Cooperative Exploration and Mapping

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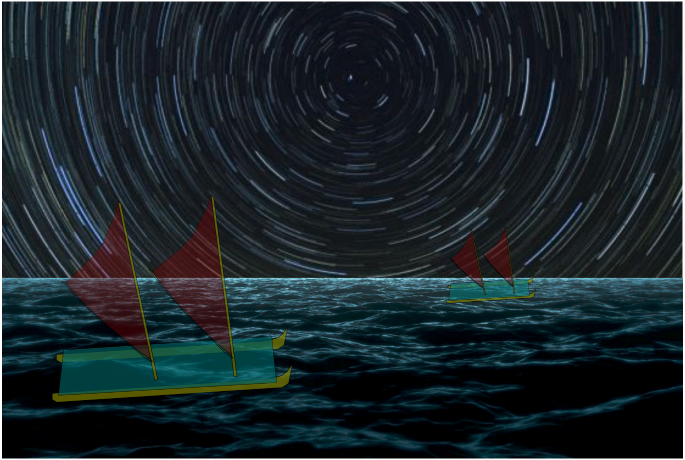
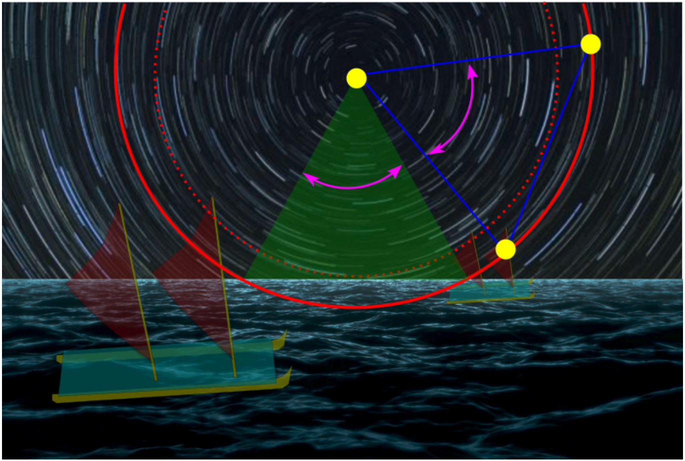
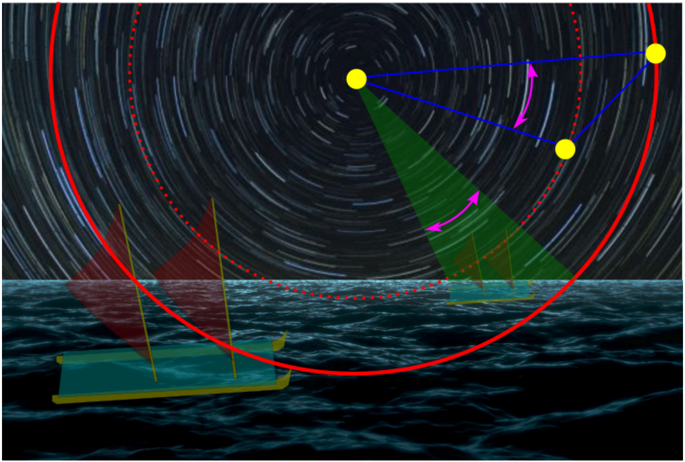
Polynesian navigation



Find the route without GPS, compass and clocks with *wa'a kaulua*[2]

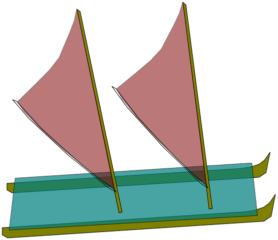


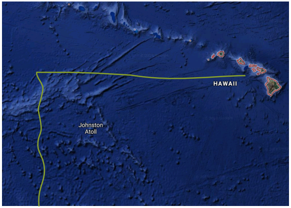
Pair of stars technique

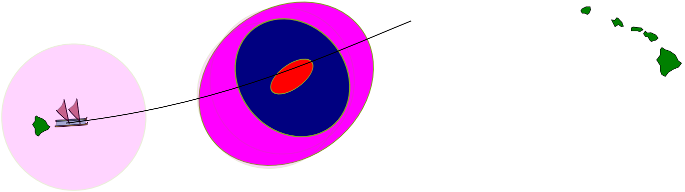


Pair of stars technique

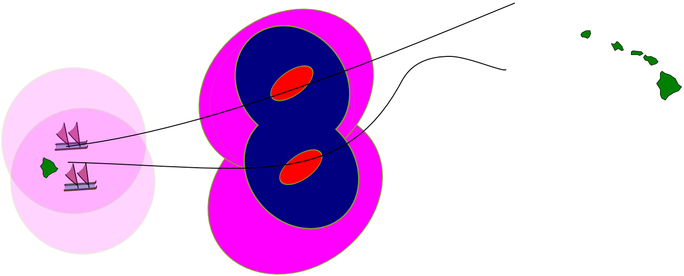

Arctic Ocean
Russia
Canada
Bering Sea
Sea of Okhotsk
Mongolia
Sea of Japan
China
South Korea
Japan
East China Sea
North Pacific Ocean
Myanmar (Burma)
Thailand
Philippine Sea
Vietnam
Malaysia
Indonesia
Banda Sea
Papua New Guinea
Arafura Sea
Coral Sea
Australia
Great Australian Bight
Tasman Sea
New Zealand
Google




HAWAII
Johnston Atoll

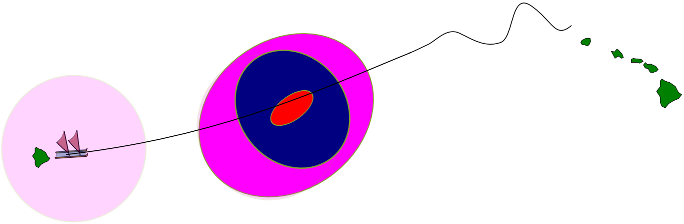


Prove that islands will be reached by one boat

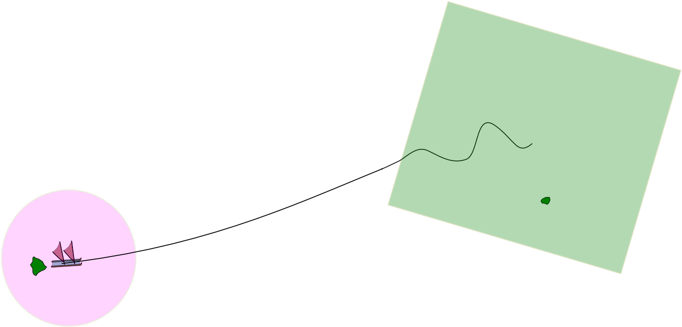


Prove that islands will be reached by the n boats

Alignment to keep the heading in case of clouds



Find a control to reach the geo-localized islands



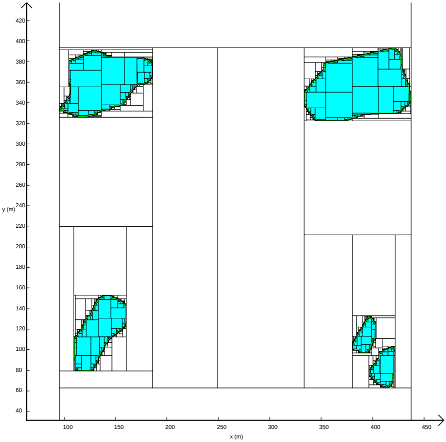
Explore a given area entirely to find new islands

Polynesian exploration problem

- Given a set of unmapped islands $\mathbf{m}_i, i \geq 0$ with coastal area:

$$\mathbb{C}_i = \{\mathbf{x} \,|\, c_i(\mathbf{x}) \leq 0\}.$$

- A robot has to map this environment without being lost.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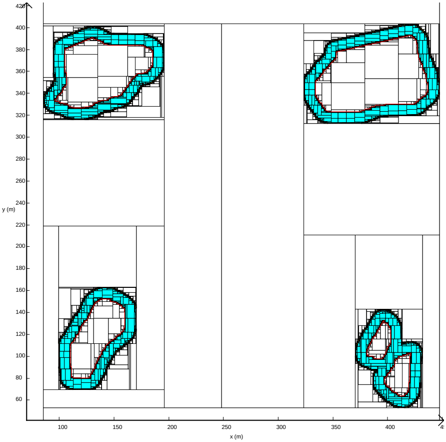
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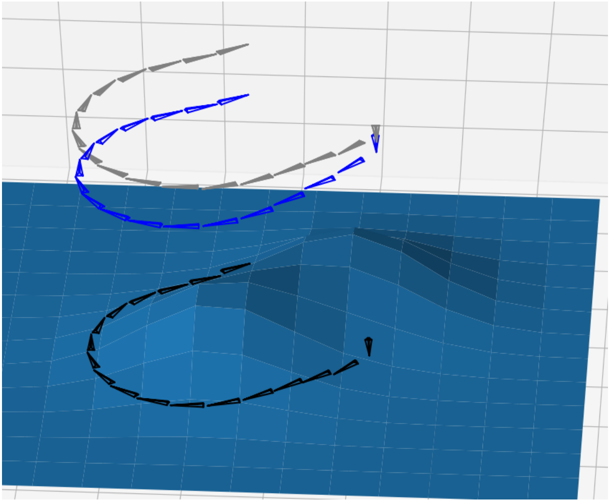
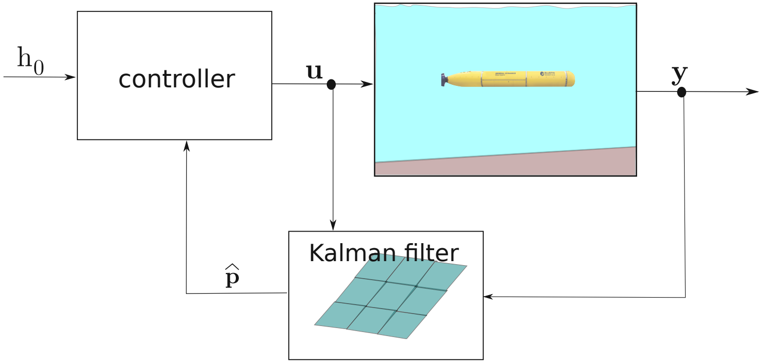
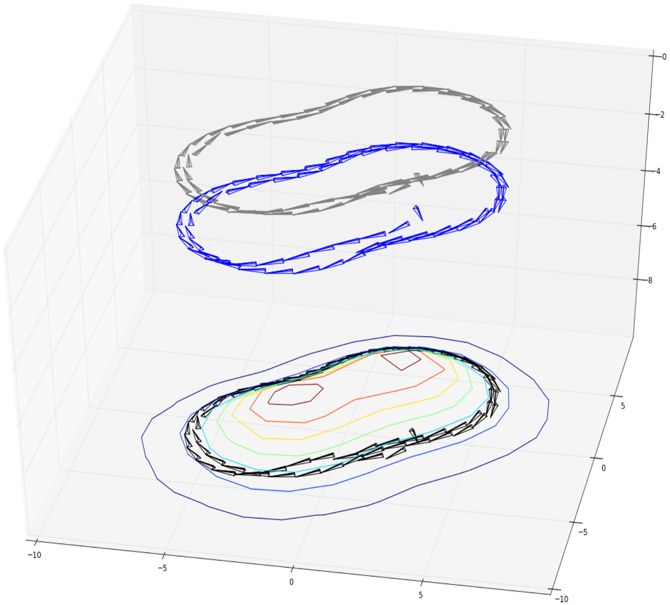
Follow isobaths




h_0
controller
$\mathbf{u}$
$\mathbf{y}$
Kalman filter
$\widehat{\mathbf{p}}$


0
2
4
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5
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−10
−10
−5
0
5

Bind Exploration

Visible area

The robot has a state $\mathbf{x}$. The visible area is $\mathbb{V}(\mathbf{x})$
Example. The robot is able to see all up to 3 meters

$$\mathbb{V}(\mathbf{x}) = \left\{ (z_1, z_2) \,|\, (z_1 - x_1)^2 + (z_2 - x_2)^2 \leq 9 \right\}.$$

Blind exploration

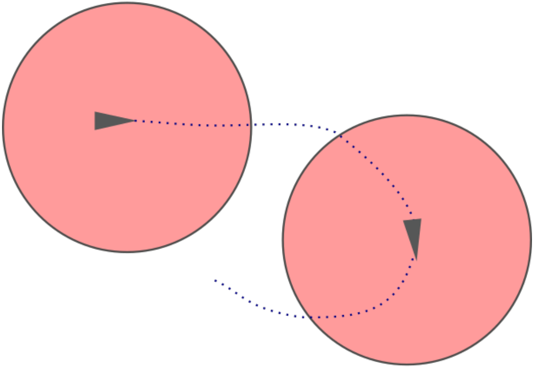
The explored zone $\mathbb{Z}$ is defined by [1]

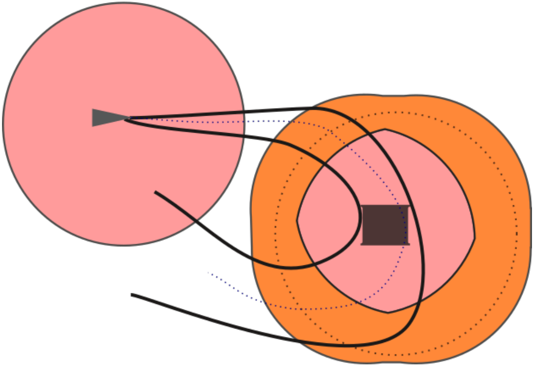
$$\left\{ \begin{array}{c} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}),\ \mathbf{u}(t) \in [\mathbf{u}](t) \\ \mathbb{Z} = \bigcup_{t \geq 0} \ \mathbb{V}(\mathbf{x}(t)) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{array} \right.$$

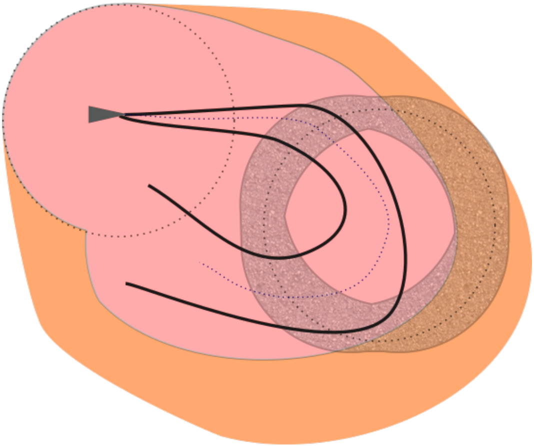
We have

$$\underbrace{\bigcap_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^-} \subset \mathbb{Z} \subset \underbrace{\bigcup_{\mathbf{x}(\cdot)\in\mathscr{X}(\cdot)} \bigcup_{t\geq 0} \mathbb{V}(\mathbf{x}(t))}_{\mathbb{Z}^+}.$$

$\mathbb{Z}^-$ is the *certainly explored zone.*
$\mathbb{Z}^+$ is the *maybe explored zone*.
$\mathbb{Z}^+\backslash\mathbb{Z}^-$ is the *penumbra.*







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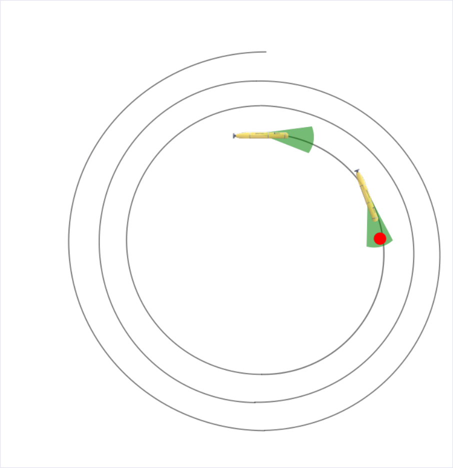
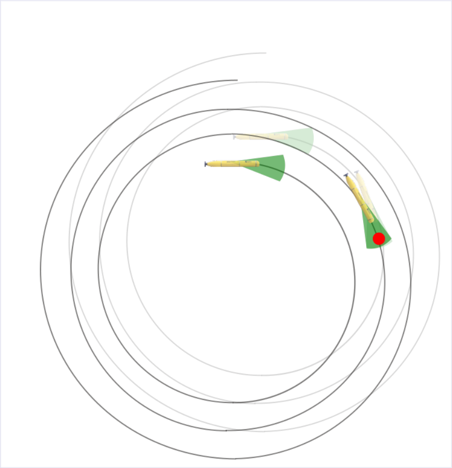
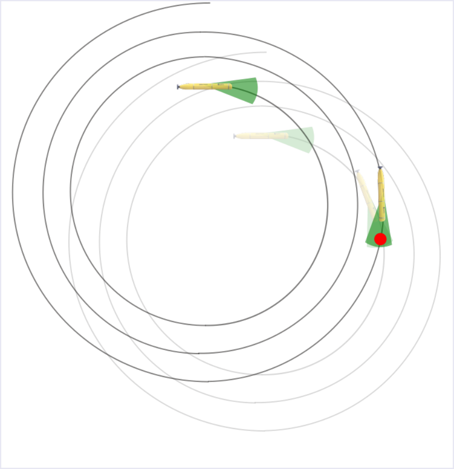
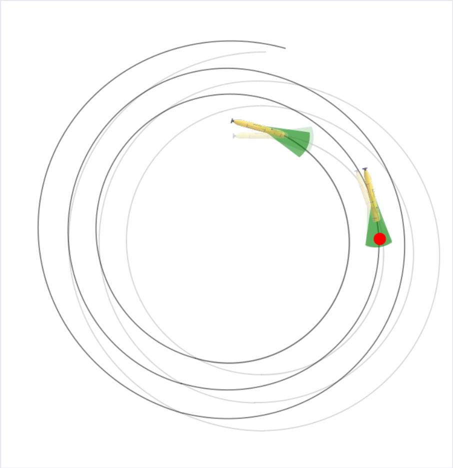
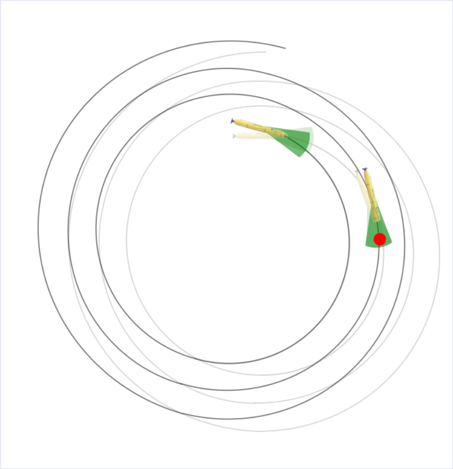
Spiral scan

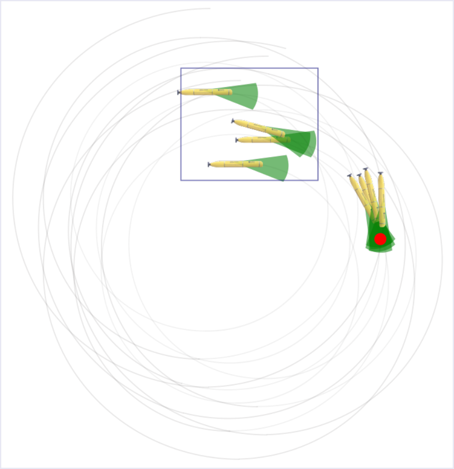






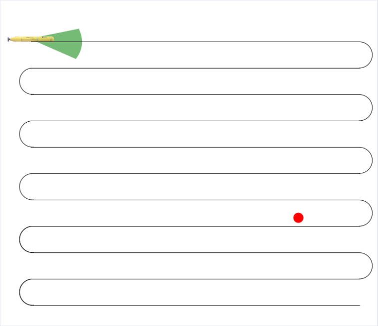


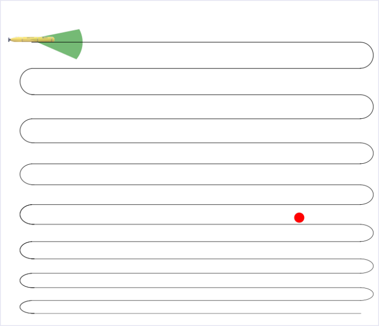




Boustrophedon

Which pattern is the best for exploration?





Reach an island

Robot

The robot is described by

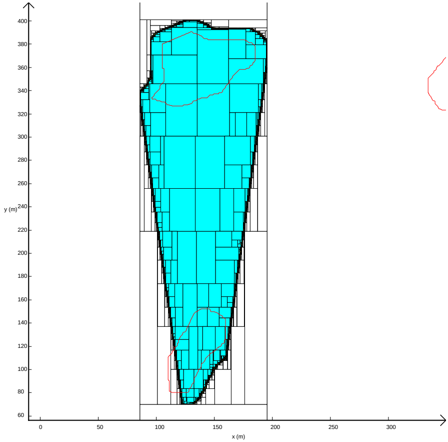
$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), & \mathbf{u}(\cdot) \in [\mathbf{u}](t) \\ \mathbf{x}(0) = \mathbf{x}_0 \end{cases}$$

Offshore, the robot is blind and an open loop strategy.

Backward reach set

Given the set $\mathbb{A}$, the backward reach set is defined by

$$\mathrm{Back}(\mathbb{A}) = \{\mathbf{x} \mid \forall \varphi \in \mathbf{\Phi}, \exists t \geq 0, \varphi(t, \mathbf{x}) \in \mathbb{A}\}.$$


400
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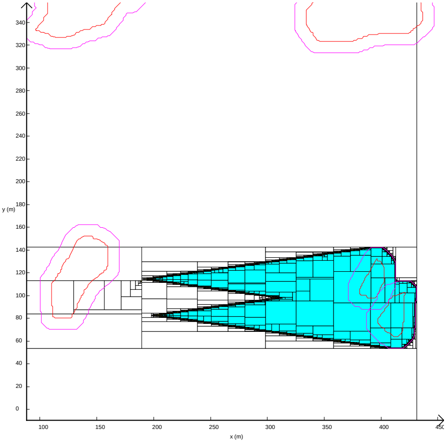
y (m)
240
220
200
180
160
140
120
100
80
60
0
50
100
150
200
250
300
x (m)

Archipelagic effect

We have

$$\mathsf{Back}(\mathbb{A} \cup \mathbb{B}) \quad \supset \quad \mathsf{Back}(\mathbb{A}) \cup \mathsf{Back}(\mathbb{B})$$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y (m)
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400
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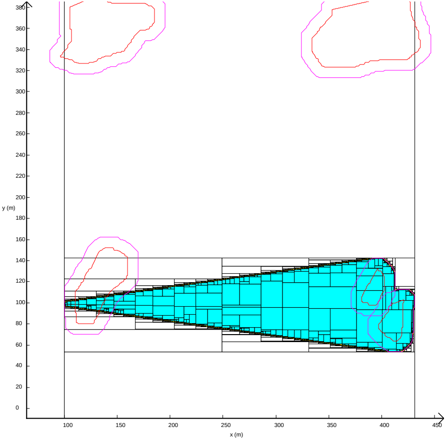
x (m)


y (m)
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250
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350
400
450
x (m)

No-lost zone

Moving between coastal zones

- We have m coastal sets $\mathbb{C}_1, \mathbb{C}_2, \ldots, i \in \{1, 2, \ldots\}$
- We have open loop control strategies $\mathbf{u}_j, j \in \{1, 2, \ldots\}$,
- Equivalently, we have set flows $\mathbf{\Phi}_j(t, \mathbf{x}_0)$.
- The control strategy cannot change offshore.

Graph

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2$ with the jth control strategy if

$$\mathbb{C}_1 \cap \mathrm{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2$ with at least one control strategy if

$$\mathbb{C}_1 \cap \bigcup_j \mathrm{Back}(j, \mathbb{C}_2) \neq \emptyset.$$

From $\mathbb{C}_1$ we can reach $\mathbb{C}_2 \cup \mathbb{C}_3$ with at least one control strategy if

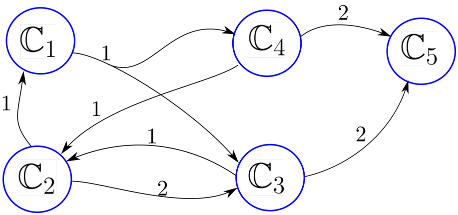
$$\mathbb{C}_1 \cap \bigcup_j \mathrm{Back}(j, \mathbb{C}_2 \cup \mathbb{C}_3) \neq \emptyset.$$

We define $\hookrightarrow$ as:

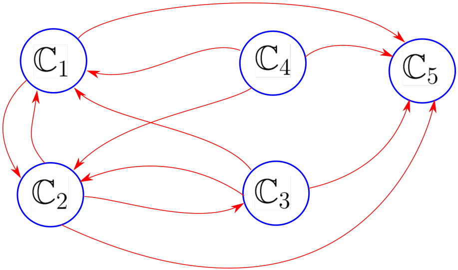
- $\mathbb{C}_a \hookrightarrow \mathbb{C}_b$ if from $\mathbb{C}_a$ we can reach $\mathbb{C}_b$ with at least one control strategy j.
- $\hookrightarrow$ is the smallest transitive relation such that

$$\left\{ \begin{array}{c} \forall k \in \mathbb{K}, \mathbb{C}_{i_k} \hookrightarrow \mathbb{C}_b \\ \exists j, \mathbb{C}_a \cap \mathrm{Back}(j, \bigcup_{k \in \mathbb{K}} \mathbb{C}_{i_k}) \neq \emptyset \end{array} \right. \quad \Rightarrow \quad \mathbb{C}_a \hookrightarrow \mathbb{C}_b$$

Graph



$$\mathbb{C}_1 \cap \mathsf{Back}(1, \mathbb{C}_3 \cup \mathbb{C}_4) \neq \emptyset \Rightarrow \mathbb{C}_1 \to (\mathbb{C}_3, \mathbb{C}_4)$$



Graph of $\hookrightarrow$

Secure a zone

Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.

Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1, \ldots, \mathscr{R}_n$ at positions $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are moving in the ocean.
- If the intruder is in the visibility zone of one robot, it is detected.[3]

Complementary approach

- We assume that a virtual intruder exists inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}(t)).$$

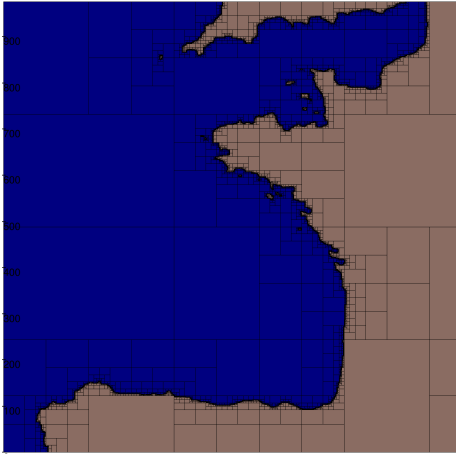
- Each robot $\mathscr{R}_i$ has the visibility zone $g_{\mathbf{a}_i}^{-1}([0, d_i])$ where d_i is the scope.

Theorem. An (undetected) intruder has a state vector $\mathbf{x}(t)$ inside the set

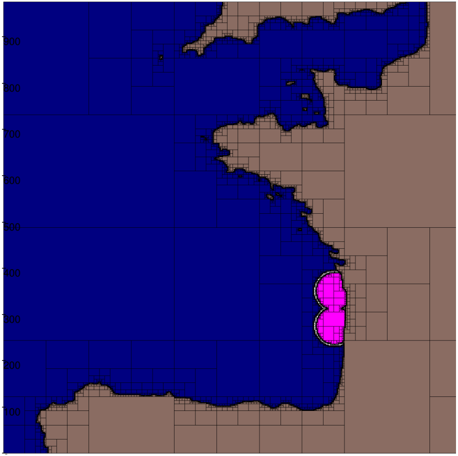
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

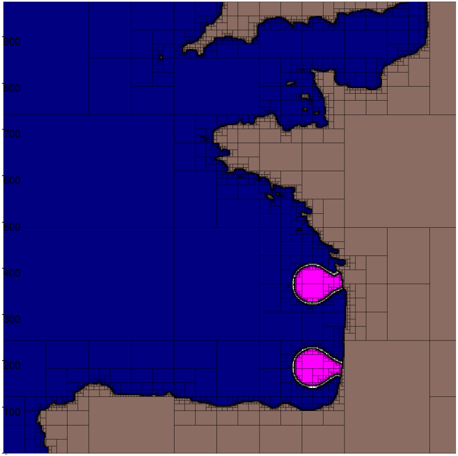
$$\mathbb{S}(t) = \overline{\mathbb{X}(t)}.$$



Set $\mathbb{G}$ in blue

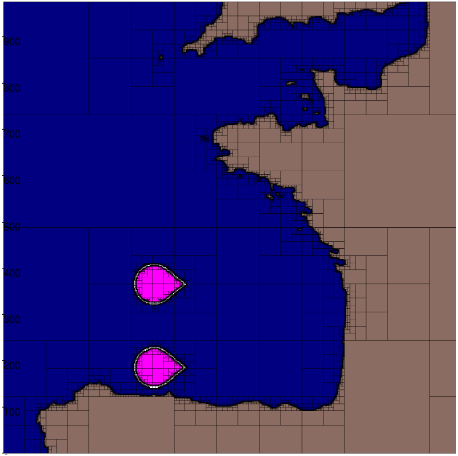


Magenta: $\mathbb{G} \cap \bigcup_i g_{\mathbf{a}_i(t)}^{-1}([0, d_i(t)])$ Blue:$\mathbb{G} \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty])$



Blue:

$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]).$$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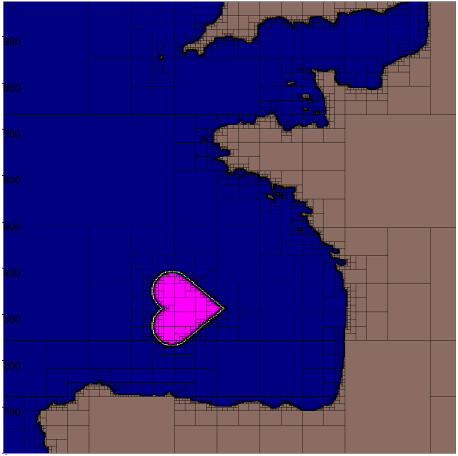
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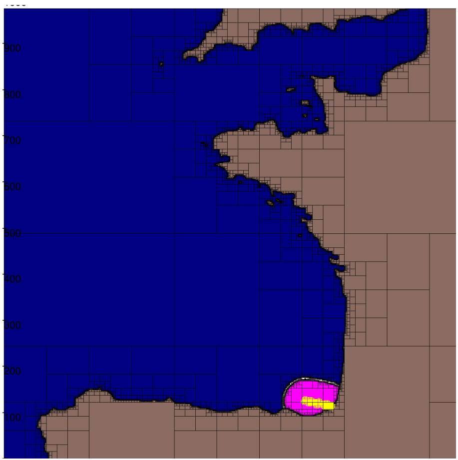
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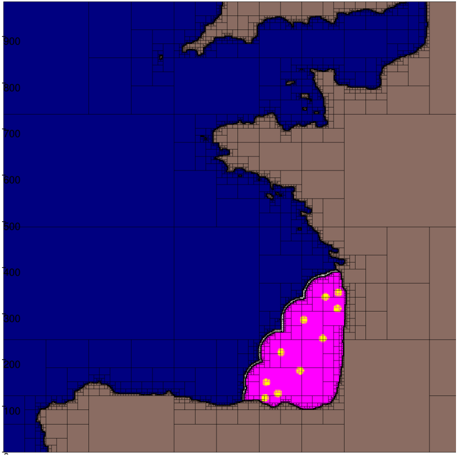
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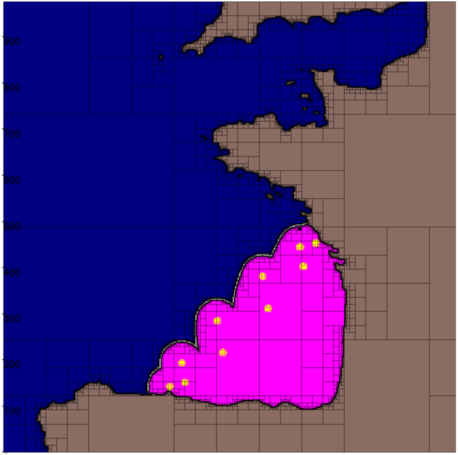
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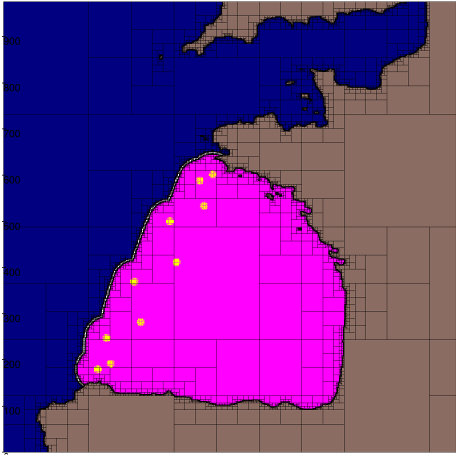

Video : https://youtu.be/rNcDW6npLfE

Smoother

Idea: Take into account the future.
The feasible set can be obtained by the following contractions

$$\begin{array}{rcl} \overrightarrow{\mathbb{X}}(t) & = & \overrightarrow{\mathbb{X}}(t) \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \\ \overleftarrow{\mathbb{X}}(t) & = & \overleftarrow{\mathbb{X}}(t) \cap (\mathbb{X}(t+dt) - dt \cdot \mathbb{F}(\mathbb{X}(t+dt))) \\ \mathbb{X}(t) & = & \overrightarrow{\mathbb{X}}(t) \cap \overleftarrow{\mathbb{X}}(t) \end{array}$$

with the initialization

$$\mathbb{X}(t) = \overrightarrow{\mathbb{X}}(t) = \overleftarrow{\mathbb{X}}(t) = \mathbb{G}.$$

B. Desrochers and L. Jaulin.
Computing a guaranteed approximation the zone explored by a robot.
IEEE Transaction on Automatic Control, 62(1):425–430, 2017.

T. Nico, L. Jaulin, and B. Zerr.
Guaranteed polynesian navigation.
In *SWIM'19, Paris, France*, 2019.

K. Vencatasamy, L. Jaulin, and B. Zerr.
Secure the zone from intruders with a group robots.
Special Issue on Ocean Engineering and Oceanography, Springer, 2018.