

Calcul par intervalles pour l'étude de la stabilité dynamique ; application au suivi de route de robots voiliers

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Journée du GT2 du GDR robotique,
Perception et localisation pour les robots marins
et sous-marins

22 novembre 2012, Paris, UPMC - Jussieu

1 Interval analysis

Problem. Given $f : \mathbb{R}^n \rightarrow \mathbb{R}$, a box $[\mathbf{x}] \subset \mathbb{R}^n$, prove that

$$\forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0.$$

Interval arithmetic can solve efficiently this problem.

Interval arithmetic

$$[-1, 3] + [2, 5] = [1, 8],$$

$$[-1, 3] \cdot [2, 5] = [-5, 15],$$

$$\text{abs}([-7, 1]) = [0, 7]$$

If f is given

Algorithm $f(\text{in: } \mathbf{x} = (x_1, x_2, x_3), \text{out: } y)$
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<pre>1 $z := x_1;$ 2 for $k := 0$ to 100 3 $z := (\cos x_2) (\sin(z) + kx_3);$ 4 next; 5 $y := \sin(zx_1);$</pre>
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Its interval extension is

Algorithm $[f]$ (in: $[x] = ([x_1], [x_2], [x_3])$, out: $[y]$)
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<pre>1 $[z] := [x_1];$ 2 for $k := 0$ to 100 3 $[z] := (\cos [x_2]) * (\sin ([z]) + k * [x_3]);$ 4 next; 5 $[y] := \sin([z] \cdot [x_1]);$</pre>

Theorem (Moore, 1970)

$$[f]([\mathbf{x}]) \subset \mathbb{R}^+ \Rightarrow \forall \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) \geq 0 \Leftrightarrow \neg(\exists \mathbf{x} \in [\mathbf{x}], f(\mathbf{x}) < 0)$$

2 Vaimos



Vaimos (IFREMER and ENSTA)

The robot satisfies a state equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{z}),$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector, $\mathbf{z} \in [\mathbf{z}]$ is the perturbation vector

With the controller $\mathbf{u} = \mathbf{g}(\mathbf{x}, \mathbf{z})$,

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{g}(\mathbf{x}, \mathbf{z}), \mathbf{z}) \\ &= \tilde{\mathbf{f}}(\mathbf{x}, \mathbf{z}).\end{aligned}$$

Define

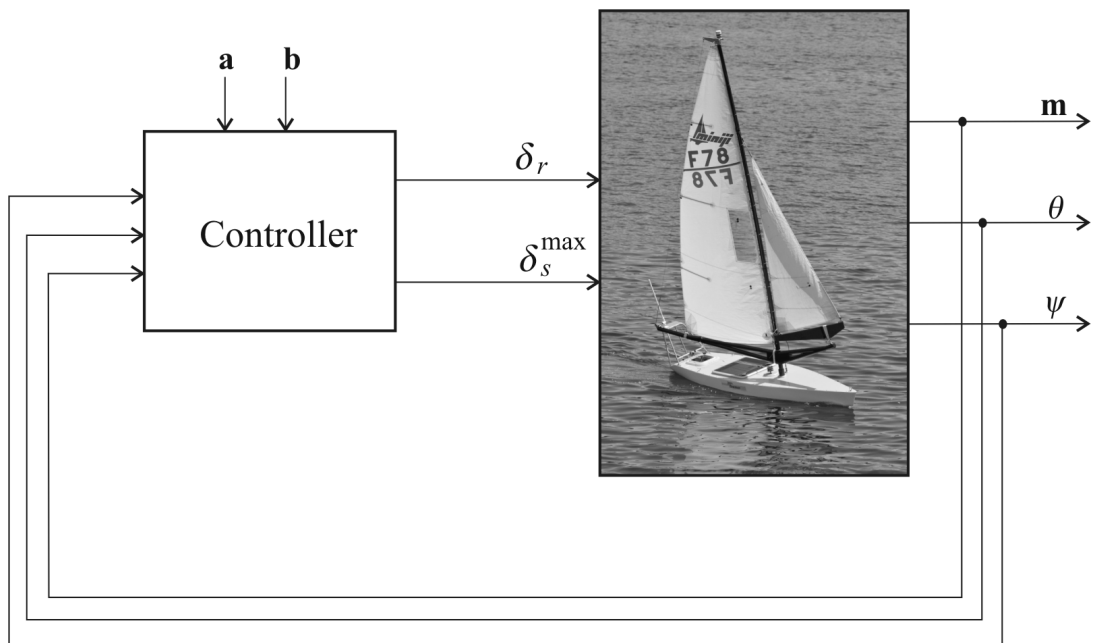
$$\mathbf{F}(\mathbf{x}) = \left\{ \tilde{\mathbf{f}}(\mathbf{x}, \mathbf{z}), \mathbf{z} \in [\mathbf{z}] \right\}.$$

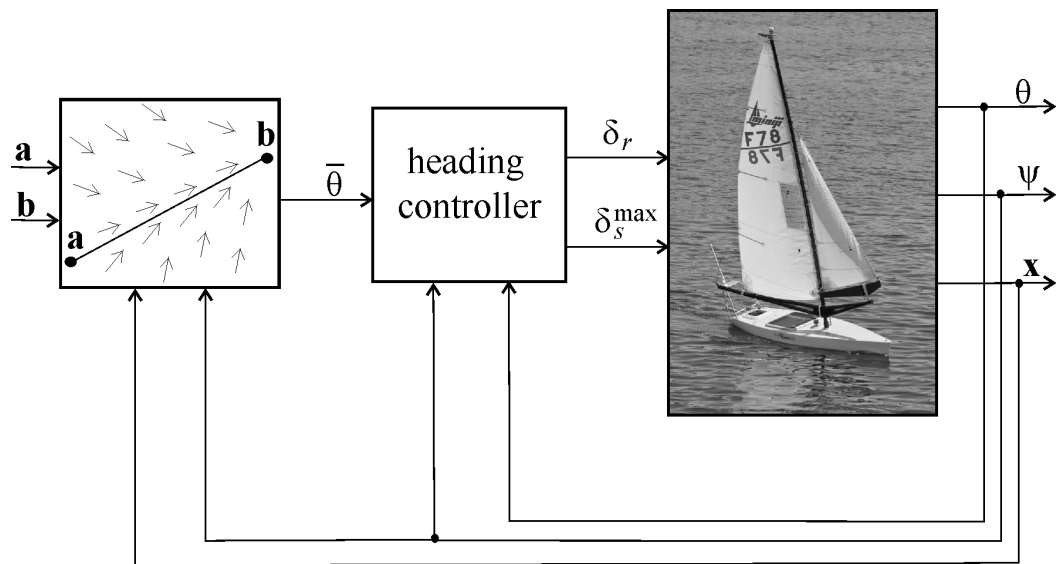
The robot satisfies.

$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$$

which is *a differential inclusion*.

3 Line following

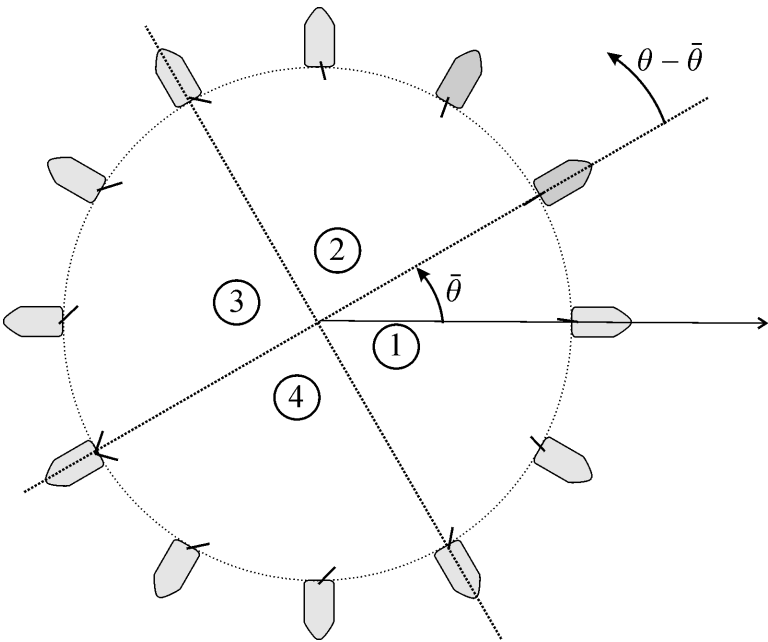




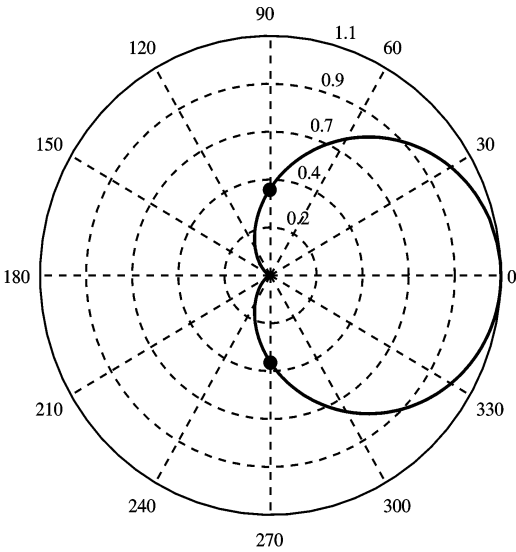
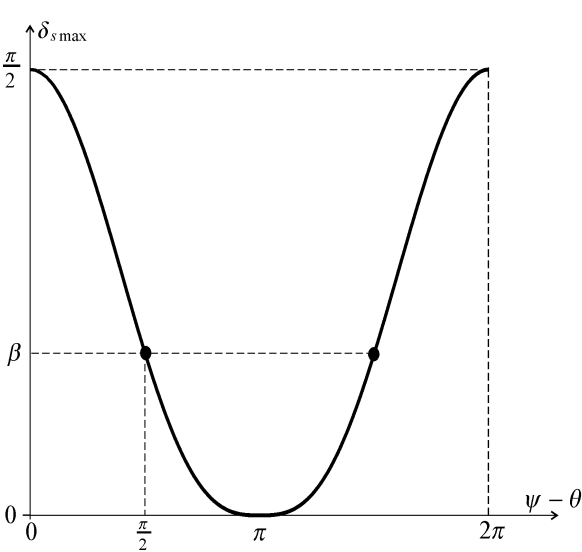
3.1 Heading controller

$$\begin{cases} \delta_r &= \begin{cases} \delta_r^{\max} \cdot \sin(\theta - \bar{\theta}) & \text{if } \cos(\theta - \bar{\theta}) \geq 0 \\ \delta_r^{\max} \cdot \text{sign}(\sin(\theta - \bar{\theta})) & \text{otherwise} \end{cases} \\ \delta_s^{\max} &= \frac{\pi}{2} \cdot \left(\frac{\cos(\psi - \bar{\theta}) + 1}{2} \right)^q . \end{cases}$$

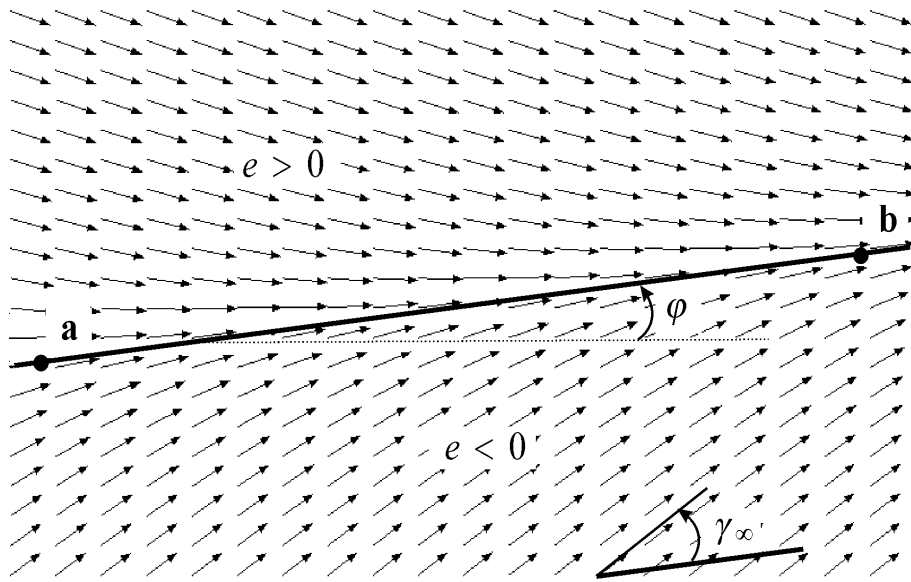
Rudder



Sail

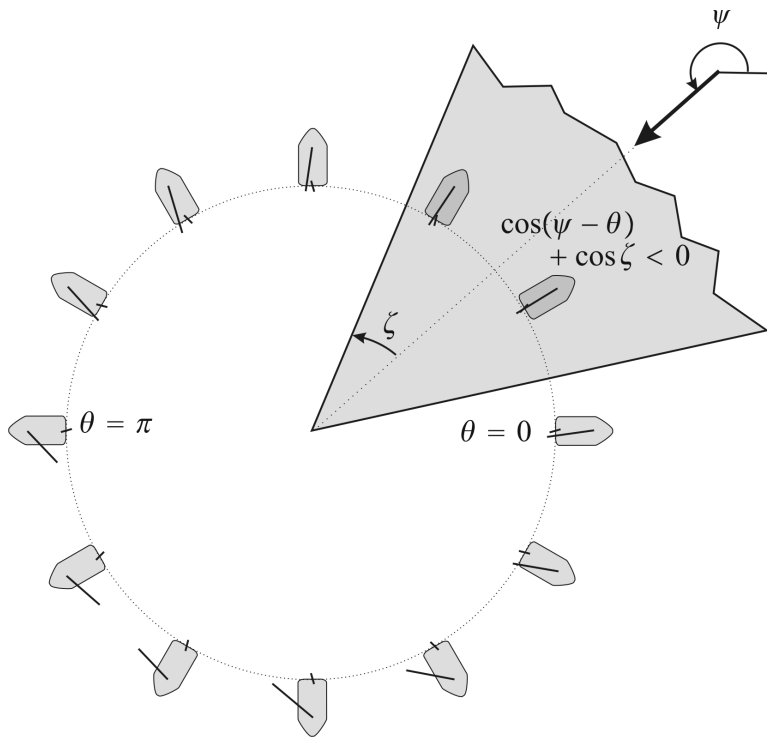


3.2 Vector field



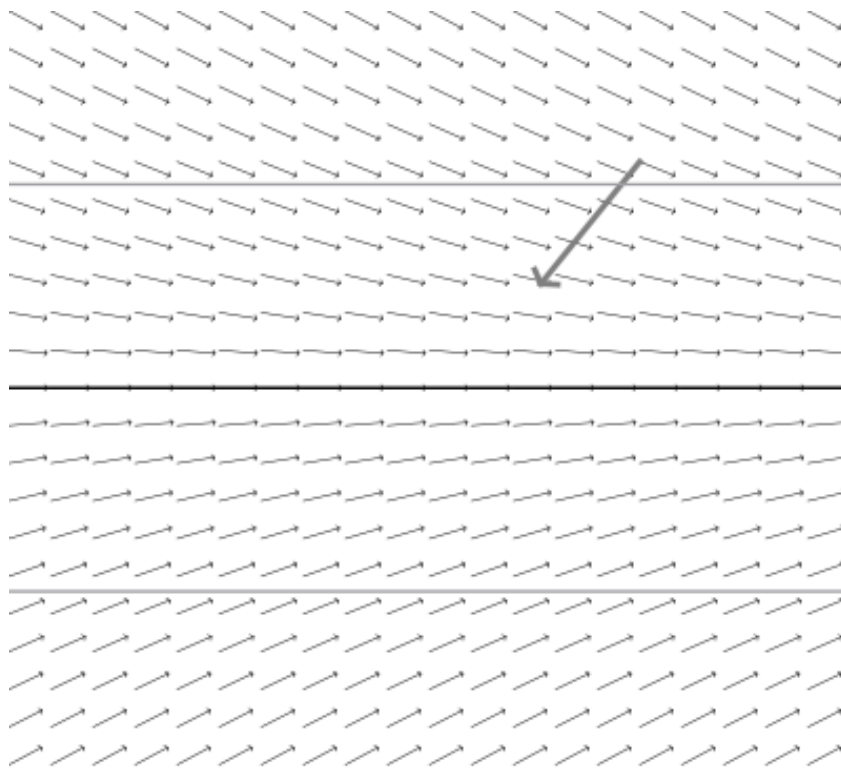
Nominal vector field: $\theta^* = \varphi - \frac{2 \cdot \gamma_{\infty}}{\pi} \cdot \text{atan} \left(\frac{e}{r} \right) .$

A course θ^* may be unfeasible

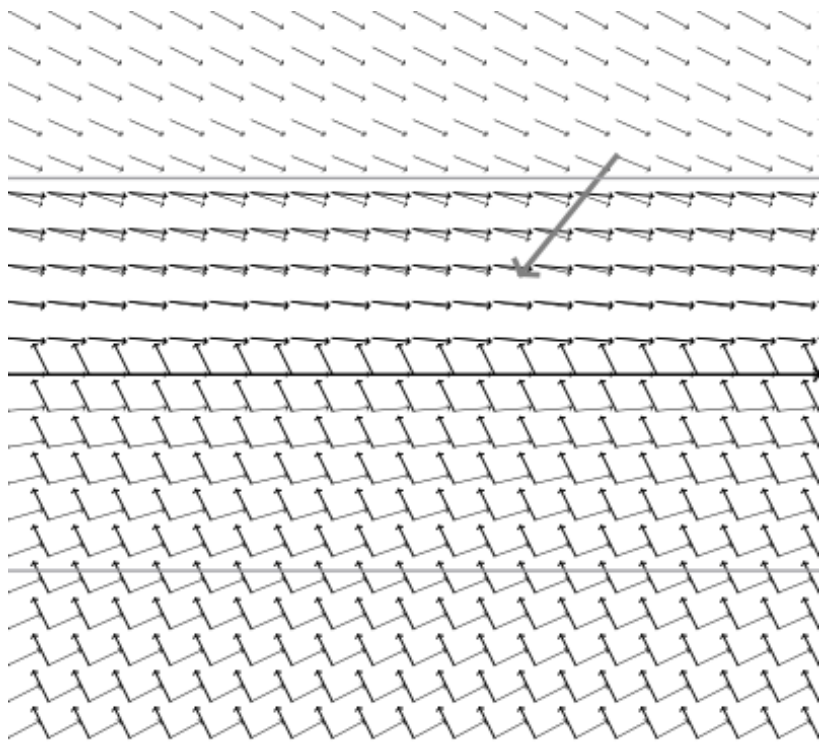


$$\cos(\psi - \bar{\theta}) + \cos \zeta < 0 \Rightarrow \bar{\theta} \text{ is unfeasible}$$

In this case, take $\bar{\theta} = \pi + \psi - \zeta \cdot \text{sign}(e)$



$$\theta^* = -\frac{2 \cdot \gamma_\infty}{\pi} \cdot \text{atan}\left(\frac{e}{r}\right)$$



Keep-close-hauled strategy.

3.3 Controller

Controller in: $\mathbf{m}, \theta, \psi, \mathbf{a}, \mathbf{b}$; out: $\delta_r, \delta_s^{\max}$; inout: q

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1   $e = \det \left( \frac{\mathbf{b} - \mathbf{a}}{\|\mathbf{b} - \mathbf{a}\|}, \mathbf{m} - \mathbf{a} \right)$ 
2  if  $|e| > \frac{r}{2}$  then  $q = \text{sign}(e)$ 
3   $\varphi = \text{atan2}(\mathbf{b} - \mathbf{a})$ 
4   $\theta^* = \varphi - \frac{2 \cdot \gamma_\infty}{\pi} \cdot \text{atan}\left(\frac{e}{r}\right)$ 
5  if  $\cos(\psi - \theta^*) + \cos \zeta < 0$ 
6    or ( $|e| < r$  and  $(\cos(\psi - \varphi) + \cos \zeta < 0)$ )
7    then  $\bar{\theta} = \pi + \psi - q \cdot \zeta$ .
8    else  $\bar{\theta} = \theta^*$ 
9  end
10 if  $\cos(\theta - \bar{\theta}) \geq 0$  then  $\delta_r = \delta_r^{\max} \cdot \sin(\theta - \bar{\theta})$ 
11 else  $\delta_r = \delta_r^{\max} \cdot \text{sign}(\sin(\theta - \bar{\theta}))$ 
12  $\delta_s^{\max} = \frac{\pi}{2} \cdot \left( \frac{\cos(\psi - \bar{\theta}) + 1}{2} \right)$ .
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The sailboat robot satisfies

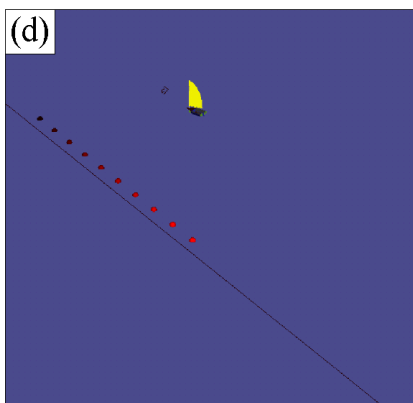
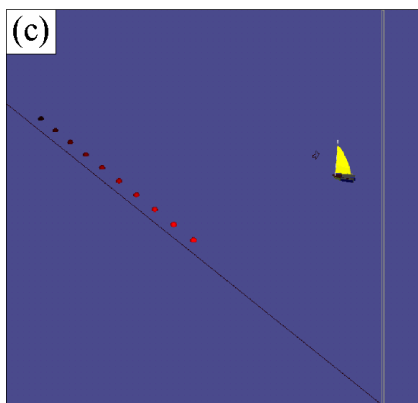
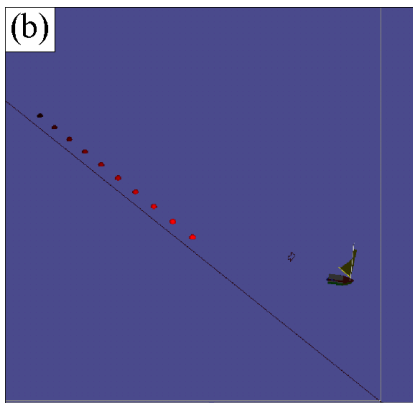
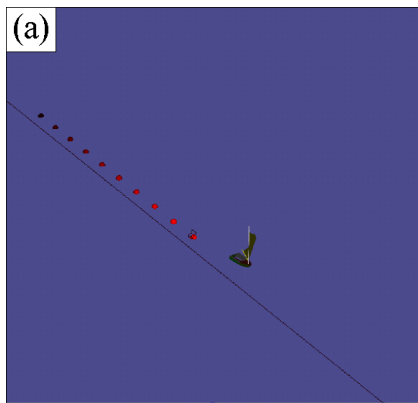
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \psi, q)$$

or equivalently

$$\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$$

which is a differential inclusion.

4 Validation by simulation



5 Theoretical validation

Le concept de V-Stabilité et sa résolution par des méthodes ensemblistes entre dans le cadre d'un contrat que nous avons avec MBDA (contact : Stéphane Le Menec).

Stability. The system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$$

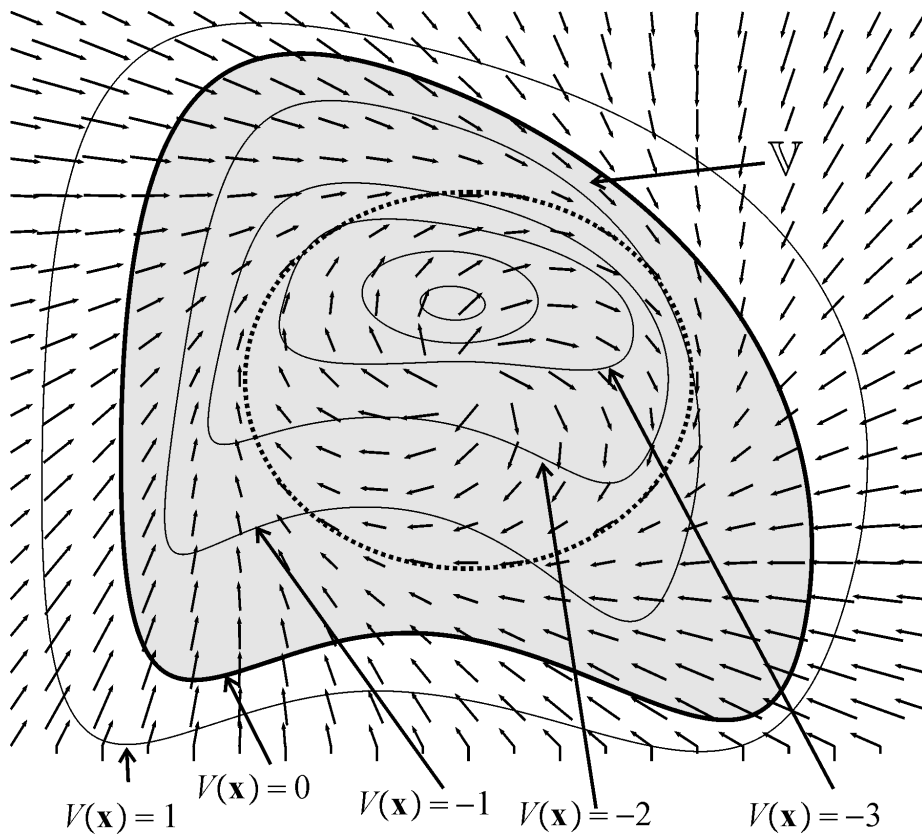
is Lyapunov-stable (1892) if there exists $V(\mathbf{x}) \geq 0$ such that

$$\dot{V}(\mathbf{x}) < 0 \text{ if } \mathbf{x} \neq \mathbf{0}.$$

$$V(\mathbf{x}) = 0 \text{ iff } \mathbf{x} = \mathbf{0}$$

Definition. Consider a differentiable function $V(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$. The system is V -stable if

$$\left(V(\mathbf{x}) \geq 0 \Rightarrow \dot{V}(\mathbf{x}) < 0 \right).$$



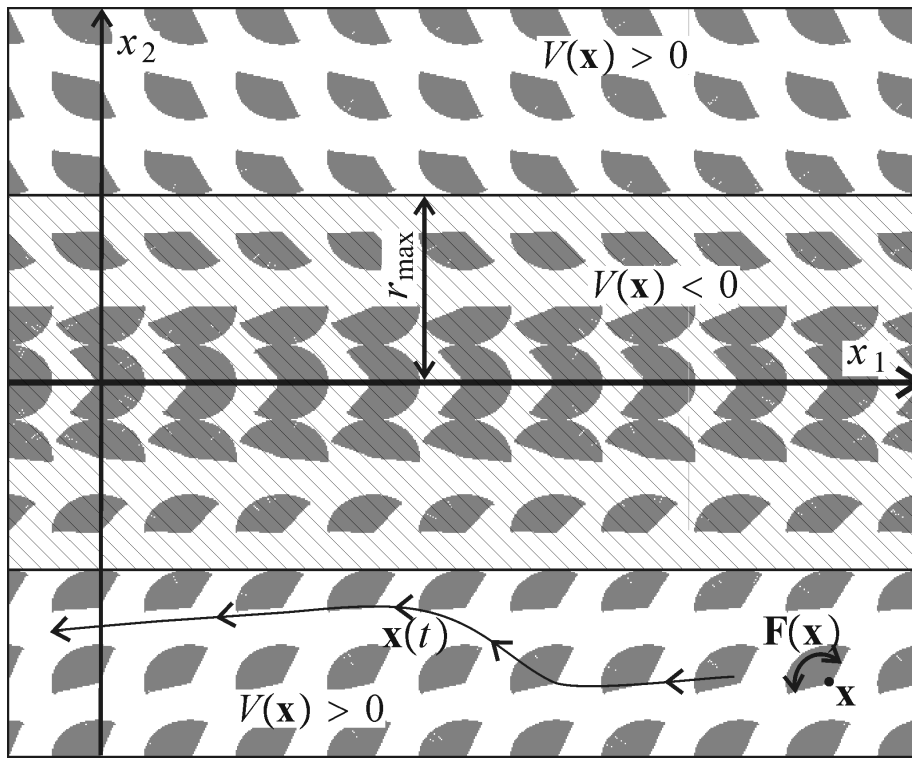
Theorem. If the system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ is V -stable then

- (i) $\forall \mathbf{x}(0), \exists t \geq 0$ such that $V(\mathbf{x}(t)) < 0$
- (ii) if $V(\mathbf{x}(t)) < 0$ then $\forall \tau > 0, V(\mathbf{x}(t + \tau)) < 0$.

Theorem. We have

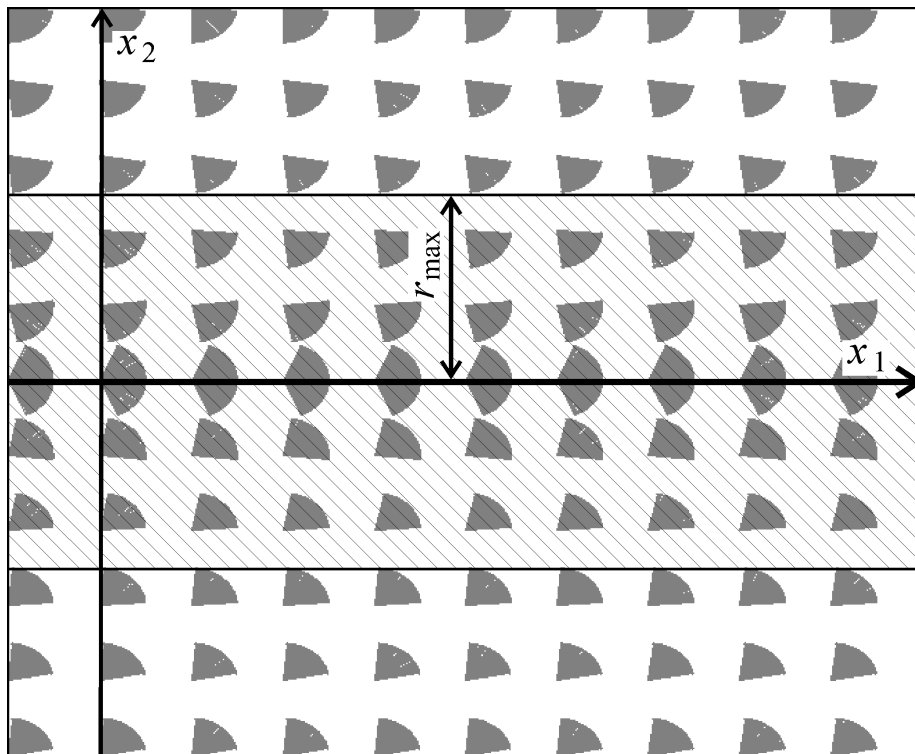
$$\left\{ \begin{array}{l} \frac{\partial V}{\partial \mathbf{x}}(\mathbf{x}) \cdot \mathbf{a} \geq 0 \\ \mathbf{a} \in \mathbf{F}(\mathbf{x}) \\ V(\mathbf{x}) \geq 0 \end{array} \right. \text{ inconsistent } \Leftrightarrow \dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x}) \text{ is } V\text{-stable}$$

Interval methods could easily prove the V -stability.



Differential inclusion $\dot{\mathbf{x}} \in \mathbf{F}(\mathbf{x})$ for the sailboat.

$$V(x) = x_2^2 - r_{\max}^2.$$



For Vaimos, we have found parameters for the controller such that

Property 1. If $|e(\mathbf{x})| < r_{\max}$ then, it will be the case for ever.

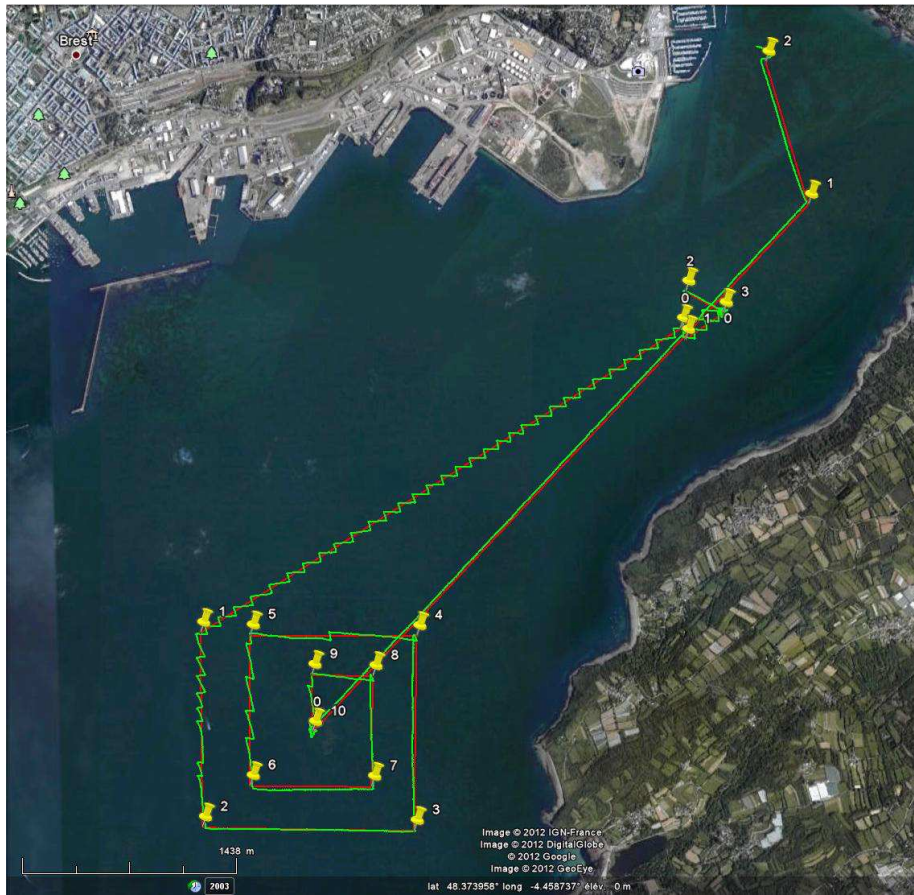
Property 2. If $|e(\mathbf{x})| > r_{\max}$ then $|e(\mathbf{x})|$ will decrease until $|e(\mathbf{x})| < r_{\max}$.

Property 3. The course is be feasible, i.e.,

$$\cos(\psi - \bar{\theta}) + \cos \zeta \geq 0.$$

6 Experimental validation

Brest



Brest: Vaimos did 27 km

Show the dashboard

Brest-Douarnenez. January 17, 2012, 8am











Middle of Atlantic ocean



350 km made by Vaimos in 53h, September 6-9, 2012.

Consequence.

It is possible for a sailboat robot to navigate inside a corridor.

Essential, to create circulation rules when robot swarms are considered.

Essential to determine who has to pay in case of accident.

Main reference: L. Jaulin, F. Le Bars (2012). An interval approach for stability analysis; Application to sailboat robotics. IEEE Transaction on Robotics, Volume 27, Issue 5.

Next year WRSC will take place in Brest, the first week of
september