Intervals and symmetries for inertial units

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Inertial Unit

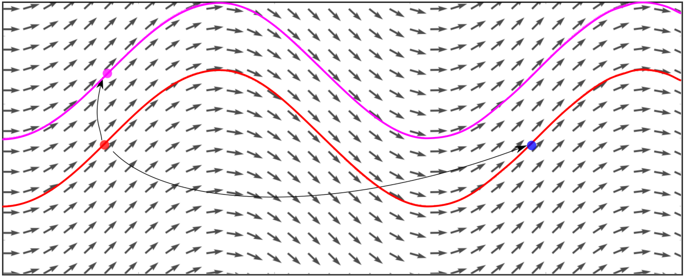
$$\begin{cases} \dot{\mathbf{p}} & = & \mathbf{R} \cdot \mathbf{v} \\ \dot{\mathbf{R}} & = & \mathbf{R} \cdot (\omega \wedge) \\ \dot{\mathbf{v}} & = & \mathbf{R}^T \cdot \mathbf{g}(\mathbf{p}) + \mathbf{a} - \omega \wedge \mathbf{v} \end{cases}$$

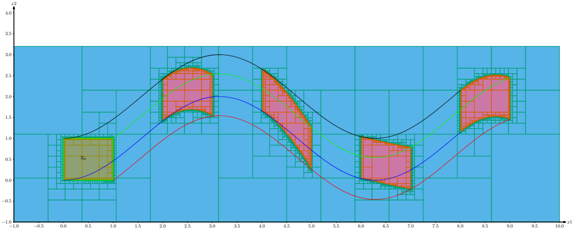
The inputs $\mathbf{a}$ and ω are collected by the accelerometers and the gyroscope.
We know the initial state and a *gravity map* $\mathbf{g}(\mathbf{p})$.

$$(\omega \wedge) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

$$\mathbf{R}(t+dt) = \mathbf{R}(t) \cdot \exp(dt \cdot \boldsymbol{\omega}(t) \wedge)$$

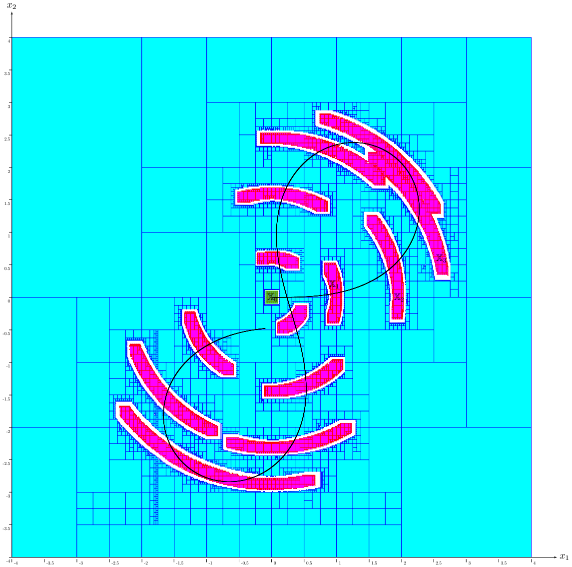
Symmetries





https://youtu.be/0fBloMWkSF8

$$\begin{cases} \dot{x}_1 = u_1 \cdot \cos x_3 \\ \dot{x}_2 = u_1 \cdot \sin x_3 \\ \dot{x}_3 = u_2 \end{cases}$$


x_2
x_1
X_0
X_1
X_2
X_3

Use Lie symmetries [2, 1] and interval integration [3],[4] to estimate the error of an inertial unit with integrity.

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