} \wedge \overrightarrow{\mathbf{r}}_3 & & \\
\overrightarrow{\mathbf{p}_2\mathbf{m}_4} \wedge \mu_4\mathbf{j} & = & 0 \\
\overrightarrow{\mathbf{r}}_1 - (\mu_1 + \mu_3)\mathbf{j} + \overrightarrow{\mathbf{f}} & = & 0 \\
\overrightarrow{\mathbf{r}}_2 - \overrightarrow{\mathbf{f}} - (\mu_2 + \mu_4)\mathbf{j} + \overrightarrow{\mathbf{r}}_3 & = & 0
\end{array}\right.$$

A scalar decomposition yields

$$
\left\{
\begin{array}{rcl}
-\mu_1 \left(m_{1x} - p_{1x}\right) + \left(c_{2x} - p_{1x}\right) f_y & & \\
-\left(c_{2y} - p_{1y}\right) f_x - \mu_3 \left(m_{3x} - p_{1x}\right) & = & 0 \\
\mu_2 \left(m_{2x} - p_{2x}\right) + \left(c_{2x} - p_{2x}\right) f_y & & \\
-\left(c_{2y} - p_{2y}\right) f_x - \left(p_{3x} - p_{2x}\right) r_{3y} & & \\
+\left(p_{3y} - p_{2y}\right) r_{3x} + \mu_4 \left(m_{4x} - p_{2x}\right) & = & 0 \\
r_{1x} + f_x & = & 0 \\
r_{1y} - \mu_1 - \mu_3 + f_y & = & 0 \\
r_{2x} - f_x + r_{3x} & = & 0 \\
r_{2y} - f_y - \mu_2 - \mu_4 + r_{3y} & = & 0
\end{array}
\right.
$$

This system can be written into a matrix form as

$$\mathbf{A}_1(x).\mathbf{y} = \mathbf{b}_1(x),$$

where

$$\mathbf{A}_1(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & p_{1y}-c_{2y} & c_{2x}-p_{1x} & -\mu_3 & 0 \\ 0 & 0 & 0 & 0 & p_{2y}-p_{3y} & p_{3x}-p_{2x} & c_{2y}-p_{2y} & p_{2x}-c_{2x} & 0 & -\mu_4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{b}_1(x) \;=\; \begin{pmatrix} \mu_1\left(m_{1x}-p_{1x}\right)-\mu_3 p_{1x} \\ \mu_2\left(m_{2x}-p_{2x}\right)-\mu_4 p_{2x} \\ 0 \\ \mu_1+\mu_3 \\ 0 \\ \mu_2+\mu_4 \end{pmatrix}$$

and

$$\mathbf{y} \;=\; \left(r_{1x}, r_{1y}, r_{2x}, r_{2y}, r_{3x}, r_{3y}, f_x, f_y, m_{3x}, m_{4x}\right)^{\mathsf{T}}.$$

We have 10 unknowns for 6 equations: our robot has a second order hyperstatic equilibrium.

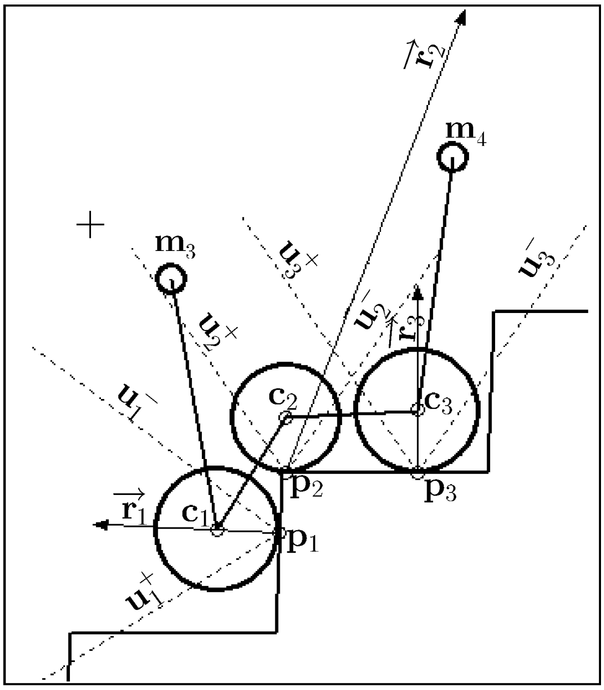
5.2 Non-sliding conditions

None of the wheels will slide if all $\overrightarrow{\mathbf{r}}_i$ belong to their Coulomb cone:

$$\mathbf{A}_2(x).\mathbf{y} \leq \mathbf{0},$$

where $\mathbf{A}_2(x)$ is given by

$$\begin{pmatrix} u_{1y}^{-} & -u_{1x}^{-} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -u_{1y}^{+} & u_{1x}^{+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & u_{2y}^{-} & -u_{2x}^{-} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -u_{2y}^{+} & u_{2x}^{+} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_{3y}^{-} & -u_{3x}^{-} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_{3y}^{+} & u_{3x}^{+} & 0 & 0 & 0 & 0 \end{pmatrix}$$



A configuration where the middle wheel is almost sliding.

5.3 Collision avoidance

The pendulums should not intersect the ground or the robot itself

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \mathbf{y} \in \begin{pmatrix} [m_{3x}^{\text{min}}, m_{3x}^{\text{max}}] \\ [m_{4x}^{\text{min}}, m_{4x}^{\text{max}}] \end{pmatrix} .$$

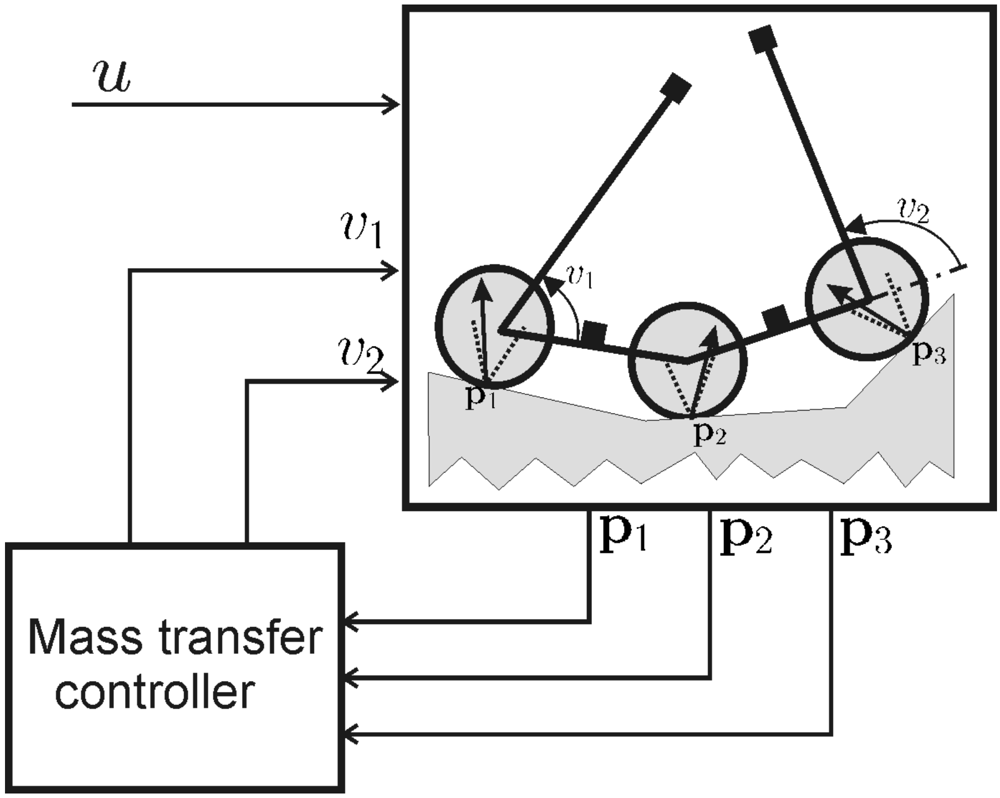
5.4 Recapitulation of the constraints

Our robot can be described by

$$\begin{array}{lccc} \text{(i)} & \dot{x} & = & u \\ \text{(ii)} & \mathbf{g}(x, v_1, v_2) & \leq & \mathbf{0} \end{array}$$

where (ii) is equivalent to

$$\exists \mathbf{y} = \begin{pmatrix} r_{1x}, r_{1y} \\ r_{2x}, r_{2y} \\ r_{3x}, r_{3y} \\ f_x, f_y \\ m_{3x}, m_{4x} \end{pmatrix}, \left\{ \begin{array}{lcc} \mathbf{A}_1(x).\mathbf{y} & = & \mathbf{b}_1(x) \\ \mathbf{A}_2(x).\mathbf{y} & \leq & \mathbf{0} \\ \mathbf{A}_3(x).\mathbf{y} & \leq & \mathbf{b}_3(x) \end{array} \right.$$



u
v_1
v_2
v_1
v_2
$\mathbf{p}_1$
$\mathbf{p}_2$
$\mathbf{p}_3$
$\mathbf{p}_1$
$\mathbf{p}_2$
$\mathbf{p}_3$
Mass transfer controller

6 Simulation

Angle friction coefficient: $\phi = 0.54$
Radius of the wheels: $\rho_1 = 85$mm, $\rho_2 = 75$mm, $\rho_3 = 85$mm
Lengths of the pendulums: $\ell_1 = \ell_2 = 350$mm
Weights of the platforms: $\mu_1 = \mu_2 = 70$N
Weights and the pendulum masses: $\mu_3 = \mu_4 = 20$N.
Height and the width of the stairs: 220mm and 280mm