Secure the Bay of Biscay with Robots

L. Jaulin, B. Zerr and 15 students

ENSTA-Bretagne, UBO, Lab-STICC

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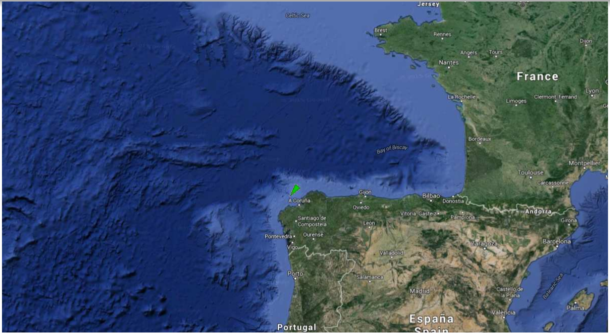
Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.



Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1, \dots, \mathscr{R}_n$ at positions $\mathbf{a}_1, \dots, \mathbf{a}_n$ are moving in a 2D or 3D world.
- If the intruder is in the visibility zone of one robot, it is detected.
- The robots collaborate to guarantee that they is no moving intruder inside a subzone of $\mathbb{G}$.

Complementary approach

- We assume that there exists a virtual intruder with a limited speed inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies a state inclusion

$$\frac{d\mathbf{x}}{dt}(t) \in \mathbb{F}(\mathbf{x}(t)).$$

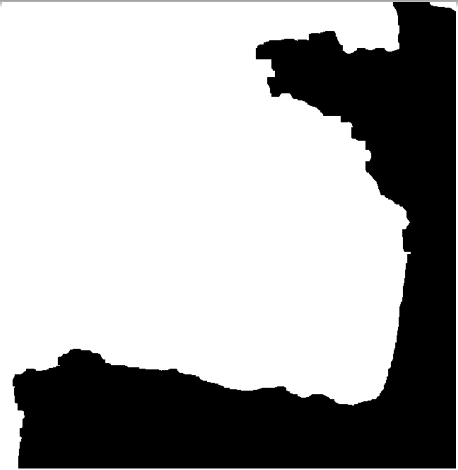
- Each robot $\mathscr{R}_i$ has a visibility zone of the form $g_{\mathbf{a}_i}^{-1}([0,d_i])$ where d_i is the scope.

Theorem. The intruder has a state vector $\mathbf{x}(t)$ inside the set

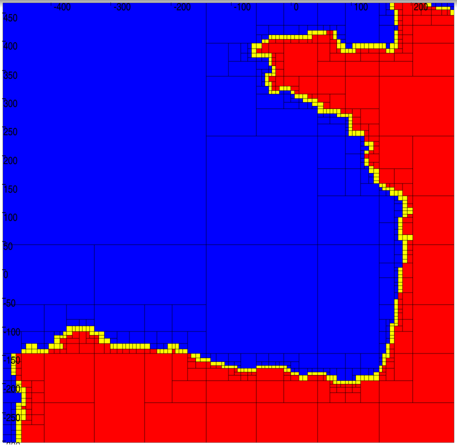
$$\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

$$\mathbb{S}(t) = \overline{proj_{world}(\mathbb{X}(t))}.$$



Set $\mathbb{G}$ in white


-400
-300
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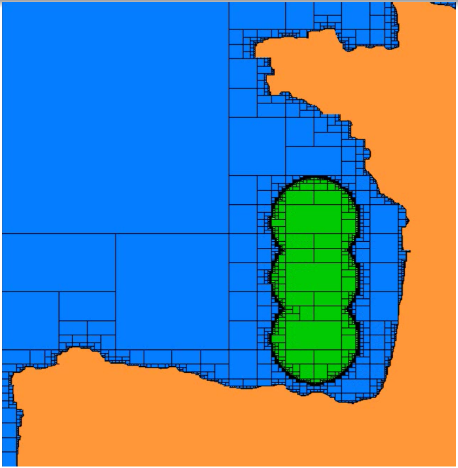
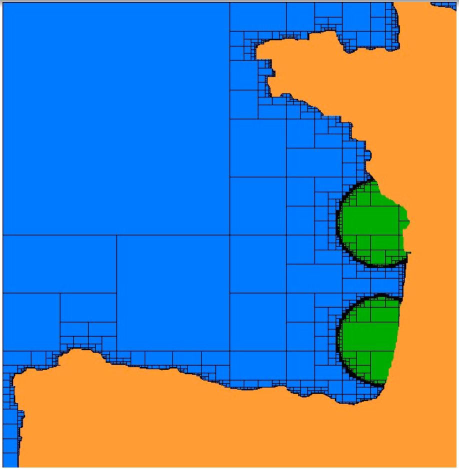
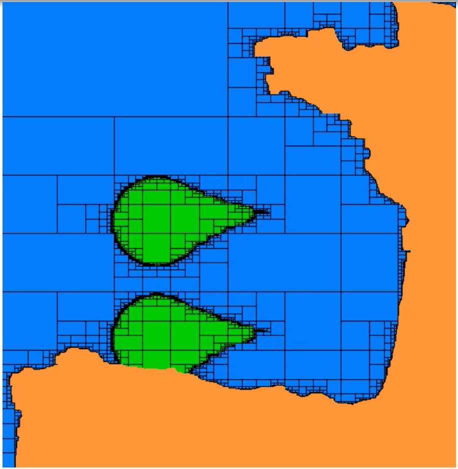
-200
-250



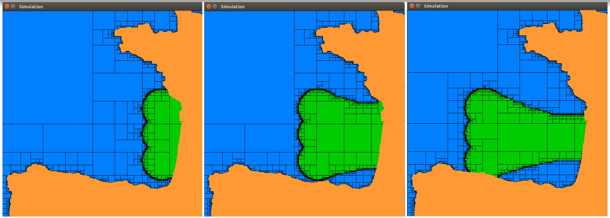
green: $\bigcup_i g^{-1}_{\mathbf{a}_i(t)}([0, d_i(t)])$

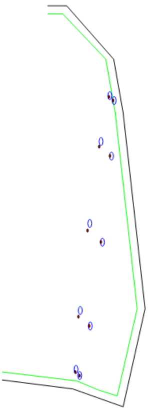


Blue: $\mathbb{G} \cap \bigcap_i g_{\mathbf{a}_i(t)}^{-1}([d_i(t), \infty])$

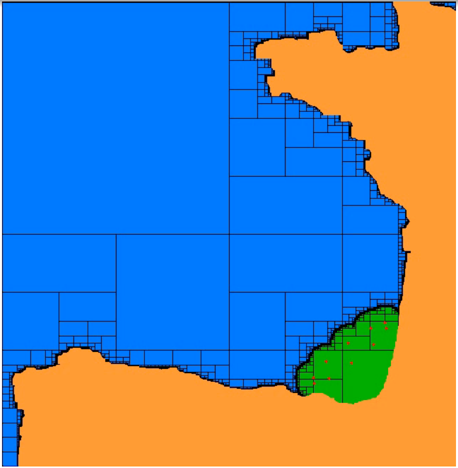


Blue: $\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t - dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty])$.


Simulation
Simulation
Simulation

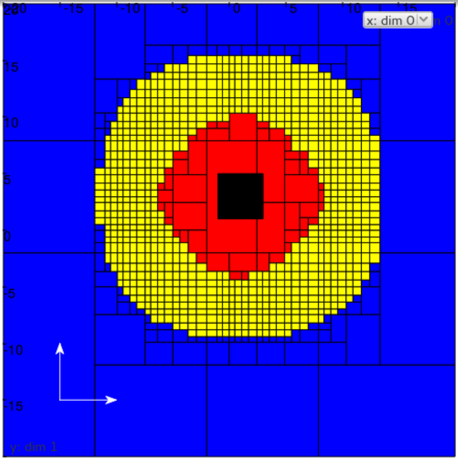


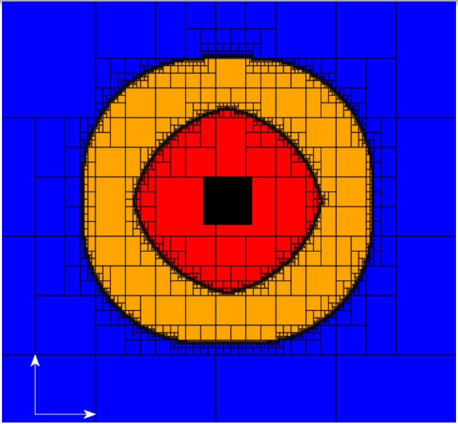
Strategy of the ellipse

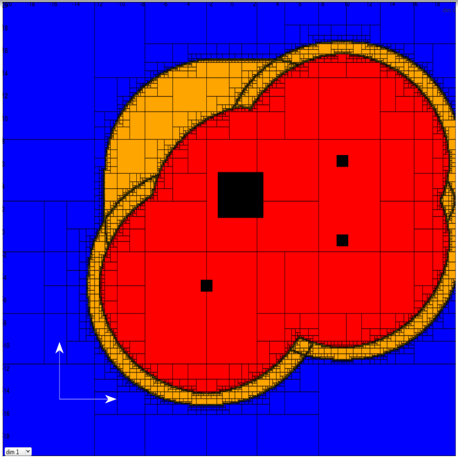


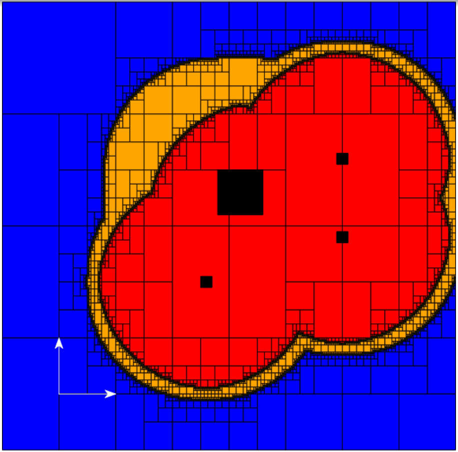
Video : https://youtu.be/rNcDW6npLfE

Thick sets


x: dim 0
y: dim 1





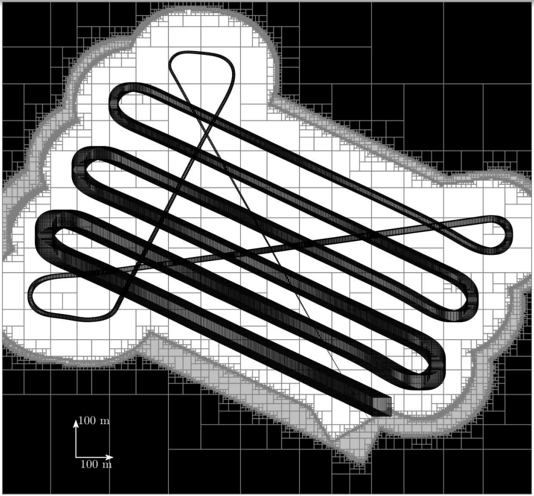


We have $d_i(t) \in [d_i(t)]$ and $\mathbf{a}_i(t) \in [\mathbf{a}_i](t)$. The thick observer is

$$[\![\mathbb{X}]\!](t) = \mathbb{G} \cap dt \cdot \mathbb{F}([\![\mathbb{X}]\!](t - dt)) \cap \bigcap_i g^{-1}_{[\mathbf{a}_i](t)}([\![[d_i(t)], \infty]\!]).$$

Formalism

B. Desrochers and L. Jaulin (2016). Computing a guaranteed approximation the zone explored by a robot. *IEEE Transaction on Automatic Control*.


100 m
100 m

Thick sets

$$
\begin{aligned}
[\![\mathbb{X}]\!] &= [\![\mathbb{X}^{\subset}, \mathbb{X}^{\supset}]\!] \\
&= \{\mathbb{X} \in \mathscr{P}(\mathbb{R}^n) \mid \mathbb{X}^{\subset} \subset \mathbb{X} \subset \mathbb{X}^{\supset}\}
\end{aligned}
$$

Intersection of thick sets

$$
\begin{array}{rcl}
[\![\mathbb{X}]\!] \cap [\![\mathbb{Y}]\!] & = & \{\mathbb{X} \cap \mathbb{Y}, \mathbb{X} \in [\![\mathbb{X}]\!], \mathbb{Y} \in [\![\mathbb{Y}]\!]\} \\
& = & [\mathbb{X}^{\subset} \cap \mathbb{Y}^{\subset}, \mathbb{X}^{\supset} \cap \mathbb{Y}^{\supset}] \\
[\![\mathbb{X}]\!] \sqcap [\![\mathbb{Y}]\!] & = & \{\mathbb{Z} \in \mathscr{P}(\mathbb{R}^n), \mathbb{Z} \in [\![\mathbb{X}]\!], \mathbb{Z} \in [\![\mathbb{Y}]\!]\}. \\
& = & [\mathbb{X}^{\subset} \cup \mathbb{Y}^{\subset}, \mathbb{X}^{\supset} \cap \mathbb{Y}^{\supset}]
\end{array}
$$

Set inversion

$$\mathbb{X} = \mathbf{f}^{-1}(\mathbb{Y}).$$

Thick set inversion

$$\mathbb{X} = \mathbf{f}^{-1}(\mathbb{Y}), \mathbf{f} \in [\mathbf{f}] \text{ and } \mathbb{Y} \in [\![\mathbb{Y}]\!]$$

where $[\![\mathbb{Y}]\!]$ and $[\mathbf{f}]$ are thick.

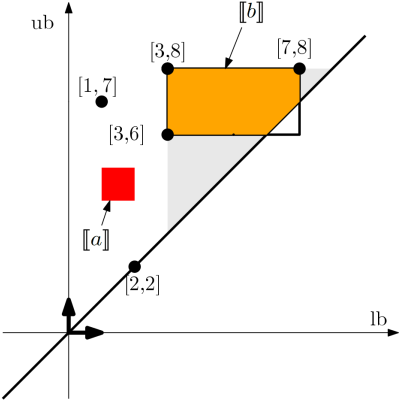
Thick intervals

Lower-upper bounds representation

$$\begin{array}{rcl} [\![x]\!] & = & [\![[x^-],[x^-]]\!] \\ & = & \left\{[x^-,x^+] \in \mathbb{IR} \mid x^- \subset [x^-] \text{ and } x^+ \subset [x^+]\right\} \end{array} .$$

Su(b-p)set representation

$$[\![x]\!] = \{[x] \in \mathbb{IR} \mid [x^\subset] \subset [x] \subset [x^\supset]\}.$$


ub
lb
[[b]]
[3,8]
[7,8]
[1, 7]
[3,6]
[[a]]
[2,2]

Exercise. The two intervals $[a] = [1,5]$ and $[b] \in [\![[2,4],[3,6]]\!]$, overlap ?

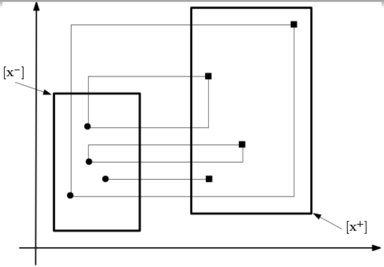
Yes, always:

$$[2,4]-5 \subset \mathbb{R}^{-} \text{and}\, 1-[3,6] \subset \mathbb{R}^{-}.$$

Using the su(b-p)set bounds, we cannot conclude:

$$\emptyset \subset [b] \subset [2,6].$$

The interval $[b]=[6,6]$ satisties this inclusion but does not intersect $[a]$.


[x⁻]
[x⁺]

Non causal secure zone

Idea: Take into account the future. If $\mathbb{H}$ is the set-membership flow, we have

$$\mathbb{X}(t) = \mathbb{G} \cap dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_{t_1 \geq t} \mathbb{H}_{t-t_1}\left(\bigcap_i g^{-1}_{\mathbf{a}_i(t_1)}([d_i(t_1), \infty]\right).$$