

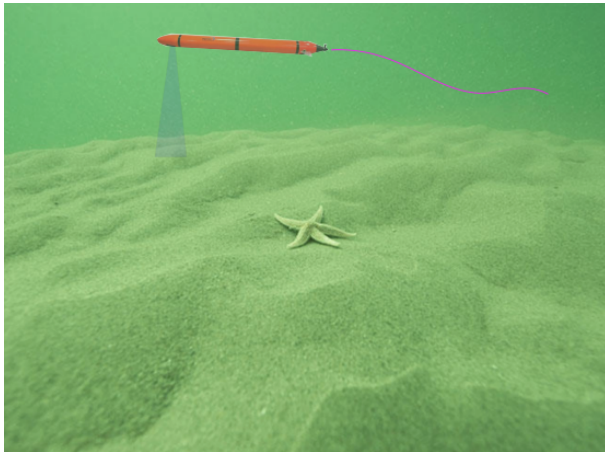
# Explore and return with underwater robots in a minimalist environment

Luc Jaulin, Quentin Brateau and Fabrice Le Bars



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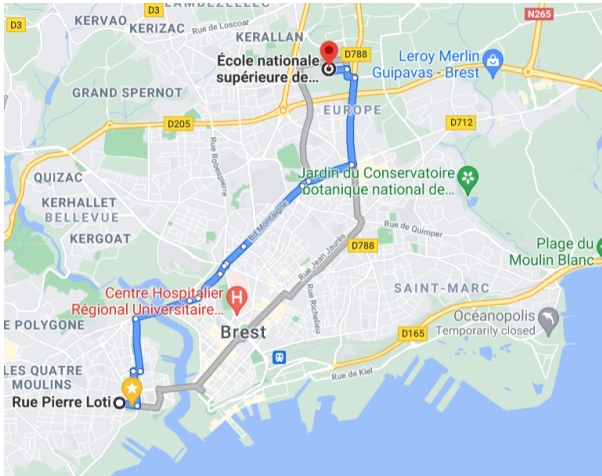
# 1. Underwater navigation



Explore and return in a minimalist environment

# Map-based navigation





Modern navigation: high cost (computation, infrastructure)

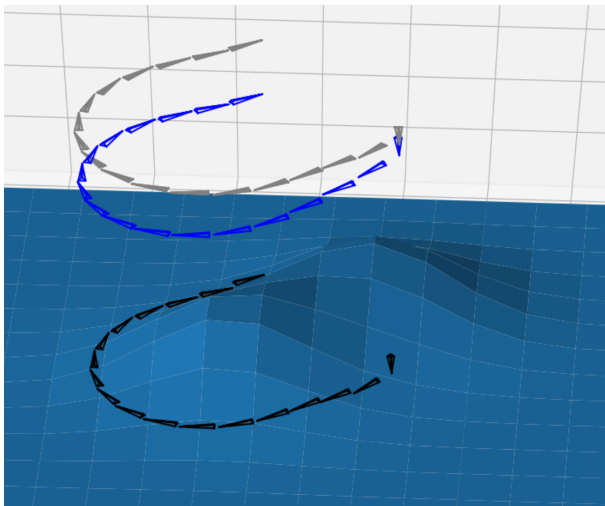
# Route-based navigation



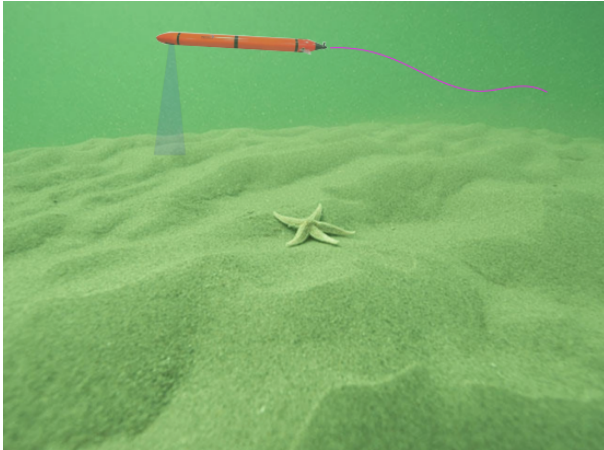
## Submeeting 2018

# Follow a route

Given a function  $h: \mathbb{R}^2 \mapsto \mathbb{R}$ , a route is defined by  $h(\mathbf{p}) = 0$ .  
 $h$  could be the temperature, the radiation, the pressure, the altitude, the time shift between two periodic events.

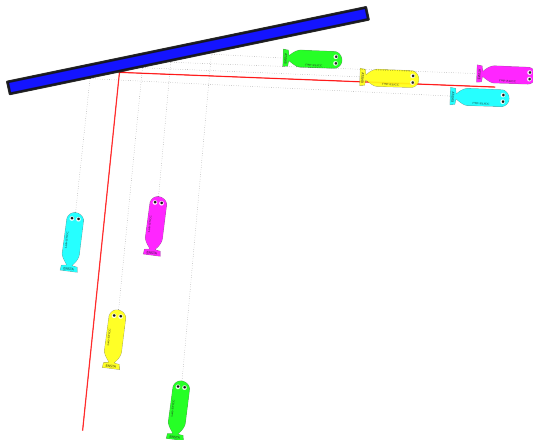


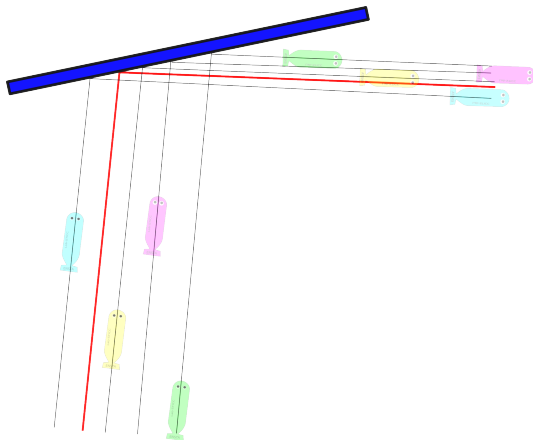
## 2. Stable bouncing (phd of Quentin Brateau)



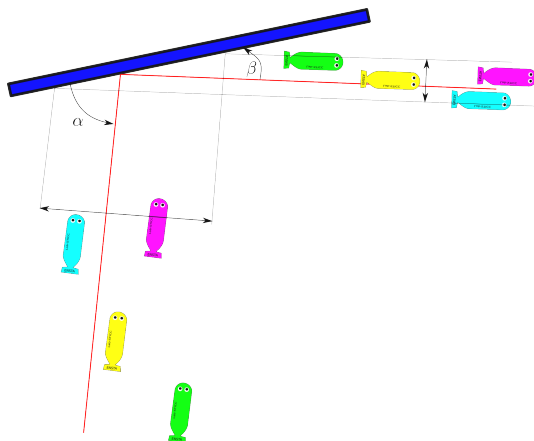
No route exists



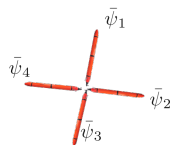
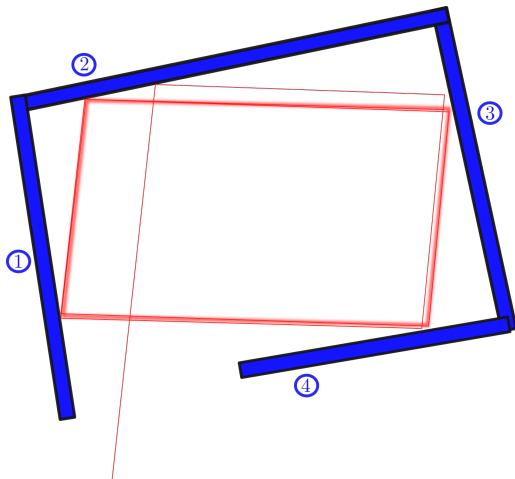


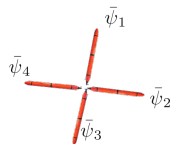
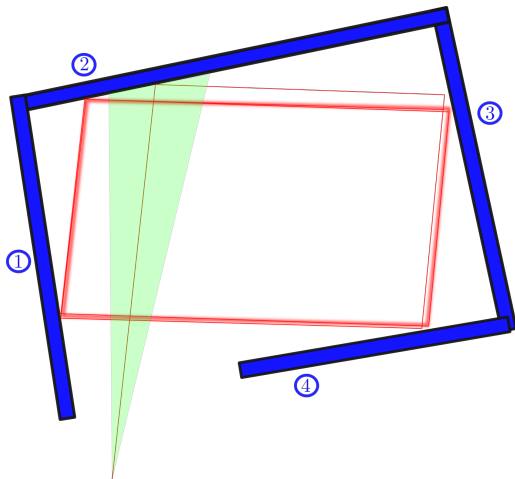


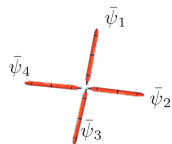
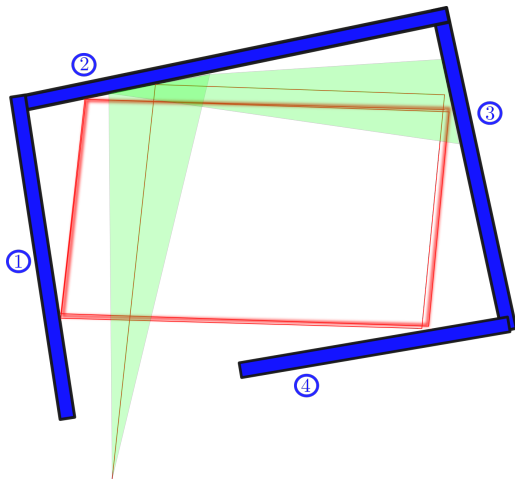
Contraction of the distance

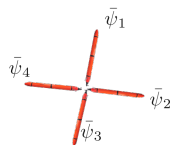
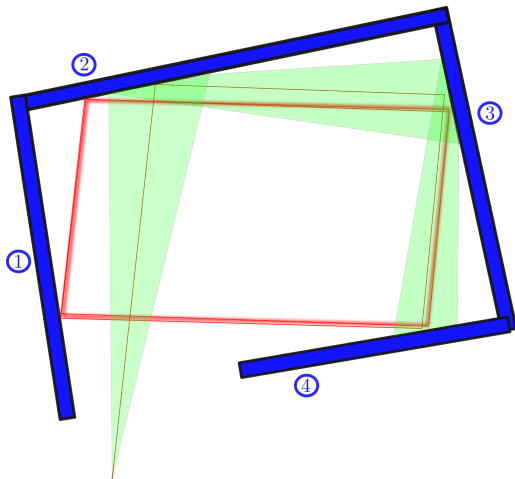


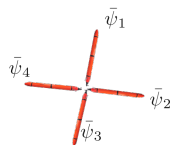
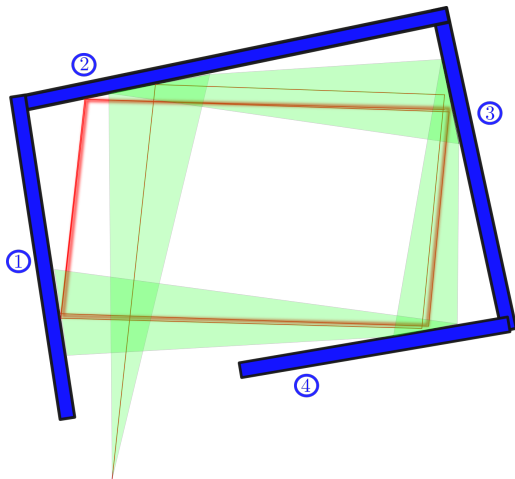
Contraction if  $\frac{\sin \beta}{\sin \alpha} < 1$



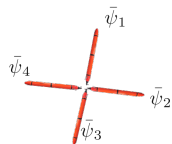
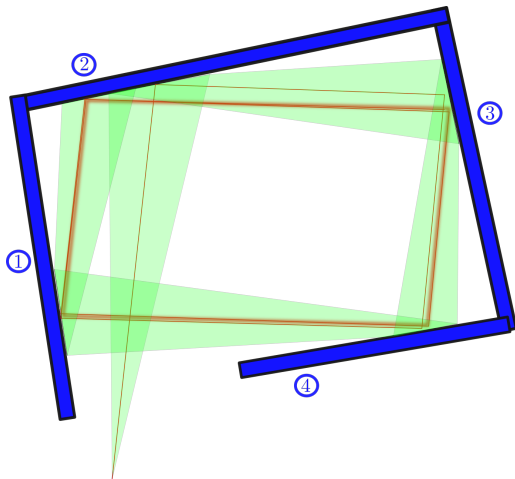


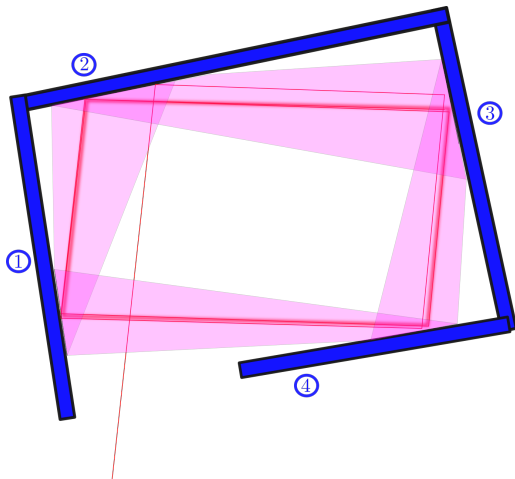


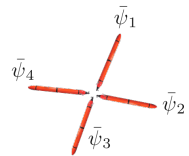
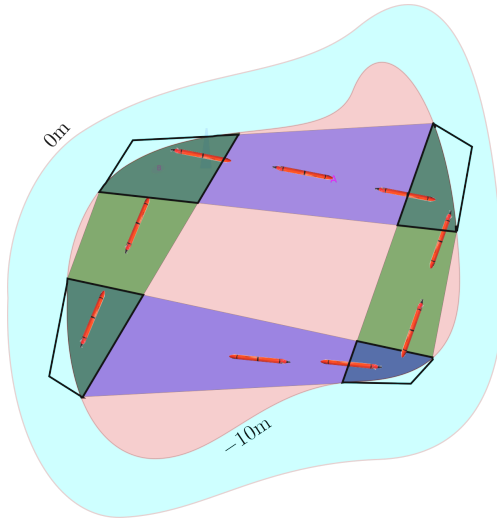


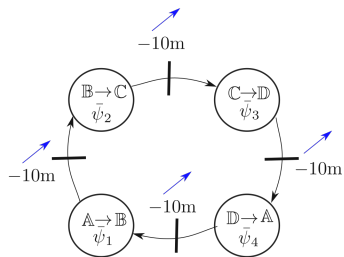
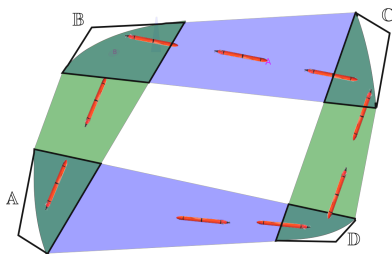


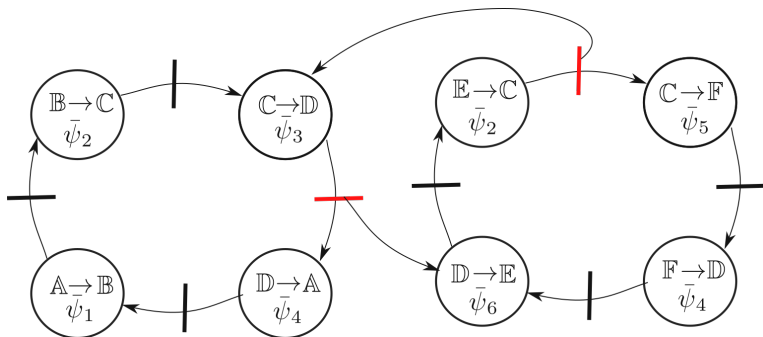




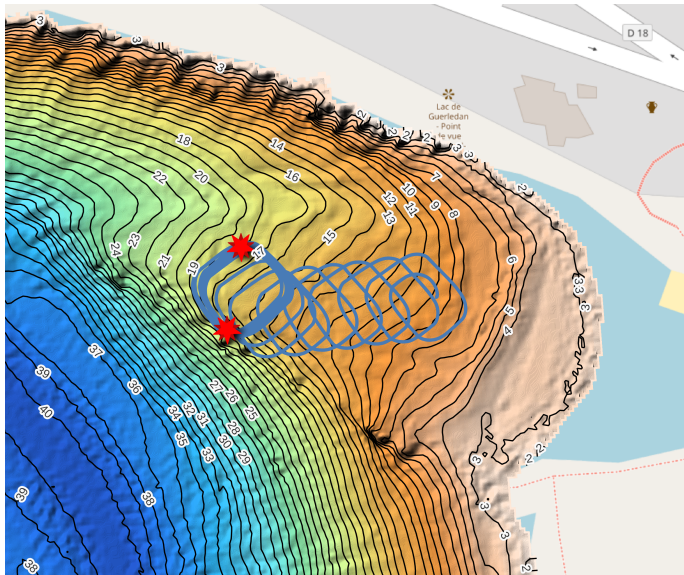








# Experiment (phd of Quentin Brateau)

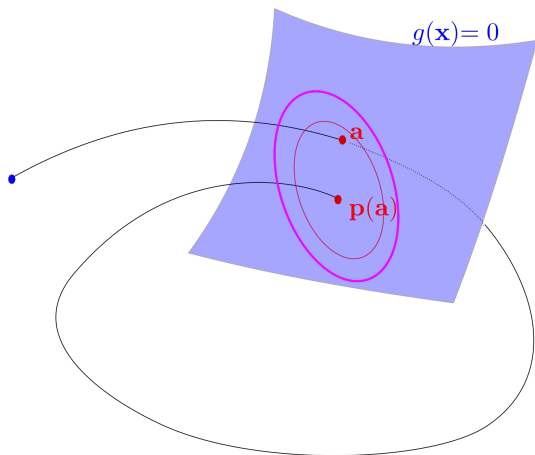


# Stability of cycles



The Poincaré first recurrence map is defined by

$$\mathbf{a}(k+1) = \mathbf{p}(\mathbf{a}(k))$$



# References

- 1 Interval and stability [2][7]
- 2 Route following [4][5]
- 3 Navigation with stable cycles [3]
- 4 Tubes [6][1]



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