

Secure a zone with robots using a set-membership state observer

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Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne



Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.



Bay of Biscay 220 000 km²



An intruder

- Several robots $\mathcal{R}_1, \dots, \mathcal{R}_n$ at positions $\mathbf{a}_1, \dots, \mathbf{a}_n$ are moving in the ocean.
- If the intruder is in the visibility zone of one robot, it is detected.

Complementary approach

- We assume that a virtual intruder exists inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}(t)).$$

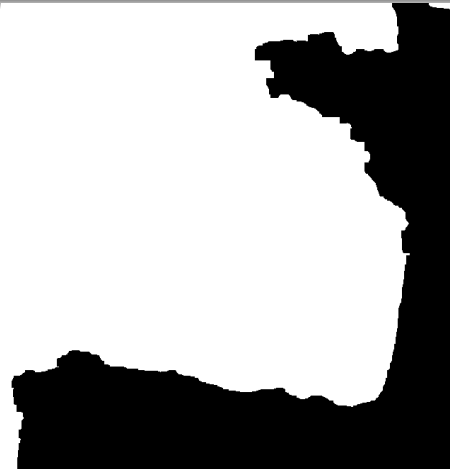
- Each robot $\mathcal{R}_i$ has the visibility zone $g_{\mathbf{a}_i}^{-1}([0, d_i])$ where d_i is the scope.

Theorem. An (undetected) intruder has a state vector $\mathbf{x}(t)$ inside the set

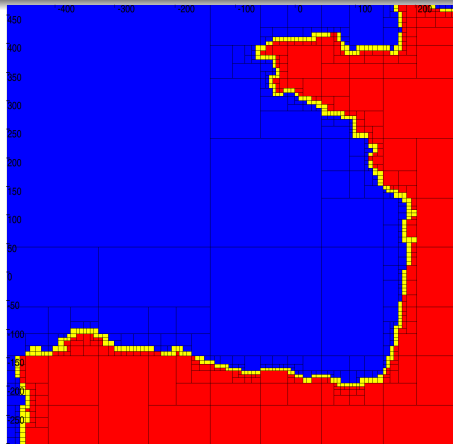
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t - dt) + dt \cdot \mathbb{F}(\mathbb{X}(t - dt))) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty]),$$

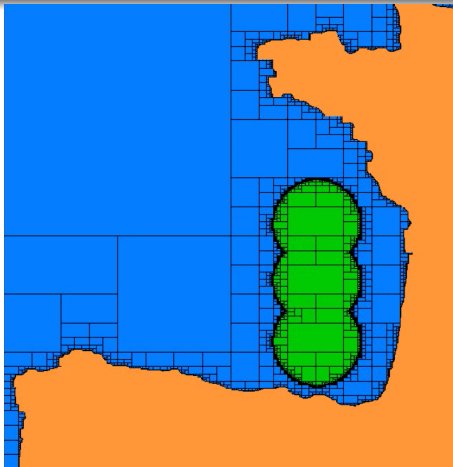
where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

$$\mathbb{S}(t) = \overline{\text{proj}_{\text{world}}(\mathbb{X}(t))}.$$

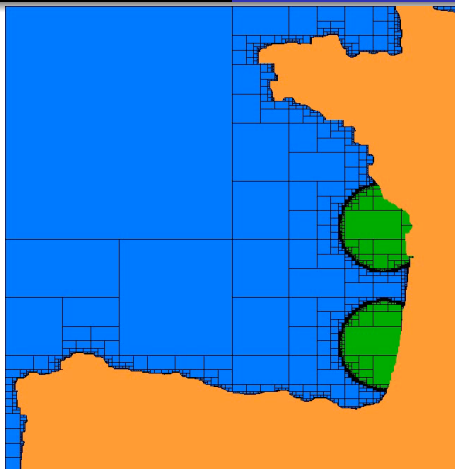


Set $\mathbb{G}$ in white

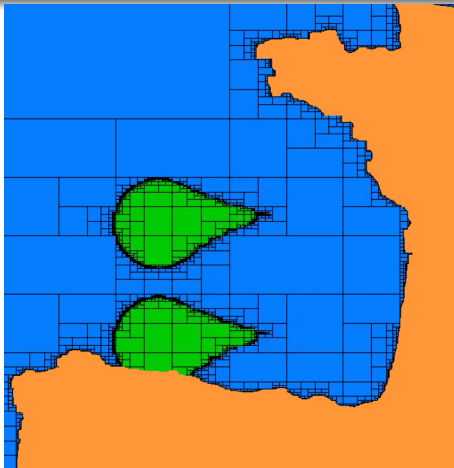




$$\text{green: } \bigcup_i g_{a_i(t)}^{-1}([0, d_i(t)])$$



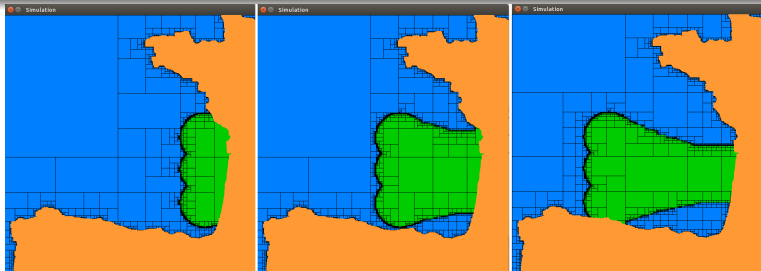
Blue: $\mathbb{G} \cap \bigcap_i g_{a_i(t)}^{-1}([d_i(t), \infty])$

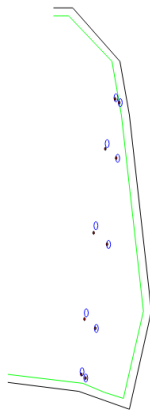


Blue:

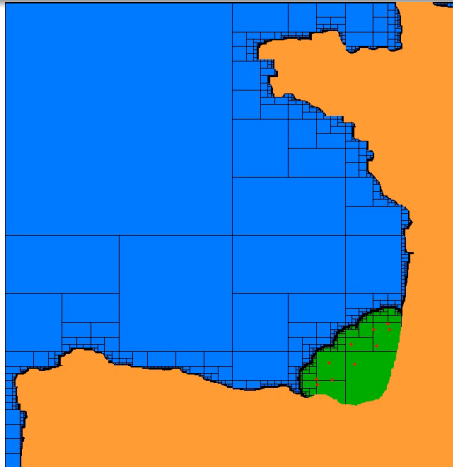
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty]).$$

Secure a zone
Complementary approach
Thick sets



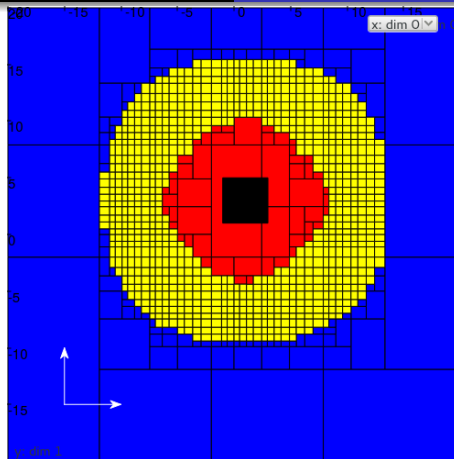


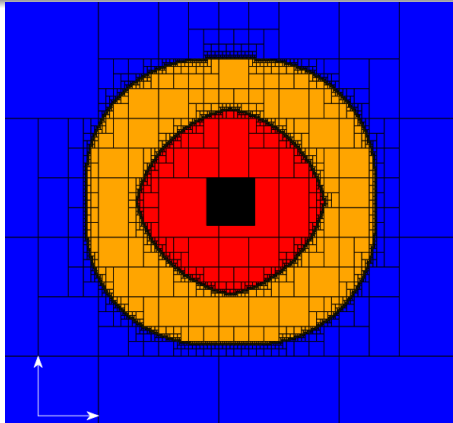
Strategy of the ellipse

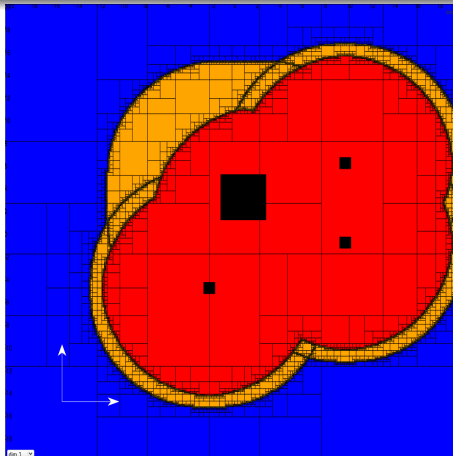


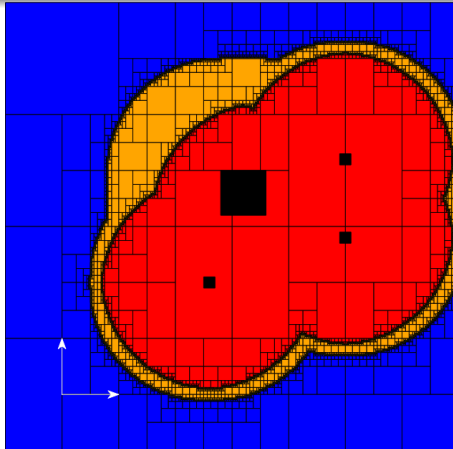
Video : <https://youtu.be/rNcDW6npLfE>

Thick sets









If $\mathbf{a}_i(t) \in [\mathbf{a}_i](t)$, the thick observer is

$$[\mathbf{X}](t) = \mathbb{G} \cap ([\mathbf{X}](t - dt) + dt \cdot \mathbb{F}([\mathbf{X}](t - dt))) \cap \bigcap_i g_{[\mathbf{a}_i](t)}^{-1}([d_i(t)], \infty).$$

Smoother

Idea: Take into account the future.

The feasible set can be obtained by the following contractions

$$\begin{aligned}\overrightarrow{\mathbb{X}}(t) &= \overrightarrow{\mathbb{X}}(t) \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \\ \overleftarrow{\mathbb{X}}(t) &= \overleftarrow{\mathbb{X}}(t) \cap (\mathbb{X}(t+dt) - dt \cdot \mathbb{F}(\mathbb{X}(t+dt))) \\ \mathbb{X}(t) &= \overrightarrow{\mathbb{X}}(t) \cap \overleftarrow{\mathbb{X}}(t)\end{aligned}$$

with the initialization

$$\mathbb{X}(t) = \overrightarrow{\mathbb{X}}(t) = \overleftarrow{\mathbb{X}}(t) = \mathbb{G}.$$